Technical Analysis Techniques versus Mathematical Models: Boundaries of Their Validity Domains

Christophette Blanchet-Scalliet[1], Awa Diop[2], Rajna Gibson[3], Denis Talay[2], and Etienne Tanré[2]

[1] Laboratoire Dieudonné, université Nice Sophia-Antipolis Parc Valrose 06108 Nice Cedex 2, France blanchet@math.unice.fr
[2] INRIA, Projet OMEGA, 2004 route des Lucioles, BP93, 06902 Sophia-Antipolis, France {Awa.Diop,Denis.Talay,Etienne.Tanre}@sophia.inria.fr
[3] NCCR FINRISK, Swiss Banking Institute, University of Zürich, Plattenstrasse 14, Zürich 8032, Switzerland rgibson@isb.unizh.ch

Abstract We aim to compare financial technical analysis techniques to strategies which depend on a mathematical model. In this paper, we consider the moving average indicator and an investor using a risky asset whose instantaneous rate of return changes at an unknown random time. We construct mathematical strategies. We compare their performances to technical analysis techniques when the model is misspecified. The comparisons are based on Monte Carlo simulations.

1 Introduction

In the financial industry, there are three main approaches to investment: the fundamental approach, where strategies are based on fundamental economic principles, the technical analysis approach, where strategies are based on past prices behavior, and the mathematical approach where strategies are based on mathematical models and studies. The main advantage of technical analysis is that it avoids model specification, and thus calibration problems, misspecification risks, etc. On the other hand, technical analysis techniques have limited theoretical justifications, and therefore no one can assert that they are riskless, or even efficient (see [LMW00]).

Consider a nonstationary financial economy. It is impossible to specify and calibrate models which can capture all the sources of instability during a long time interval. Thus it might be useful to compare the performances obtained by using erroneously calibrated mathematical models and the performances obtained by technical analysis techniques.

To our knowledge, this question has not yet been investigated in the literature. The purpose of this paper is to present its mathematical complexity and preliminary results.

Here we consider the case of an asset whose instantaneous expected rate of return changes at an unknown random time. We compare the performances of traders who respectively use:

- a strategy which is optimal when the model is perfectly specified and calibrated,
- mathematical strategies for misspecified situations,
- a technical analysis technique.

In all this paper, we limit ourselves to the case in which the trader's utility function is logarithmic. Of course, it is a severe limitation from a financial point of view. This choice is also questionable from a numerical point of view because logarithmic utilities tend to smoothen the effects of the different strategies. However, we will see that, even in this case and within a simplified model, the analytical formulae are rather cumbersome and that our analysis requires nonelementary mathematical and numerical tools. See also the Remark 1 below.

Our study is divided into two parts: a mathematical part which, when possible, provides analytical formulae for portfolios managed by means of mathematical and technical analysis strategies; a numerical part which provides quantitative comparisons between all these various strategies.

2 Description of the Setting

The financial market consists of two assets which are traded continuously. The first one is an asset without systematic risk, typically a *bond* (or a bank account), whose price at time t evolves according to

$$\begin{cases} dS_t^0 = S_t^0 r dt, \\ S_0^0 = 1. \end{cases} \tag{1}$$

The second asset is a stock subject to systematic risk. We model the evolution of its price at time t by the linear stochastic differential equation

$$\begin{cases} dS_t = S_t \left(\mu_2 + (\mu_1 - \mu_2)\mathbb{1}_{(t \leq \tau)}\right) dt + \sigma S_t dB_t, \\ S_0 = S^0, \end{cases} \tag{2}$$

where $(B_t)_{0 \leq t \leq T}$ is a one-dimensional Brownian motion on a given probability space $(\Omega, \mathcal{F}, \mathbb{P})$. At the random time τ, which is neither known, nor directly observable, the instantaneous return rate changes from μ_1 to μ_2. A simple computation shows that

$$S_t = S^0 \exp\left(\sigma B_t + (\mu_1 - \frac{\sigma^2}{2})t + (\mu_2 - \mu_1)\int_0^t \mathbb{1}_{(\tau \leq s)} ds\right) =: S^0 \exp(R_t),$$

where the process $(R_t)_{t\geq 0}$ is defined as

$$R_t = \sigma B_t + \left(\mu_1 - \frac{\sigma^2}{2}\right) t + (\mu_2 - \mu_1) \int_0^t \mathbb{1}_{(\tau \leq s)} ds. \tag{3}$$

This model was considered by Shiryaev [Shi63] who studied the problem of detecting the change time τ as early and reliably as possible when one only observes the process $(S_t)_{t\geq 0}$.

Assumptions and Notation

- The σ algebra generated by the observations at time t is denoted by

$$\mathcal{F}_t^S := \sigma\left(S_u, 0 \leq u \leq t\right), \; t \in [0, T].$$

  Note that the Brownian motion $(B_t)_{0\leq t\leq T}$ is not adapted to the filtration $(\mathcal{F}_t^S)_{t\geq 0}$.
- The Brownian motion $(B_t)_{t\geq 0}$ and the random variable τ are independent.
- The change time τ follows an exponential law [4] of parameter λ:

$$\mathbb{P}\left(\tau > t\right) = e^{-\lambda t}, \quad t \geq 0. \tag{4}$$

- The value of the portfolio at time t is denoted by W_t.
- We denote by F_t the conditional *a posteriori probability* (constructed by means of the observation of the process S) that the change time has occurred within the interval $[0, t]$:

$$F_t := \mathbb{P}\left(\tau \leq t / \mathcal{F}_t^S\right). \tag{5}$$

- We denote by $(L_t)_{t\geq 0}$ the following exponential likelihood-ratio process :

$$\begin{aligned} L_t = \exp\Bigg\{ & \frac{1}{\sigma^2}(\mu_2 - \mu_1) R_t \\ & -\frac{1}{2\sigma^2}\left((\mu_2 - \mu_1)^2 + 2(\mu_2 - \mu_1)(\mu_1 - \frac{\sigma^2}{2})\right) t \Bigg\}. \end{aligned} \tag{6}$$

- Finally, the parameters μ_1, μ_2, $\sigma > 0$ and $r \geq 0$ are such that

$$\mu_1 - \frac{\sigma^2}{2} < r < \mu_2 - \frac{\sigma^2}{2}.$$

[4] Any other law is allowed to derive our main results.

3 A Technical Analysis Detection Strategy

3.1 Introduction

Technical analysis is an approach which is based on the prediction of the future evolution of a financial instrument price using only its price history. Thus, technical analysts compute indicators which result from the past history of transaction prices and volumes. These indicators are used as signals to anticipate future changes in prices (see, e.g., the book by Steve Achelis [Ach00]).

Here, we limit ourselves to the moving average indicator because it is simple and often used to detect changes in return rates. To obtain its value, one averages the closing prices of the stock during the δ most recent time periods.

3.2 Moving Average Based on the Prices

Our trader takes decisions at discrete times. We thus consider a regular partition of the interval $[0, T]$ with step $\Delta t = \frac{T}{N}$:

$$0 = t_0 < t_1 < \ldots < t_N = T, \quad t_n = n\Delta t.$$

We denote by $\pi_t \in \{0, 1\}$ the proportion of the agent's wealth invested in the risky asset at time t, and by M_t^δ the moving average of the prices. Therefore,

$$M_t^\delta = \frac{1}{\delta} \int_{t-\delta}^{t} S_u \, du. \tag{7}$$

We suppose that, at time 0, the agent knows the history of the risky asset prices before time 0 and has enough data to compute M_0^δ.

At each t_n, $n \in [1 \cdots N]$, the agent invests all his/her wealth into the risky asset if the price S_{t_n} is larger than the moving average $M_{t_n}^\delta$. Otherwise, he/she invests all the wealth into the riskless asset. Consequently,

$$\pi_{t_n} = \mathbb{1}_{\left(S_{t_n} \geq M_{t_n}^\delta\right)}. \tag{8}$$

Denote by x the initial wealth of the trader. The wealth at time t_{n+1} is

$$W_{t_{n+1}} = W_{t_n} \left(\frac{S_{t_{n+1}}}{S_{t_n}} \pi_{t_n} + \frac{S^0_{t_{n+1}}}{S^0_{t_n}} (1 - \pi_{t_n}) \right),$$

and therefore, since $S^0_{t_{n+1}} / S^0_{t_n} = \exp(r\Delta t)$,

$$W_T = x \prod_{n=0}^{N-1} \left[\pi_{t_n} \left(\exp(R_{t_{n+1}} - R_{t_n}) - \exp(r\Delta t) \right) + \exp(r\Delta t) \right]. \tag{9}$$

3.3 The Particular Case of the Logarithmic Utility Function

One of our key results is

Proposition 1. *The expectation of the logarithmic utility function of the agent's wealth is*

$$\begin{aligned}\mathbb{E}\log(W_T) &= \log(x) + rT + \left(\mu_2 - \frac{\sigma^2}{2} - r\right) T p_\delta^{(1)} \\ &+\Delta t \left(\mu_2 - \frac{\sigma^2}{2} - r\right) \frac{1-e^{-\lambda T}}{1-e^{-\lambda \Delta t}} \left((p_\delta^{(2)} - p_\delta^{(1)}) e^{\lambda\delta} + p_\delta^{(3)} \right) \\ &-\Delta t(\mu_2 - \mu_1)(e^{-\lambda\Delta t} - \lambda\Delta t) \frac{1-e^{-\lambda T}}{1-e^{-\lambda\Delta t}} p_\delta^{(3)},\end{aligned}$$

where we have set

$$p_\delta^{(1)} = \int_0^\infty \int_y^\infty \frac{z^{\mu_2 - 3/2}}{2y} e^{-\frac{(\mu_2/\sigma - \sigma/2)^2\delta}{2} - \frac{(1+z^2)}{2\sigma^2 y}} i_{\sigma^2\delta/2}\left(\frac{z}{\sigma^2 y}\right) dz dy, \tag{10}$$

$$\begin{aligned}p_\delta^{(2)} = \int_0^\delta \int_{\mathrm{R}^4} \mathbb{1}_{\left\{\delta y_2 \geq \frac{z_1}{y_1} + z_2\right\}} & \frac{z_2^{\mu_2-3/2}}{2y_2} e^{-\frac{(\mu_2/\sigma - \sigma/2)^2(\delta - v)}{2} - \frac{(1+z_2^2)}{2\sigma^2 y_2}} \\ & i_{\sigma^2(\delta-v)/2}\left(\frac{z_2}{\sigma^2 y_2}\right) \frac{z_1^{\mu_1 - 3/2}}{2y_1} e^{-\frac{(\mu_1/\sigma - \sigma/2)^2 v}{2} - \frac{(1+z_1^2)}{2\sigma^2 y_1}} i_{\sigma^2 v/2}\left(\frac{z_1}{\sigma^2 y_1}\right) \\ & e^{-\lambda v} dy_1 dz_1 dy_2 dz_2 dv,\end{aligned} \tag{11}$$

$$p_\delta^{(3)} = \int_0^\infty \int_y^\infty \frac{z^{\mu_1 - 3/2}}{2y} e^{-\frac{(\mu_1/\sigma - \sigma/2)^2\delta}{2} - \frac{(1+z^2)}{2\sigma^2 y}} i_{\sigma^2\delta/2}\left(\frac{z}{\sigma^2 y}\right) dz dy, \tag{12}$$

$$i_y(z) = \frac{z e^{\pi^2/4y}}{\pi\sqrt{\pi y}} \int_0^\infty e^{-z\cosh(u) - u^2/4y} \sinh(u) \sin(\pi u/2y) du. \tag{13}$$

Proof. The tedious calculation involves an explicit formula, due to Yor [Yor01], for the density of $(\int_0^t \exp(2B_s)ds, B_t)$. See [BSDG+05] for details.

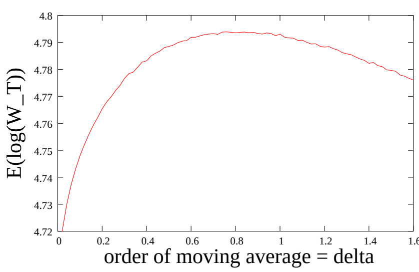
3.4 Empirical Determination of a Good Windowing

One can optimize the choice of δ by using Proposition 1 and deterministic numerical optimization procedures, or by means of Monte Carlo simulations. In this subsection we present results obtained from Monte Carlo simulations, which show that bad choices of δ may weaken the performance of the technical analyst strategy. For each value of δ we have simulated 500,000 trajectories of the asset price and computed the expectation $\mathbb{E}\log(W_T)$ by a Monte Carlo method. The parameters used to obtain Figure 1(a) and Figure 1(b) are all equal but the volatility. It is clear from the figures that the optimal choice of δ varies. When the volatility is 5 percent, the optimal choice of δ is around 0.3

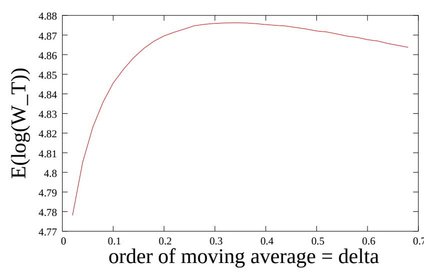
whereas, when the volatility is 15 percent, the optimal choice of δ is around 0.8.

The parameters used to obtain Figure 1(b) and Figure 1(c) are all identical but the maturity. The optimal choice of δ is around 0.3 when the maturity is 2 years, and is around 0.4 when the maturity is 3 years.

The empirical variance of $\log(W_T)$ is around 0.04. Thus, the Monte Carlo error on $\mathbb{E}\log(W_T)$ is of order 5.10^{-4} with probability 0.99. The number of trajectories used for these simulations seems to be too large; however, considered as a function of δ, the quantity $\mathbb{E}\log(W_T)$ varies very slowly, so that we really need a large number of simulations to obtain the smooth curves (Figure 1).



(a)



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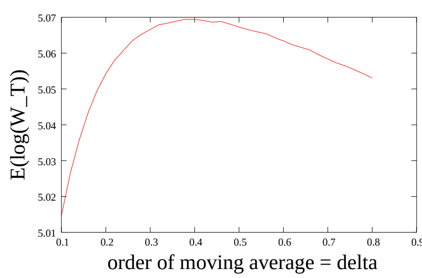

(c)

Fig. 1. $\mathbb{E}(\log(W_T))$ as a function of δ.

Parameter	μ_1	μ_2	λ	σ	r	T
Figure 1(a)	-0.2	0.2	2	0.15	0.0	2.0
Figure 1(b)	-0.2	0.2	2	0.05	0.0	2.0
Figure 1(c)	-0.2	0.2	2	0.05	0.0	3.0

Figure 2 below illustrates the impact of the parameters μ_1, μ_2 and σ on the optimal choice of δ.

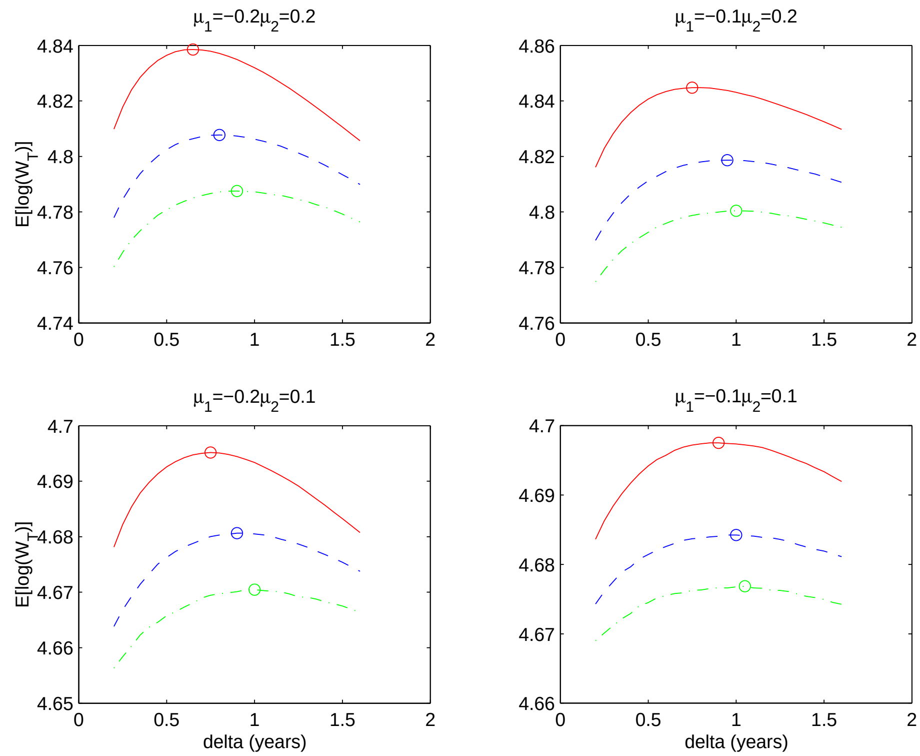

Fig. 2. Volatility and Optimal Moving Average Window Size: Plot of Expected Value of the Log of Terminal Wealth vs. Window size with $T = 2$, $\lambda = 2$, - $\sigma = 0.1$, - - $\sigma = 0.15$, and -. $\sigma = 0.2$

Remark 1. One can observe that the empirical optimal choices of δ are close to the classical values used by the technical analysts, that is, around 200 days or 50 days. One can also observe from Monte Carlo simulations that these optimal values also hold when the trader's utility function belongs to the HARA family: see [BSDG$^+$05].

4 The Optimal Portfolio Allocation Strategy

4.1 A General Formula

In this section our aim is to make explicit the optimal wealth and strategy of a trader who perfectly knows all the parameters μ_1, μ_2, λ and σ. Of course, this situation is unrealistic. However it is worth computing the best financial performances that one can expect within our model. To be able to compare this optimal strategy to a technical analyst strategy, we impose constraints on the portfolio. Indeed, a technical analyst is only allowed to invest all his/her wealth in the stock or the bond. Therefore the proportions of the trader's wealth invested in the stock are constrained to lie within the interval $[0,1]$.

To compute the constrained optimal wealth we use the martingale approach to stochastic control problems as developed by Karatzas, Shreve, Cvitanić, etc. More precisely, we follow and carefully adapt the martingale approach to the celebrated Merton problem [Mer71]. We emphasize that our situation differs from the Merton problem by two aspects:

- The drift coefficient of the dynamics of the risky asset is not constant over time (since it changes at the random time τ).
- Here we must face some subtle measurability issues since the trader's strategy needs to be adapted with respect to the filtration generated by (S_t): as already noticed, the drift change at the random time τ makes this filtration different from the filtration generated by the Brownian motion (B_t).

Let π_t be the proportion of the trader's wealth invested in the stock at time t; the remaining proportion $1-\pi_t$ is invested in the bond. For a given nonrandom initial capital $x>0$, let $W^{x,\pi}_{\cdot}$ denote the wealth process corresponding to the portfolio $(\pi_{\cdot})$. Let $\mathcal{A}(x)$ denote the set of admissible portfolios, that is,

$$\begin{aligned}\mathcal{A}(x) := \{\pi_{\cdot} - \mathcal{F}_t^S - &\text{ progressively measurable process s.t.}\\ &W_0^{x,\pi}=x,\ W_t^{x,\pi}>0 \text{ for all } t>0,\ \pi_{\cdot}\in[0,1]\}.\end{aligned}$$

The investor's objective is to maximize his/her expected utility U of wealth at the terminal time T. The value function thus is

$$V(x) := \sup_{\pi_{\cdot}\in\mathcal{A}(x)} \mathbb{E}\, U(W_T^{\pi}).$$

As in Karatzas-Shreve [KS98], we introduce an auxiliary unconstrained market defined as follows. We first decompose the process R in its own filtration as

$$dR_t = \left((\mu_1-\frac{\sigma^2}{2}) + (\mu_2-\mu_1)F_t\right)dt + \sigma d\overline{B}_t,$$

where $\overline{B}_{\cdot}$ is the innovation process, i.e., the $\mathcal{F}_t^S$- Brownian motion defined as

$$\overline{B}_t = \frac{1}{\sigma}\left(R_t - (\mu_1-\frac{\sigma^2}{2})t - (\mu_2-\mu_1)\int_0^t F_s ds\right),\quad t\geq 0,$$

where F is the conditional a posteriori probability (5).

Let $\mathcal{D}$ the subset of the $\{\mathcal{F}_t^S\}-$ progressively measurable processes $\nu : [0,T]\times\Omega\to \mathrm{R}$ such that

$$\mathbb{E}\int_0^T \nu^-(t)dt < \infty \text{ , where } \nu^-(t) := -\inf(0, \nu(t)).$$

The bond price process $S^0(\nu)$ and the stock price $S(\nu)$ satisfy

$$\begin{aligned} S_t^0(\nu) &= 1 + \textstyle\int_0^t S_u^0(\nu)(r + \nu^-(u))du, \\ S_t(\nu) &= S_0 + \textstyle\int_0^t S_u(\nu)\left((\mu_1 + (\mu_2 - \mu_1)F_u + \nu(u)^- + \nu(u))du + \sigma d\overline{B}_u\right). \end{aligned}$$

For each auxiliary unconstrained market driven by a process ν, the value function is

$$V(\nu, x) := \sup_{\pi.\in\mathcal{A}(\nu,x)} \mathbb{E}_x U(W_T^\pi(\nu)),$$

where

$$dW_t^\pi(\nu) = W_t^\pi(\nu)\left((r + \nu^-(t))dt + \pi_t\left(\nu(t)dt + (\mu_2 - \mu_1)F_t dt + (\mu_1 - r)dt + \sigma d\overline{B}_t\right)\right).$$

Proposition 2. *If there exists $\widetilde{\nu}$ such that*

$$V(\widetilde{\nu}, x) = \inf_{\nu\in\mathcal{D}} V(\nu, x) \tag{14}$$

then there exists an optimal portfolio π^* for which the optimal wealth is

$$W_t^* = W_t^{\pi^*}(\widetilde{\nu}). \tag{15}$$

An optimal portfolio allocation strategy is

$$\pi_t^* := \sigma^{-1}\left(\frac{\mu_1 - r + (\mu_2 - \mu_1)F_t + \widetilde{\nu}(t)}{\sigma} + \frac{\phi_t}{H_t^{\widetilde{\nu}} W_t^* \ e^{-rt - \int_0^t \widetilde{\nu}^-(s)ds}}\right), \tag{16}$$

where F_t defined in (5) satisfies

$$F_t = \frac{\lambda e^{\lambda t} L_t \int_0^t e^{-\lambda s} L_s^{-1} ds}{1 + \lambda e^{\lambda t} L_t \int_0^t e^{-\lambda s} L_s^{-1} ds},$$

and $H_t^{\widetilde{\nu}}$ is the exponential process defined by

$$\begin{aligned} H_t^{\widetilde{\nu}} = \exp\Bigg(&- \int_0^t \left(\frac{\mu_1 - r + \widetilde{\nu}(s)}{\sigma} + \frac{(\mu_2 - \mu_1)F_s}{\sigma}\right) d\overline{B}_s \\ &- \frac{1}{2}\int_0^t \left(\frac{\mu_1 - r + \widetilde{\nu}(s)}{\sigma} + \frac{(\mu_2 - \mu_1)F_s}{\sigma}\right)^2 ds\Bigg), \end{aligned}$$

and ϕ is a $\mathcal{F}_t^S$ adapted process which satisfies

$$\mathbb{E}\left(H_T^{\widetilde{\nu}} e^{-rT - \int_0^T \widetilde{\nu}^-(t)dt} (U')^{-1}(\upsilon H_T^{\widetilde{\nu}} e^{-rT - \int_0^T \widetilde{\nu}^-(t)dt}) \ / \ \mathcal{F}_t^S\right) = x + \int_0^t \phi_s d\overline{B}_s.$$

Here, υ is the Lagrange multiplier which makes the expectation of the left hand side equal to x for all x.

Proof. See Karatzas-Shreve [KS98, p. 275] to prove (15). We obtain (16) by solving the classical unconstrained problem for $\widetilde{\nu}$.

4.2 The Particular Case of the Logarithmic Utility Function

Proposition 3. *If $U(\cdot) = \log(\cdot)$ and the initial endowment is x, then the optimal wealth process and strategy are*

$$\boxed{\begin{aligned} W_t^{*,x} &= \frac{x e^{r(T-t)+\int_t^T \widetilde{\nu}^-(t)dt}}{H_t^{\widetilde{\nu}}}, \\ \pi_t^* &= \left(\frac{\mu_1 - r + (\mu_2-\mu_1)F_t + \widetilde{\nu}(t)}{\sigma^2}\right), \end{aligned}} \tag{17}$$

where

$$\widetilde{\nu}(t) = \begin{cases} -(\mu_1 - r + (\mu_2-\mu_1)F_t) & \textit{if } \dfrac{\mu_1 - r + (\mu_2-\mu_1)F_t}{\sigma^2} < 0, \\ 0 & \textit{if } \dfrac{\mu_1 - r + (\mu_2-\mu_1)F_t}{\sigma^2} \in [0,1], \\ \sigma^2 - (\mu_1 - r + (\mu_2-\mu_1)F_t) & \textit{otherwise}, \end{cases} \tag{18}$$

and, as above,

$$\widetilde{\nu}^-(t) = -\inf(0, \widetilde{\nu}(t)).$$

Remark 2. The optimal strategies for the constrained problem are the projections on $[0,1]$ of the optimal strategies for the unconstrained problem.

Remark 3. In the case of the logarithmic utility function, when t is small and thus before the change time τ with high probability, one has F_t close to 0; as, by hypothesis, one also has $\frac{\mu_1 - r}{\sigma^2} \leq 0$, the optimal strategy is close to 0 ; after the change time τ, one has F_t close to 1, and the optimal strategy is close to $\min(1, \frac{\mu_2 - r}{\sigma^2})$. In both cases, we approximately recover the optimal strategies of the constrained Merton problem with drift parameters equal to μ_1 or μ_2 respectively.

Using (18) one can obtain an explicit formula for the value function corresponding to the optimal strategy:

$$\begin{aligned} \mathbb{E}\log(W_T) &= \log(x) + rT \\ &+ \int_0^T \int_0^\infty \Bigg[\left(\mu_1 - r + (\mu_2-\mu_1)\frac{a}{1+a} - \frac{\sigma^2}{2}\right) \mathbb{1}_{\left\{a > \frac{\sigma^2 - \mu_1 + r}{\mu_2 - \sigma^2 + r}\right\}} \\ &\frac{1}{\sigma^2}\left(\mu_1 - r + (\mu_2-\mu_1)\frac{a}{1+a}\right)^2 \mathbb{1}_{\left\{-\frac{\mu_1 - r}{\mu_2 - r} < a < \frac{\sigma^2 - \mu_1 + r}{\mu_2 - \sigma^2 + r}\right\}} \Bigg] \\ &e^{-\lambda t}(1+a) g(\lambda a, t) \lambda da dt. \end{aligned} \tag{19}$$

Here, $g(a,t)da$ is the density function of

$$\int_0^t \exp\left(\frac{\mu_2-\mu_1}{\sigma}\tilde{B}_s - \frac{(\mu_2-\mu_1)^2}{2\sigma^2}s + \lambda s\right) ds,$$

where $\tilde{B}$ is a Brownian Motion. This density admits a rather complex explicit expression which involves the function $i_y(z)$ defined as in (13): see Yor [Yor01] and Borodin and Salminen [BS02].

5 A Model and Detect Strategy

The optimal strategy of the preceding section assumes that the trader has chosen a mathematical model and controls the investment policy to optimize the expected utility function of his/her wealth, and that the investment policy is time continuous, whereas the technical analyst does not control the policy and invests at discrete times according to a rupture detection rule. We now consider the case of a trader who chooses a mathematical model and wants to reinvest the portfolio only once, namely at the time where the change time τ is optimally detected owing to the price history. In this section we describe the wealth of such a trader, supposing that the reinvestment rule is the same as the technical analyst's one: at the detected change time from μ_1 to μ_2, all the portfolio is reinvested in the risky asset. We also continue to suppose for a while that the trader perfectly knows all the parameters of the model.

In the full report [BSDG$^+$05], we consider two detection methods proposed by Karatzas [Kar03] and by Shiryaev [Shi02]. Here, we limit ourselves to consider Karatzas' optimal stopping rule Θ^K which minimizes the *expected miss*

$$\mathcal{R}(\Theta) := \mathbb{E}|\Theta - \tau| \tag{20}$$

over all stopping rules Θ, where τ is a positive random variable. In view of the results in [Kar03] one has here

Proposition 4. *The stopping rule Θ^K that minimizes the expected miss $\mathbb{E}|\Theta - \tau|$ over all the stopping rules Θ with $\mathbb{E}(\Theta) < \infty$ is*

$$\Theta^K = \inf\left\{t \geq 0 \;\middle|\; \lambda e^{\lambda t} L_t \int_0^t e^{-\lambda s} L_s^{-1} ds \geq \frac{p^*}{1-p^*}\right\},$$

where L_t is defined as in (6), and p^* is the unique solution in $(\frac{1}{2}, 1)$ of the equation

$$\int_0^{1/2} \frac{(1-2s)e^{-\beta/s}}{(1-s)^{2+\beta}} s^{2-\beta} ds = \int_{1/2}^{p^*} \frac{(2s-1)e^{-\beta/s}}{(1-s)^{2+\beta}} s^{2-\beta} ds$$

with $\beta = 2\lambda\sigma^2/(\mu_2-\mu_1)^2$.

We thus are in a position to compute the wealth of the trader who uses the strategy consisting in investing all of his/her money in the bond until Θ^K and in the stock after Θ^K. The value of the portfolio at maturity T is

$$W_T = \frac{xS^0_{\theta^K}}{S_{\theta^K}} S_T \mathbb{1}_{(\theta^K \leq T)} + xS^0_T \mathbb{1}_{(\theta^K > T)}.$$

In the particular case of the logarithmic utility function, one can exhibit an exact formula for $\mathbb{E}(\log(W_T))$. Unfortunately this new formula has a complexity similar to (19): see [BSDG+05]. However we can numerically compare the performances of the two change time detection strategies (one based on technical analysis, the other one based on a mathematical model), and the optimal portfolio allocation strategy. Figure 3 illustrates, based on the typical results that we have obtained so far, that the methods using mathematical models have better performances than the technical analyst method.

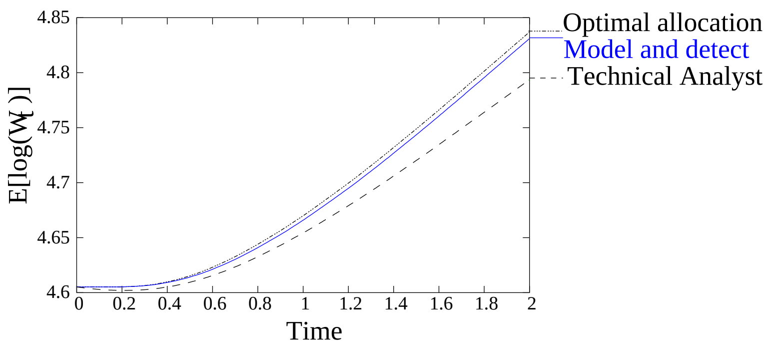

Fig. 3. Comparison

6 The Performances of the Strategies Based on Misspecified Models

6.1 Introduction

In practice, it is extremely difficult to know parameters exactly. If one may hope to calibrate μ_1 and σ relatively well owing to historical data, the value of μ_2 cannot be determined a priori (i.e. before the occurrence of the drift change), and the law of τ cannot be calibrated accurately because of the lack of data concerning τ.

Consider a trader who believes that the stock price is

$$dS_t = S_t \left(\overline{\mu}_2 + (\overline{\mu}_1 - \overline{\mu}_2) \mathbb{1}_{(t \leq \overline{\tau})} \right) dt + \overline{\sigma} S_t dB_t, \tag{21}$$

where the law of $\overline{\tau}$ is exponential with parameter $\overline{\lambda}$. We suppose that the true stock price is given by (2). Our aim is to study the misspecified optimal allocation strategy and the misspecified model and detect strategy.

Notation. As above we set $R_t = \log(S_t)$, where S_t is the actual price. We define $\overline{L}_t$ and $\overline{F}_t$ as follows:

$$\overline{L}_t = \exp\left\{\frac{1}{\overline{\sigma}^2}(\overline{\mu}_2 - \overline{\mu}_1)R_t - \frac{1}{2\overline{\sigma}^2}\left((\overline{\mu}_2 - \overline{\mu}_1)^2 + 2(\overline{\mu_2} - \overline{\mu}_1)(\overline{\mu}_1 - \frac{\overline{\sigma}^2}{2})\right)t\right\},$$
$$\overline{F}_t = \frac{\overline{\lambda}e^{\overline{\lambda}t}\overline{L}_t \int_0^t e^{-\overline{\lambda}s}\overline{L}_s^{-1}ds}{1 + \overline{\lambda}e^{\overline{\lambda}t}\overline{L}_t \int_0^t e^{-\overline{\lambda}s}\overline{L}_s^{-1}ds}.$$

6.2 On the Misspecified Optimal Allocation Strategy

Observing the stock price S_t, the trader computes a pseudo optimal allocation by using the erroneous parameters $\overline{\mu}_1$, $\overline{\mu}_2$, $\overline{\sigma}$ and $\overline{\tau}$. Thus the value of his/her misspecified optimal allocation strategy is

$$\overline{\pi}_t^* = \mathrm{proj}_{[0,1]}\frac{(\overline{\mu}_1 - r + (\overline{\mu}_2 - \overline{\mu}_1)\overline{F}_t)}{\overline{\sigma}^2},$$

and the corresponding wealth is

$$\overline{W}_t^* = e^{rt}\exp\left(\int_0^t \overline{\pi}_u^* d(e^{-ru}S_u)\right).$$

Numerical Example

In this section, we compare numerically the performance of two traders who respectively use a misspecified model and the true model. We fix the value of $\mu_1 = -0.2$, $\sigma = 0.15$, $r = 0.0$ and $\lambda = 2.0$, and we assume that they are perfectly known by the trader. A contrario μ_2 is misspecified. Its true value is $\mu_2 = 0.2$. Figure 4 shows the functions $t \to \mathbb{E}(\log(W_t))$ for three values of $\overline{\mu}_2$. It suggests that it is better to overestimate μ_2 ($\overline{\mu}_2 > \mu_2$) than to underestimate it ($\overline{\mu}_2 < \mu_2$).

6.3 On Misspecified Model and Detect Strategies

The erroneous stopping rule is

$$\overline{\Theta}^K = \inf\left\{t \geq 0,\ \ \overline{\lambda}e^{\overline{\lambda}t}\overline{L}_t\int_0^t e^{-\overline{\lambda}s}\overline{L}_s^{-1}ds \geq \frac{\overline{p}^*}{1-\overline{p}^*}\right\}$$

where $\overline{p}^*$ is the unique solution in $(\frac{1}{2}, 1)$ of the equation

$$\int_0^{1/2}\frac{(1-2s)e^{-\overline{\beta}/s}}{(1-s)^{2+\overline{\beta}}}s^{2-\overline{\beta}}ds = \int_{1/2}^{\overline{p}^*}\frac{(2s-1)e^{-\overline{\beta}/s}}{(1-s)^{2+\overline{\beta}}}s^{2-\overline{\beta}}ds$$

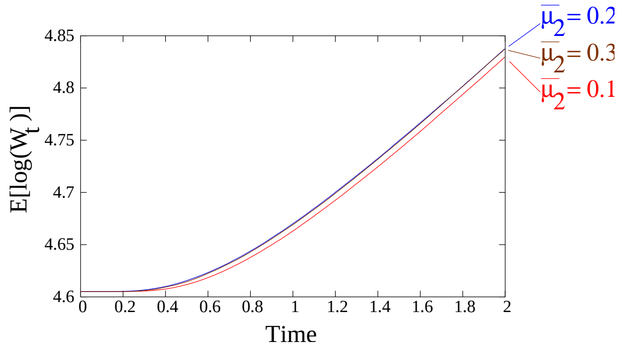

Fig. 4. Error on μ_2 for the optimal trader

with $\overline{\beta} = 2\overline{\lambda}\overline{\sigma}^2/(\overline{\mu}_2 - \overline{\mu}_1)^2$.

The value of the corresponding portfolio is

$$\overline{W}_T = xS^0_{\overline{\Theta}^K} \frac{S_T}{S_{\overline{\Theta}^K}} \mathbb{1}_{(\overline{\Theta}^K \le T)} + xS^0_T \mathbb{1}_{(\overline{\Theta}^K > T)}.$$

6.4 A Comparison Between Misspecified Strategies and the Technical Analysis Technique

Our main question is: Is it better to invest according to a mathematical strategy based on a misspecified model, or according to a strategy which does not depend on any mathematical model? Because of the analytical complexity of all the explicit formulae that we have obtained for the various expected utilities of wealth at maturity, we have not yet succeeded to find a mathematical answer to this question, even in asymptotic cases (when $\frac{\mu_2 - \mu_1}{\sigma^2}$ is large, e.g.). As this part of our work is still in progress, we present here a few numerical results obtained from Monte Carlo simulations. Consider the following study case.

Parameters of the model	μ_1	μ_2	λ	σ	r
True values	-0.2	0.2	2	0.15	0.0

Parameters used by the trader	$\overline{\mu}_1$	$\overline{\mu}_2$	$\overline{\lambda}$	$\overline{\sigma}$	r
Misspecified values (case I)	-0.3	0.1	1.0	0.25	0.0
Misspecified values (case II)	-0.3	0.1	3.0	0.25	0.0

Figure 5 shows that the technical analyst overperforms misspecified optimal allocation strategies when the parameter λ is underestimated.

We have looked for other cases where the technical analyst is able to overperform the misspecified optimal allocation strategies. Consider the case where the true values of the parameters are in Table 1. Table 2 summarizes our results. It must be read as follows. For the misspecified values $\overline{\mu}_2 = 0.1$,

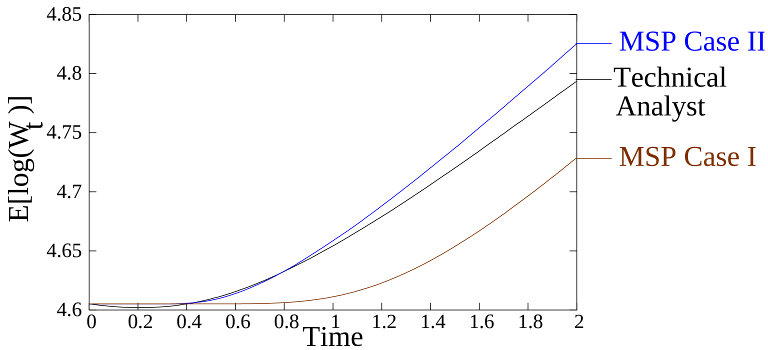

Fig. 5. A technical analyst may overperform misspecified optimal allocation strategies

$\overline{\sigma} = 0.25$, $\overline{\lambda} = 1$, if the trader chooses $\overline{\mu}_1$ in the interval $(-0.5, -0.05)$ then the misspecified optimal strategy is worse than the technical analyst's one. In fact, other numerical studies show that a single misspecified parameter is not sufficient to allow the technical analyst to overperform the Model and Detect traders. Astonishingly, other simulations show that the technical an-

Table 1. True values of the parameters

Parameter	True Value
μ_1	-0.2
μ_2	0.2
σ	0.15
λ	2

Table 2. Misspecified values and range of the parameters

$\overline{\mu}_1$	(-0.5,-0.05)	$\overline{\mu}_1$	-0.3	$\overline{\mu}_1$	-0.3	$\overline{\mu}_1$	-0.3
$\overline{\mu}_2$	0.1	$\overline{\mu}_2$	(0,0.13)	$\overline{\mu}_2$	0.1	$\overline{\mu}_2$	0.1
$\overline{\sigma}$	0.25	$\overline{\sigma}$	0.25	$\overline{\sigma}$	(0.2,$\rightarrow$)	$\overline{\sigma}$	0.25
$\overline{\lambda}$	1	$\overline{\lambda}$	1	$\overline{\lambda}$	1	$\overline{\lambda}$	(0,1.5)

alyst may overperform the misspecified optimal allocation strategy but not the misspecified model and detect strategy. One can also observe that, when μ_2/μ_1 decreases, the performances of well specified and misspecified model and detect strategies decrease. See [BSDG$^+$05].

7 Conclusions and Remarks

We have compared strategies designed from possibly misspecified mathematical models and strategies designed from technical analysis techniques. We have made explicit the trader's expected logarithmic utility of wealth in all the cases under study. Unfortunately, the explicit formulae are not propitious to mathematical comparisons. Therefore we have used Monte Carlo numerical experiments, and observed from these experiments that technical analysis techniques may overperform mathematical techniques in the case of severe misspecifications. Our study also brings some information on the range of misspecifications for which this observation holds true.

Jointly with M. Martinez (INRIA) and S. Rubenthaler (University of Nice Sophia Antipolis) we are now considering the infinite time case where the instantaneous expected rate of return of the stock changes at the jump times of a Poisson process and the values after each change time are unknown. We also plan to consider technical analysis techniques different from the moving average considered here.

Acknowledgment

This research is part of the Swiss national science foundation research program NCCR Finrisk which has funded A. Diop during her postdoc studies at INRIA and University of Zürich.

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