Overlapping domain decomposition method for microscanning framework

Daniele Graziani, Laure Blanc-Féraud*, Gilles Aubert†
*ARIANA CNRS/INRIA/UNSA SOPHIA ANTIPOLIS
Inria, 2004 route des Lucioles BP 93 06902 SOPHIA ANTIPOLIS CEDEX, FRANCE
e.mail: Daniele.Graziani@inria.fr, Laure.Blanc_Féraud@inria.fr
†Université de Nice SOPHIA ANTIPOLIS, parc valrose 06108 Nice CEDEX 2, FRANCE
e.mail: gaubert@unice.fr

Abstract—[1] **We investigate a convex variational framework to compute high resolution images from a low resolution video. We analyze the image formation process to provide a well designed model for shifting, blurring, downsampling and restoration. The microscanning super-resolution is modeled as a convex minimization problem, which is solved with a decomposition domain technique, which allows for parallel computing and real time algorithms.**

I. Introduction

The paper is concerned with the classical image processing problem of reconstructing highly resolved images from several multiple smaller images. Improvements in the resolution and fidelity of digital imaging systems have substantial value for remote sensing, military surveillance and other applications. Microscanning is a systematic approach to acquiring images with slightly different samplescene phases; between successive images the system is shifted slightly in a pre-determined controlled pattern. This makes an important difference with respect to general supersolution framework, where the system is shifted in a random pattern. First of all we describe the image formation model, by providing defintions of each standard acquisition operator: shifting, blurring, downsampling. Then in line with recent works (see, for instance [1], [3]), we introduce a convex energy used to restore highly resolved image out from low resolution frames. Such energy is made up by TV-regularization term and L^2-discrepancy term, which takes into account the acquisition process. Then we focus on the main issue of this paper, that is to address real or at least accetable computation time. For instance in aerosurveillance the imaging system is embedded on board, making crucial to restore acquired images in real time. To do this we adapt to our setting the overalapping domain decomposition algorithm for total varation minimzation proposed in [4]. So that we are able to reduce the minimazion of the energy to a finite sequence of sub-problems of a smaller size, so allowing, at least in principle, for parallel computation. Finally we show applications and results on real and synthetic data.

[1]The research of Daniele Graziani is supported by CNRS under the research project FUI "Gyrovision".

II. Preliminaries

We define the discrete rectangular domain Ω of step size $\delta x = 1$ and dimension $d_1 d_2$. $\Omega = \{1, ..., d_1\} \times \{1, ..., d_2\} \subset \mathbb{Z}^2$. In order to simplify the notations we set $X = R^{d_1 \times d_2}$. $u \in X$ denotes a matrix of size $d_1 \times d_2$. For $u \in X$, $u_{i,j}$ denotes its (i, j)-th component, with $(i, j) \in \{1, ..., d_1\} \times \{1, ..., d_2\}$. For $g \in Y$, $g_{i,j}$ denotes the (i, j)-th component of with $g_{i,j} = (g^1_{i,j}, g^2_{i,j})$ and $(i, j) \in \{1, ..., d_1\} \times \{1, ..., d_2\}$ We endowed the space X and Y with standard scalar product and standard norm. For $u, v \in X$:

$$\langle u, v \rangle_X = \sum_{i=1}^{d_1} \sum_{j=1}^{d_2} u_{i,j} v_{i,j}.$$

For $u \in X$ and $p \in [1, +\infty)$ we set:

$$|u|_p := (\sum_{i=1}^{d_1} \sum_{j=1}^{d_2} |u_{i,j}|^p)^{\frac{1}{p}}.$$

If G, F are two vector spaces and $H : G \to F$ is a linear operator the norm of H is defined by

$$\|H\| := \max_{\||u\|_G \leq 1} (\|Hu\|_F).$$

If $u \in X$ the gradient $\nabla u \in Y$ is given by:

$$(\nabla u)_{i,j} = ((\nabla u)^1_{i,j}, (\nabla u)^2_{i,j})$$

where

$$(\nabla u)^1_{i,j} = \begin{cases} u_{i+1,j} - u_{i,j} & \text{if } i < d_1 \\ 0 & \text{if } i = d_1, \end{cases}$$

and

$$(\nabla u)^2_{i,j} = \begin{cases} u_{i,j+1} - u_{i,j} & \text{if } j < d_2 \\ 0 & \text{if } j = d_2. \end{cases}$$

III. The model

In what follows HR and LR stand for high and low resolution respectively.

A. The acquisition process

As in [1] we assume the following acquisition process:

$$u(x,y) \text{ Real world scene}$$
$$\downarrow$$
$$F_k u(x,y) \text{ shifted image.}$$
$$\downarrow$$
$$H_k F_k u(x,y) \text{ blurred shifted image}$$
$$\downarrow$$
$$D_k H_k F_k u(x,y) \text{ downsampled blurred shifted image}$$
$$\downarrow$$
$$u_0(m,n) = D_k F_k H_k u(x,y) + b \text{ noisy blurred down-sampled image.}$$

The microscanning super-resolution reconstruction problem is the following: *Given a set* $\{u_k^0\}_1^K$ *of* $K = r^2$ *LR images (where* r^2 *is the resolution enhancement factor between the LR and the HR image) , find* u. For any k we have the

$$u_k^0 = D_k H_k F_k u + b$$

where:

- u_k^0 is the LR frame: a vector of size $[N^2 \times 1]$
- D_k is the down-sampling operator: a matrix of size $[N^2 \times r^2 N^2]$
- H_k is the PSF: a matrix of size $[r^2 N^2 \times r^2 N^2]$
- F_k are the shifting operators: each F_k is a matrix of size $[r^2 N^2 \times r^2 N^2]$
- u is HR image: a vector of size: $[r^2 N^2 \times 1]$.

Considering that the frames are acquired with a unique camera we assume the following facts: $H_k = H$ and $D_k = D$ for each frame.

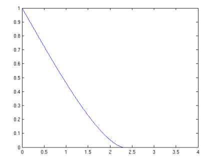
B. Assumption on the acquisition operators

We define the blur operator H by means of its Fourier transform $\hat{h}$ (see Figure 1 (a)). If f denotes a frequence in the Fourier space and f_c is the cutting frequency of the acquisiton system.

$$\hat{h}(f) = \begin{cases} \left| \frac{2}{\pi} x \Big(\arcsin(\frac{f}{f_c} - \frac{x}{\gamma} \sqrt{1 - (\frac{f}{f_c})^2} \Big) \right| & \text{if } f \leq f_c \\ 0 & \text{if } f \geq f_c \end{cases} \tag{1}$$

In particular we have that $\|\hat{h}\|_2 \leq 1$, which ensures that $\|H\| \leq 1$. Given a continuous image u, we define the pixel value $u_{i,j}$ of the corresponding discrete image at the position (i,j) by computing the mean in the pixel region $\Delta_{i,j} = (i,j) + [-\frac{1}{2}, \frac{1}{2})^2$. Then we define the downsampling operator as:

$$\begin{aligned} & u_{i,j} \mapsto D u_{i,j} = \\ = \quad & u_{k,l} = \frac{1}{r^2 A(\Delta^r_{k,l})} \sum_{0 \leq i,j \leq MN} A(\Delta_{i,j} \cup \Delta^r_{k,l}) u_{i,j}), \end{aligned} \tag{2}$$



(a) PSF profile $\hat{h}$, $f_c = 2.27$

Figure 1.

where r is the scale factor, $1 \leq k, l \leq \frac{M}{r}$, $\frac{M}{r}$ and A stands for Lebesgue measure. It is not difficult to check that $\|D\| \leq \frac{1}{r^2}$. Then the following bound holds:

$$\sum_{k=1}^{r^2} \|T_k\| \leq 1,$$

where $T_k = DHF_k$. Moreover, up to rescaling, we can always assume

$$\sum_{k=1}^{r^2} \|T_k\| < 1. \tag{3}$$

C. Trace operator

We define the trace operator as the restriction to a boundary Γ_i of some subdomains.

$$Tr\lfloor_{\Gamma_i} \colon V_i \mapsto \mathbb{R}^{\Gamma_i}, \quad i = 1, 2,$$

with $Tr\lfloor_{\Gamma_i} v_i = v_i \lfloor_{\Gamma_i}$, the restriction of v_i on Γ_i. $\mathbb{R}^{\Gamma_i}$ denotes the set of maps from Γ_i into $\mathbb{R}$.

D. Convex minimization problem

To retrieve the super resolution image u we wish to minimize the following energy:

$$\mathcal{F}(u) = \sum_{K=1}^{r^2} \|T_k u - u_k^0\| + 2\lambda \|\nabla u\|_1 \tag{4}$$

where $T_k = DHF_k$ is a linear operator belonging to $\mathcal{L}(\mathbb{R}^{MN}, \mathbb{R}^{\frac{MN}{r^2}})$. In order to obtain a fast minimization we follow the decompostion overlapping domain method used in [4]. So that instead of minimizing (4) on the whole image domain, we split Ω in two or more overlapping subdomains Ω_1 and Ω_2 such that $\Omega_1 = \Omega_1 \cap \Omega_2 \neq \emptyset$ (see figure 2). For simplicity and without loss of generality we limit the presentation to two subdomains. We denote by Γ_1 the interface between Ω_1 and $\Omega_2 \setminus \Omega_1$ and byΓ_2 the interface between Ω_2 and $\Omega_1 \setminus \Omega_2$. Due to the decomposition $\mathbb{R}^{MN}$ is decomposed in two closed subspaces $V_j = \{u \in \mathbb{R}^{MN} : supp(u) \subset \Omega_j\}$. Then we wish to minimize energy (4) by using the following alternating algorithm: pick an initial data $u^0 = \tilde{u}_1^0 + \tilde{u}_2^0 \in V_1 + V_2$ and iterate the following procedure:

$$\begin{cases} u_1^{n+1} = \arg\min_{v_1 \in V_1} \mathcal{F}(v_1 + u_2^n) \\ u_2^{n+1} = \arg\min_{v_2 \in V_2} \mathcal{F}(u_1^{n+1} + v_2) \\ u^{n+1} := u_1^{n+1} + u_2^{n+1} \end{cases} \tag{5}$$

Figure 2. *The overlapping decomposition.*

IV. The algorithm

A. Subspace minimization

Let us consider the minimization on Ω_1. The minimum problem is the following:

$$\begin{aligned} & \underset{\substack{v_1 \in V_1 \\ Tr\lfloor_{\Gamma_1} v_1 = 0}}{\arg\min} \ \mathcal{F}(v_1 + u_2) \\ = & \underset{\substack{v_1 \in V_1 \\ Tr\lfloor_{\Gamma_1} v_1 = 0}}{\arg\min} \sum_{k=1}^{r^2} \|T_k v_1 - (u_0^k - T_k u_2)\|_2^2 \\ + & \ 2\lambda \|\nabla (v_1 + u_2 \lfloor_{\Omega_1}\|_1. \end{aligned} \quad (6)$$

As in [4] we introduce the surrogate functional to separate the variable u_1 from the action of the operators T_k. For $a, u_1 \in V_1$, $u_2 \in V_2$ we define

$$\mathcal{F}_1^s(u_1 + u_2, a) := \mathcal{F}(u_1 + u_2) + r^2 \|u_1 - a\|_2^2 - \sum_{k=1}^{r^2} \|T_k(u_1 - a)\|_2^2.$$

By same computation of [4] we obtain that

$$\begin{aligned} & \mathcal{F}_1^s(u_1 + u_2, a) \\ = & \sum_{k=1}^{r^2} \|u_1 - (a + (T_k^*(u_0^k - T_k u_2 - T_k a))\lfloor \Omega_1\|_2^2 \\ + & \ 2\lambda \|\nabla (u_1 + u_2)\| + \Phi(a, u_k^0, u_2), \end{aligned} \quad (7)$$

where Φ does not depend on u_1. Then we can compute an approximate solution of problem (6) by using the following algorithm:

$$\begin{cases} u_1^{l+1} = \arg\min_{\substack{u_1 \in V_1 \\ Tr\lfloor_{\Gamma_1} u_1 = 0}} \mathcal{F}_1^s(u_1 + u_2, u_1^l), & l \geq 0 \\ u_1^0 = \tilde{u}_1^0 \in V_1 & . \end{cases} \quad (8)$$

As in [4], to control the solutions on the overlapping parts we fix a bounded uniform partition of unity, that is $\{\chi_1, \chi_2\}$ such that

1) $Tr\lfloor_{\Gamma_i} \chi_i = 0$ for $i = 1, 2$.
2) $\chi_1 + \chi_2 = 1$
3) $supp \chi_i \subset \Omega_i$ for $i = 1, 2$
4) $\max\{\|\chi_1\|_\infty, \chi_2\|_\infty\} = c < +\infty$.

We rewrite the algortithm as follows: Pick as initial data $u^0 = \tilde{u}_1^0 + \tilde{u}_2^0 \in V_1 + V_2$ and iterate:

$$\begin{cases} \begin{cases} u_1^{(n+1,0)} = \tilde{u}_1^n \\ u_1^{(n+1,l+1)} = \arg\min_{\substack{u_1 \in V_1 \\ Tr\lfloor_{\Gamma_1} = 0}} \mathcal{F}_1^s(u_1 + \tilde{u}_2^n, u_1^{(n+1),l)}) \end{cases} \\ \begin{cases} u_2^{(n+1,0)} = \tilde{u}_2^n \\ u_2^{(n+1,l+1)} = \arg\min_{\substack{u_2 \in V_2 \\ Tr\lfloor_{\Gamma_2} = 0}} \mathcal{F}_2^s(u_1^{(n+1,L)} + u_2^n, u_2^{(n+1),m)}) \end{cases} \\ u^{(n+1)} := u_1^{(n+1,L)} + u_2^{(n+1,M)} \\ \tilde{u}_1^{n+1} := \chi_1 u^{n+1} \\ \tilde{u}_2^{n+1} := \chi_2 u^{n+1} \end{cases} \quad (9)$$

B. Pararell version

Pick as initial data $u^0 = \tilde{u}_1^0 + \tilde{u}_2^0 \in V_1 + V_2$ and iterate:

$$\begin{cases} \begin{cases} u_1^{(n+1,0)} = \tilde{u}_1^n \\ u_1^{(n+1,l+1)} = \arg\min_{\substack{u_1 \in V_1 \\ Tr\lfloor_{\Gamma_1} = 0}} \mathcal{F}_1^s(u_1 + \tilde{u}_2^n, u_1^{(n+1),l)}) \end{cases} \\ \begin{cases} u_2^{(n+1,0)} = \tilde{u}_2^n \\ u_2^{(n+1,l+1)} = \arg\min_{\substack{u_2 \in V_2 \\ Tr\lfloor_{\Gamma_2} = 0}} \mathcal{F}_2^s(u_1^{(n+1,L)} + u_2^n, u_2^{(n+1),m)}) \end{cases} \\ u^{(n+1)} := \frac{u_1^{(n+1,L)} + u_2^{(n+1,M)} + u^n}{2} \\ \tilde{u}_1^{n+1} := \chi_1 u^{n+1} \\ \tilde{u}_2^{n+1} := \chi_2 u^{n+1} \end{cases} \quad (10)$$

C. Convergence properties

By using the bound $\sum_{k=1}^{r^2} \|T_k\| < 1$, we have as in [4] the following proposition. (see Proposition 5.4 and Theorem 5.7 of [4]). The proof, up to minor changes, is the same.

Proposition 4.1: The algorithms (9) and (10) produces a sequence (u^n) with the following properties:

1) $\mathcal{F}(u^{(n)}) > \mathcal{F}(u^{(n+1)})$ for all $n \in \mathbb{N}$ (unless $u^{(n)} = u^{(n+1)}$)
2) $\lim_{n \to +\infty} \|u^{(n+1)} - u^{(n)}\|_2 = 0$;
3) the sequence $(u^{(n)})$ has convergent subsequences.
4) The accumulation points of the sequence $(u^{(n)})$ are minimizers of $\mathcal{F}$. If $\mathcal{F}$ has a unique minimizer, then the sequence $(u^{(n)})$ converges to it.

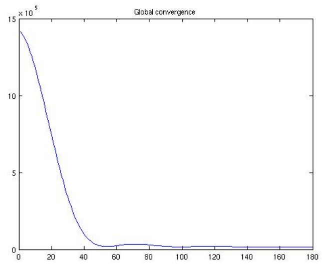
V. Experiments

Figure 3,4 show a controlled simulated experiment. In this experiment we create a sequence of 4 LR frames by using one HR image (Fig. 3 (a)). We shifted the image by a pixel in the horizontal vertical diagonal, direction (we taken as fourth shifted image the original one). Then to simulate the effect of camera we applied the blur operator H to the 4 shifted images. Finally the shifted images are downsampled by a factor $r^2 = 4$ and a gaussian noise with two different variances is added. To retrieve the HR image we implemented the sequential algorithm (9) with initial data $u^0 = 0$, by using

(a) Original image

(b) Noisy frame $\sigma = 0.01$

(c) Restored HR image



(d) Convergence time in seconds

Figure 3.

a decomposition with 8 subdomains. The original image is of size 512×512 and is rescaled in $[0, 1]$. The global CPU time is about 3 minutes running on an Intel (R) Xeon(R) CPU 5120 at 1.86GHz. The local CPU time on every subdomains, without disposing parallel processors, is approximately 22 seconds. We precise that algorithm is implemented with Matlab software and the matlab code is not optimized. We also remark that without decomposition the algorithm takes about 100 seconds. In figure 5 we execute the same experiments, but with an original image of size 256×256. The global CPU time is about 1 minutes ans 25 seconds running on an Intel (R) Xeon(R) CPU 5120 at 1.86GHz. The local CPU time on every subdomains, without disposing of parallel processors, is approximately 10 seconds.

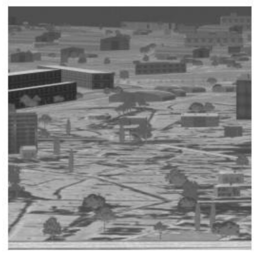
VI. Conclusion

In this paper we presented a fast algorithm to enhance the quality of a finite sequence of noisy and blurred frames. At least in principle our method allows for real time computation, which is one of the main problems in microscanning super-resolution framework. One important extension, we are now investigating, is the generalization of our algorithm to high resolution color image and demosaicing. In this direction several analogous model have been proposed (see, among others, [2], [5]), but no decompositon method domain is available in the literature for such a problem.

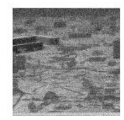
(a) Noisy frame $\sigma = 0.1$

(b) Restored HR image

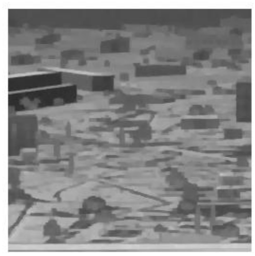
Figure 4.



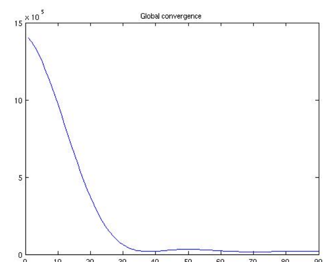
(a) Original image



(b) Noisy frame $\sigma = 0.05$



(c) Restored HR image



(d) Convergence time in seconds

Figure 5.

References

[1] S. Farsiu, M. Dirk robinson, M .Elad, P. Milanfar. *Fast and Robust Multiframe Super Resolution*, **13** IEEE Transactions on image processing (2004), 1327-1344.

[2] S.Farsiu, M. Elad, P. Milanfar. *Multiframe demosaicing and super-resolution of color images*. IEEE Transactions on image processing (2006), **15** 141-159.

[3] M. Unger, T. Pock, M. Werlberger, H. Bischof *A convex approach for variational super-resolution*, Proceedings of the 32nd DAGM conference on Pattern recognition.

[4] M. Fornasier, A. Langer, C. Schonlieb. *A convergent overlapping decomposition method for total variation Minimization"*, Numeriske Math. to appear.

[5] P. Vandewalle, K. Krichane, D. Alleysson, Ssstrunk, *Joint Demosaicing and Super-Resolution Imaging from a Set of Unregistered Aliased Images*, Proc. IST/SPIE Electronic Imaging: Digital Photography III, Vol. 6502, 2007.