ADMM algorithm for demosaicking deblurring denoising

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Abstract

The paper is concerned with the definiton and adaption, in the context of color demosaicking, of ADMM method to perform at the same time: demosaicking, deblurring and denoising.

1 Introduction

In this paper we address another classical challenging problem in image processing: the so called demosaicking, used in digital cameras for reconstructing color images. Let us give a short description.

Most digital cameras use a single sensor which is placed in front of color filter array: the Bayer Matrix. The sensor therefore sampled only one color per spatial position and the observed image is degraded by the effect of mosaic generation. It is therefore necessary to implement, possibly fast, algorithms to define an image with three color components by spatial position. The set of techniques used in the literature, to resolve this problem, is huge. Without claiming of being exhaustive we refer the reader to [1] for a general dissertation.

The originality of our research, in this context, is to define a variational method well suited to take into account all possible degradation effects due to: mosaic effect, blur and noise. Looking at the literature in this direction it is worth mention the work of Condat (see [3]) where a demosaicking-denoising method is proposed, but without taking into account blur effects. In [6, 7] all the degradation effects are considered, but with regularization energies, which does not allow for fast convex optimization technique. Indeed in these works, in order to take into account the correlation between the RGB components, the prior regularization term has a complicated expression.

Here we analyze and test a new method to perform in a more direct and possibly faster way demosaicking-deblurring-denoising problem. Our approach is based on two steps. The first one, as in [3], is working in a suitable basis where the three color components are statistically decorrelated. Then we are able to write our problem as a convex minimization problem. Finally to solve such a problem and to restore the image, we adapt to our framework the, well known, ADMM (Alternating Direction Multipliers Minimization) convex optimization technique. The ADMM method and its variants are largely used to solve convex minimization problems in image processing. We refer the reader to [5], for a general dissertation on convex optimization techniques, such as ADMM methods or others, and their applications to image processing.

Organization of the paper

The paper is organized as follows. Section 2 is devoted to notation in discrete setting. In section 3 we give a short description of the general ADDM method. In section 4 we define the new basis for which the channels of the color image are decorrelated. and we introduce the Bayer matrix and the blur operator. Section 5 is concerned with the definition of our variational model. We also show how to adapt the classical ADMM algorithm to our case.

Finally in the last section we give some applications of our algorithm on color images of big sizes.[1].

2 Discrete setting

We define the discrete rectangular domain Ω of step size $\delta x = 1$ and dimension $d_1 d_2$. $\Omega = \{1, ..., d_1\} \times \{1, ..., d_2\} \subset \mathbb{Z}^2$. In order to simplify the notations we set $X = R^{d_1 \times d_2}$ and $Y = X \times X$. $u \in X$ denotes a matrix of size $d_1 \times d_2$. For $u \in X$, $u_{i,j}$ denotes its (i, j)-th component, with $(i, j) \in \{1, ..., d_1\} \times \{1, ..., d_2\}$. For $g \in Y$, $g_{i,j}$ denotes the (i, j)-th component of with $g_{i,j} = (g^1_{i,j}, g^2_{i,j})$ and $(i, j) \in \{1, ..., d_1\} \times \{1, ..., d_2\}$ We endowed the space X and Y with standard scalar product and standard norm. For $u, v \in X$:

$$\langle u, v\rangle_X = \sum_{i=1}^{d_1} \sum_{j=1}^{d_2} u_{i,j} v_{i,j}.$$

For $g, h \in Y$:

$$\langle g, h\rangle_Y = \sum_{i=1}^{d_1} \sum_{j=1}^{d_2} \sum_{l=1}^{2} g^l_{i,j} h^l_{i,j}.$$

For $u \in X$ and $p \in [1, +\infty)$ we set:

$$\|u\|_p := (\sum_{i=1}^{d_1} \sum_{j=1}^{d_2} |u_{i,j}|^p)^{\frac{1}{p}}.$$

For $g \in Y$ and $p \in [1, +\infty)$:

$$\|g\|_p := (\sum_{i=1}^{d_1} \sum_{j=1}^{d_2} \sum_{l=1}^{2} |g^l_{i,j}|_2^{p})^{\frac{1}{p}}.$$

[1] a version of the matlab code is available at the I3S laboratory (CNRS/UNS)

If G, F are two vector spaces and $H : G \to F$ is a linear operator the norm of H is defined by

$$\|H\| := \max_{\| \|u\|_G \leq 1} (\|Hu\|_F).$$

3 Demosaicking-Deblurring-Denoising

We describe here a new algorithm to perform, in the same time, demosaicking deblurring, denoising. To this purpose we will adapt to our context an ADMM type algorithm. We recall the relevant features necessary to illustrate the application of such a method to our setting. We refer the reader to [5] and references therein, for a general dissertation on convex optimization techniques in image processing and recent developments on this matter.

3.1 ADMM algorithm for constrained minimization problem

In this section we describe the optimization method, we will adapt to our setting. The so called Alternating Direction Minimization Multipliers method ADMM. This particular optimization technique is well suited for constrained minimization problem of the following form:

$$\min_{u,z} F(z) + G(u) \quad \text{subject to } Bz + Au = b \tag{1}$$

where $F, G : \mathbb{R}^d \to \mathbb{R}$ and A and B matrix.

To solve problem (1) one considers the augmented Lagrangian and seeks its stationary points.

$$L_\alpha(z, u, \lambda) = F(z) + G(u) + \langle \lambda, Au + Bz - b \rangle + \frac{\alpha}{2}\|Au + Bz - b\|^2. \tag{2}$$

Then one iterate as follows:

$$\begin{cases} (z^{k+1}, u^{k+1}) = \operatorname{argmin}_{z,u} L_\alpha(z, u, \lambda^k) \\ \lambda^{k+1} = \lambda^k + \alpha(Au^{k+1} + Bz^{k+1} - b), \qquad \lambda^0 = 0 \end{cases} \tag{3}$$

The following result has been proven in [4].

Theorem 3.1 (Eckstein, Bertsekas) *Suppose B has full column rank and $G(u)+\|A(u)\|^2$ is strictly convex. Let λ_0 and u_0 arbitrary and let $\alpha > 0$. Suppose we are also given sequences $\{\mu_k\}$ and $\{\nu_k\}$ with $\sum_k^\infty \mu_k < \infty$ and $\sum_k^\infty \nu_k < \infty$. Assume that*

1. $\|z^{k+1} - \operatorname{argmin}_{z\in\mathbb{R}^N} F(z) + \langle \lambda^k, Bz\rangle + \frac{\alpha}{2}\|Au^k + Bz - b\|^2\| \leq \mu_k$
2. $\|u^{k+1} - \operatorname{argmin}_{z\in\mathbb{R}^M} G(u) + \langle \lambda^k, Au\rangle + \frac{\alpha}{2}\|Au + Bz^{k+1} - b\|^2\| \leq \nu_k$
3. $\lambda^{k+1} = \lambda^k + \alpha(Au^{k+1} + Bz^{k+1} - b)$.

If there exists a saddle point of $L_\alpha(z, u, \lambda)$ then $(z^k, u^k, \lambda^k) \to (z^*, u^*, \lambda^*)$ which is such a saddle points. If no such saddle point exists, then at least one of the sequences $\{u^k\}$ or $\{\lambda_k\}$ is unbounded.

4 Decorrelation and acquisition operators

4.1 Decorrelation

It is well known the RGB components of a color image $u^c = (u^R, u^G, u^B)^T$ are strongly statistically correlated. It is possible to show, from an experimental point of view, (Alleyson et al. [2]), that there exists a basis $L, C^{G/M}, C^{R/B}$ in which the image $u^d = (u^L, u^{G/M}, u^{R/B})^T$ is now approximately decorrelated.

This new orthonormal basis $L, C^{G/M}, C^{R/B}$ with decorrelation is given by:

- $L = \frac{1}{\sqrt{3}}[1,1,1]^T$ is the luminance
- $C^{G/M} = \frac{1}{\sqrt{2}}[-1,2,-1]^T$ is the green magenta chrominance
- $C^{R/B} = \frac{1}{\sqrt{2}}[1,0,-1]^T$ the red blue chrominance

Moreover the change of basis matrices have the following expression:

$$(4) \qquad u^d = \begin{bmatrix} u^L \\ u^{G/M} \\ u^{R/B} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} u^R \\ u^G \\ u^B \end{bmatrix} = T(u^c)$$

and

$$(5) \qquad u^c = \begin{bmatrix} u^R \\ u^G \\ u^B \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 0 \\ 1 & -10 & 2 \end{bmatrix} \begin{bmatrix} u^L \\ u^{G/M} \\ u^{R/B} \end{bmatrix} = T^{-1}(u^d).$$

Hereafter u^c denotes the image in the canonical basis R, G, B, while u^d is the image in the basis $L, C^{G/M}, C^{R/B}$.

4.2 Bayer filter and blur operator

For every $(i,j) \in Z^2$ we define the color image $u^c = (u^c(i,j))_{(i,j)\in Z^2}$ where

$$u^c(i,j) = [u^R(i,j), u^G(i,j), u^B(i,j)]^T$$

is the color of the pixel of u^c at location (i,j) in the canonical R, G, B base. We define the following Bayer filter

(6)

$$u^c = [u^R(i,j), u^G(i,j), u^B(i,j)]^T \to B(u^c) = (u^c)^{X(i,j)} \quad \text{with } X(i,j) \in \{R, G, B\}\ \forall (i,j),$$

So that the image $(u^c)^{X(i,j)}$ has only one of the components RGB per spatial position.

Concerning the blur operator we assume that it is the same for every components. In particular we suppose the following form (with abuse of notation):

$$H = \begin{bmatrix} H & 0 & 0 \\ 0 & H & 0 \\ 0 & 0 & H \end{bmatrix}$$

Where H is a matrix representing standard convolution with some Gaussian kernel.

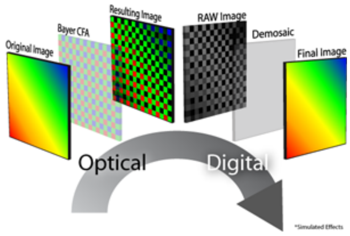

Figure 1: Bayer filter

5 The variational model

Let us start by recalling the acquisition camera sequence. We have as usual:

$$u^c \mapsto Hu^c \mapsto BHu^c \to BHu^c + b = u_0. \tag{7}$$

On the other hand from (5) we have $u^c = T^{-1}(u^d)$. So that from (7) we get an ideal acquisition process for u^d

$$u^d \mapsto T^{-1}(u^d) \mapsto HT^{-1}(u^d) \mapsto BHT^{-1}(u^d) \to BHT^{-1}(u^d) + b = u_0. \tag{8}$$

The idea is then to restore u^d by working with the, much more convenient, decorrelation basis $L, C^{G/M}, C^{R/B}$. Finally, at once u^d is restored, simply set $u^c = T(u^d)$.

In order to retrieve u^d, we have to solve an ill posed inverse problem. So that as usual we seek for minimizer of an energy given by an L^2-discrepancy term plus a regularization penalty.

Now the key point is that, since we are working in the decorrelation basis, it makes to consider the following minimization problem:

$$\arg\min_{u^d} \|\nabla u^L\|_1 + \|\nabla u^{G/M}\|_1 + \|\nabla u^{R/B}\|_1 + \mu\|BHT^{-1}(u^d) - u_0\|_2^2.$$

5.1 Application of ADMM method to our problem

In order to apply the ADMM method, we must rewrite the problem

$$\arg\min_{u^d} \|\nabla u^L\|_1 + \|\nabla u^{G/M}\|_1 + \|\nabla u^{R/B}\|_1 + \mu\|BHT^{-1}(u^d) - u_0\|_2^2, \tag{9}$$

in the form (1), which was

$$\min_{u,z} F(z) + G(u^d) \quad \text{subject to } Bz + Au^d = b.$$

Then we set

(10)

$$z = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ v \end{bmatrix} = \begin{bmatrix} \nabla u^L \\ \nabla u^{G/M} \\ \nabla u^{R/B} \\ BHT^{-1}(u^d) - u_0 \end{bmatrix}, \quad B = -I, \quad A = \begin{bmatrix} \nabla^L \\ \nabla^{G/M} \\ \nabla^{R/B} \\ BHT^{-1} \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ u_0 \end{bmatrix}$$

We also need the dual variable

$$\lambda = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ q \end{bmatrix}.$$

To simplify the notation we write

$$z = \begin{bmatrix} w \\ v \end{bmatrix} = \begin{bmatrix} \nabla u^d \\ BHT^{-1}(u^d) - u_0 \end{bmatrix} A = \begin{bmatrix} \nabla \\ K \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ u_0 \end{bmatrix} \tag{11}$$

an finally

$$\lambda = \begin{bmatrix} p \\ q \end{bmatrix}$$

We can now write down the corresponding augmented lagrangian as:

$$\begin{aligned} & L_\alpha(z, u^d, \lambda) = \|w\|_1 + \mu\|v\|_1^2 + \langle p, \nabla u^d - w\rangle + \langle q, Ku^d - u_0 - v\rangle \\ + \quad & \frac{\alpha}{2}\|v - Ku^d + u_0\|^2. \end{aligned} \tag{12}$$

The ADMM iterations are then given by:

$$\begin{aligned} w^{k+1} &= \operatorname*{argmin}_w \|w\|_1 + \frac{\alpha}{2}\|w - \nabla(u^d)^k - D(u^d)^k - \frac{p^k}{\alpha}\|_2^2 \\ v^{k+1} &= \operatorname*{argmin}_w \mu\|v\|_1 + \frac{\alpha}{2}\|v - K(u^d)^k + u_0 - \frac{q^k}{\alpha}\|_2^2 \\ (u^d)^{k+1} &= \operatorname*{argmin}_u \frac{\alpha}{2}\|\nabla u^d - w^{k+1} + \frac{p^k}{\alpha}\|_2^2 + \frac{\alpha}{2}\|Ku - v^{k+1} - u_0 + \frac{q^k}{\alpha}\|_2^2 \\ p^{k+1} &= p^k + \alpha(\nabla(u^d)^{k+1} - w^{k+1}) \\ q^{k+1} &= q^k + \alpha(K(u^d)^{k+1} - u_0 - v^{k+1}), \end{aligned}$$

with $p^0 = q^0 = 0$ $\alpha > 0$.

The standard explicit formulas for w^{k+1}, v^{k+1} and $(u^d)^{k+1}$ are:

$$\begin{aligned} w^{k+1} &= S_{\frac{1}{\alpha}}(\nabla(u^d)^k + \frac{p^k}{\alpha}) \\ v^{k+1} &= S_{\frac{\mu}{\alpha}}(K(u^d)^k - u_0 + \frac{q^k}{\alpha} \\ (u^d)^{k+1} &= (-\Delta + K^*K)^{-1}\big(\nabla^* w^{k+1} + K^*(v^{k+1} + u_0)\big) \end{aligned} \tag{13}$$

where $S_{\frac{1}{\alpha}}(t)$ is the standard soft thresholding, that is

$$S_{\frac{1}{\alpha}}(t) = \begin{cases} t - \frac{1}{\alpha} sign(t) & |t| > \frac{1}{\alpha} \\ 0 & \text{otherwise.} \end{cases}$$

$S_{\frac{\mu}{\alpha}}$ is defined in the same way, up to the obvious replacement of $\frac{1}{\alpha}$ with $\frac{\mu}{\alpha}$. K^* denotes the adjoint matrix of the matrix $K = BHT^{-1}$ given by $K^* = (T^{-1})^* H^* B^*$. Note that one can compute all of these adjoint operators. Δ denotes the usual Laplace's operator.

While, concerning the last iteration of system (13), we used a classical conjugate gradient method.

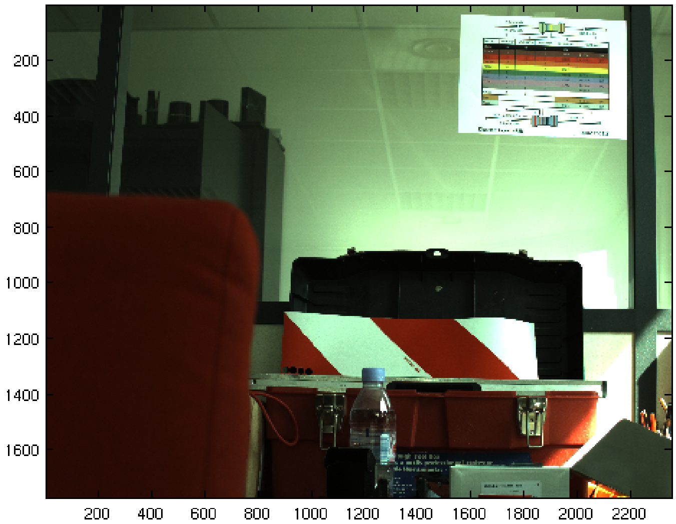

Figure 2: Original image $u^c = T^{-1}(u^d)$. Size image 2200x2000

6 Numerics

We test our method on images of big size (number of pixels $1550 \leq P \leq 4000$). In order to have a blurred mosaicked image to test, we follow the following standard procedure:

1 we pick a color image as a reference u^c, which is a good approximation of a color image to without mosaicking effect.;

2 we apply in the right order the acquisition operator to get the observed degraded image u_0:

$$u_0 = BHu^c + b;$$

3 we formally write $u^c = T^{-1}u^d$ and we work with the new basis $(u^L, u^{\frac{G}{M}}, u^{\frac{R}{B}})$. So we have

$$u_0 = BHT^{-1}(u^d) + b;$$

4 We apply the ADMM algorithm to restore u^d;

5 We set $u^c = T(u^d)$.

As blur operator we always have considered a standard Gaussian low pass filter of size $h = 11$, with standard deviation $\epsilon = 1$. In figures 2 3,4 we restore an image of size 2.200 with a low level of noise. When the level noise is high, μ cannot be too small otherwise, the algorithm does not perform a good demosaicking. In this case the parameter μ is chosen in order to have a good balancing between denoising and demosaicking. In figure 5 we show the restoration results of an image reference detail with different value of the parameter μ. Then in figures 7,8 we show the restoration result obtained on the whole image.

We deal with rescaled images in $[0, 1]$. We made run the Matlab code on an Intel(R) Xeon(R) CPU 5120 @ 1.86GHz.

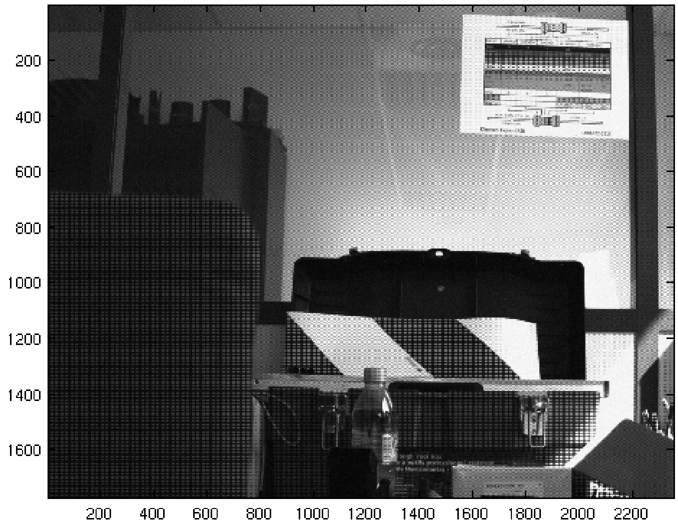

Figure 3: Observed mosaicked blurred and noisy image $u_0 = BHT^{-1}(u^d) + b$. $\sigma = 0.01$.

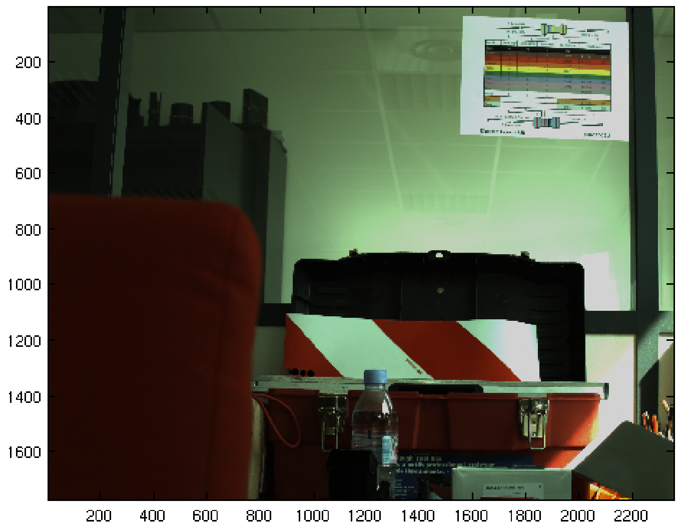

Figure 4: Restored image $u^c = T(u^d)$. CPU time about 30mn, number of iterations 30 $\mu = 30$

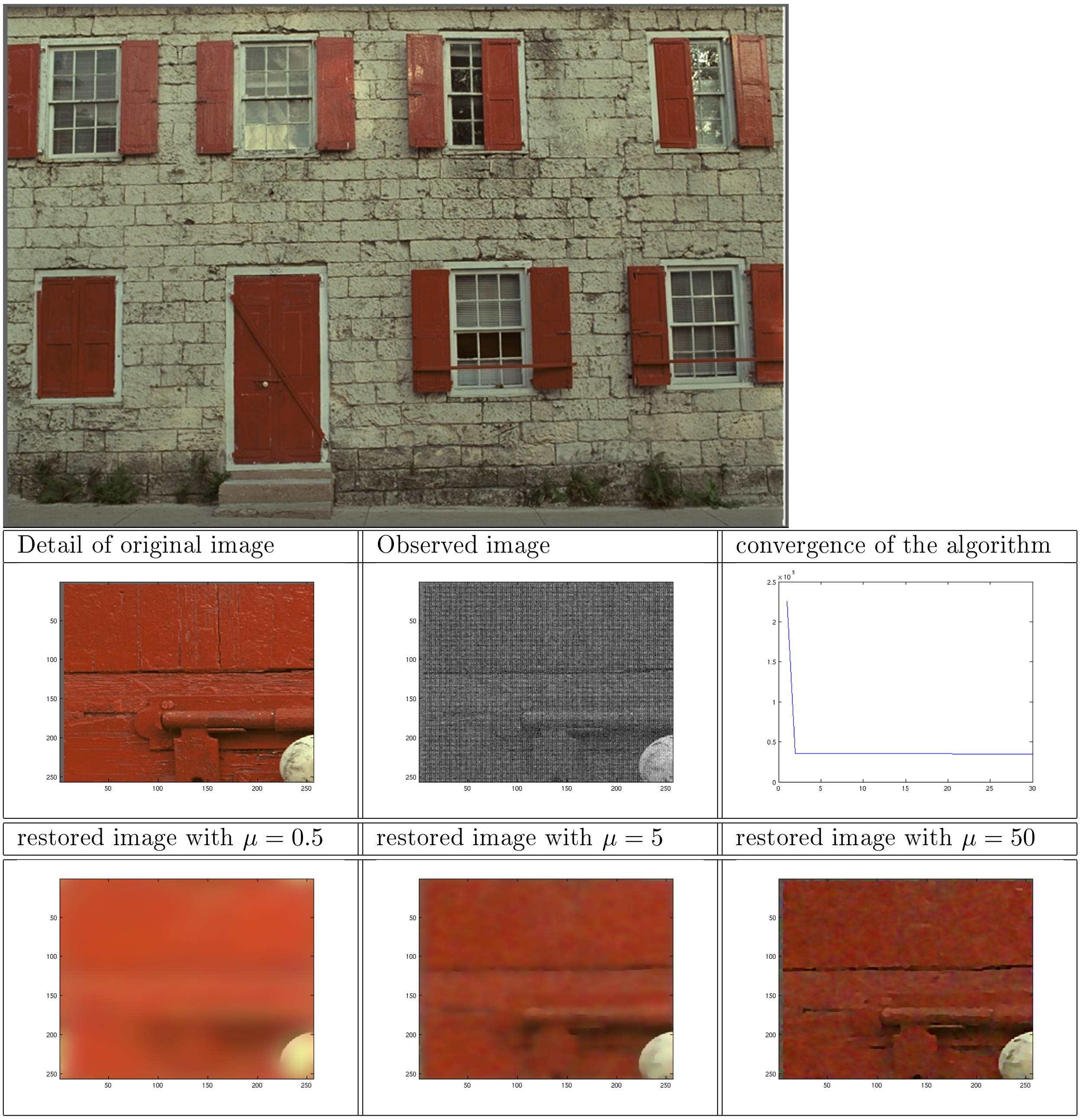

Figure 5: Top left: crop of the original image. Crop size 256x256. Top center: blurred mosaicked noisy image. Top right: convergence of the algorithm. Down left: restored image with a small μ to promote the denoising against the demosaicking. Number of iterations 30. Down center and down right : restored image with a greater value of μ to promote demosaicking against the denoising. Number of iterations 30. Cpu time 4mn.

Figure 6: Original image of size 768x512

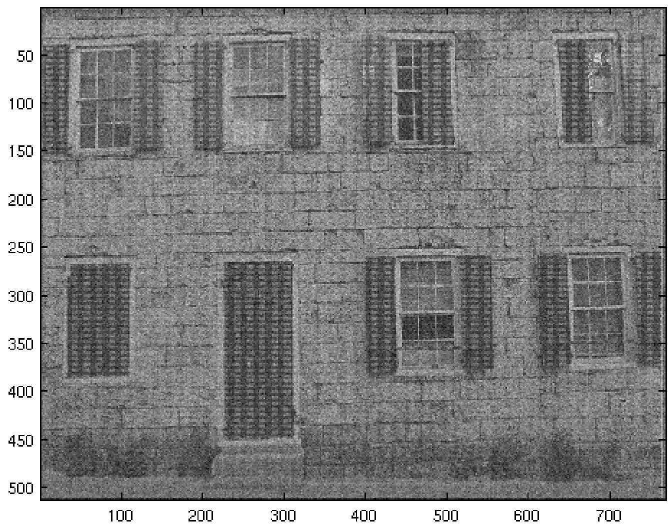

Figure 7: Observed mosaicked blurred and noisy image. $\sigma = 0.5$

Figure 8: Restored image $u^c = T(u^d)$. CPU time about 20 mn, number of iterations 30 $\mu = 20$

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