Performance Analysis of the IEEE 802.11 MAC and Physical Layer Protocol

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Abstract

We present in this paper an analytical model that accounts for the positions of stations with respect to the Access Point (AP) while evaluating the performance of 802.11 MAC layer. Our work is based on the Bianchi's model where the performance of 802.11 MAC layer is computed using a discrete time Markov chain, but where all stations are implicitly assumed to be located at the same distance to the AP. In our model, given the position of one station, we compute its saturation throughput while conditioning on the positions of the other concurrent stations. Further, our model provides the total saturation throughput of the medium. We solve the model numerically and we show that the saturation throughput per station is strongly dependent not only on the station's position but also on the positions of the other stations. Results confirm that a station achieves a higher throughput when it is closer to the AP but bring out that there is a distance threshold above which the throughput decrease is fast and significant. When a station is far from the AP compared to the other stations, it will end up by contending for the bandwidth not used by the other stations. We believe that our model is a good tool to dimension 802.11 wireless access networks and to study their capacities and their performances.

1 Introduction

Nowadays, the IEEE 802.11 WLAN technology offers the largest deployed wireless access to the Internet. This technology specifies both the Medium Access Control (MAC) and the Physical Layers (PHY) [1]. The PHY layer selects the correct modulation scheme given the channel conditions and provides the necessary bandwidth, whereas the MAC layer decides in a distributed manner on how the

offered bandwidth is shared among all stations (STAs). This standard allows the same MAC layer to operate on top of one of several PHY layers.

Different analytical models and simulation studies have been elaborated the last years to evaluate the 802.11 MAC layer performance. These studies mainly aim at computing the saturation throughput of the MAC layer and focus on its improvement. One of the most promising models has been the so-called Bianchi model [2]. It provides closed-form expressions for the saturation throughput and for the probability that a packet transmission fails due to collision.

The modeling of the 802.11 MAC layer is an important issue for the evolution of this technology. One of the major shortcomings in existing models is that the PHY layer conditions are not considered. The existing models for 802.11 assume that all STAs have the same physical conditions at the receiving STA (same power, same coding,. . .), so when two or more STAs emit a packet in the same slot time, all their packets are lost, which may not be the case in reality when for instance one STA is close to the receiving STA and the other STAs far from it [3]. This behavior, called the *capture effect*, can be analyzed by considering the spatial positions of the STAs. In [4] the spatial positions of STAs are considered for the purpose of computing the capacity of wireless networks, but only an ideal model for the MAC layer issued from the information theory is used. The main contribution of this paper is considering both PHY and MAC layer protocols to analyze the performance of exciting IEEE 802.11 standard. Our work reuses the model for 802.11 MAC layer from [6], and extends it to consider interference from other STAs. We compute, for a given topology, the throughput of any wireless STA using the 802.11 MAC protocol with a specific PHY layer protocol.

Without losing the generality of the approach, we only consider in this paper traffic flows sent from the mobile STAs in direction to the AP. The case of bidirectional traffic is a straight forward extension; we omit it to ease the exposition of our contribution. Further, we assume that all STAs use the Distributed Coordination Function (DCF) of

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802.11 and they always have packets to send (case of saturated sources). We present an evaluation of our approach for 802.11b with data rates equal to 1 and 2 Mbps and the results indicate that it leads to very accurate results.

In the next section an overview of the IEEE 802.11 MAC and PHY specifications and the calculation of probability of packet and bit error in an additive white gaussian channel are presented. Section 3 addresses some related works on MAC and PHY layer modeling in IEEE 802.11. In Section 4 we present our model and derive the characterizing equations for it. The numerical and simulation results obtained are presented in Section 5. Section 6 concludes the paper with some pointers to our future work.

2   Background

Two forms of MAC layer have been defined in IEEE 802.11 standard specification named, Distributed Coordination Function (DCF) and Point Coordination Function (PCF). The DCF protocol uses Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) mechanism and is mandatory, while PCF is defined as an option to support time-bounded delivery of data frames. The DCF protocol in IEEE 802.11 standard defines how the medium is shared among stations. DCF which is based on CSMA/CA, consists of a basic access method and an optional channel access method with request-to-send (RTS) and clear-to-send (CTS) exchanged as shown in Fig. 1.
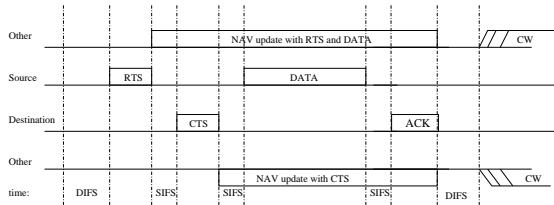

Figure 1. CSMA/CA with RTS/CTS exchange.

If the channel is busy for the source STA, a backoff time (measured in slot times) is chosen randomly in the interval $[0, CW)$, where CW is called the contention window. This timer is decremented by one as long as the channel is sensed idle for a DIFS (Distributed Inter Frame Space) time. It stops when the channel is busy and resumes when the channel is idle again for at least DIFS time. CW is an integer with the range determined by PHY layer characteristics: CW_{min} and CW_{max}. CW will be doubled after each unsuccessful transmission, up to the maximum value which is determined by $CW_{max} + 1$.

When the backoff timer reaches zero, the source transmits the data packet. The ACK is transmitted by the receiver immediately after a period of time called SIFS (Short Inter

Frame Space) which is less than DIFS. When a data packet is transmitted, all other stations hearing this transmission adjust their Network Allocation Vector (NAV), which is used for virtual CS at the MAC layer. In optional RTS/CTS access method, an RTS frame should be transmitted by the source and the destination should accept the data transmission by sending a CTS frame prior to the transmission of actual data packet. Note that STAs in the sender's range that hear the RTS packet update their NAVs and defer their transmissions for the duration specified by the RTS. Nodes that overhear the CTS packet update their NAVs and refrain from transmitting. This way, the transmission of data packet and its corresponding ACK can proceed without interference from other nodes (hidden nodes problem).

Table 1 shows the main characteristics of the IEEE 802.11a/b/g physical layers. 802.11b radios transmit at $2.4GHz$ and send data up to 11 Mbps using Direct Sequence Spread Spectrum (DSSS) modulation; whereas 802.11a radios transmit at $5GHz$ and send data up to 54 Mbps using Orthogonal Frequency Division Multiplexing (OFDM) [1]. The IEEE 802.11g standard [1], extends the data rate of the IEEE 802.11b to 54 Mbps in an upgraded PHY layer named extended rate PHY layer (ERP).

Table 1. PHY layer Characteristics in 802.11.

PHY Layer Characteristic	Available in 802.11/a/b/g
Frequency	5, 2.4 GHz
Data Rates	1, 2, 5.5, 6, 9, 11, 12, 18, 22, 24, 33, 36, 48, 54 Mbps
Modulation	BPSK, DBPSK, QPSK, DQPSK, 16-QAM, 64-QAM, CCK
Error Correction Code	Convolutional codes 1/2, 2/3, 3/4

In each physical layer, there is a basic transmission mode (usually used to send ACK, RTS, CTS and PLCP header) which has the maximum coverage range among all transmission modes. This maximum range is obtained using BPSK or DBPSK modulation which have the minimum probability of bit error for a given SNR compared to other modulation schemes. It has the minimum data rate as well. As shown in Fig. 2, each packet may be sent using two different rates; the PLCP header is sent at the basic rate while the rest of the packet might be sent at a higher rate. The basic rate is 1 Mbps (with DBPSK modulation and CRC 16 bits) for 802.11b and 6 Mbps (with BPSK and FEC rate equal to 1/2) for 802.11a. The higher rate used to transmit the physical-layer payload (which includes the MAC header) is indicated in the PCLP header.

The PLCP Protocol Data Unit (PPDU) frame includes PLCP preamble, PLCP header, and MPDU. Fig. 3 shows the format for long preamble in 802.11b. The PLCP preamble contains the following fields: Synchronization (Sync)
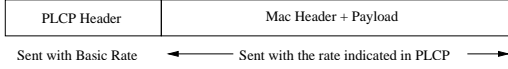
PLCP Header	Mac Header + Payload
Sent with Basic Rate	Sent with the rate indicated in PLCP

Figure 2. Packet format in IEEE 802.11.

and Start Frame Delimiter (SDF). The PLCP Header contains the following fields: Signal, Service, Length, and CRC. The short PLCP preamble and header may be used to minimize overhead and thus maximize the network data throughput. Note that the short PLCP header uses the 2 Mbps with DQPSK modulation and a transmitter using the short PLCP only can interoperate with the receivers which are capable of receiving this short PLCP format. In this paper we suppose that all stations use the long PPDU format in 802.11b. We evaluate our model in 802.11b where STAs use transmission rate equal to 1 and 2 Mbps. Our model can be employed for all other transmission modes for all standards if the packet error rate is calculated.
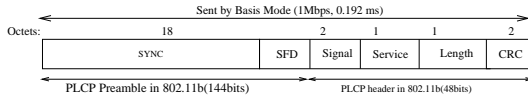

Figure 3. 802.11b long preamble frame format.

In this paper, we assume that the noise over the wireless channel is white Gaussian with spectral density equal to $N_0/2$. In our model we define N_0 as the power of the thermal noise,

$$N_0 = N_f \cdot N_t = N_f \cdot kTW \qquad (1)$$

where N_f denotes the circuit noise value, k the Boltzmann constant, T the temperature in Kelvin and W is the frequency bandwidth. For the BPSK modulation[1], the bit error probability is given by [11]:

$$P_b^{BPSK} = Q\left(\sqrt{2 \cdot \frac{E_b}{N_0}}\right) = Q\left(\sqrt{2 \cdot \frac{E_b}{N_0}}\right) \qquad (2)$$

and for QPSK (4-QAM) is:

$$P_b^{QPSK} = Q\left(\sqrt{2 \cdot \frac{E_b}{N_0}}\right) - \frac{1}{2}Q^2\left(\sqrt{2 \cdot \frac{E_b}{N_0}}\right) \qquad (3)$$

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[1]Our expressions for probability of bit error rates are those of BPSK and QPSK, although 802.11 standards use differentially encoded versions DBPSK and DQPSK for 1 and 2 Mbps, respectively. For coherent detection for high SNR values, above BER expressions are applicable to both classical and differential modulation schemes.

where $\frac{E_b}{N_0}$ is the average signal to noise ratio per bit. The $\frac{E_b}{N_0}$ of the received signal is derived from SNR using the following relationship:

$$\frac{E_b}{N_0} = SNR \cdot \frac{W}{R_b} \qquad (4)$$

where R_b (1 and 2 Mbps) is the maximum bit rate of transmission mode and W (2 MHz) is the unspread bandwidth of the signal. Considering the data packet format shown in Figure 2 the probability of error for packet is:

$$p = PER = 1 - (1 - P_e^{PLCP}) \cdot (1 - P_e^{Payload}) \qquad (5)$$

Where P_e is the probability of error for PLCP (or Payload) and is given by:

$$P_e = 1 - (1 - P_b)^{Length} \qquad (6)$$

P_b is derived from equation 2 and 3 for 1 Mbps and 2 Mbps data rate respectively.

3 Related Works

There have been various attempts to model and analyze the saturation throughput and delay of the IEEE 802.11 DCF protocol since the standards have been proposed. As explained in the introduction there are different analytical models and simulation studies that analyze the performance of 802.11 MAC layer. As an example Foh and Zuckerman present the analysis of the mean packet delay at different throughputs for IEEE 802.11 MAC in [5]. Kim and Hou [7] analyze the protocol capacity of IEEE 802.11 MAC with the assumption that the number of active stations having packets ready for transmission is large. In [8] and [9] they have suggested some extensions to the model proposed in [2] to evaluate packet delay, the packet drop probability and the packet drop time. Since in our model we have used the Bianchi's model [2] and its extension proposed in [6], we will detail these models in this section.

Bianchi's model uses a simple and elegant discrete-time Markov chain to analyze the case of saturated STAs, i.e. STAs that always have packets to send. Wu et al. [6] proposed a scheme named $DCF+$ to enhance the performance of reliable transport protocol over WLAN and analyzed it with an extension of Bianchi's considering finite packet retry limits as defined in the IEEE 802.11 standard.

The retransmission limit is defined in the IEEE 802.11 MAC standard specification with the help of two following counters: Short Retry Count (SRC) and Long Retry Count (LRC). These counters are incremented and reset independently. SRC is incremented every time an RTS fails and LRC is incremented when data transmission fails. Both SRC and LRC

are reset to zero after a successful data transmission. Data frames are discarded when LRC (SRC) reaches dot11LongRetryLimit (dot11ShortRetryLimit). The default values for dot11LongRetryLimit and dot11ShortRetryLimit are 4 and 7 respectively. Considering this limitation, the Markov chain proposed by Bianchi is modified in [6] as shown in Fig. 4. Unlike paper [2], in Wu model m is the maximum backoff stage (retransmission count) which is different for data frame and RTS. In Wu model, m' represents the maximum contention window, i.e. $2^{m'}(CW_{min} + 1) = (CW_{max} + 1)$. In fact the key difference between Bianchi model [2] and Wu model [6], is that the Markov chain models are different, which is because Wu model considers the effects of frame retransmitting limit.
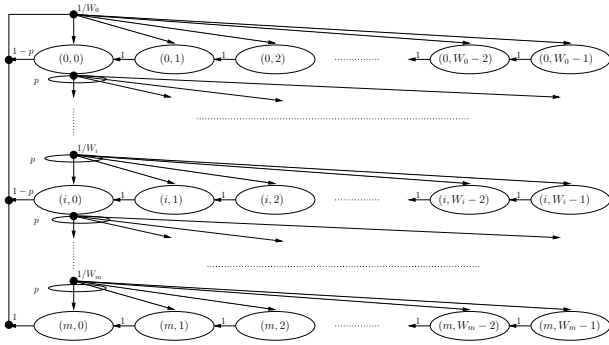

Figure 4. Markov model for backoff window.

In this Bianchi model, the time is divided into slots of variable duration based on what happens during a slot: no transmission, correct transmission, collision. The model computes among others the probability that a station transmits in a slot τ, the probability that a transmitted packet collides with other transmissions p, and the saturation throughput of a station $Z(p, \tau)$, which is a function of τ and p as well as other physical parameters. The packet loss probability is computed as:

$$p = 1 - (1 - \tau)^{n-1} \qquad (7)$$

where n is the total number of STAs. The model also gives the expression of τ as a function of the packet loss probability p using the Markov chain that describes the system, see Fig. 4. Let B be the function relating p and τ in this model, then:

$$\tau = B(p) = \frac{1 - p^{m+1}}{1 - p} b_{0,0} \qquad (8)$$

and $b_{0,0}$ (Which is the stationary probability to find the Markov chain in state $(0,0)$) can be obtained from solving the Markov chain as shown in Equation (9).

Equations (7) and (8) are solved for the values of p and τ. Once these probabilities are obtained, this model computes the saturation throughput $Z(p, \tau)$ of a station. The expression of the throughput is given in Section 4.3, where we adapt this model to our context. None of the above models have considered the channel characteristics (PHY layer). There are a few studies that consider the PHY layer characteristics in 802.11 WLANs. We overview them in what follows.

In [10], the impact of an error-prone channel over all performance measures is analytically analyzed for a traffic saturated IEEE 802.11 WLAN. A modified Markov chain is used to compute the transmission probability per station: the backoff window size that considers the frame-error rates and the maximal allowable number of retransmission attempts. The transition probability from one stage to another (in Bianchi's Markov model) is denoted by p. It is also the probability of an unsuccessful (re)transmission attempt perceived by a test station as its frame is being transmitted on the channel. They supposed that the unsuccessful (re)transmission attempt can happen due to: *collision* of this station with at least one of the $n - 1$ remaining stations, occurring with probability: $p_1 = 1 - (1 - \tau)^{n-1}$; and/or an *error frame*, occurring with probability P_f (due to the channel fading and/or noise). Then they supposed that both events are independent and the probability p can be expressed as: $p = 1 - (1 - p_1)(1 - P_f)$

Other calculations are similar to the Markov model (see Equation (8)). Similar to [2] they express the normalized saturation throughput of IEEE 802.11 DCF within a single WLAN cell in an error-prone channel.

In [12], an improved analytical model that calculates IEEE 802.11 DCF performance taking into account both packet retry limits and transmission errors for the IEEE 802.11a protocol is proposed. Their analysis is very similar to the model presented in [10].

Finally, [13] proposes an analytical model to compute the throughput for the single and multi-user cases with a non ideal channel. This model is validated with system simulations in realistic deployment scenarios. Some changes in Bianchi's Markov chain model are proposed as well. In addition they have assumed that the probability P_j, (the failure probability viewed by the station when a packet is transmitted) is due to either a collision on the channel (with probability P_{cj}) or without collision but error transmission (with probability PER). Hence: $p_j = P_{cj} + (1 - P_{cj})PER$

None of these models have considered specific physical aspects like modulation, FEC, PLCP format in IEEE 802.11 or the channel characteristics (e.g. the distance between the source and the destination). Our approach tries to provide more precise results considering these characteristics in IEEE 802.11.

$$b_{0,0} = \begin{cases} \dfrac{2(1-2p)(1-p)}{W(1-(2p)^{m+1})(1-p)+(1-2p)(1-p^{m+1})} & m \leq m^{'} \\[2em] \dfrac{2(1-2p)(1-p)}{W(1-(2p)^{m^{'}+1})(1-p)+(1-2p)(1-p^{m+1})+W2^{m^{'}}p^{(m^{'}+1)}(1-2p)(1-p^{m-m^{'}})} & m > m^{'} \end{cases} \quad (9)$$

4 Distance Aware Model

Our model considers the interference from the other STAs and the background noise to compute the packet loss probability p. We call it the *DAW (Distance AWare)* model. The expression for the transmission probability τ remains the same as that in Equation (8). The computation of packet error rate (p) is done under the assumptions we presented in Section 2.

Let consider an STA k that transmits a packet to the AP and compute the probability that this packet is lost (i.e., cannot be decoded correctly). We suppose that this STA is located at distance d_k from the AP. We denote its packet loss probability by $p_k(d_k)$. Using Equation (5), the packet loss probability can be computed as follow;

$$P_k = 1 - (1 - P_b^{PLCP})^{L_{PLCP}} \cdot (1 - P_b^{Payload})^{L_{Payload}}, \quad (10)$$

where L_{PLCP} and $L_{Payload}$ are PLCP and Payload length respectively, and P_b^x is bit error probability for part x of the packet (PLCP or Payload). P_b^x can be computed using Equation (2) and (3) considering transmission mode. This expression for P_k assumes that the bit error process is *iid* during the reception of the packet and that the data is not protected by any channel coding scheme. As we need the packet loss probability averaged over all values of bit errors, we will focus on the computation of the expected value $p_k(d_k) = \mathbb{E}[P_k]$.

To do so, we need to decompose SNR into identically distributed elements for which a *pdf* can be defined. Once such pdf is found, we can obtain $p_k(d_k)$ by substituting in Equation (10) and taking the expectation. In order to decompose SNR, we first introduce the Bernoulli random variables Y_i ($i = 1, \dots, n$), being equal to 1 when STA i transmits a packet in a slot time, and equal to 0 otherwise. Next step we look for the power of signal transmitted by STA i at the AP. We denote such power with X_i and we define it as;

$$X_i = Y_i \cdot L(D_i), \quad (11)$$

where $L(D_i)$ expresses the power with which the signal of STA i arrives at the AP after being attenuated over distance D_i and calculated using simple path loss model;

$$L(D_i) = \frac{P_0}{D_i^\alpha}. \quad (12)$$

In this expression, P_0 denotes the STA transmission power and α, the path loss exponent, determines the loss rate. We use $\alpha = 3$ which is commonly used to model loss in an urban environment [14]. Note that this model for the power only considers the attenuation caused by the distance between the emitting terminal and the AP, and ignores other factors such as mobility, shadowing, multi-path fading, etc. Having the power of each STA at the AP, we can compute the interfering power a packet transmitted by STA k faces. We denote this power by I_k and write it as;

$$I_k = \sum_{i \neq k}^{n} Y_i \cdot L(D_i). \quad (13)$$

This allows to write the following expression for the SNR at the AP of a packet/signal coming from STA k at the given distance d_k:

$$SNR_k = \frac{L(d_k)}{N_0 + I_k} = \frac{L(d_k)}{N_0 + \sum_{i \neq k}^{n} Y_i \cdot L(D_i)}. \quad (14)$$

N_0 is the background noise (see Equation (1)). We can see that to compute P_k using Equation (10), the only random variable is I_k. Hence, having the *pdf* of I_k, which we denote by $f_{I_k}(x)$, we can compute $p_k(d_k) = \mathbb{E}[P_k]$. Assuming independence of Y_i, as in the Bianchi model, $f_{I_k}(x)$ can be expressed as an $n - 1$ convolution:

$$f_{I_k}(x) = f_{X_1} \otimes \cdots \otimes f_{X_{k-1}} \otimes f_{X_{k-2}} \otimes \cdots \otimes f_{X_n}(x). \quad (15)$$

In the analysis above, we kept the distance from STA i to the AP random denoted by D_i, except for STA k for which we are computing p_k. Then we compute p_k for two cases. First, we compute it when the stations' positions are known (the D_i are deterministic): the only randomness in this case lies in the dynamics of the MAC layer. Second, we compute p_k for a more general case where nodes are uniformly distributed in the plane.

4.1 Fixed Topologies

Suppose we are given the distance vector $\overline{D} = \{d_1, \dots, d_n\}$, where d_i describes the distance of STA i to the access point. Since all distances are fixed, we omit in this section the index of distance from loss and transmission probabilities. For an STA k, we aim at finding the *pdf* of I_k. I_k gives the interfering power produced by all the

other STAs at the AP. To compute I_k, we need $f_X(x)$, the *pdf* of the power at the AP of an individual STA. For an STA i, $f_{X_i}(x)$ can be written as:

$$f_{X_i}(x) = \delta_x(0)(1 - \tau_i) + \delta_x(L(d_i))\tau_i, \qquad (16)$$

where $\delta_x(x_0)$ is a Dirac pulse at $x = x_0$ and τ_i denotes the transmission probability of STA i. $f_{I_k}(x)$ can be computed using Equation (15). Note that the values τ_i in $f_{I_k}(x)$ are left unknown.

Using Equation (10) and taking expectation, we get the packet loss probability of STA k:

$$p_k = \mathbb{E}[1 - (1 - P_b^{PLCP})^{L_{PLCP}} \cdot (1 - P_b^{Payload})^{L_{Payload}}] \qquad (17)$$

For example when the packet is sent with 1 Mbps, the equation can be simplified as shown in Equation (18) (Since the PLCP and the payload are sent with the same modulation and data rate).

This expression of p_k is a function of the transmission probabilities of the other STAs via the *pdf* functions f_{X_i}. From the Bianchi model, the transmission probability of an STA is related to its collision probability via the function in Equation (8) (by substituting p by p_k and τ by τ_k). Thus, using Equations (8) and (17), we set up a non linear system of equations, which can be solved numerically for all p_k and τ_k. Having the p_k and τ_k, the throughput of any STA k can be computed. This computation is shown in Section 4.3.

4.2 Random Topologies

We consider now the case where the STAs are uniformly distributed in a disk of radius r around the AP. Thus, the *pdf* of D (the distance to the AP of an STA) has the following form:

$$f_D(d) = \{1_{0 \le d \le 1}\}\frac{2d}{r^2}. \qquad (19)$$

Consider an STA k located at distance d_k from the AP, and let us focus at computing its average performance over all possible positions of the concurrent STAs. As for fixed topology case, we have to find the *pdf* of I_k (The interference caused by the other STAs at the AP.).

However the computation of $f_{X_i}(x)$ (the *pdf* of signal power at the AP of a random STA i) becomes more complex. We first write the cumulative distribution function of X_i (for $x \ge 0$), see Equation (20). In this equation, $\mathbb{E}[\tau_i(D_i)]$ is the transmission probability of an STA i averaged over all its possible locations. By differentiation and using the expression of $L(D_i)$, we find the *pdf* of X_i, see Equation (21).

Assume that the transmission probability of a random STA i is only dependent on its own position and independent of that of the others. Only the number of the other STAs is supposed to influence the transmission probability of STA i. This is the case when the number of STAs is large. Under this assumption, the variables X_i are independent of each other. We can therefore compute the *pdf* of I_k using Equation (15) and (21).

Note that f_{I_k} is a function of one unknown $\mathbb{E}[\tau_i(D_i)]$. The packet collision probability can be obtained by plugging f_{I_k} in Equation (17). We substitute then the expression of $p_k(d_k)$ in Equation (8) to get $\tau_k(d_k)$, the probability with which STA k transmits a packet in a slot time averaged over all possible positions of the other STAs. Finally, the throughput of STA k averaged over all locations of the other STAs can be computed in a similar way to the fixed case as will be discussed in Section 4.3.

Now we explain how to find the expression of $\mathbb{E}[\tau_i(D_i)]$. To solve for this expectation, we write an implicit equation with $\mathbb{E}[\tau_i(D_i)]$ as a variable, then we solve this equation numerically. Equations (21) and (17) give us the expression of $p_k(d_k)$ for STA k as a function of $\mathbb{E}[\tau_i(D_i)]$. Denote by $p_k(d_k) = G(d_k, \mathbb{E}[\tau_i(D_i)])$ this expression. Using 17, we can write;

$$\tau_k(d_k) = B(p_k(d_k)) = B(G(d_k, \mathbb{E}[\tau_i(D_i)])) \qquad (22)$$

We get our implicit equation in $\mathbb{E}[\tau_i(D_i)]$ by summing over all the values of d_k, as shown in Equation (23).

$$\mathbb{E}[\tau_i(D_i)] = \mathbb{E}[B(p_k(d_k))] = \int_0^r \frac{2\pi\rho d\rho}{\pi r^2} B(G(\rho, \mathbb{E}[\tau_i(D_i)])). \qquad (23)$$

Once we obtain $\mathbb{E}[\tau_i(D_i)]$, all transmission probabilities, collision probabilities and throughput can be obtained using our above analysis. In summary, for an STA located at distance d_k:

- The packet collision probability $p_k(d_k)$ can be obtained by plugging Equation (22) in (17), where the value of $\mathbb{E}[\tau_i(D_i)]$ is computed numerically with the implicit Equation (23).

- The packet transmission probability $\tau_k(d_k)$ is computed by substituting p by $p_k(d_k)$ in Equation (8).

- Given $p_k(d_k)$ and $\tau_k(d_k)$, the throughput of STA k can be obtained in a similar way to the Bianchi model. The throughput of a random STA can be computed as well.

4.3 Throughput Calculation

We now derive the throughput of a single STA k at a given distance d_k. In the case of a fixed topology (Section 4.1) this throughput depends on the position of all other STAs and their transmission probabilities τ_i, whereas in the

$$P_k = 1 - \int_{x=0}^{\infty} \left(1 - Q\left(\sqrt{\frac{2L(d_k)W}{(N_0 + x)R}} \right) \right)^{L_{PLCP} + L_{Payload}} f_{I_k}(x)dx. \tag{18}$$

$$F_{X_i}(x) = (1 - \mathbb{E}[\tau_i(D_i)]) + 1_{\{P_0/r^\alpha \le x \le P_0\}} \mathbb{E}[\tau_i(D_i)] \left(1 - \frac{(L^{-1}(x))^2}{r^2} \right). \tag{20}$$

$$f_{X_i}(x) = \delta_x(1 - \mathbb{E}[\tau_i(D_i)]) + 1_{\{P_0/r^\alpha \le x \le P_0\}} \frac{2}{\alpha r^2 x} \left(\frac{P_0}{x} \right)^{2/\alpha} \mathbb{E}[\tau_i(D_i)]. \tag{21}$$

case of random topologies (Section 4.2), the throughput depends on the other STAs average location and their average transmission probability $\mathbb{E}[\tau_i(D_i)]$.

Consider first the case of fixed topology. The throughput of an STA k is given by the function $Z(p_k, \tau_k)$, which has the following form:

$$Z(p_k, \tau_k) = \frac{\tau_k(1 - p_k)L}{(1 - P_{tr})\sigma + P_{tr}P_sT_s + P_{tr}(1 - P_s)T_c}. \tag{24}$$

In the numerator of the throughput expression, we put the average number of useful bits transmitted in a slot time whereas the denominator corresponds to the average duration of a slot. σ is the physical slot time of 802.11 MAC layer. T_s and T_c are respectively the duration of a slot (following the slot definition in the Bianchi model) when a packet is successfully transmitted and the duration of a slot when two or more packets collide. L is the payload size. We consider MAC, IP and UDP headers in our calculation for packet length (The sum of these headers is denoted by H.). In addition, as explained in Section 2, the PHY layer adds to each transmission a constant PLCP preamble and header of total duration t_{PLCP}. Similar to [6], the slot time duration T_s and T_c for basic access mode considering ACK timeout, will become:

$$\begin{cases} T_s^{bas} = 2t_{PLCP} + DIFS + \frac{H+L}{R_d} + SIFS \\ \quad + \frac{ACK}{R_b} + 2\delta \\ T_c^{bas} = 2t_{PLCP} + DIFS + \frac{H+L}{R_d} + SIFS + \frac{ACK}{R_b} \end{cases} \tag{25}$$

R_b is the data rate for the basic transmission mode (i.e., 1 Mbps for 802.11b) and R_d is the data rate for payload (which is 1 or 2 Mbps in our simulations). Note that for the RTS/CTS access mode, all calculation regarding to packet error rate should be done for RTS packets. δ is the propagation delay.

We come now to the definition of P_{tr} and P_s in the denominator of Equation (24). With P_{tr}, we denote the probability that at least one of the n STAs is transmitting, which can be formulated as $1 - \prod_{i=1}^{n}(1 - \tau_i)$. Further, P_s, the

probability that such a transmission is successful, is equal to $\frac{\sum_{i=1}^{n} \tau_i(1-p_i)}{P_{tr}}$. In the case of random positions, only the expressions of P_{tr} and P_s change. These expressions are shown in Equation (26) and (27) for an STA k located at distance d_k to the AP.

$$P_{tr} = 1 - (1 - \tau_k(d_k))(1 - \mathbb{E}[\tau_i(D_i)])^{n-1} \tag{26}$$

$$P_s = \frac{\tau_k(d_k)(1 - p_k(d_k)) + (n-1)\mathbb{E}[\tau_i(D_i)](1 - \mathbb{E}[p_i(D_i)])}{P_{tr}} \tag{27}$$

5 Model Verification and Simulation Results

We implemented the *DAW* analytical model in MATLAB and compared the results with *ns-2* simulation. Our *ns-2* simulations are based on the package described in [15]. In this package we consider the effect of wireless physical layer while simulating mobile networks. Physical layer parameters like path loss, fading, interference and noise computation are added in this *ns-2* simulation package. The *ns-2* simulation results presented later are averaged over 10 runs with different random seeds.

We first consider a fixed topology, i.e the D_i values are deterministically set. For this scenario, we used 2 network configurations as shown in Fig. 5. The first configuration consists of one AP and several STAs which send CBR packets at saturation rate to the AP using UDP connections. All STAs are located at 5 meters from the AP. In this configuration, we calculate the total throughput while varying the number of STAs. The second configuration consists of one AP and 6 STAs: 5 STAs are placed at 5 meters from the AP and the 6th STA is moving away the AP from 1 meter to 25 meters. Each meter, it held fixed for 20 minutes and transmits CBR data over a UDP connection. The fixed STAs send continuously the same traffic as the mobile STA to the AP. We calculate the throughput of both moving and fixed STAs for each position of the mobile node. Further, all simulations are done with two transmission modes (i.e., BPSK 1Mbps and QPSK 2 Mbps in IEEE 802.11b).
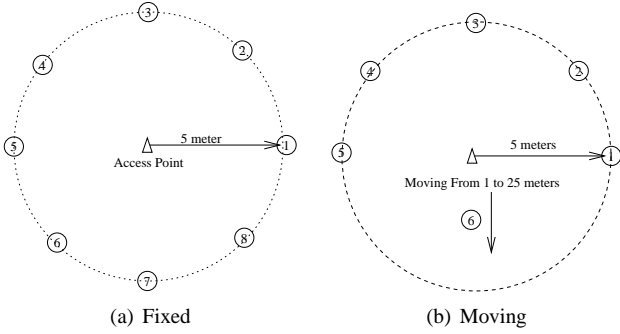
(a) Fixed   (b) Moving

Figure 5. Network topologies.

For the first configuration, we compare the throughput obtained with the *DAW* model to the ones computed from Bianchi's model and *ns-2* simulations. We expect to obtain very similar results since the probability that a packet is erroneous, and hence dropped, is very close to one when parallel transmissions occur (all STAs are positioned at the same distance from the AP). The corresponding total throughput of our distance-aware model and the Bianchi's model along with ns-2 simulations are shown in Fig. 6. We can observe a very close match between our model and *ns-2* simulation results.

We now use the *DAW* model to investigate the throughput of the STA that moves through a fixed topology of wireless STAs using the second configuration explained above. Fig. 7 shows the throughput of a fixed STA (at 5 meters from the AP) and the throughput of the moving STA. We also plot in the same figure the throughput obtained by an STA if the Bianchi's model was used. The results are very interesting. When the moving STA is close to the AP (less that 5 meters), its throughput is greater than the one of fixed stations. When the moving station and fixed stations are near each other (i.e., around 5 meters) they all have the same throughput and it is equal to the one given by the Bianchi's model (which supposes that two colliding packets are automatically lost). Finally, when the moving STA is far from the AP (more than 5 meters), its receiving power level at the AP starts to become lower than that of the close and fixed STAs and so its packets are lost when they collide with the ones from fixed STAs. The fixed STAs get then a higher throughput than the moving STA and the difference is approximately equal to the bandwidth not used by the close and fixed STAs. The Bianchi's model is no longer good in such moving case as shown in Fig. 7. The result is also confirmed with *ns-2* simulation as shown in the Figures.

To better illustrate the above results which are obtained for throughput of fixed and moving STA, we evaluate and calculate two conditional probabilities, shown in Fig. 8:

1. the probability that the moving STA loses its packet

when it contends for the medium with one or more fixed STAs ($P_{pkt-loss-mov}$).

2. the probability that one fixed STA loses its packet when it contends for the medium with the moving STA ($P_{pkt-loss-fix}$).

These two conditional probabilities illustrate the behavior of *DAW* model in the second configuration where one node moves. As an example, we calculate the second probability ($P_{pkt-loss-fix}$) for the case where data is transmitted with 1 Mbps. Let τ_{fix} (τ_{mv}) be the transmission probability of a fixed (moving) STA and d_{fix} (d_{mv}) be its distance to the AP. Equation (28) shows this probability ($P_{pkt-loss-fix}$), where bino($i, 5, \tau_{fix}$) denote the *pdf* of a binomial RV with parameters 5 and τ_{fix} that is shown in Equation (29). The sum in Equation (28) accounts for the different possible values of the number of fixed stations which are transmitting at the same as the moving station. Other probability calculations ($P_{pkt-loss-mov}$ and for different transmission rates) are straight forward.

As shown in Figure 8 both probabilities ($P_{pkt-loss-mov}$ and $P_{pkt-loss-fix}$) are equal to 1 when all STAs have equal distance from AP (same as assumed by the Bianchi's model). $P_{pkt-loss-mov}$ remains equal to 1 when the moving STA is far from the AP since its power level at the AP is low comparing to closed and fixed stations. On the other hand, $P_{pkt-loss-fix}$ drops to 0 at around 5 meters, which means that closed and fixed STAs always win when their packets collide with the packets from the moving STA at more than 5 meters from the AP. On the other hand $P_{pkt-loss-mov}$ drops to 0, when the moving station is near the AP (less than 5 meters) and so wins when its packets collide with packets from fixed stations.

We now consider the random topology case, where STAs are uniformly distributed in a disk of radius 10 meters centered at the AP. We select one STA and move it from 1 to 10 meters, and we compute its throughput averaged over all the possible locations of the other 9 STAs using the method explained in Section 4.2. We also compute the average throughput of any other fixed STA. To validate these results, we have run 250 numerical simulations (with *DAW* model) for 250 realizations of the fixed topology using above scenario. We then use the fixed topology method to find the throughput per realization, and we average over all realizations.

The results are shown in Fig. 9. When the moving STA is close to the AP, it gets a higher throughput than the average throughput of the others, since with a high probability the other STAs are far from the AP. However, this throughput decreases when the STA moves farther from the AP until it drops below the average throughput of the others. The results for 250 realizations, shown in Fig. 9, validate our analysis in random case (Section 4.2) as well.
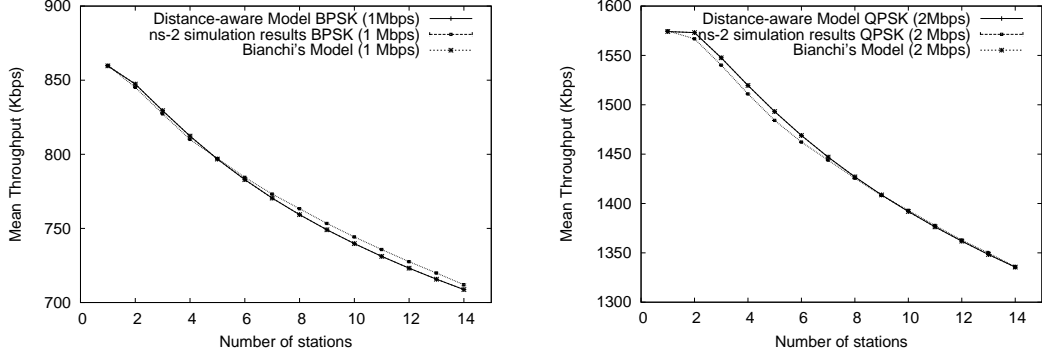
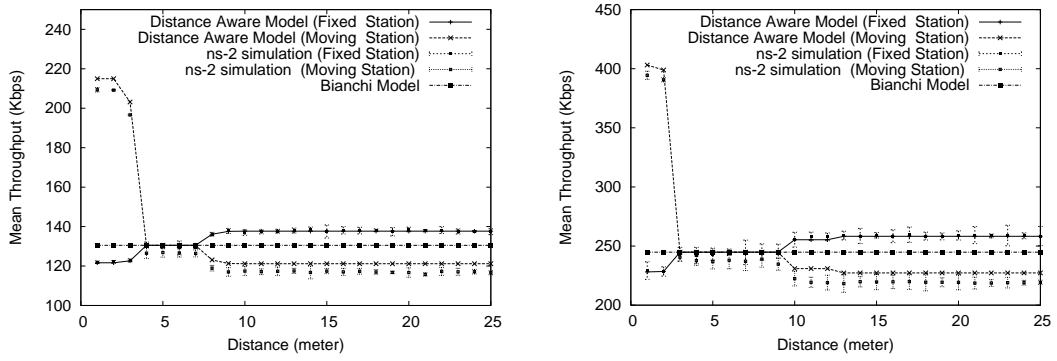**Figure 6. Simulation and Model comparison for fixed scenarios for** *left***: 1 Mbps and** *right***: 2 Mbps.**

Figure 7. Simulation and Model comparison for moving scenarios, *left***: 1 Mbps and** *right***: 2 Mbps.**

$$P_{pkt-loss-fix} = 1 - \frac{\sum_{i=1}^{5} \mathrm{bino}(i, 5, \tau_{fix}) \left(1 - Q \left(\sqrt{\frac{2L(d_{mv})W}{(N_0 + i \cdot L(d_{fix}))R}} \right) \right)^{L_{PLCP} + L_{Payload}}}{1 - (1 - \tau_{fix})^5} \tag{28}$$

$$\mathrm{bino}(i, 5, \tau_{fix}) = \sum_{x=0}^{5} \binom{5}{i} \tau_{fix}^i (1 - \tau_{fix})^{5-i} \delta_x(i). \tag{29}$$
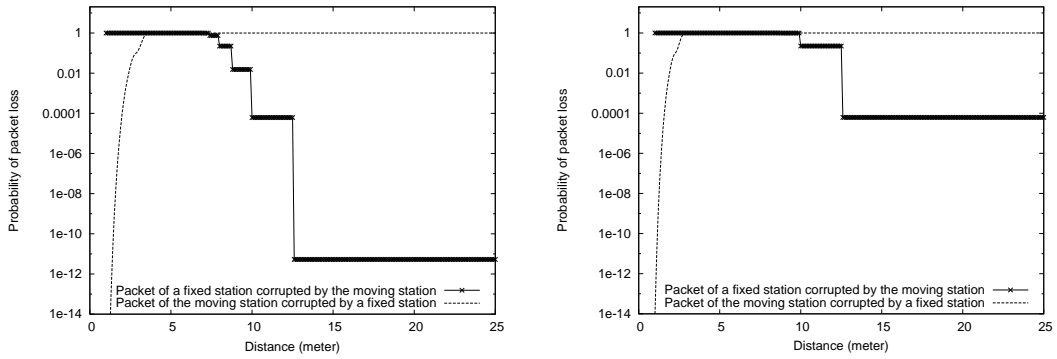

Figure 8. Packet corruption probability of the moving station, *left***: 1 Mbps and** *right***: 2 Mbps.**
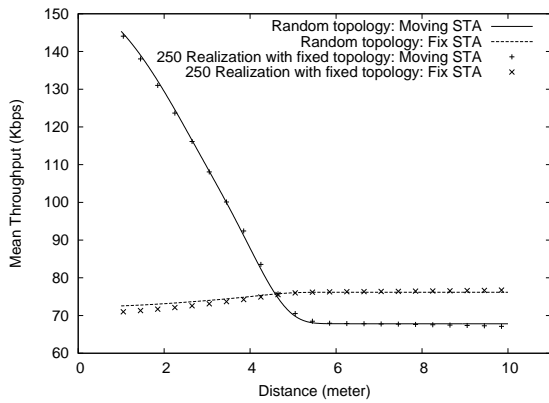
9

Figure 9. Throughput calculation for random topology, BPSK(1 Mbps).

6 Conclusion and Future Works

We presented in this paper an analytical model that accounts for the positions of STAs when evaluating the performance of 802.11 MAC layer. Our model achieves more realistic results comparing to the models which do not consider PHY channel conditions (e.g., in the scenarios where the STAs move).

This model has different extensions on which we are working on. One extension considers an AP that transmits packets, which would allow us to find the optimal AP placement for a given topology. Further extensions could consider fading channel models as well. An ad-hoc mode evaluation to numerically approach the wireless capacity found in [4] could be one of its extensions.

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