Tensor Decomposition by Vector Bundles tools

This PhD is funded by the Marie Curie program of European Union through the innovative training network (ITN) POEMA on polynomial optimization.

More info and positions at https://easychair.org/cfp/POEMA-19-22.
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Scientific context and Goals. Tensors with their different notions of rank make a common language in Signal Processing, in Phylogenetics, in Computer Vision and in several Engineering models and applications. Tensors are the multilinear extension of matrices and the intriguing connection between tensors and matrices poses several mathematical challenges. The optimization problem of matrices, with respect to the rank, is linked to the spectral theory of matrices. The analogous questions for tensors are well understood only in the rank one case, a first goal of this project is the understanding of the higher rank case.

Exact tensor decomposition algorithms, based on algebraic geometry techniques, and in particular using vector bundles, have been introduced recently. A second goal of this individual project is to convert these algorithms as numerical approximate algorithms. With this goal, the first step is to consider the kernel of Hankel or Moment matrices. When the tensor is close to a tensor of small rank, we wish to construct the kernel of the closest low rank matrix, of the same structure than the given one, developing the algebraic analysis further.

The second step is to extend the range of ranks where the algorithm can work, by considering more general contraction maps, defined by suitable vector bundles. The matrices obtained in this way are called Young flattening and they have a block structure. The kernel of such matrices is again a linear space of tensors. The main result is that the common tensor eigenvectors of this kernel space give the exact tensor decomposition of the original tensor. This algorithm has been implemented with algebraic tools. A numerical implementation is desirable and it is one important goal of this individual project.

Working Context. The PhD candidate will be hosted by the Department of Mathematics and Computer Science, located in Florence. The team in Algebraic Geometry and Tensors, led by Giorgio Ottaviani, has strong connections with Department of Mathematics in Siena (Luca Chiantini) and in Ferrara (Massimiliano Mella) with whom we organize a joint seminar. The team has a strong expertise in effective algebraic geometry, symbolic-numeric computation, algorithms and software for tensor decomposition. The team has a strong tradition in hosting international students.

Planned secondments. The PhD candidate will have a research stay (secondments) at Sorbonne University in Paris, France (Mohab Safey el Din) and at INRIA in Sophia Antipolis, France (Bernard Mourrain).

Required Skills. Motivated candidate should hold — at the date of recruitment — a Master's degree in Mathematics, Computer Science or Engineering (or any equivalent

diploma). The applicant should have a solid background in either real or complex algebraic geometry, computer algebra or optimization. Some practice with vector bundle language (sections, operations with bundles) and good programming skills are also a plus. Knowledge of Italian does not constitute a pre-requisite.

The candidates are kindly asked to send an e-mail with "POEMA candidate" in the title, a CV and motivation letter to giorgio.ottaviani@unifi.it and to submit their documents at https://easychair.org/cfp/POEMA-19-22.