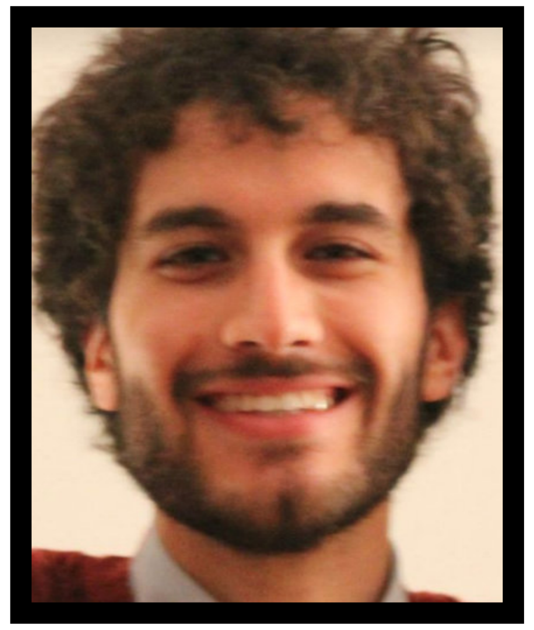
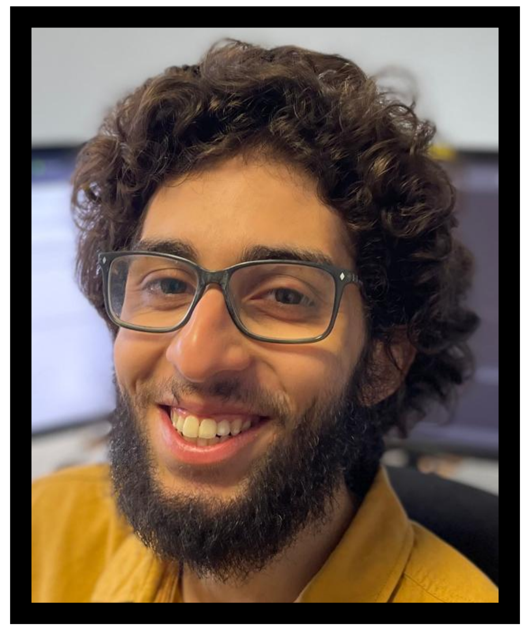
FL under heterogeneous and correlated client availability

Angelo Rodio

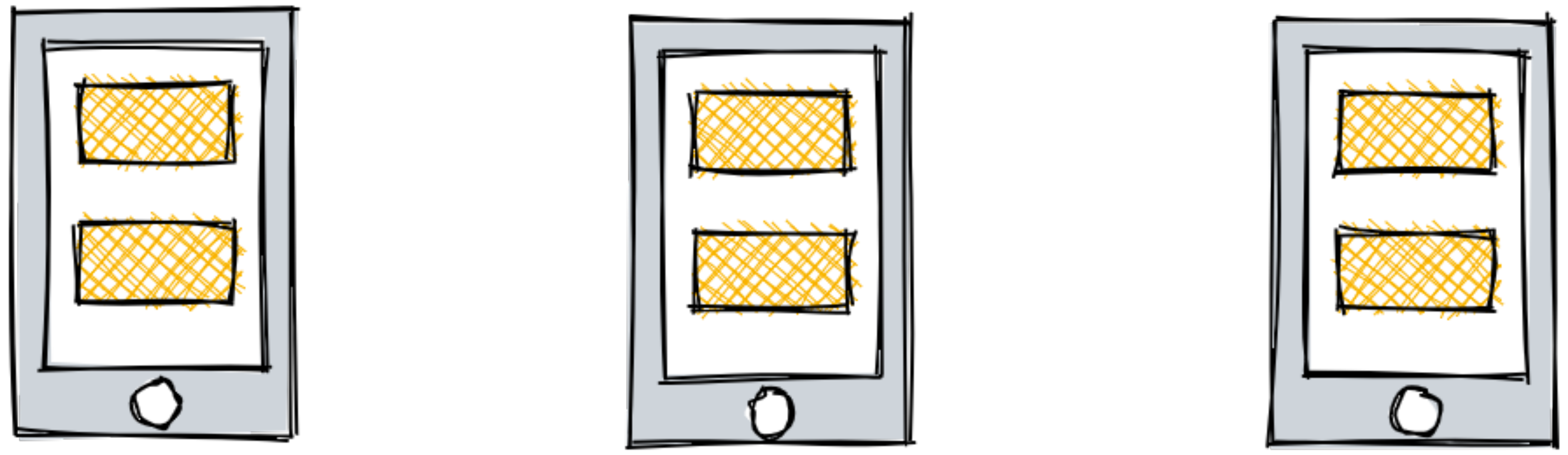
Francescomaria Faticanti

Othmane Marfoq

Giovanni Neglia

Emilio Leonardi

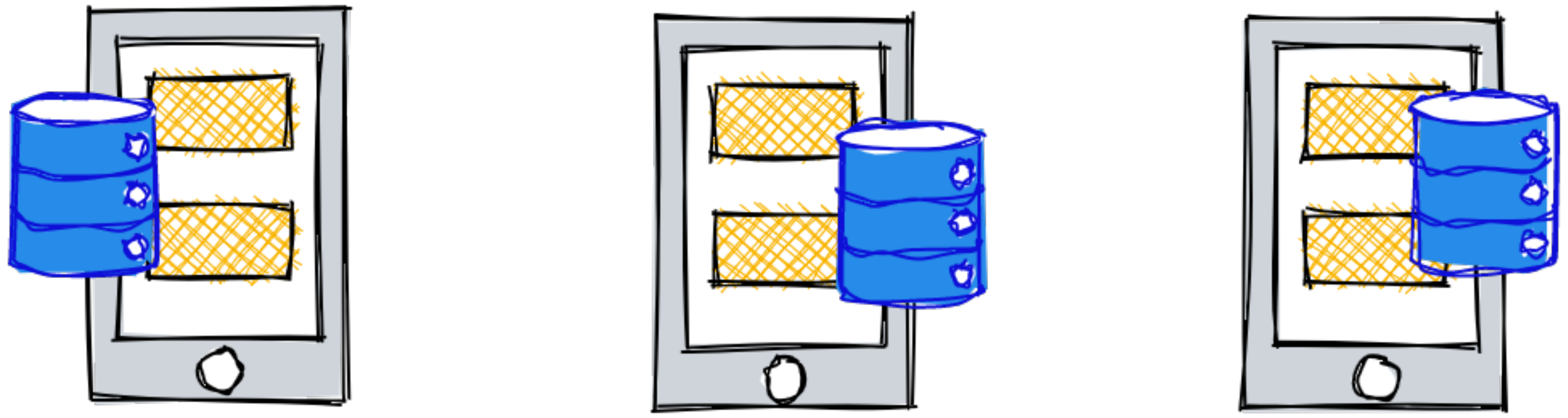
Federated Learning



Clients $k = 1, \ldots, K$

Federated Learning

Dataset $D_k = \{\xi_{kl}\}_{l=1}^{n_k}$



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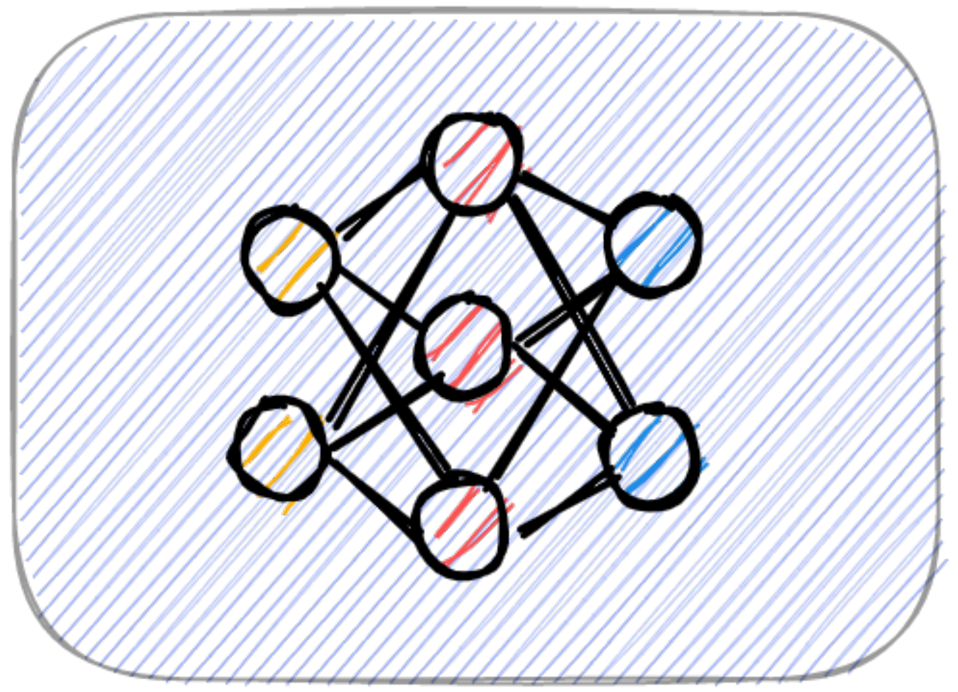
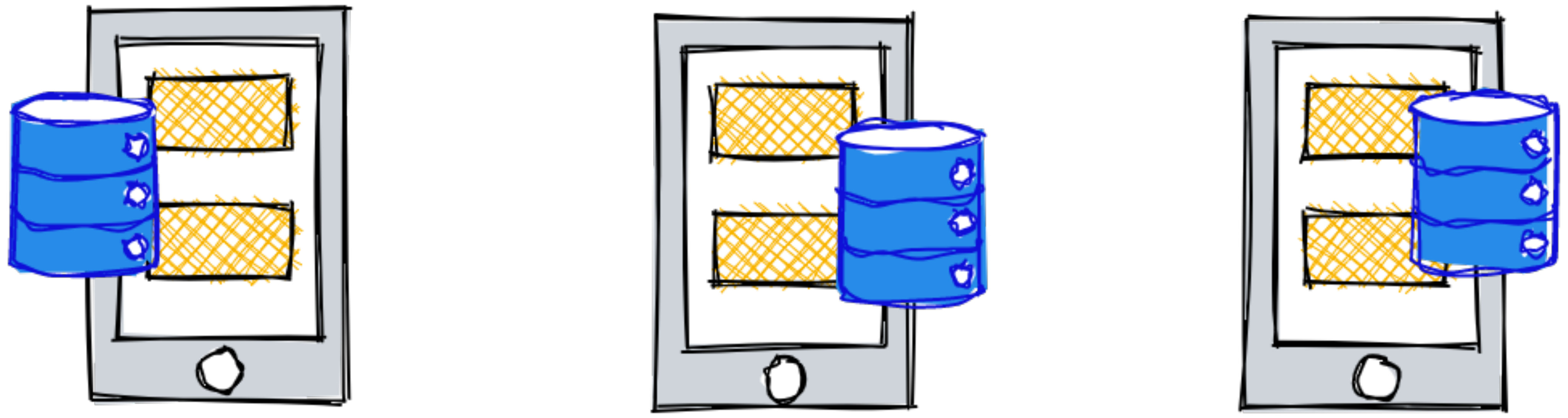


Global model

$\boldsymbol{w} \in \mathbb{R}^d$

Dataset $D_k = \{\xi_{kl}\}_{l=1}^{n_k}$

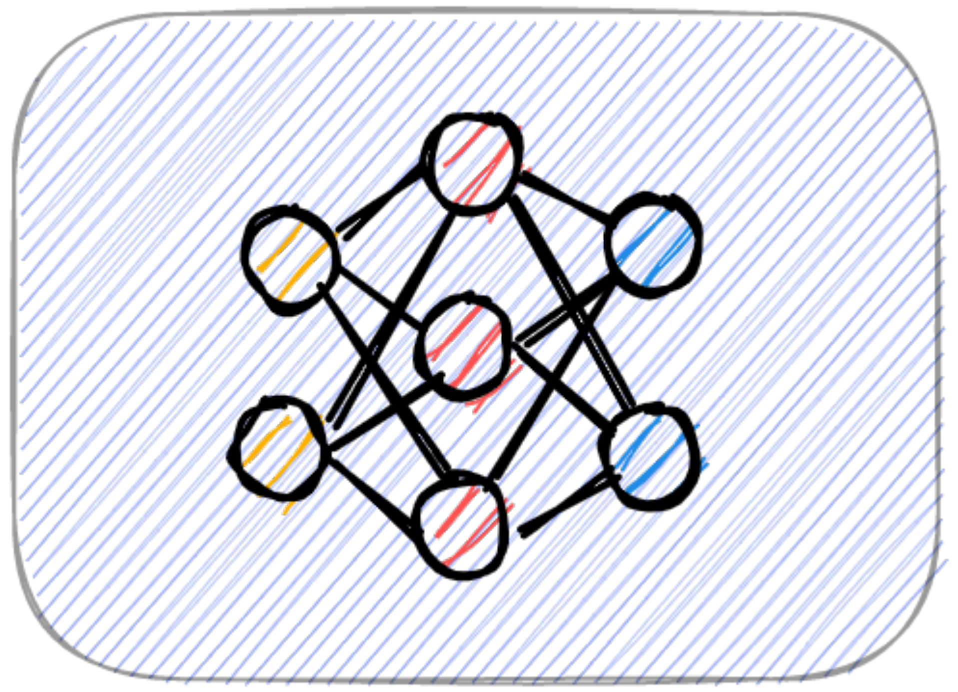


Clients $k = 1, \ldots, K$

Federated Learning

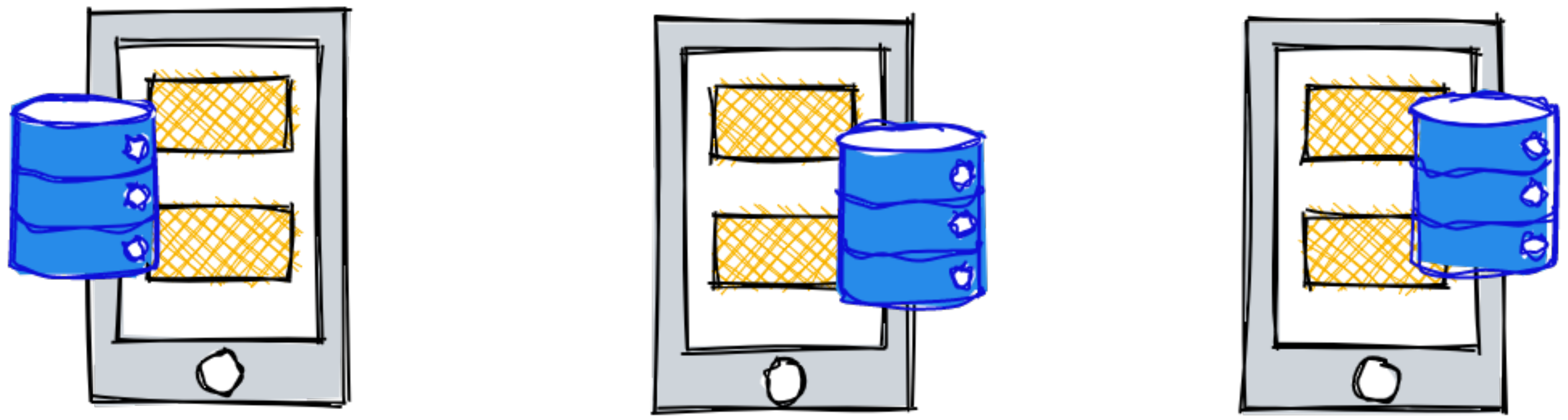
Solve the optimization problem

$$\min_{\boldsymbol{w}} F(\boldsymbol{w}) = \sum_{k=1}^{K} \alpha_k \, F_k(\boldsymbol{w})$$



where

$$F_k(\boldsymbol{w}) = \frac{1}{n_k} \sum_{l=1}^{n_k} \ell(\boldsymbol{w}, \xi_{kl})$$



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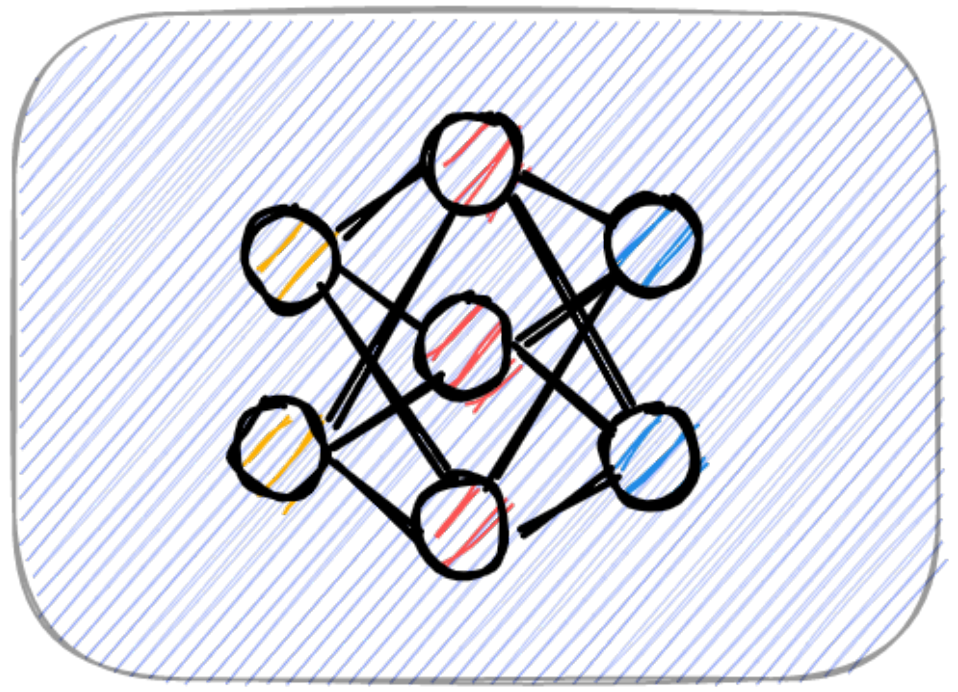
Solve the optimization problem

$$\min_{\boldsymbol{w}} F(\boldsymbol{w}) = \sum_{k=1}^{K} \alpha_k \, F_k(\boldsymbol{w})$$

$\boldsymbol{\alpha}$: target importance



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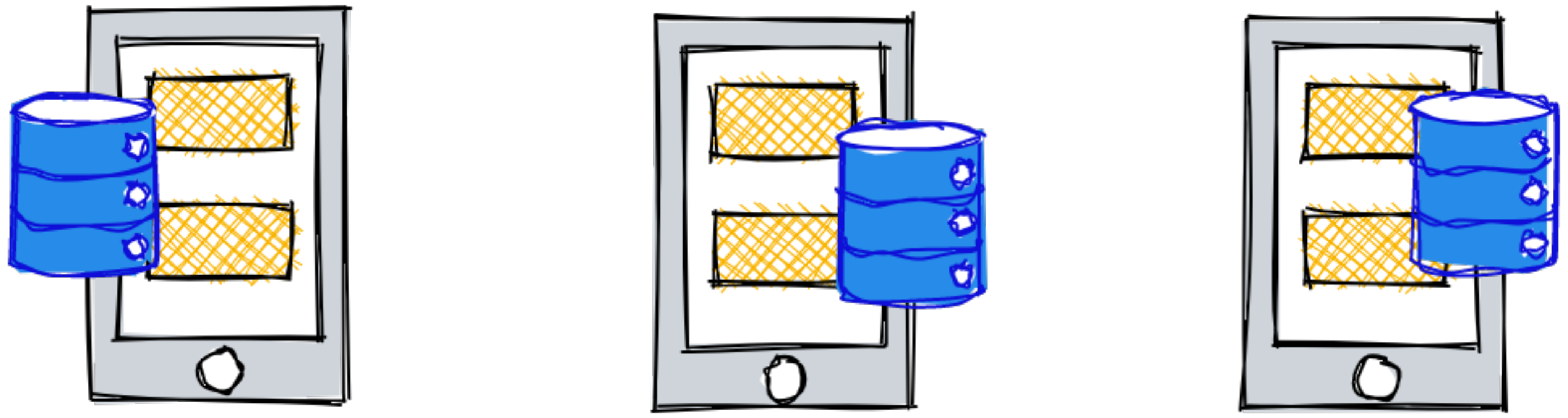
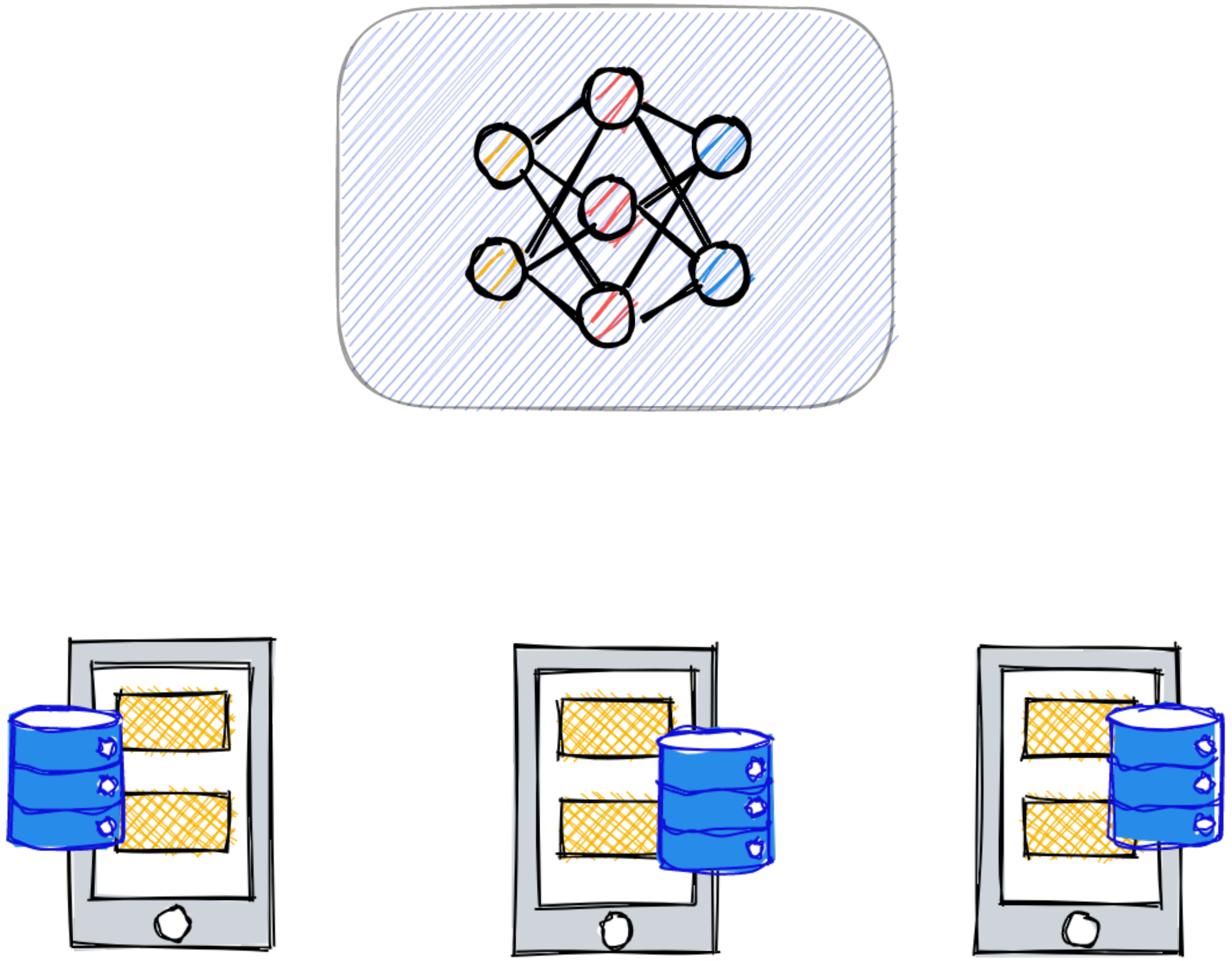


General FL Algorithm

for $t \in \{0, \ldots, T-1\}$ **do**:

A_t: set of active clients at time t



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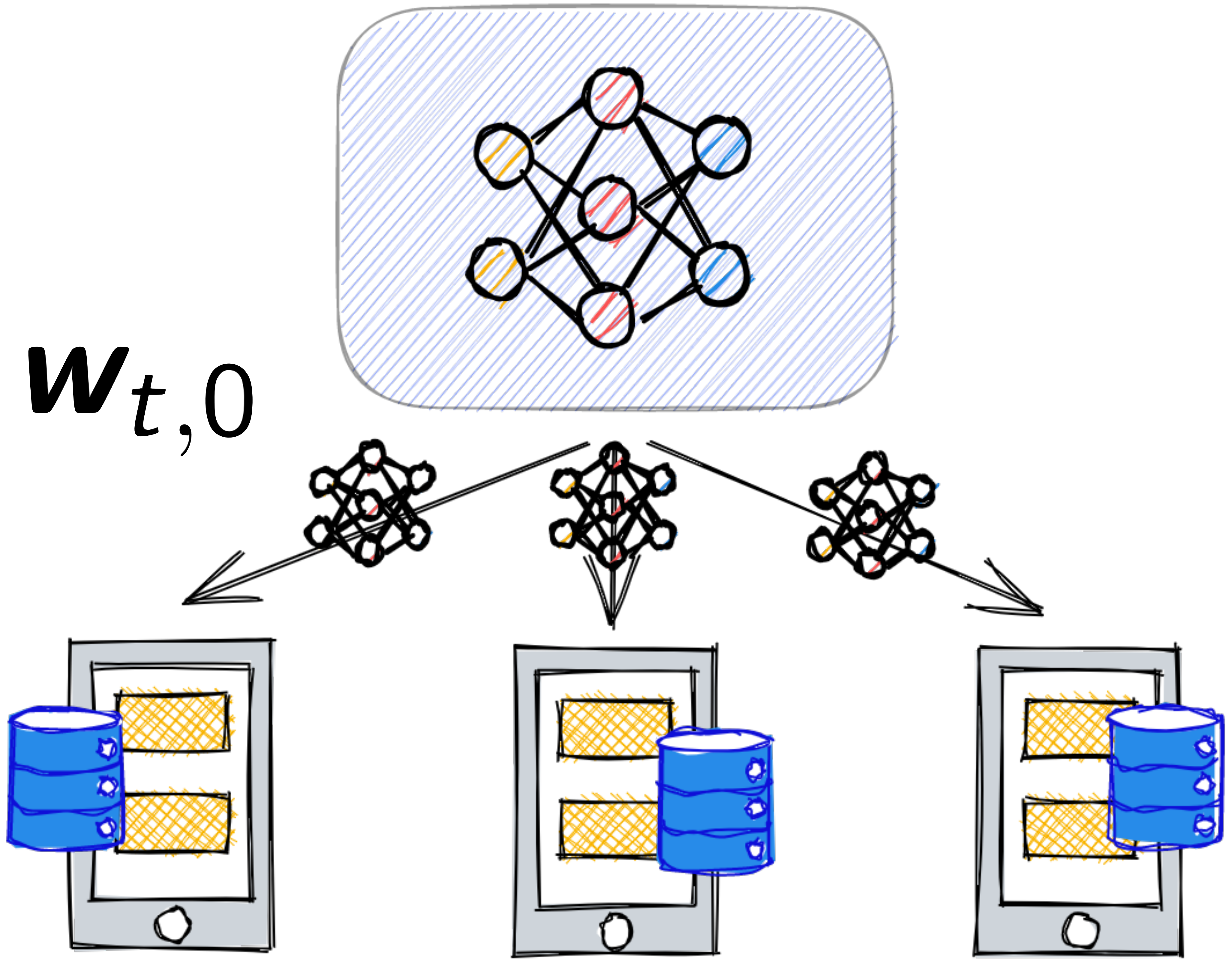
(1)



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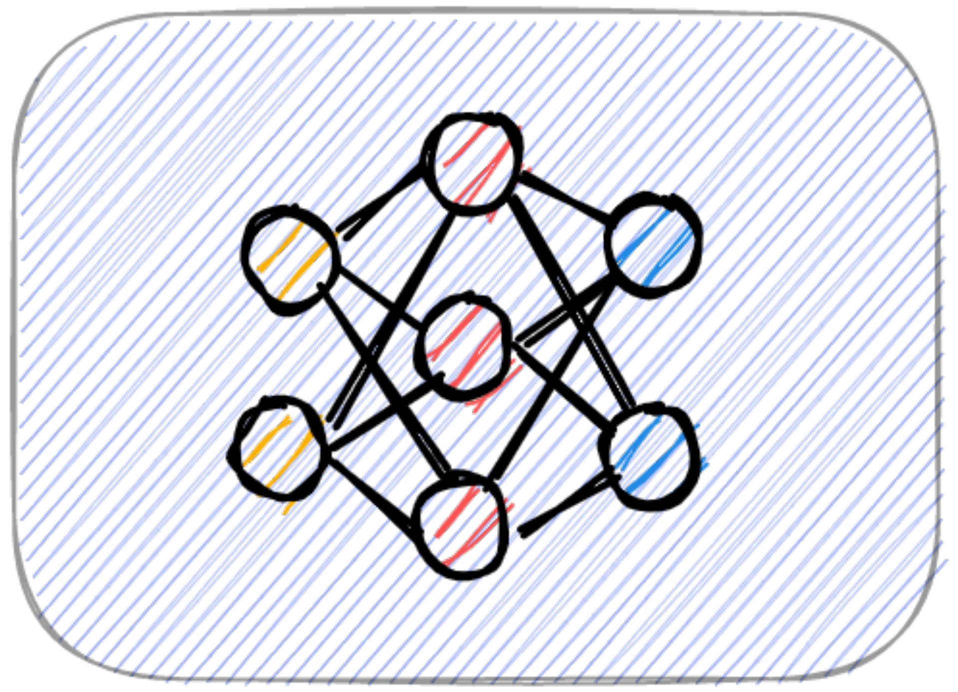
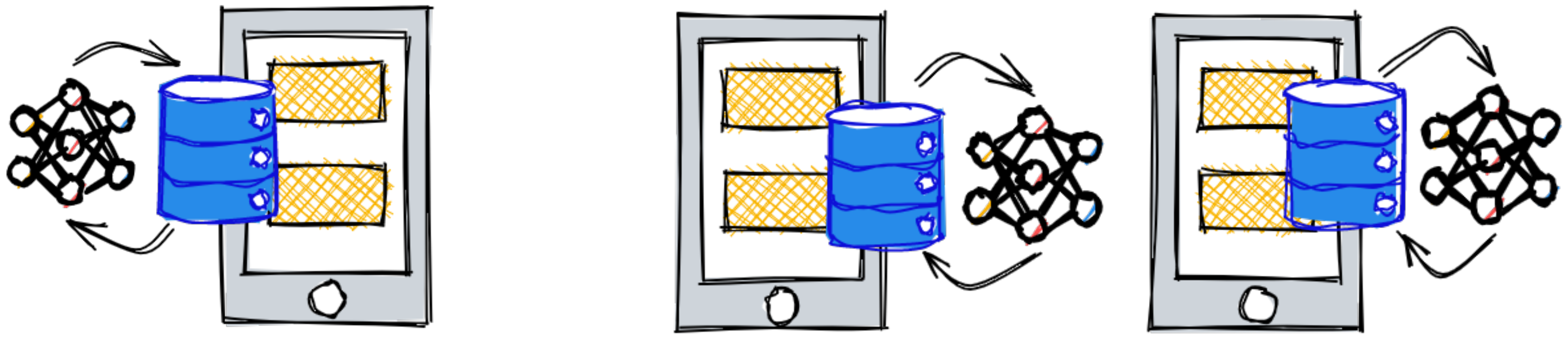


(2) **For** $j = 0, \ldots, E-1$ **do** :

$$\boldsymbol{w}_{t,j+1}^{k} = \boldsymbol{w}_{t,j}^{k} - \eta_t \nabla F_k(\boldsymbol{w}_{t,j}^{k}, \mathcal{B}_{t,j}^{k})$$

$\boldsymbol{w}_{t,E}^{k}$



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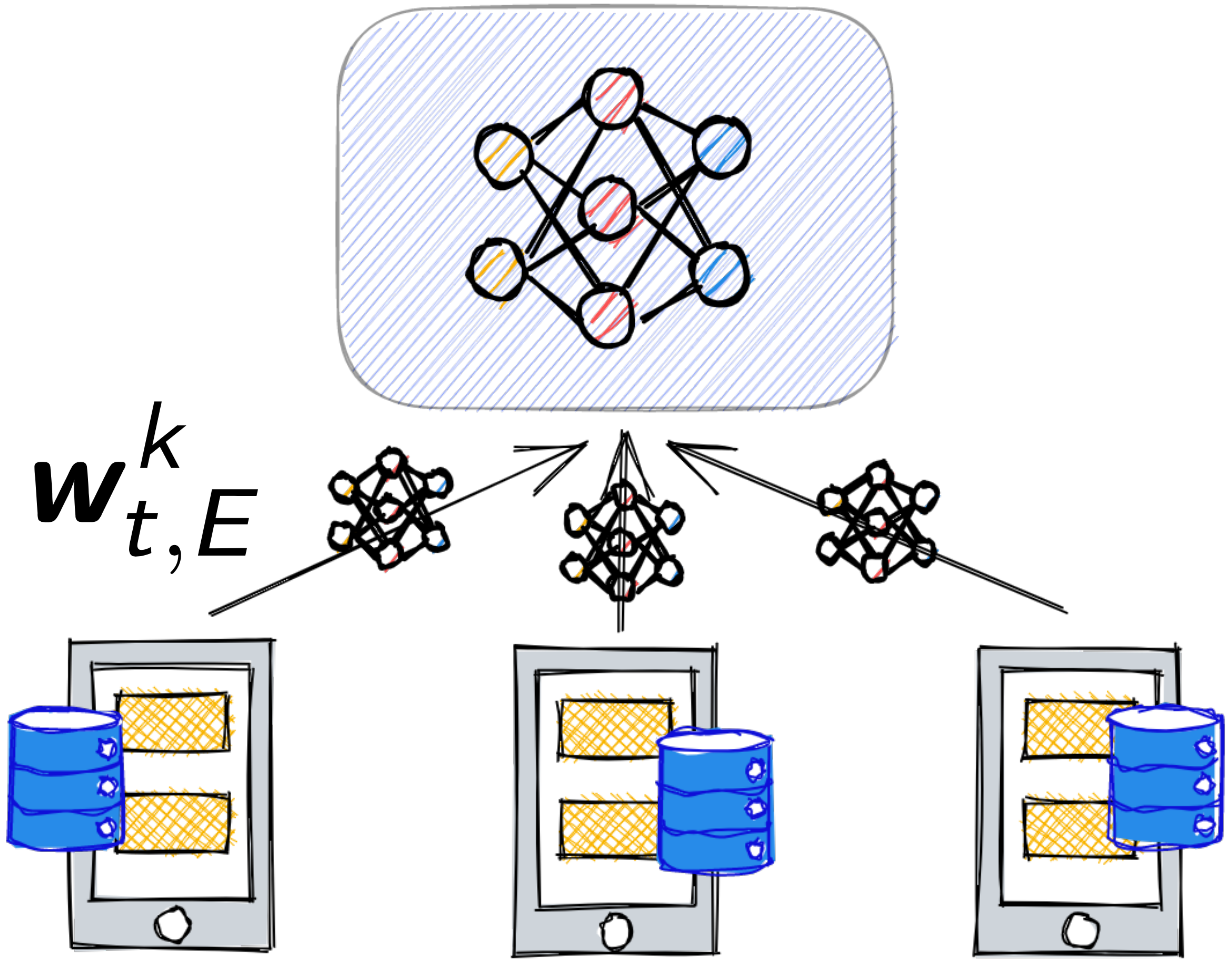


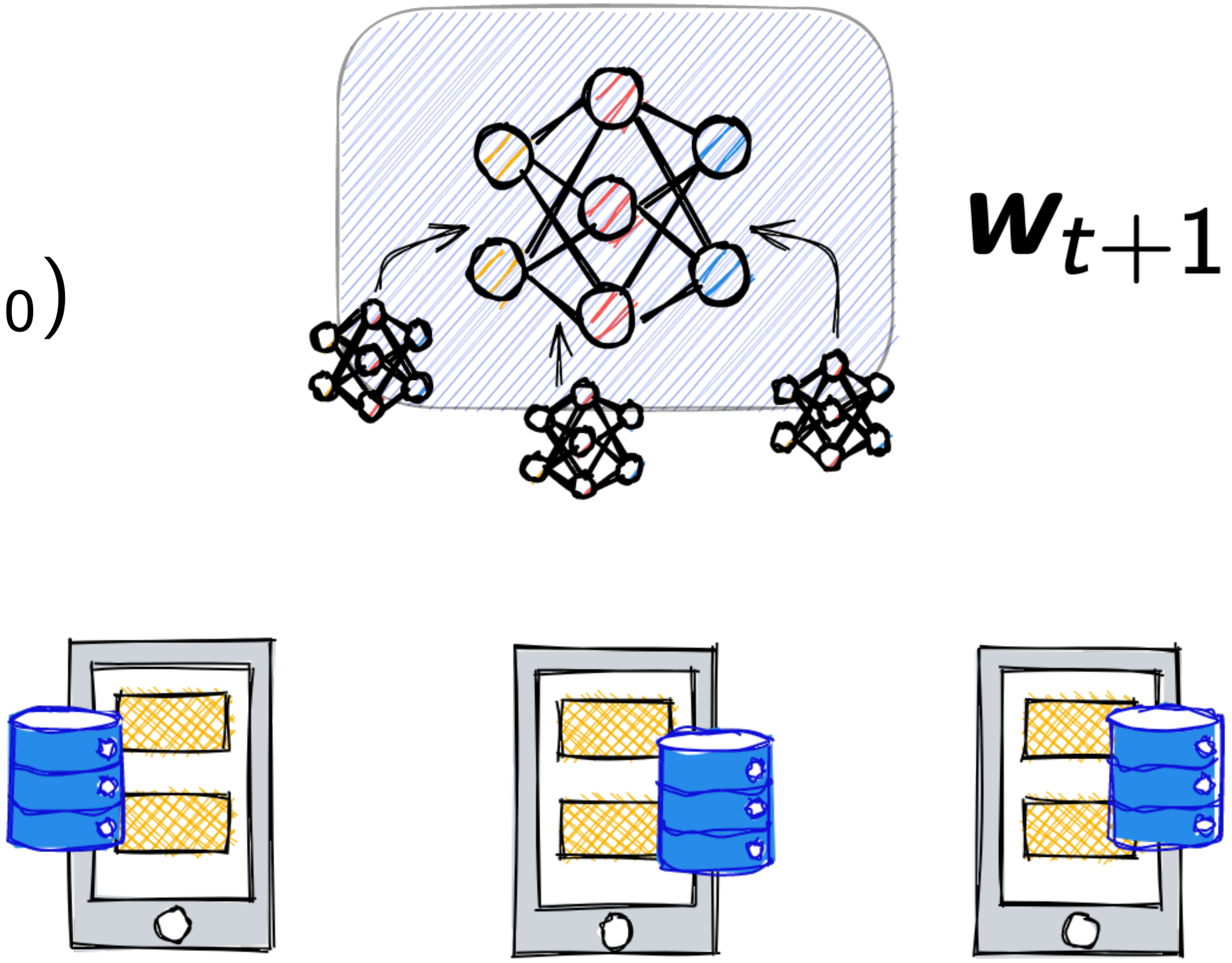
General FL Algorithm

for $t \in \{0, \ldots, T-1\}$ **do**:

A_t: set of active clients at time t

(4) $\boldsymbol{w}_{t+1,0} = \boldsymbol{w}_{t,0} + \sum_{k \in A_t} q_k (\boldsymbol{w}_{t,E}^k - \boldsymbol{w}_{t,0})$

$\boldsymbol{q}$: aggregation weights



$\boldsymbol{w}_{t+1,0}$

Client Availability in Federated Learning

Previous work in FL commonly assumed:

- Clients are always active or have uniform availability

We take into account the Heterogeneity and Correlation in Client Availability

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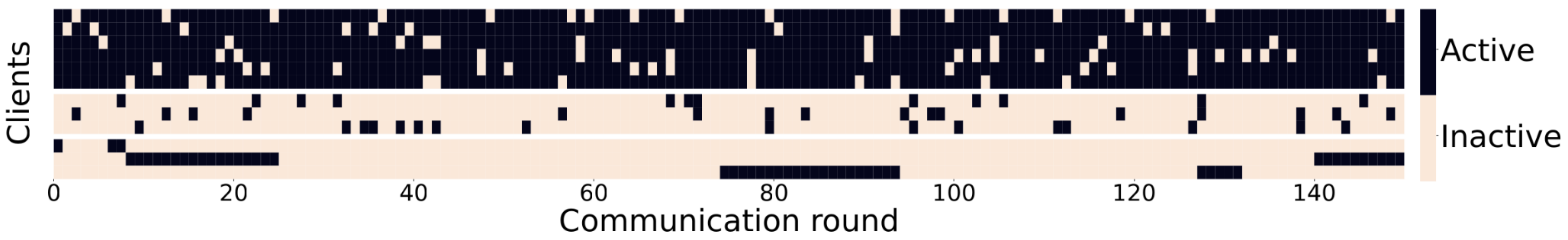

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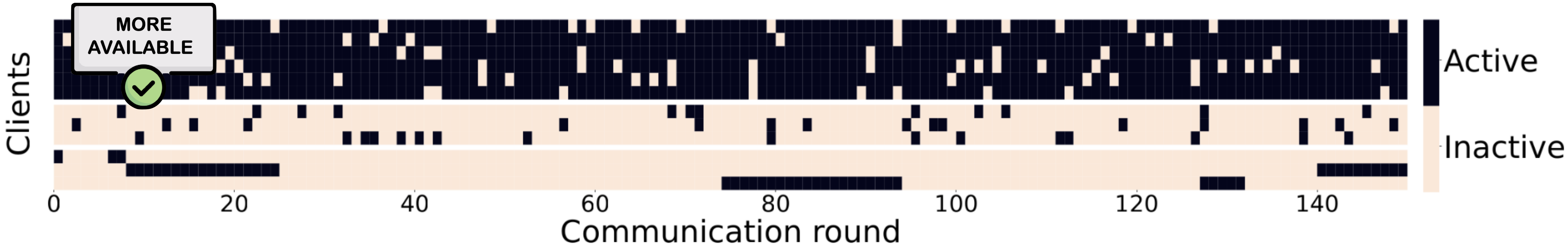

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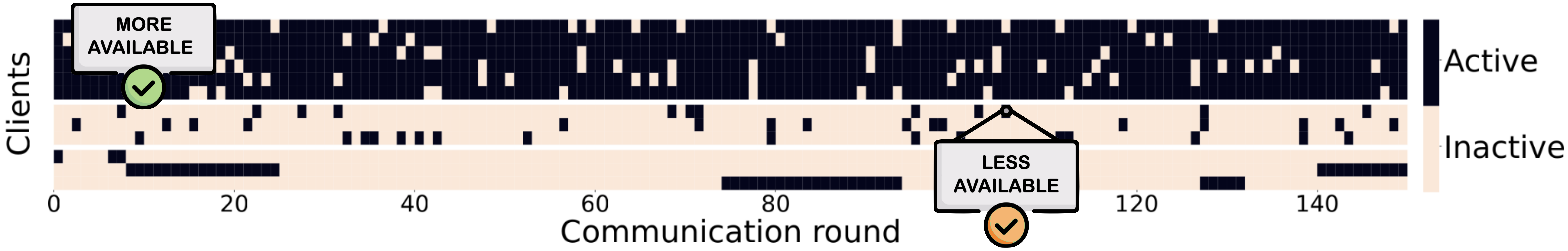

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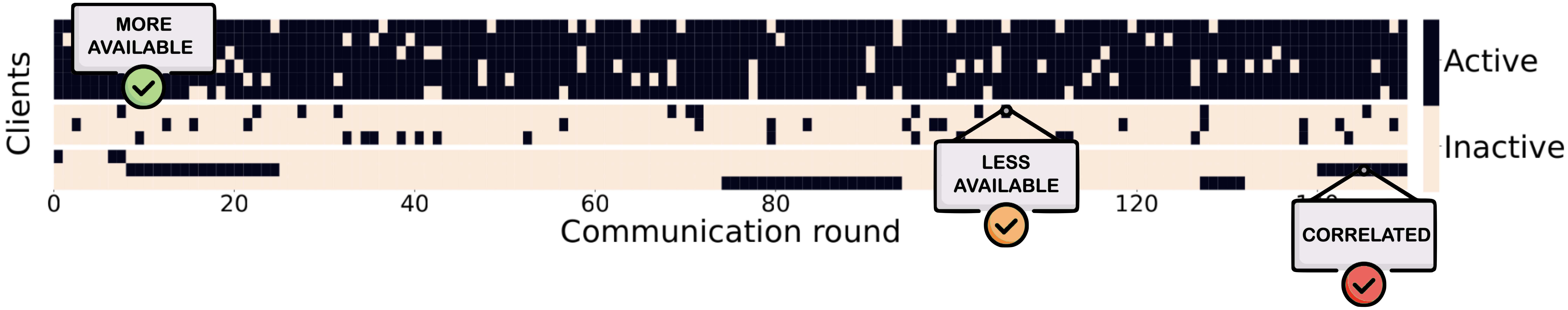

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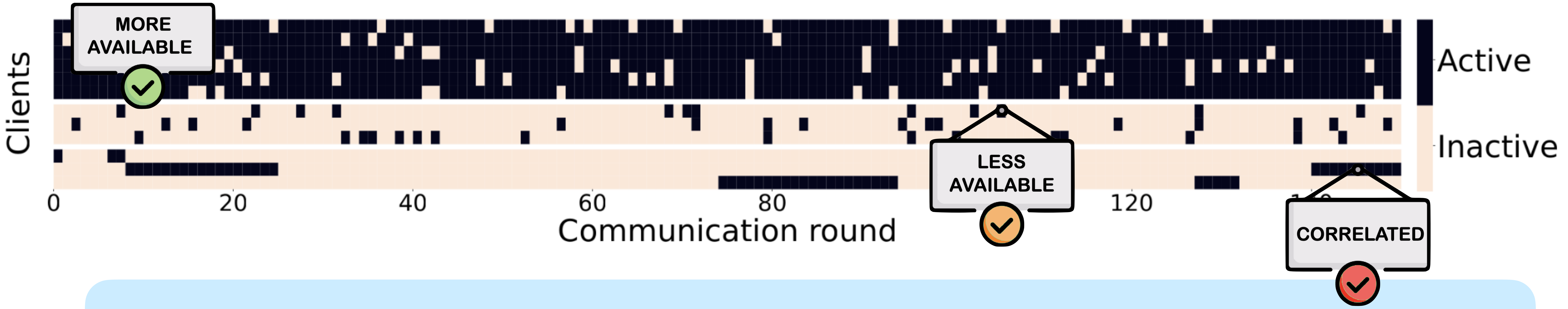
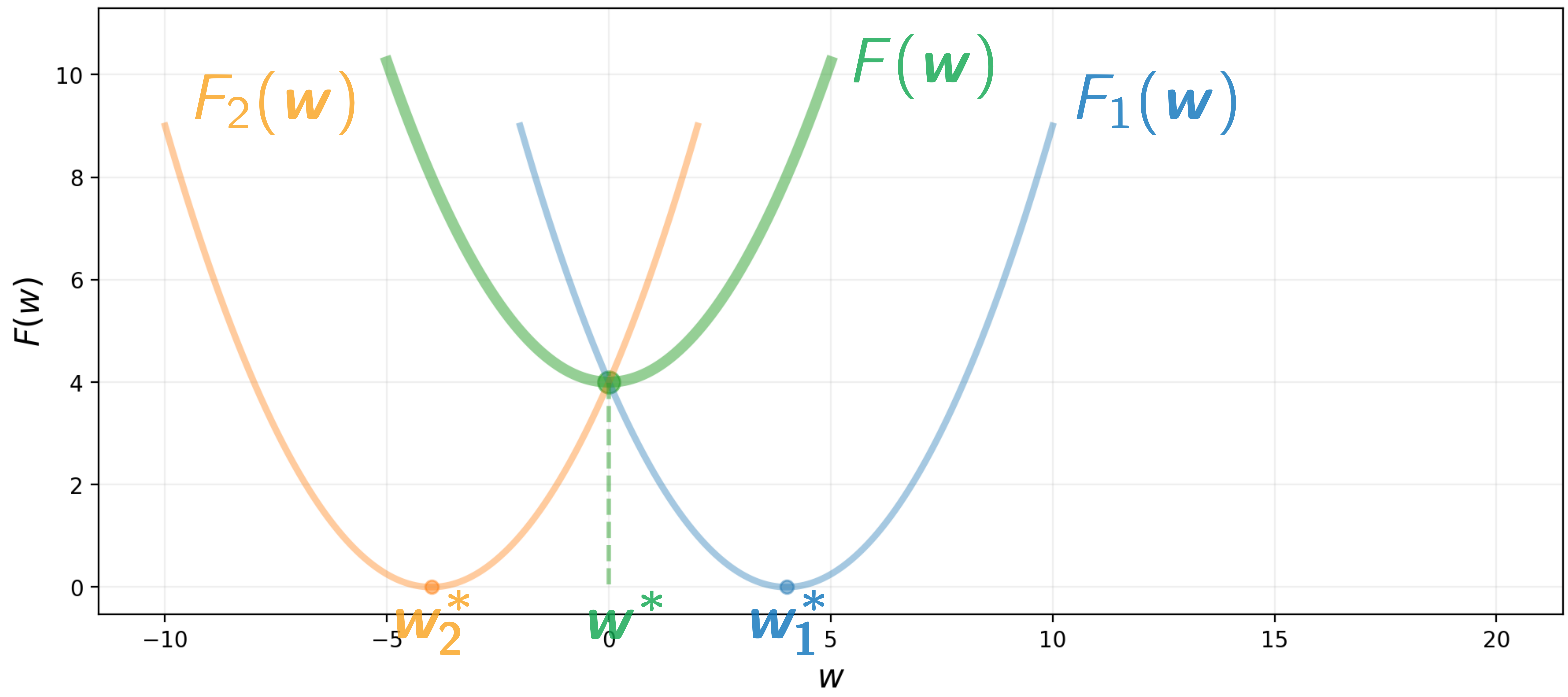
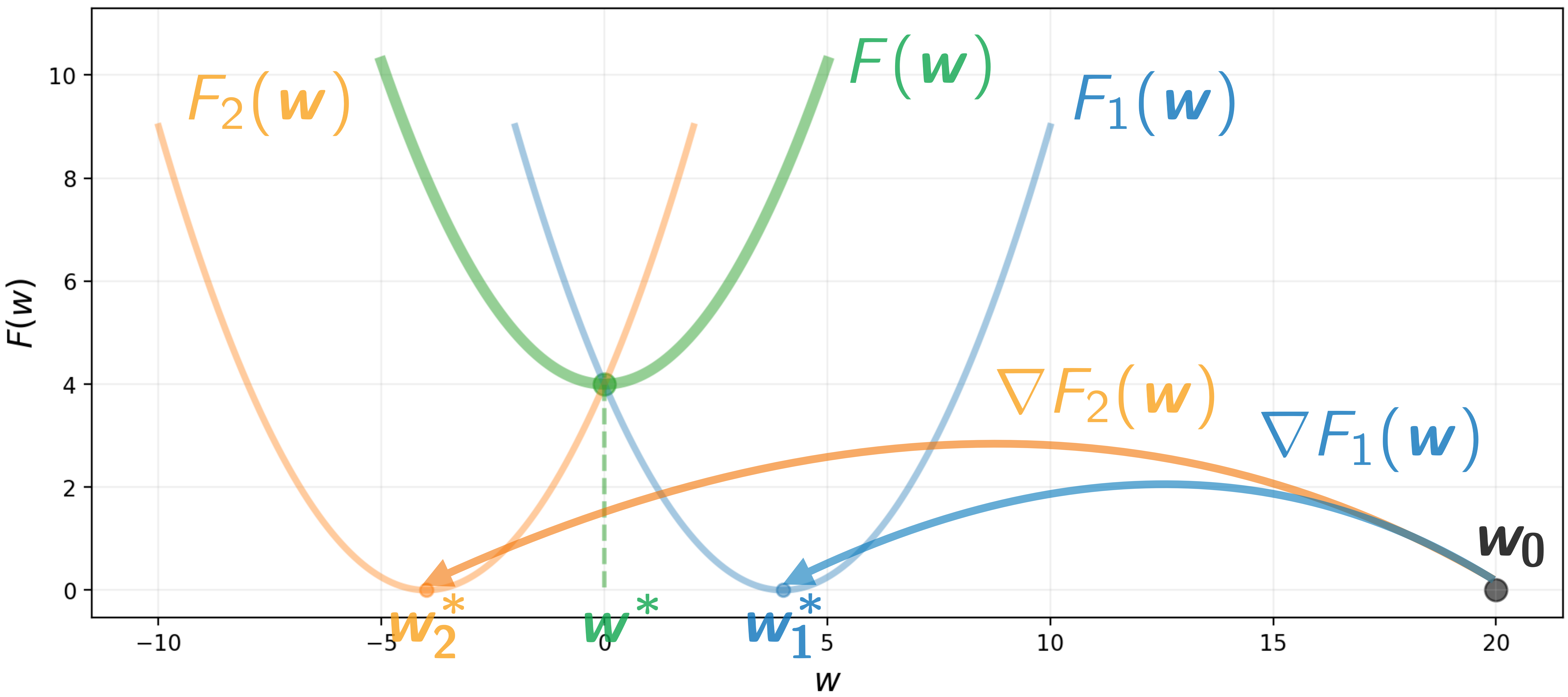


- Optimize training to make the best use of available client resources
- Minimize the impact of less available and correlated clients

Quadratic Example



1) All Clients Available



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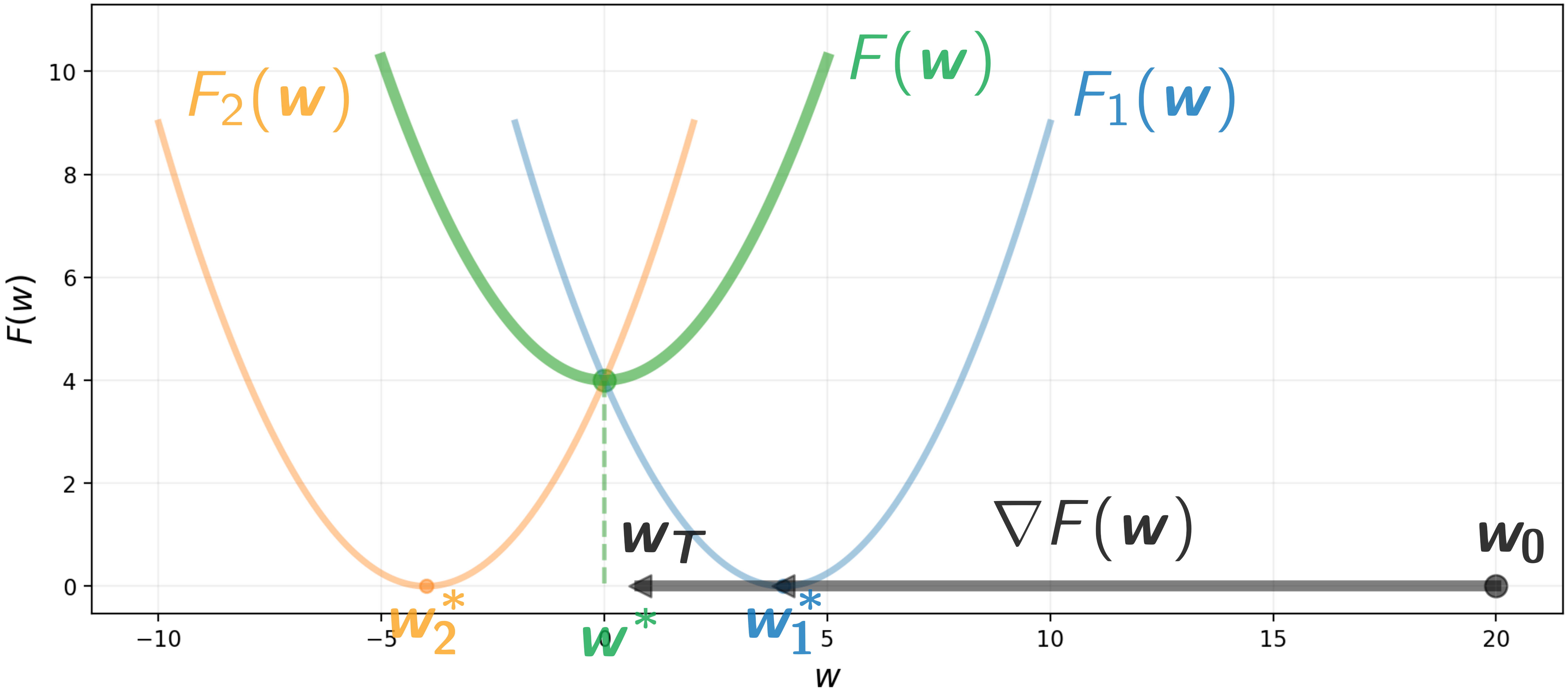
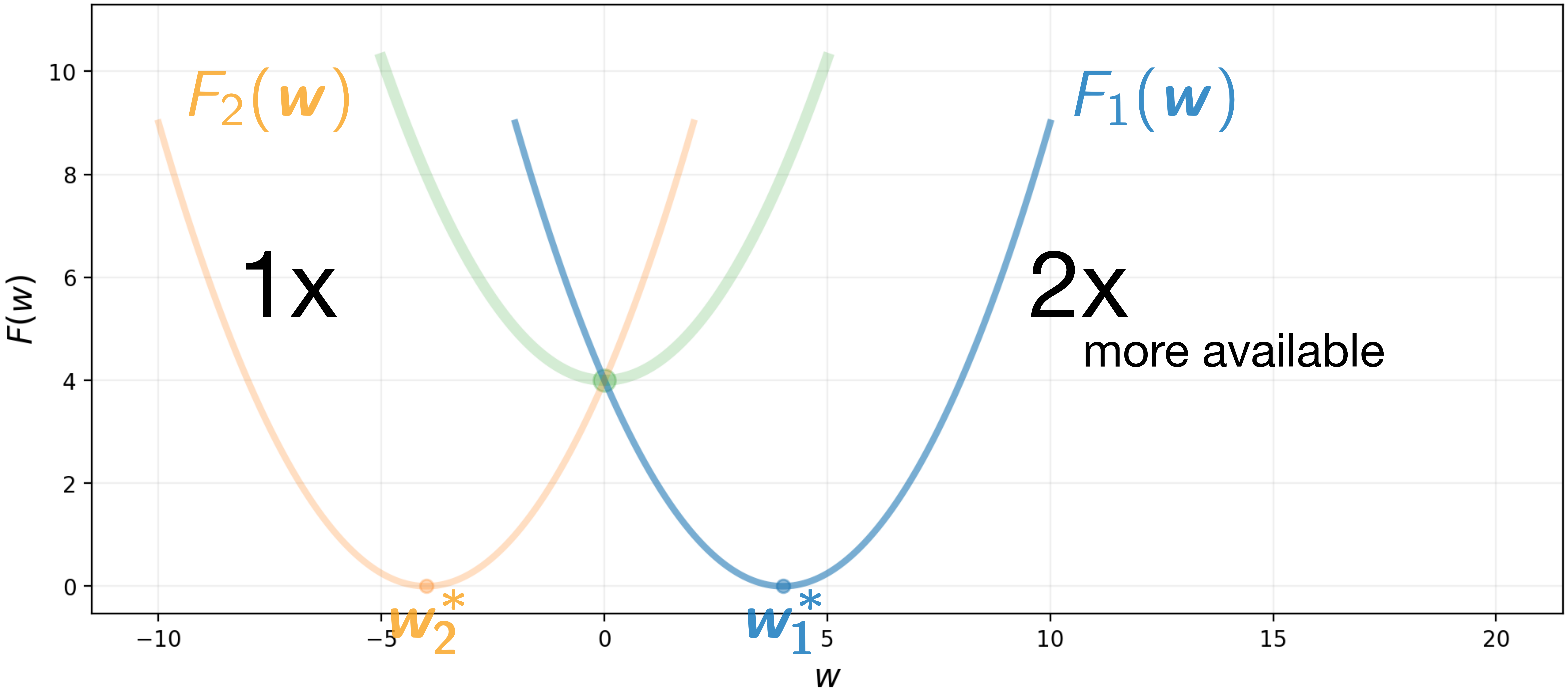


2) Heterogeneous Client Availability



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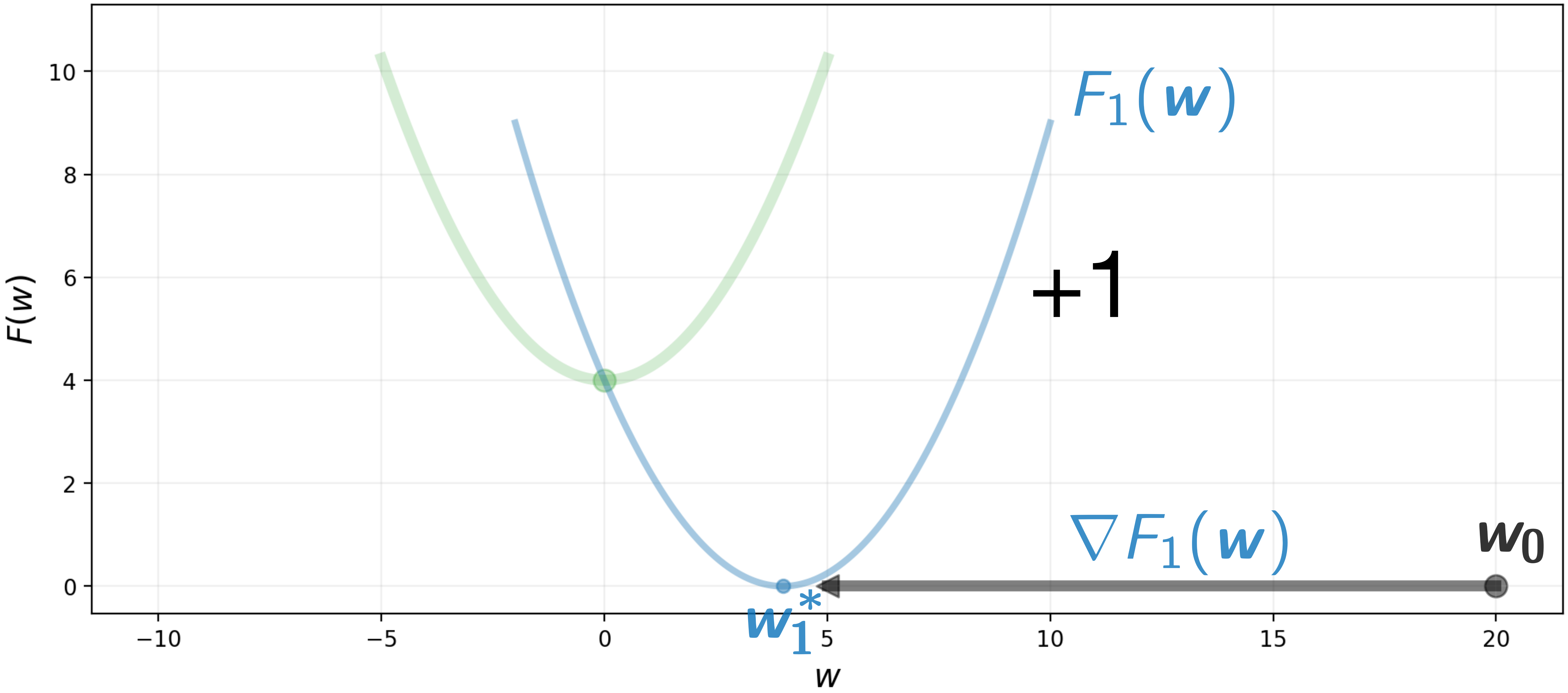

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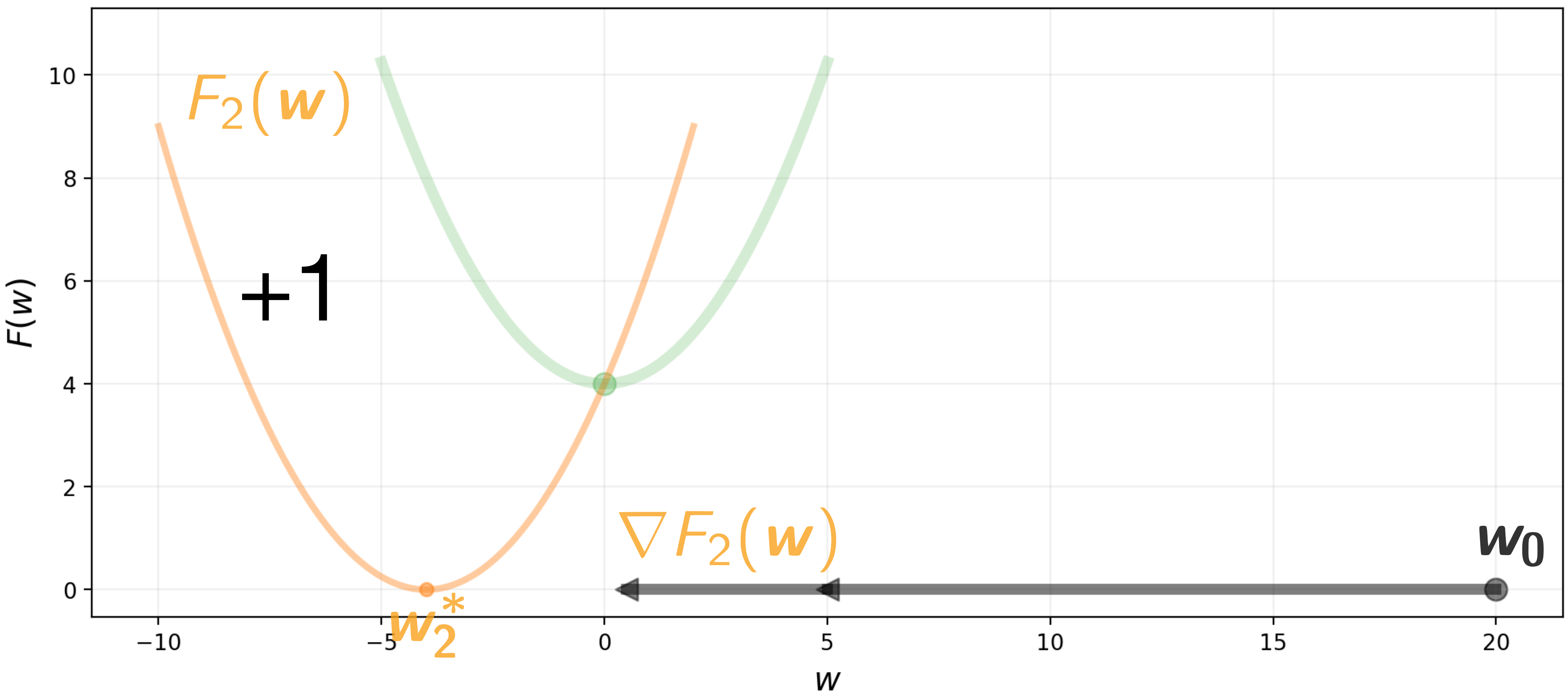

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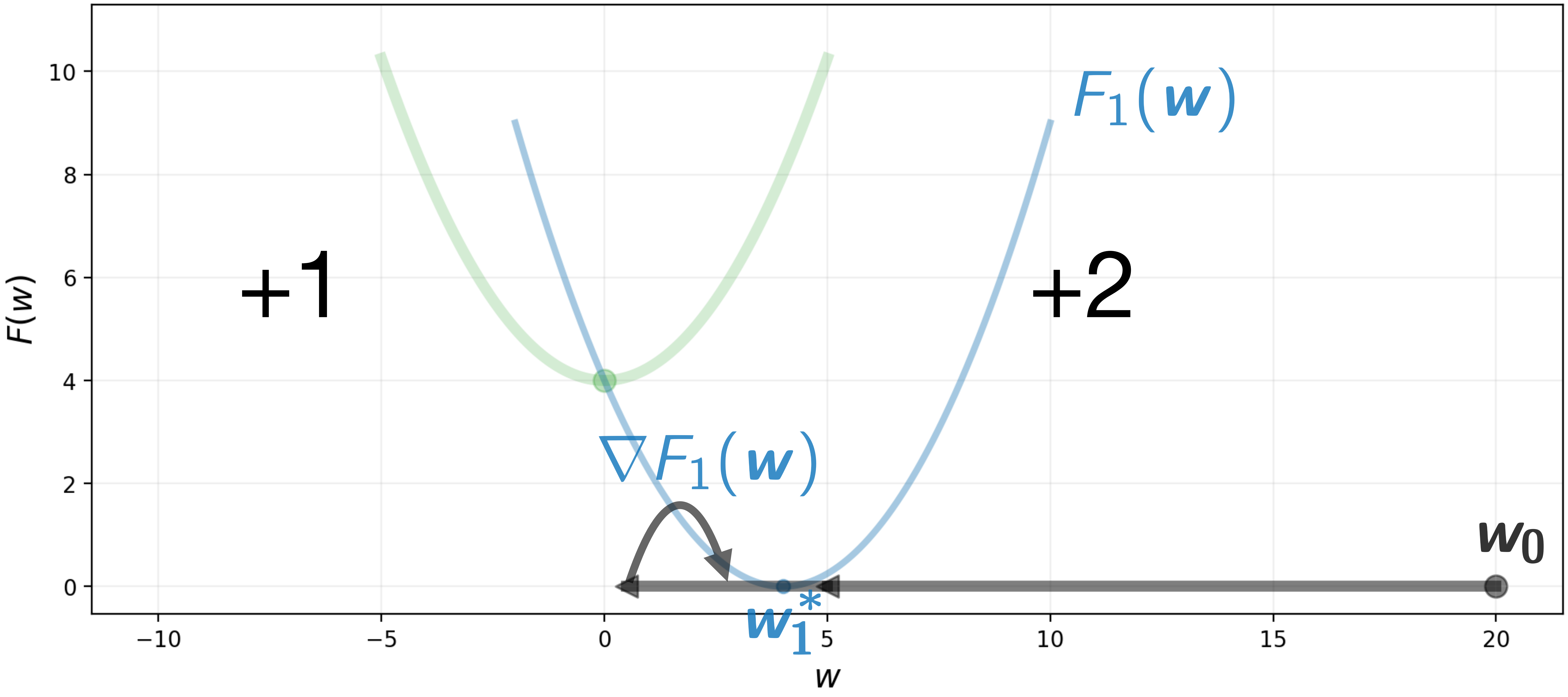

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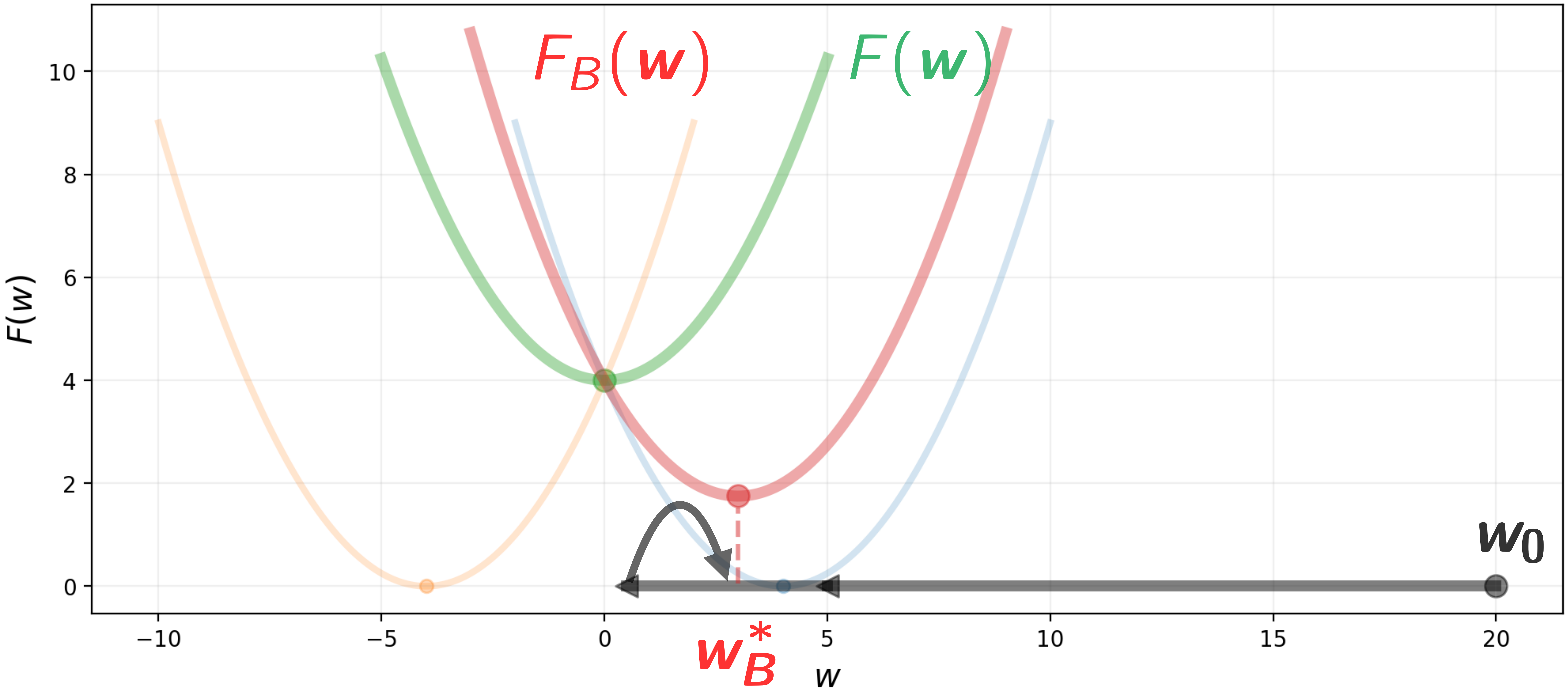

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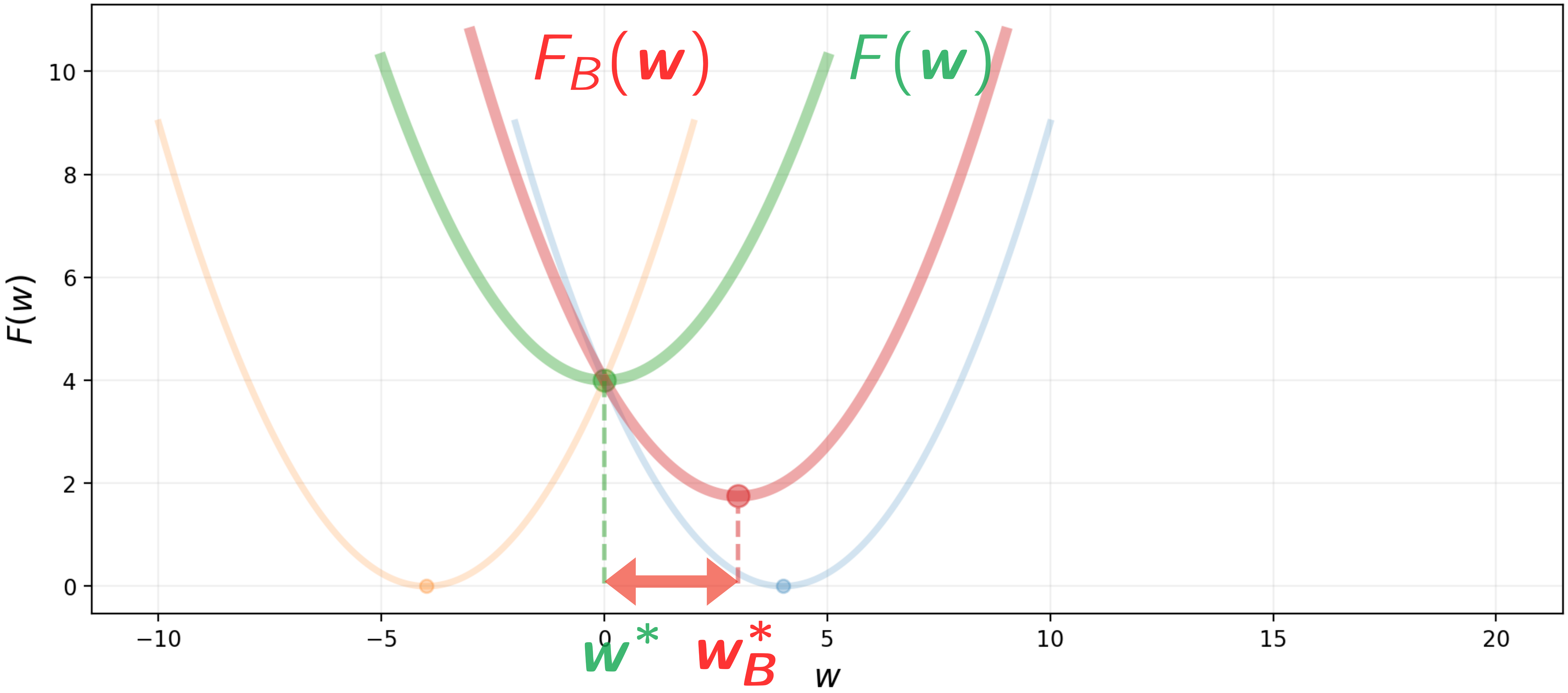

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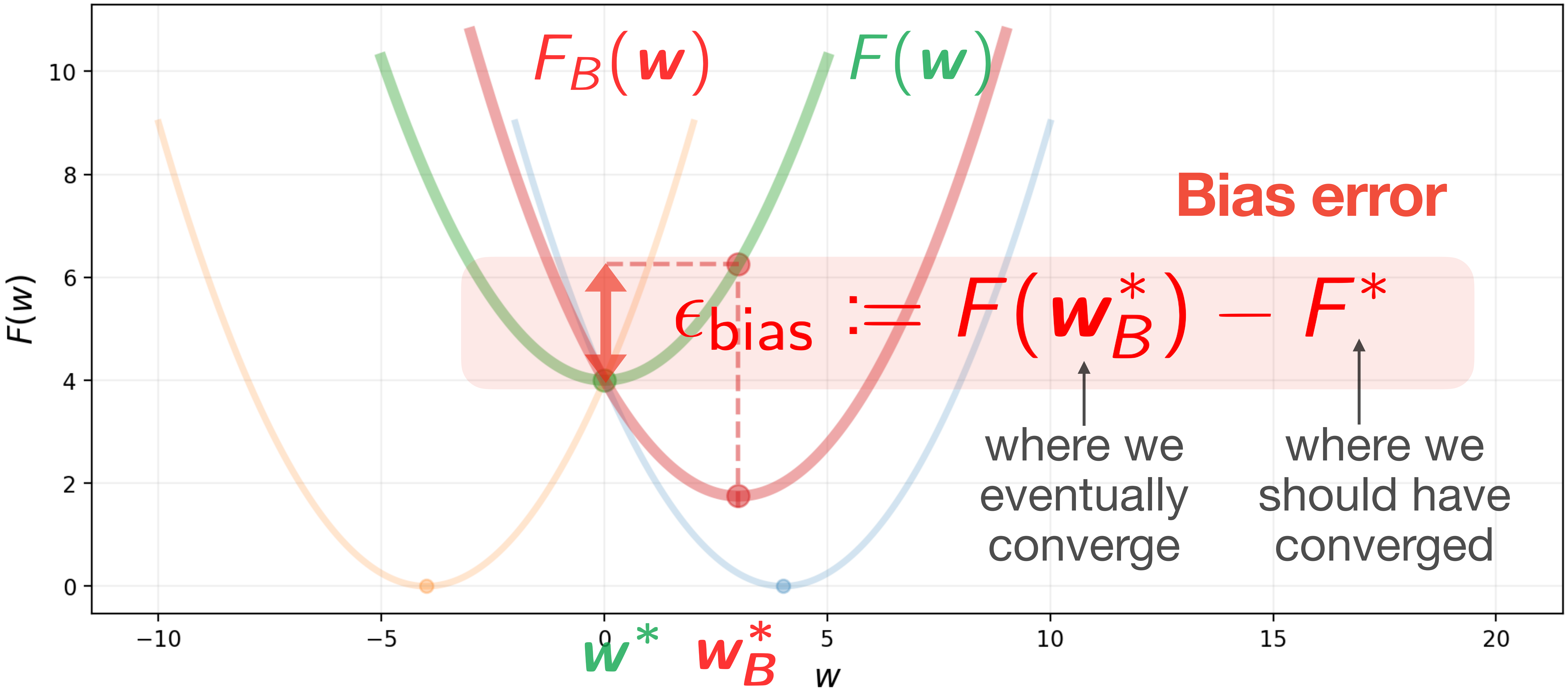

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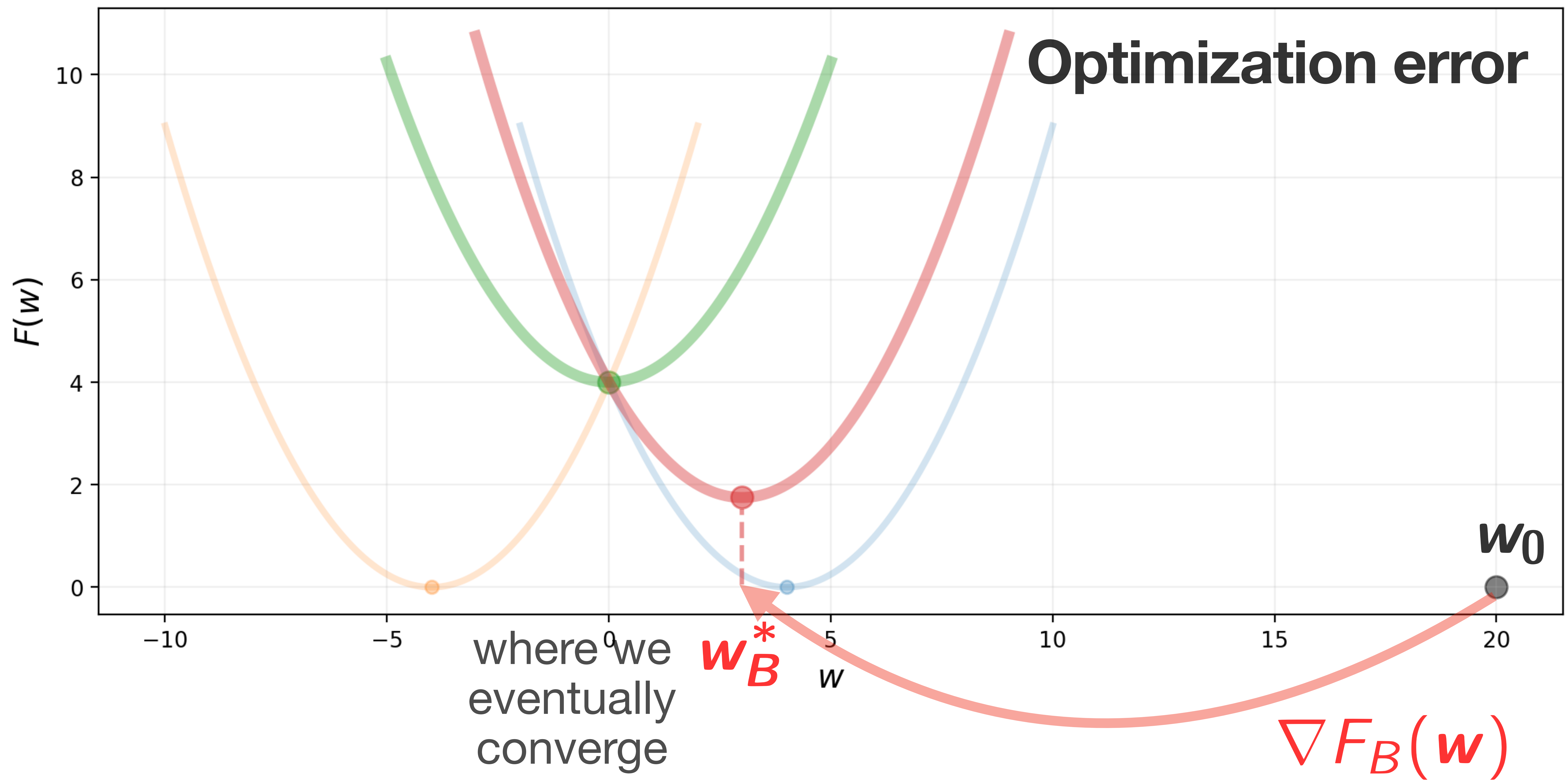

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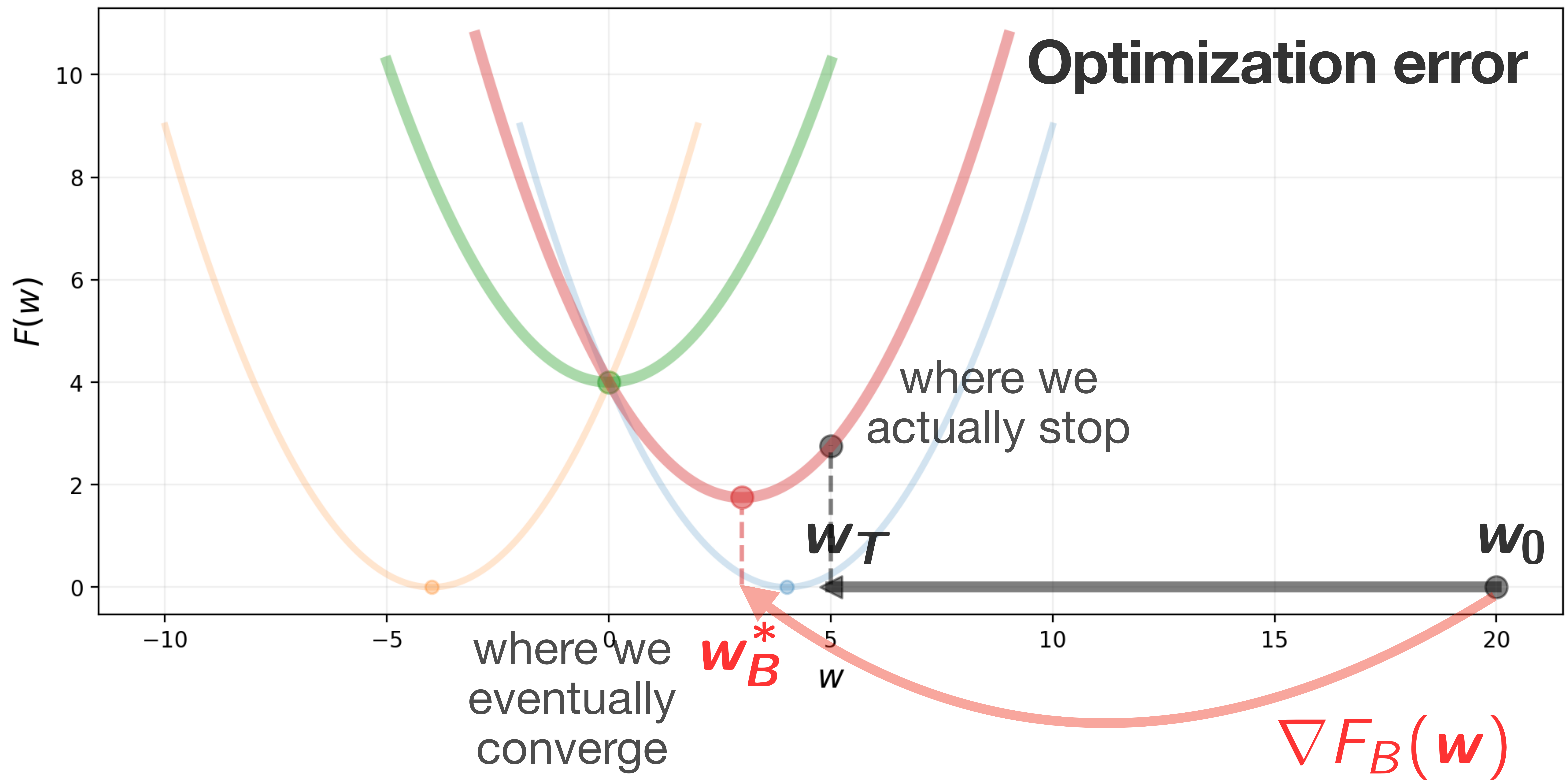

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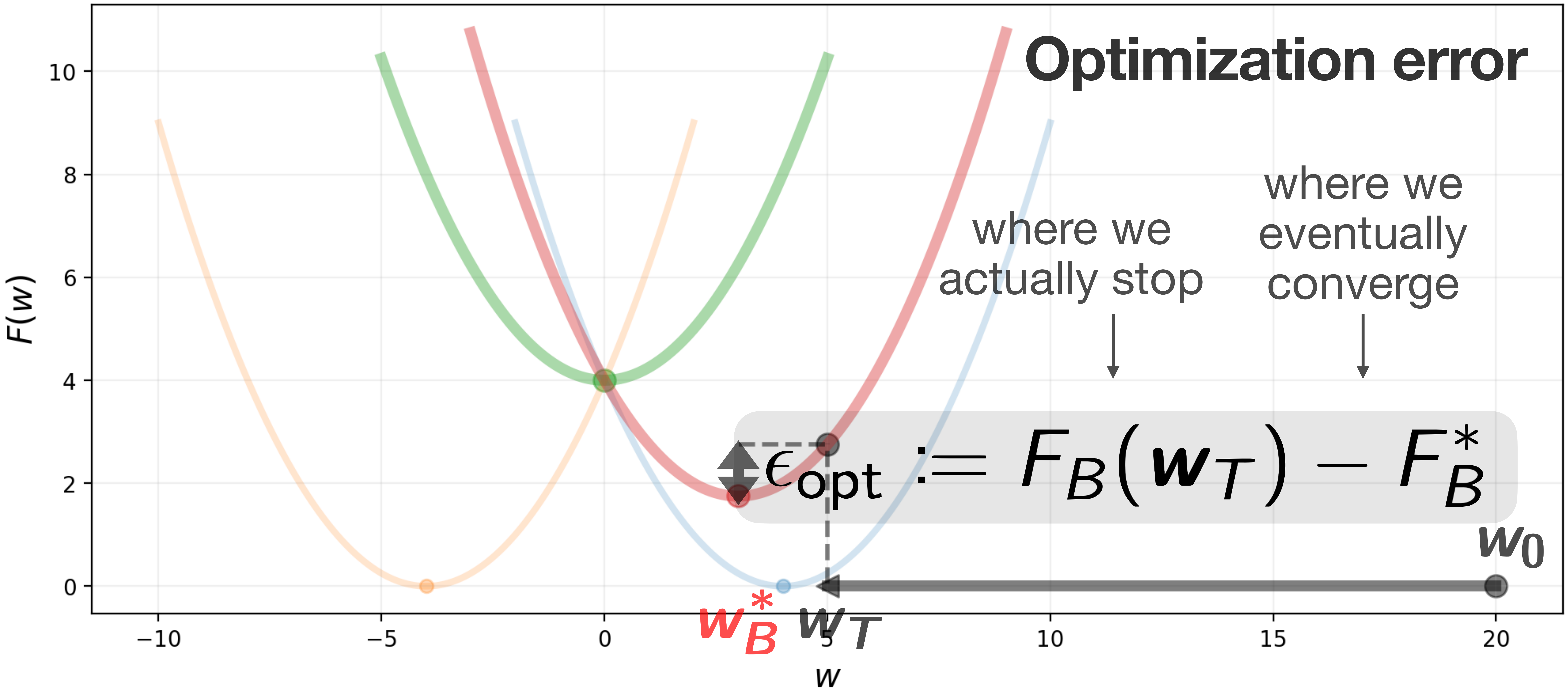

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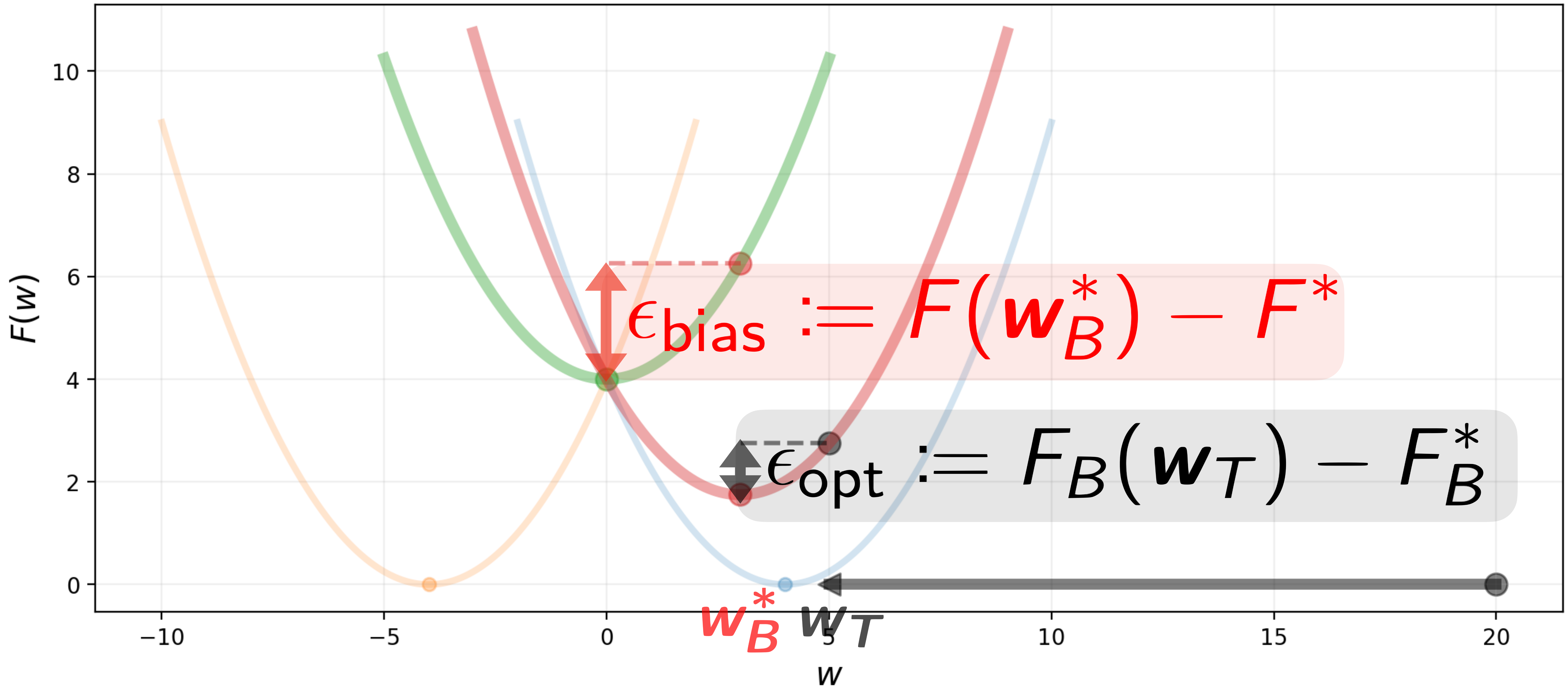
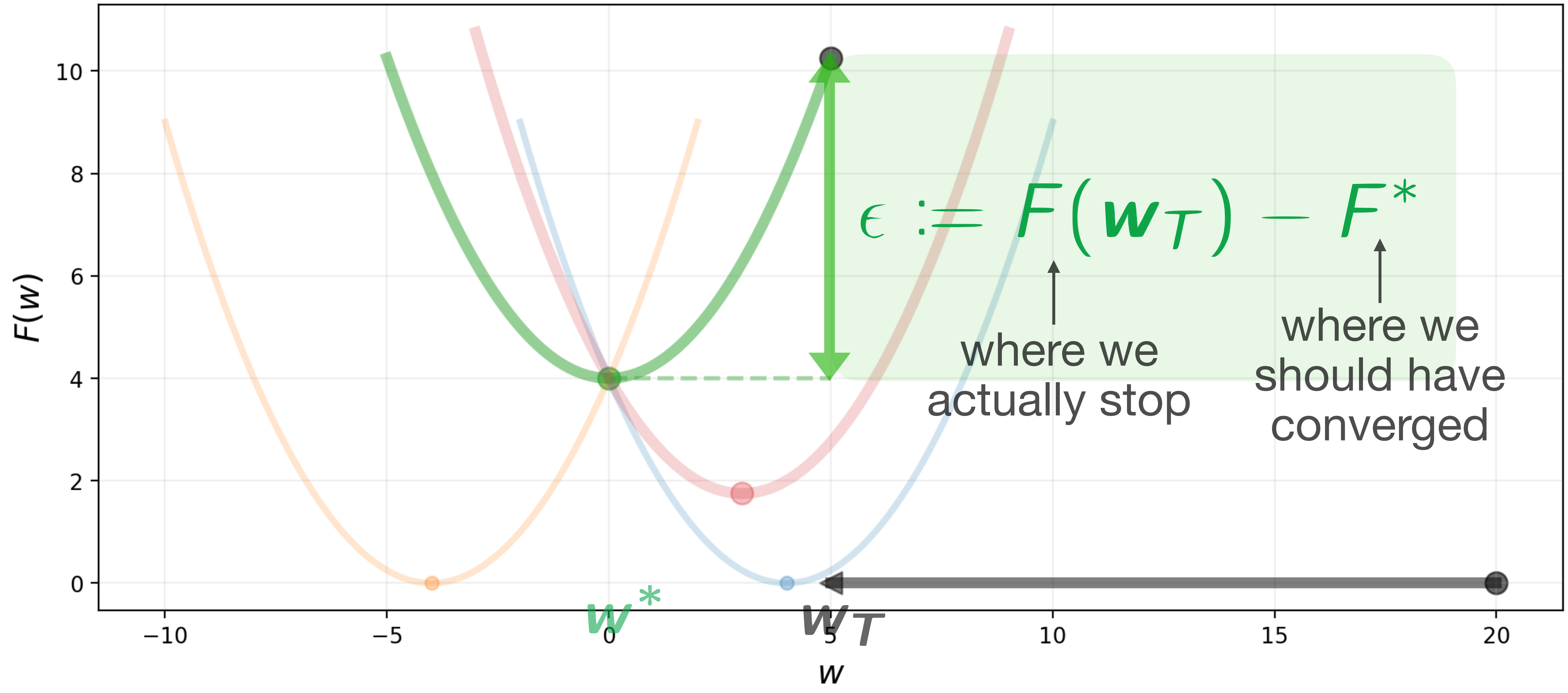


2) Heterogeneous Client Availability

Total error



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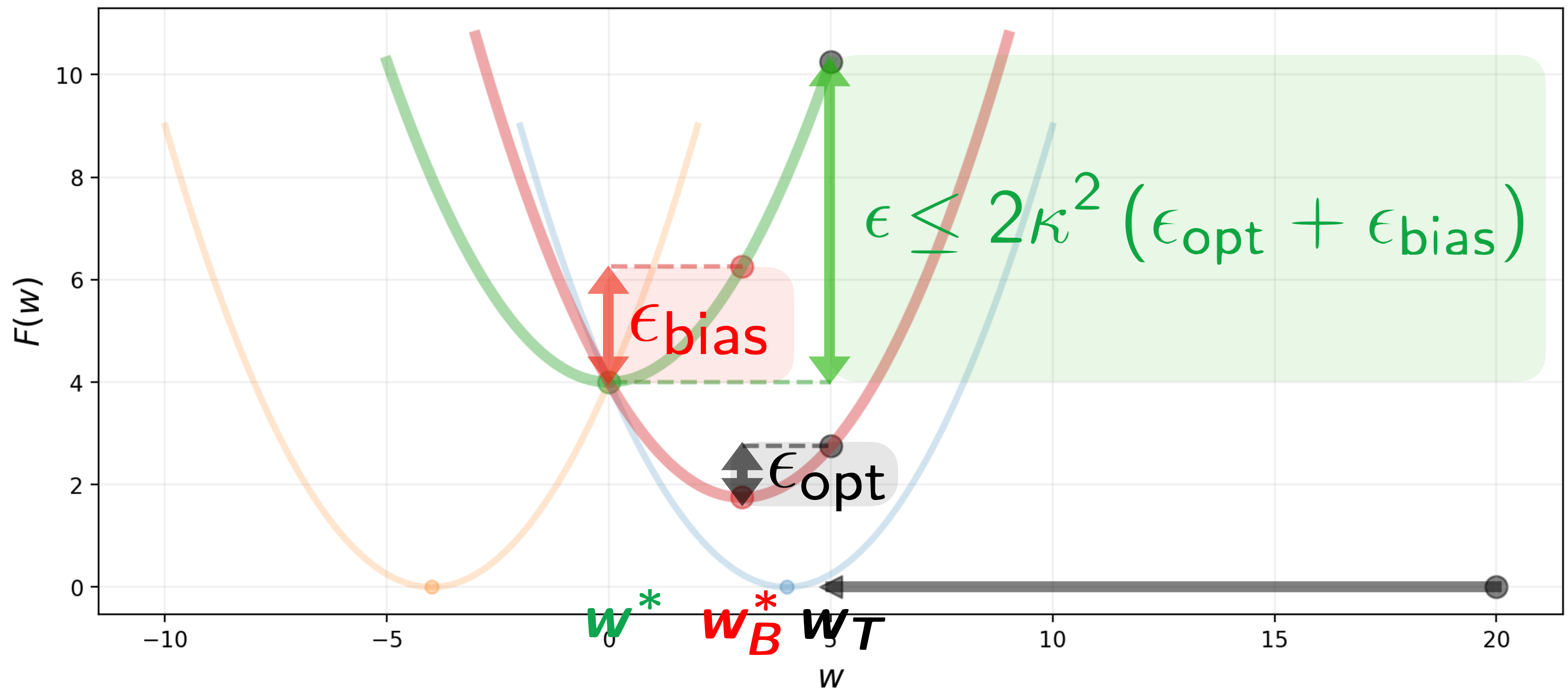
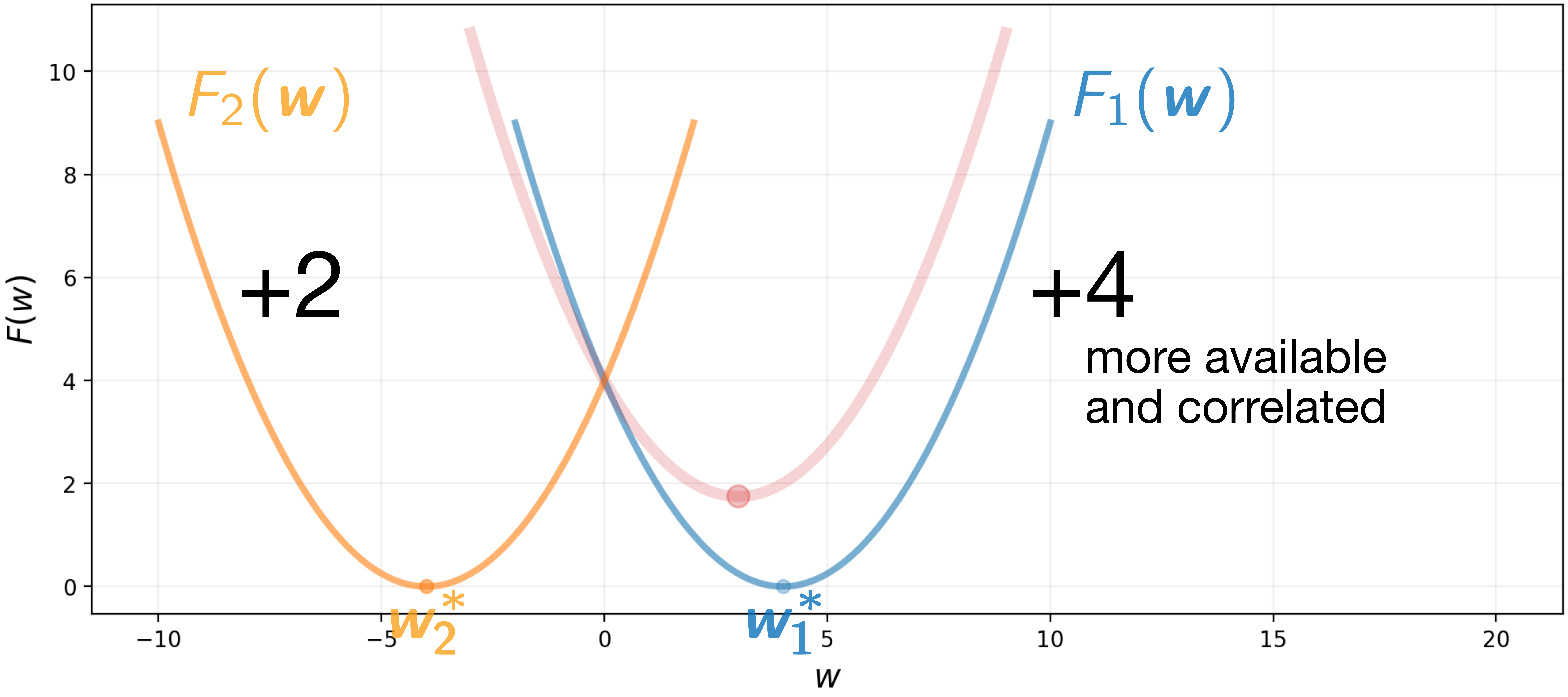


3) Heterogeneous and Correlated Client Availability



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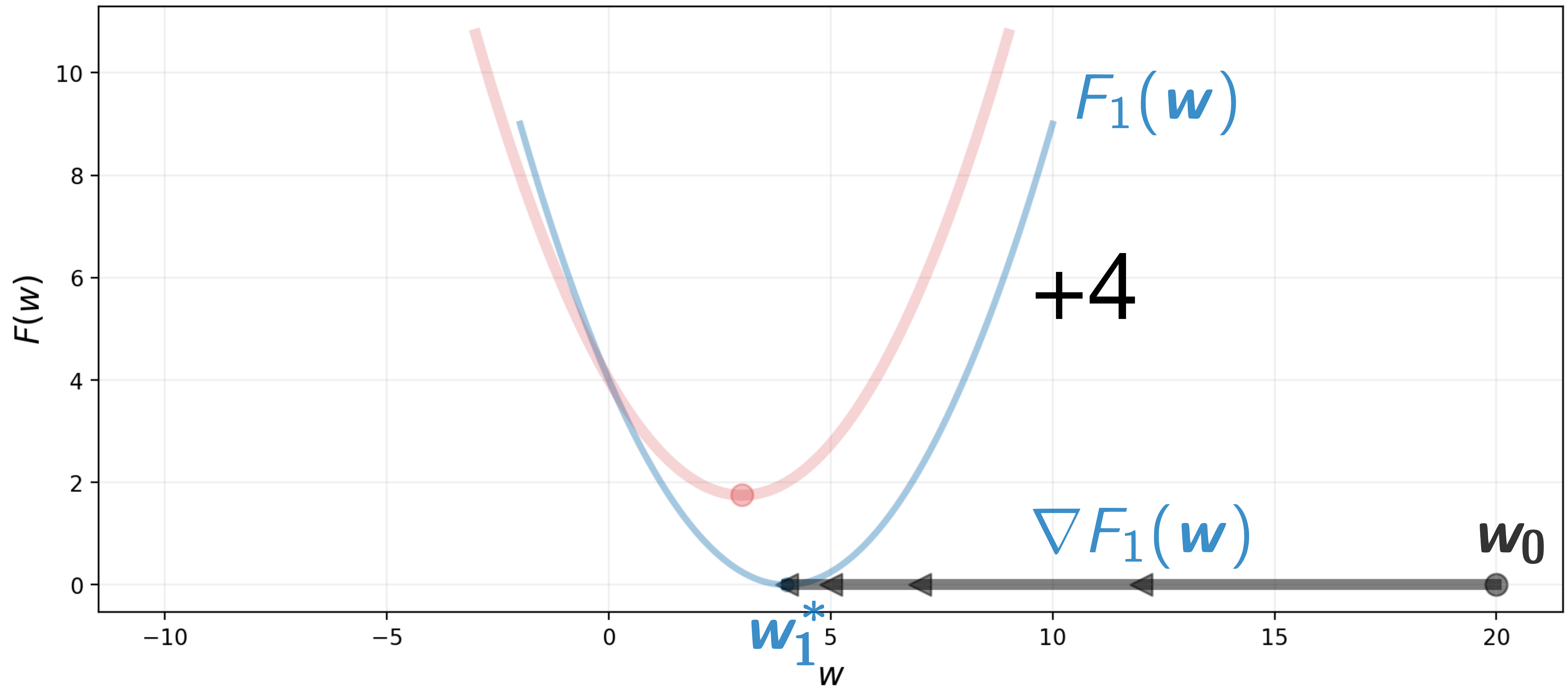

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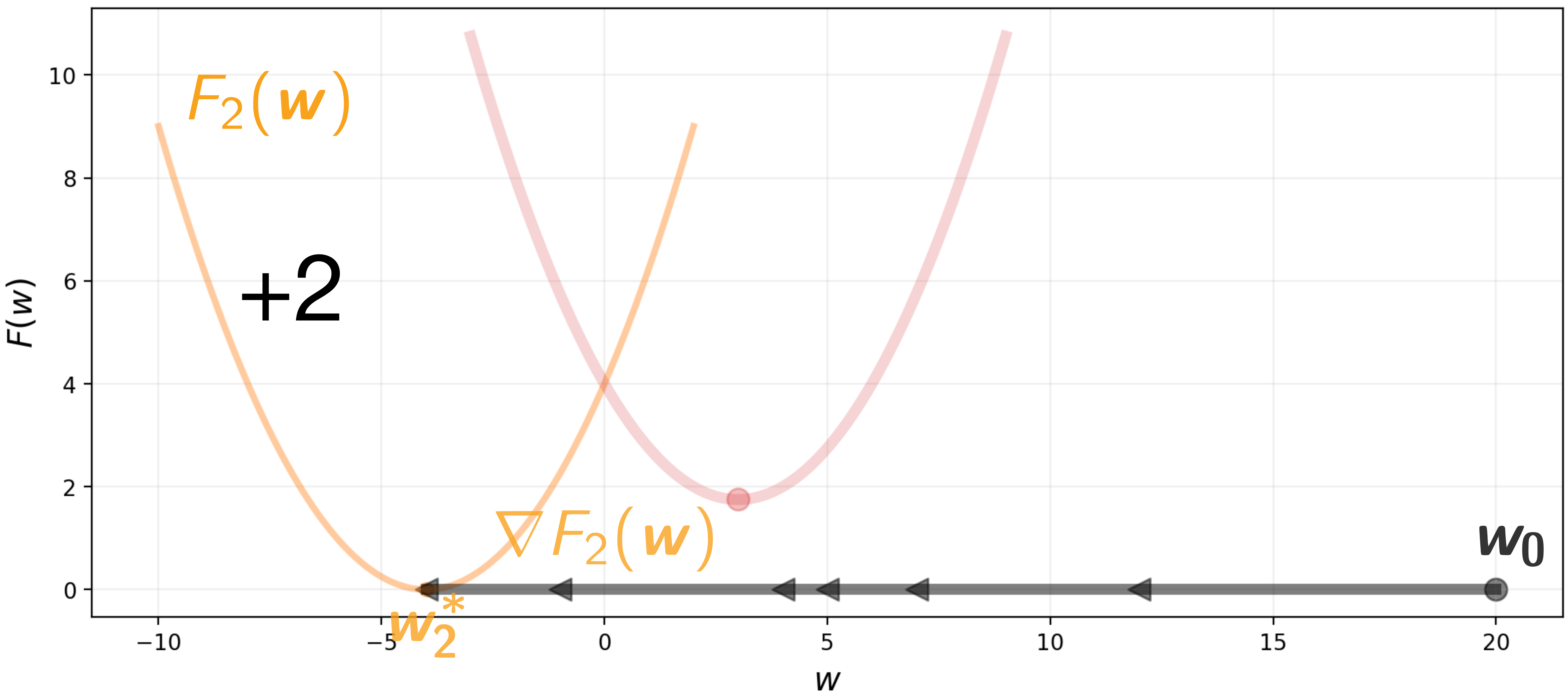

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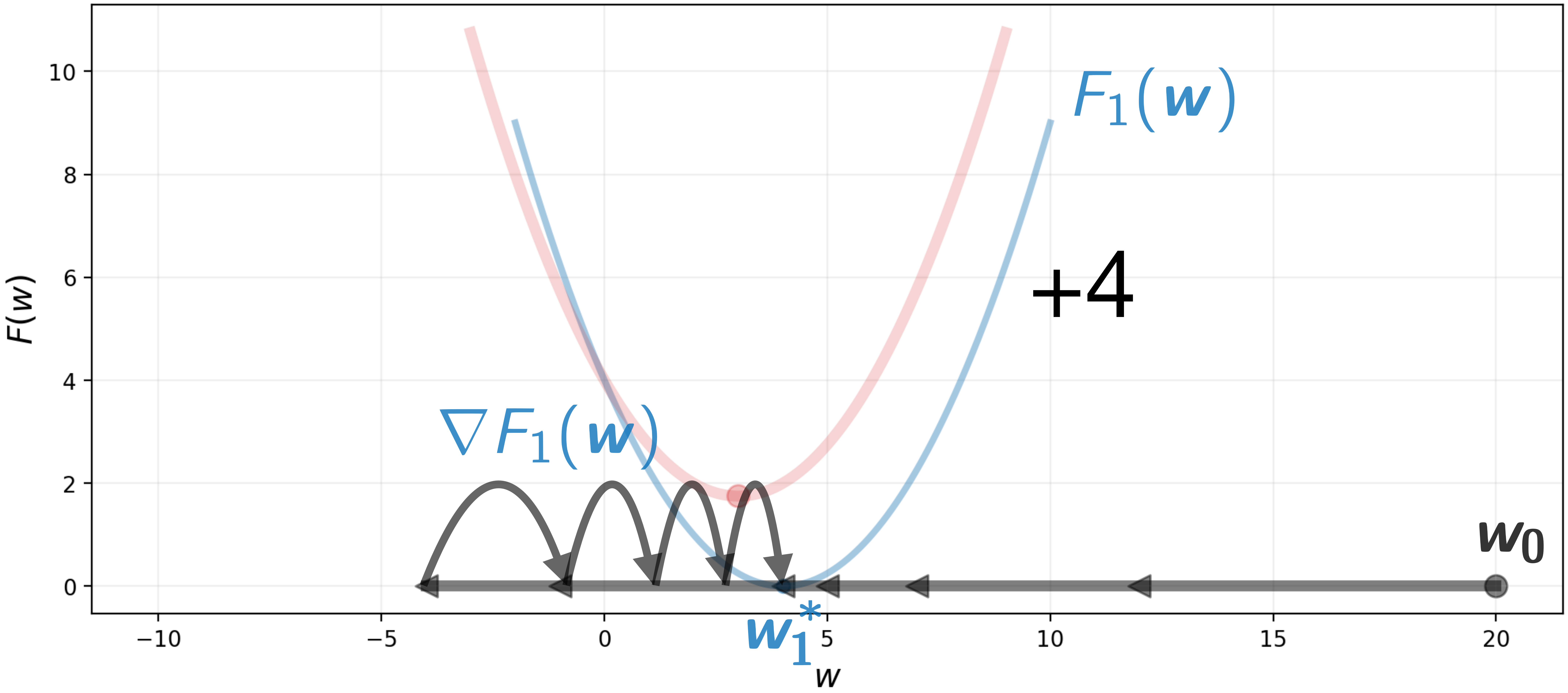

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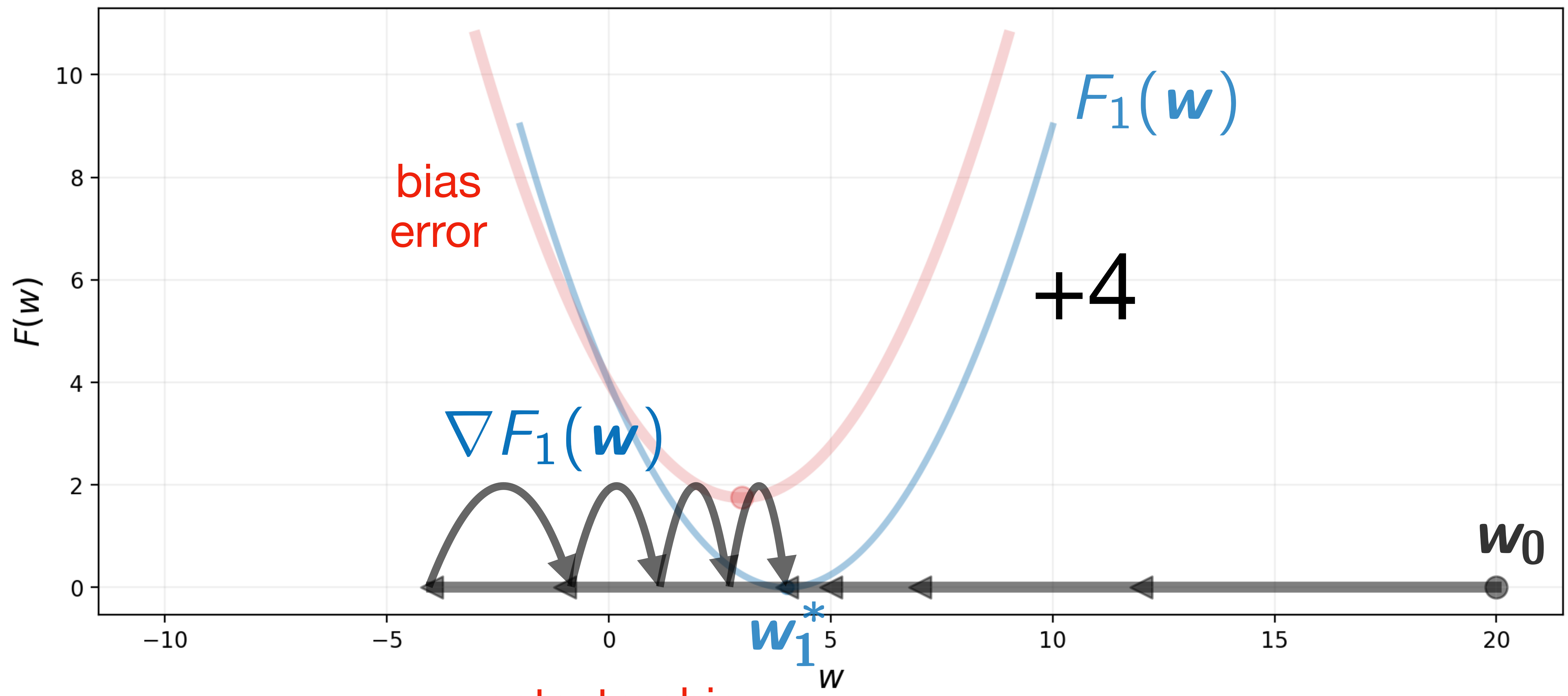

catastrophic forgetting : forget previously learned models

Assumption to model the heterogeneous and correlated client availability

- A_t: set of active clients at time t
- $(A_t)_{t \geq 0}$ is a discrete-time Markov chain
    - transition matrix $\boldsymbol{P}$
    - stationary distribution $\boldsymbol{\pi}$ $\leftrightarrow$ avg. availability
    - largest 2nd eigenvalue $\lambda(\boldsymbol{P})$ $\leftrightarrow$ correlation

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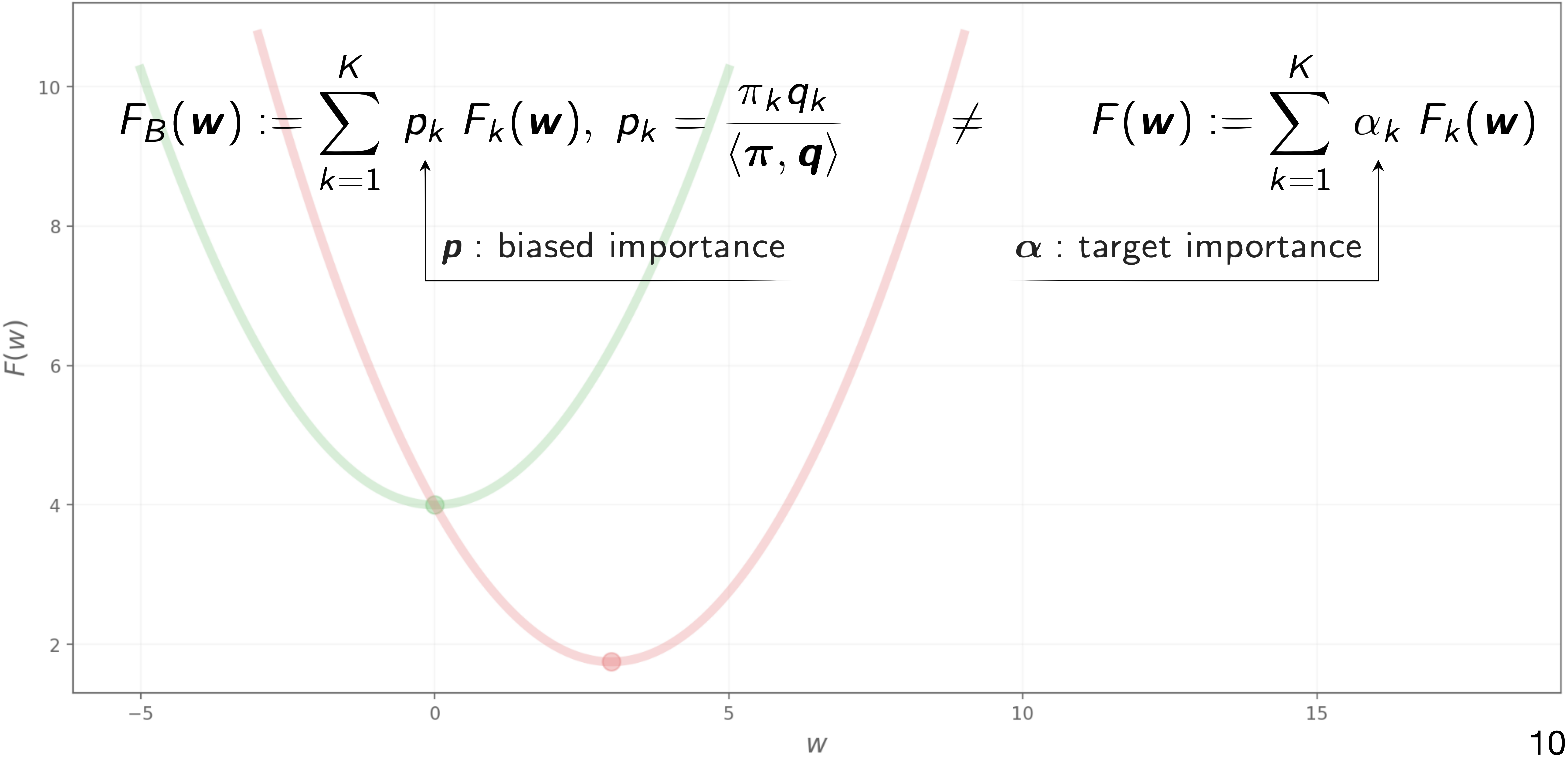
If clients' availabilities are independent: $\lambda(\boldsymbol{P}) = \max_{k \in [K]} \lambda(\boldsymbol{P_k})$

Bias error

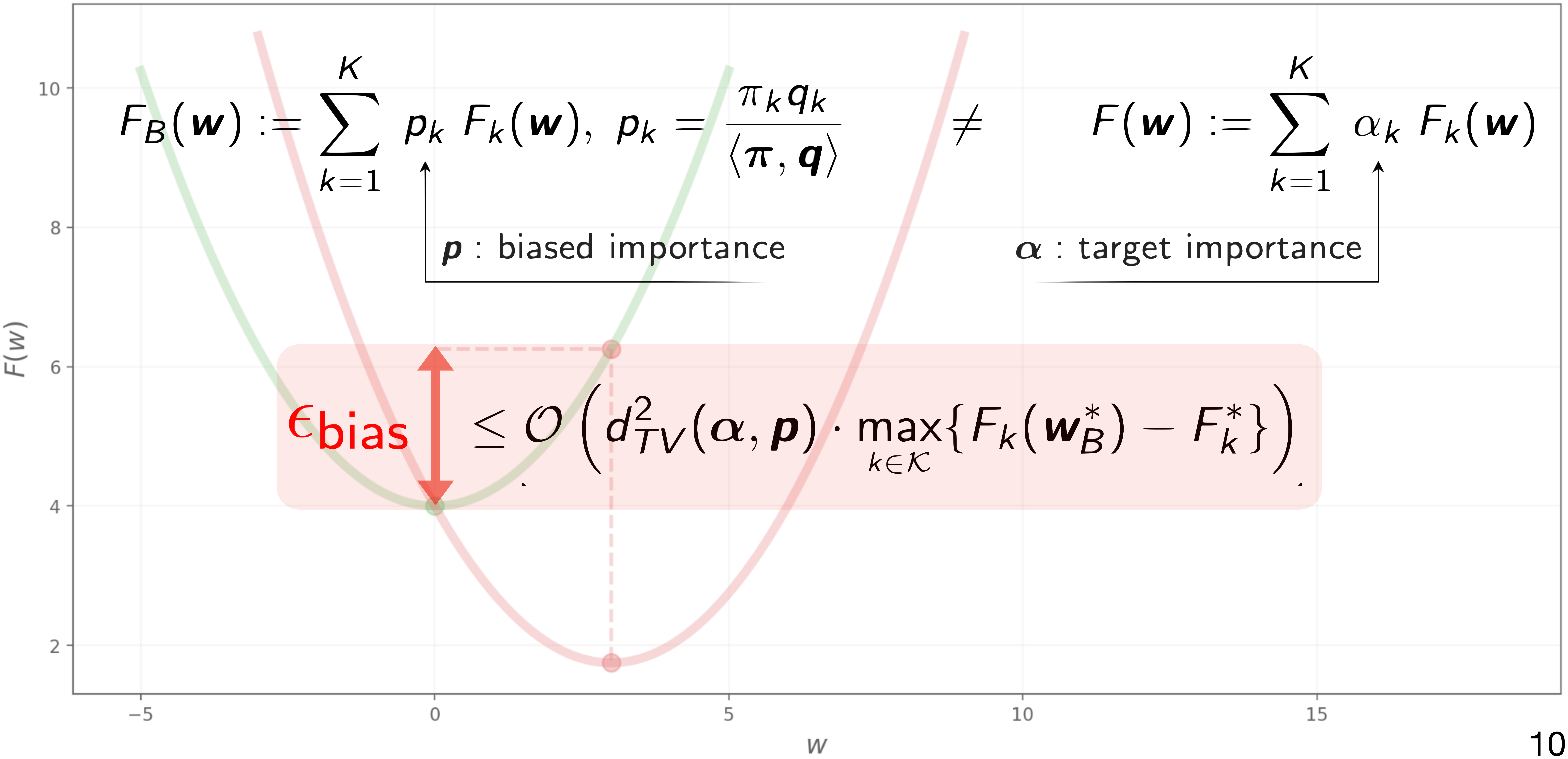


$$F_B(\boldsymbol{w}) := \sum_{k=1}^{K} p_k \, F_k(\boldsymbol{w}), \; p_k = \frac{\pi_k q_k}{\langle \boldsymbol{\pi}, \boldsymbol{q} \rangle} \neq F(\boldsymbol{w}) := \sum_{k=1}^{K} \alpha_k \, F_k(\boldsymbol{w})$$

$\boldsymbol{p}$: biased importance

$\boldsymbol{\alpha}$: target importance

Bias error



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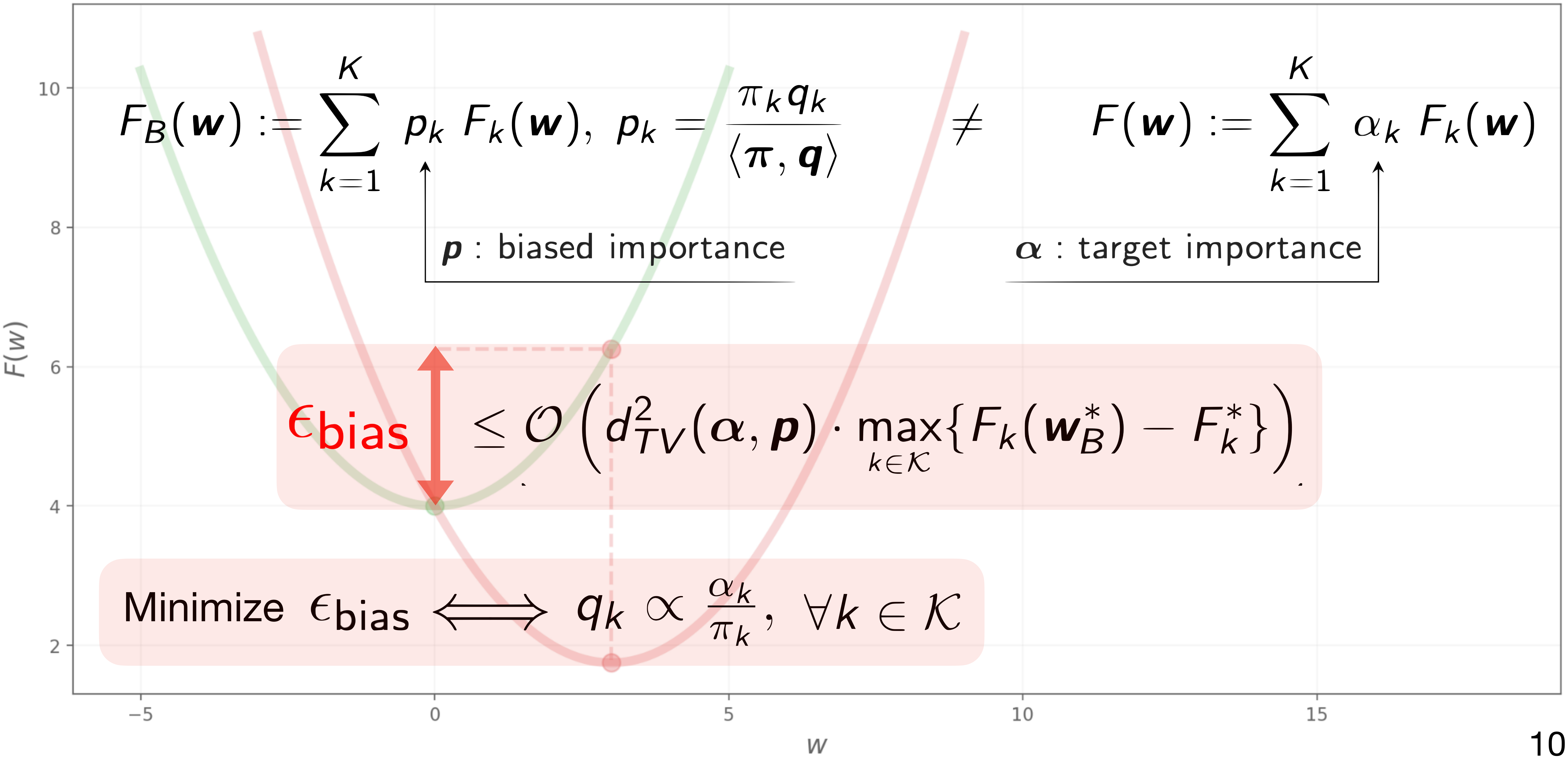
$$\epsilon_{\text{bias}} \leq \mathcal{O}\left(d_{TV}^2(\boldsymbol{\alpha}, \mathbf{p}) \cdot \max_{k \in \mathcal{K}} \{ F_k(\mathbf{w}_B^*) - F_k^* \} \right)$$

Bias error



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$$\text{Minimize } \epsilon_{\text{bias}} \iff q_k \propto \frac{\alpha_k}{\pi_k}, \; \forall k \in \mathcal{K}$$

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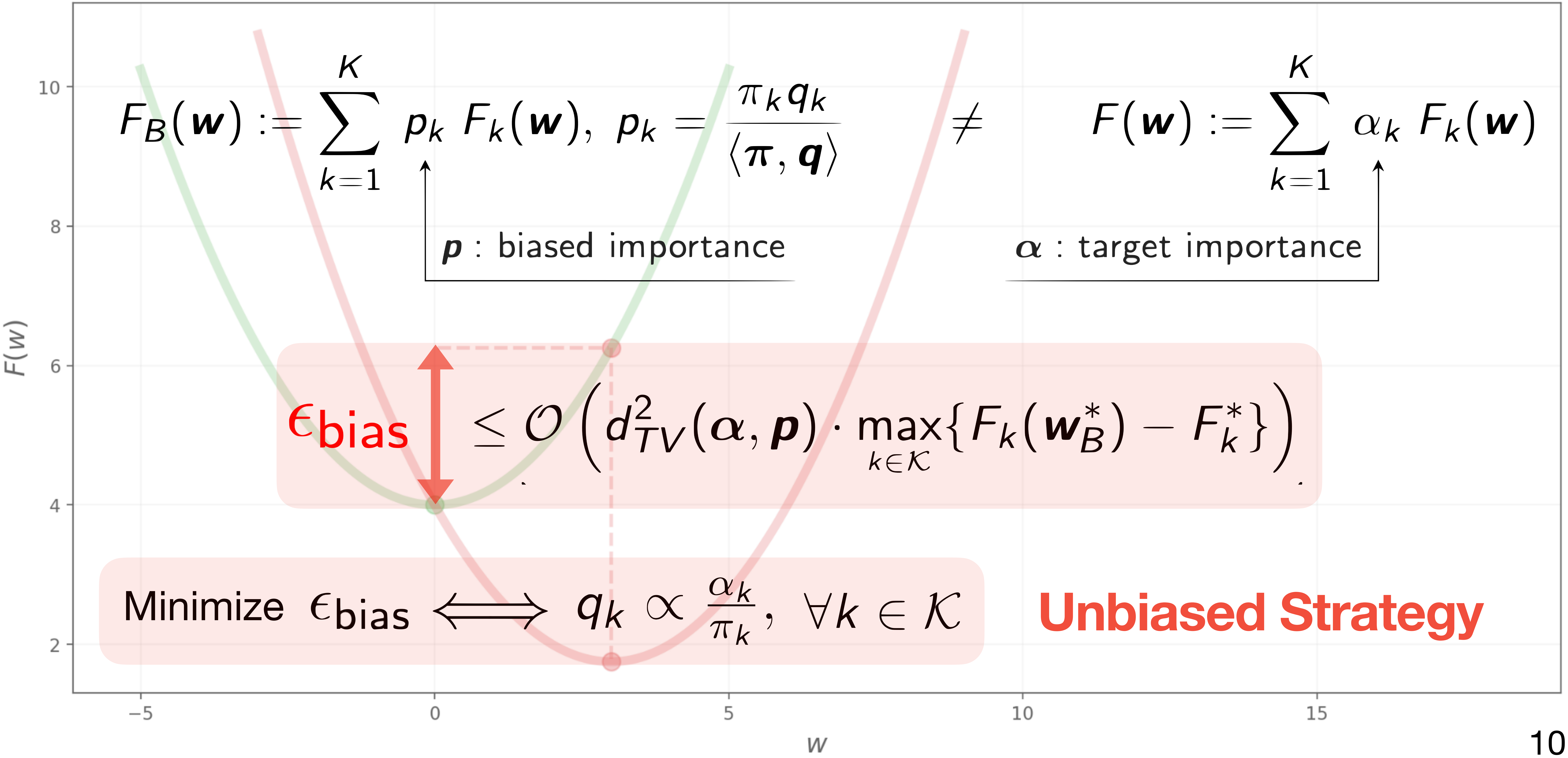

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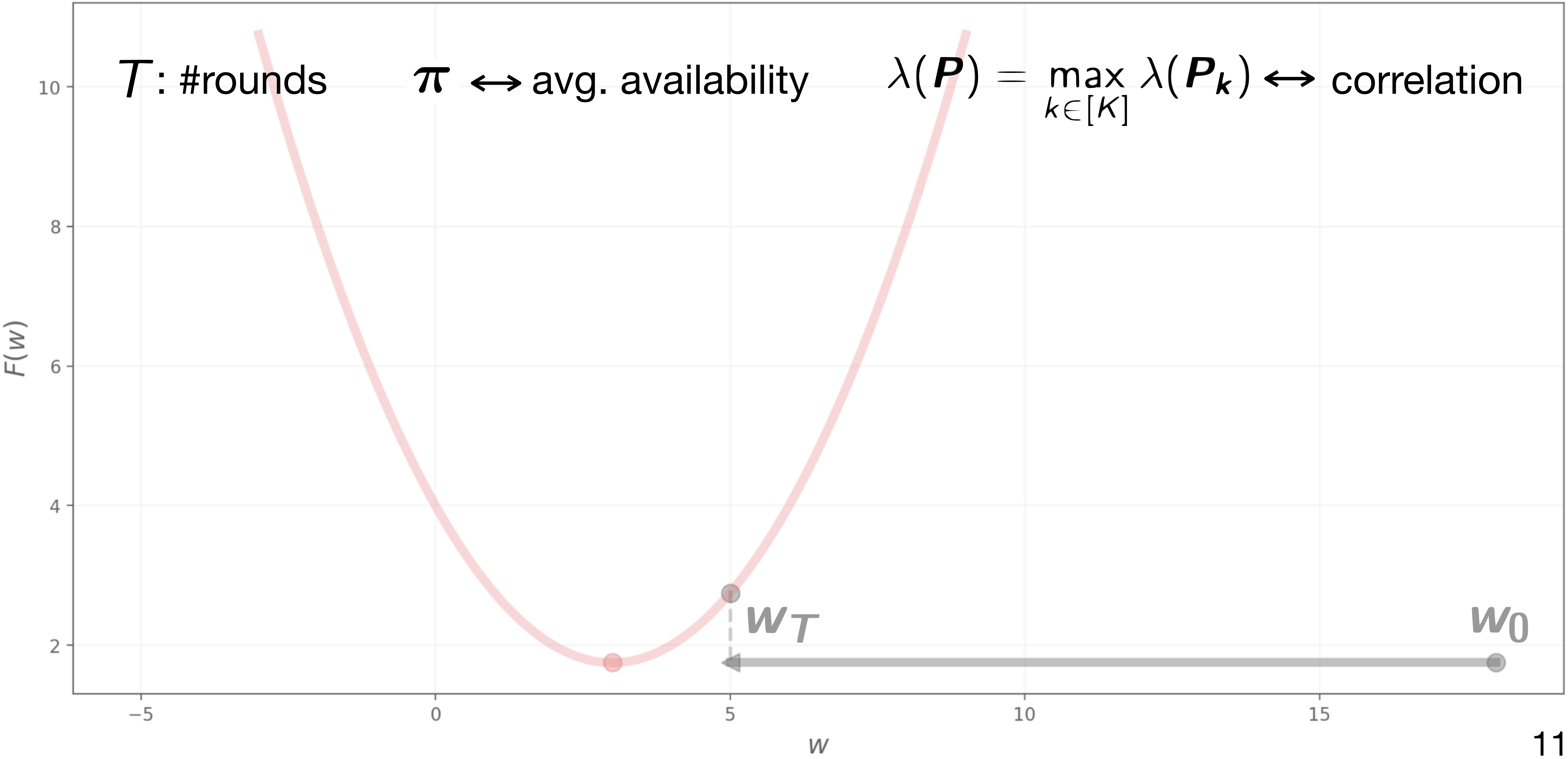
Unbiased Strategy

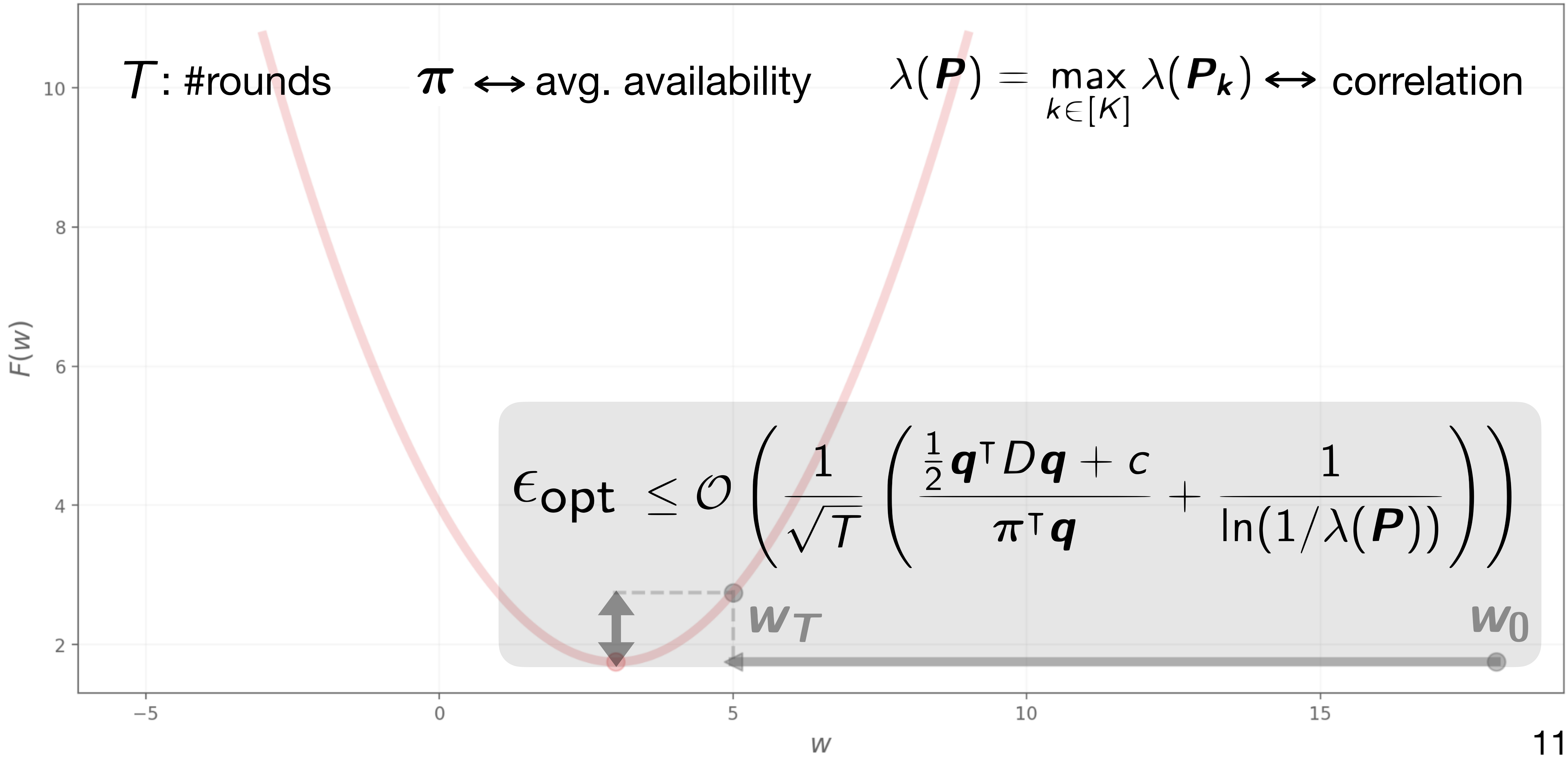
Optimization error



Optimization error

T: #rounds $\quad \boldsymbol{\pi} \leftrightarrow$ avg. availability $\quad \lambda(\boldsymbol{P}) = \max_{k\in[K]} \lambda(\boldsymbol{P_k}) \leftrightarrow$ correlation

$$\epsilon_{\text{opt}} \le \mathcal{O}\left(\frac{1}{\sqrt{T}}\left(\frac{\frac{1}{2}\boldsymbol{q}^\intercal D\boldsymbol{q} + c}{\boldsymbol{\pi}^\intercal \boldsymbol{q}} + \frac{1}{\ln(1/\lambda(\boldsymbol{P}))}\right)\right)$$



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Minimize the blue term $\Longleftrightarrow$ large q_k for large π_k

Minimize the green term $\Longleftrightarrow$ $q_k = 0$ for large $\lambda(\boldsymbol{P_k})$

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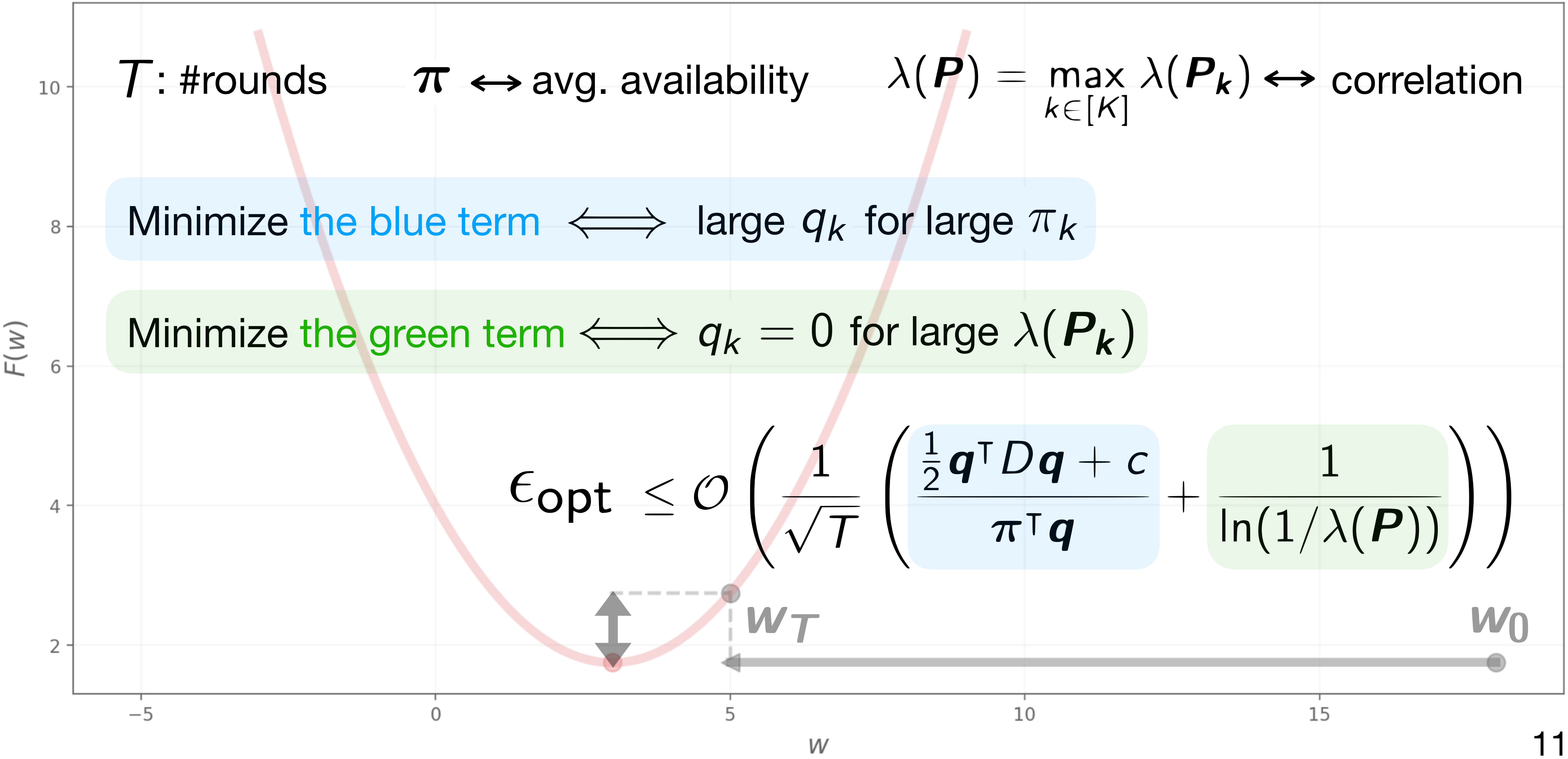

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More available strategy

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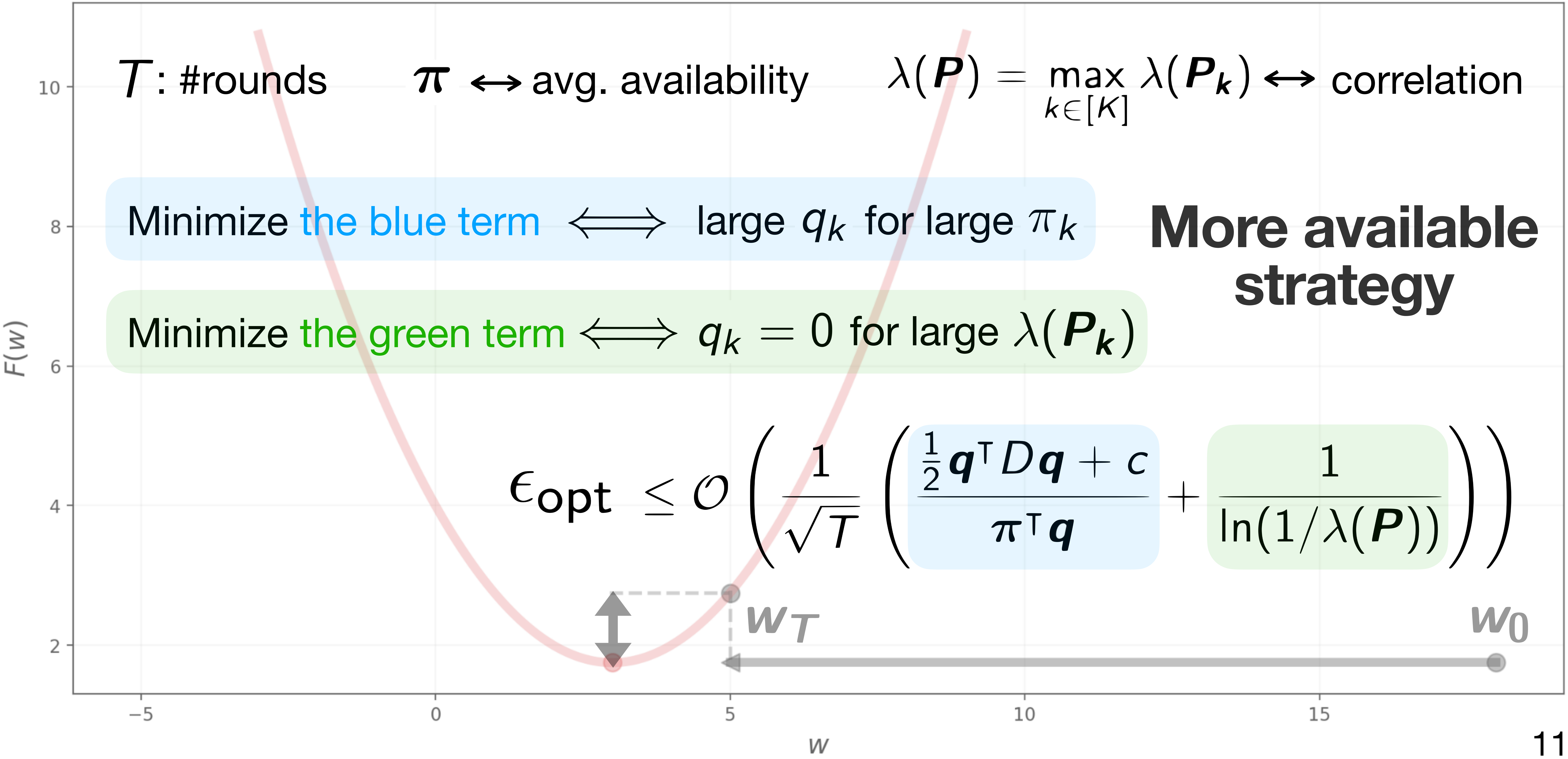
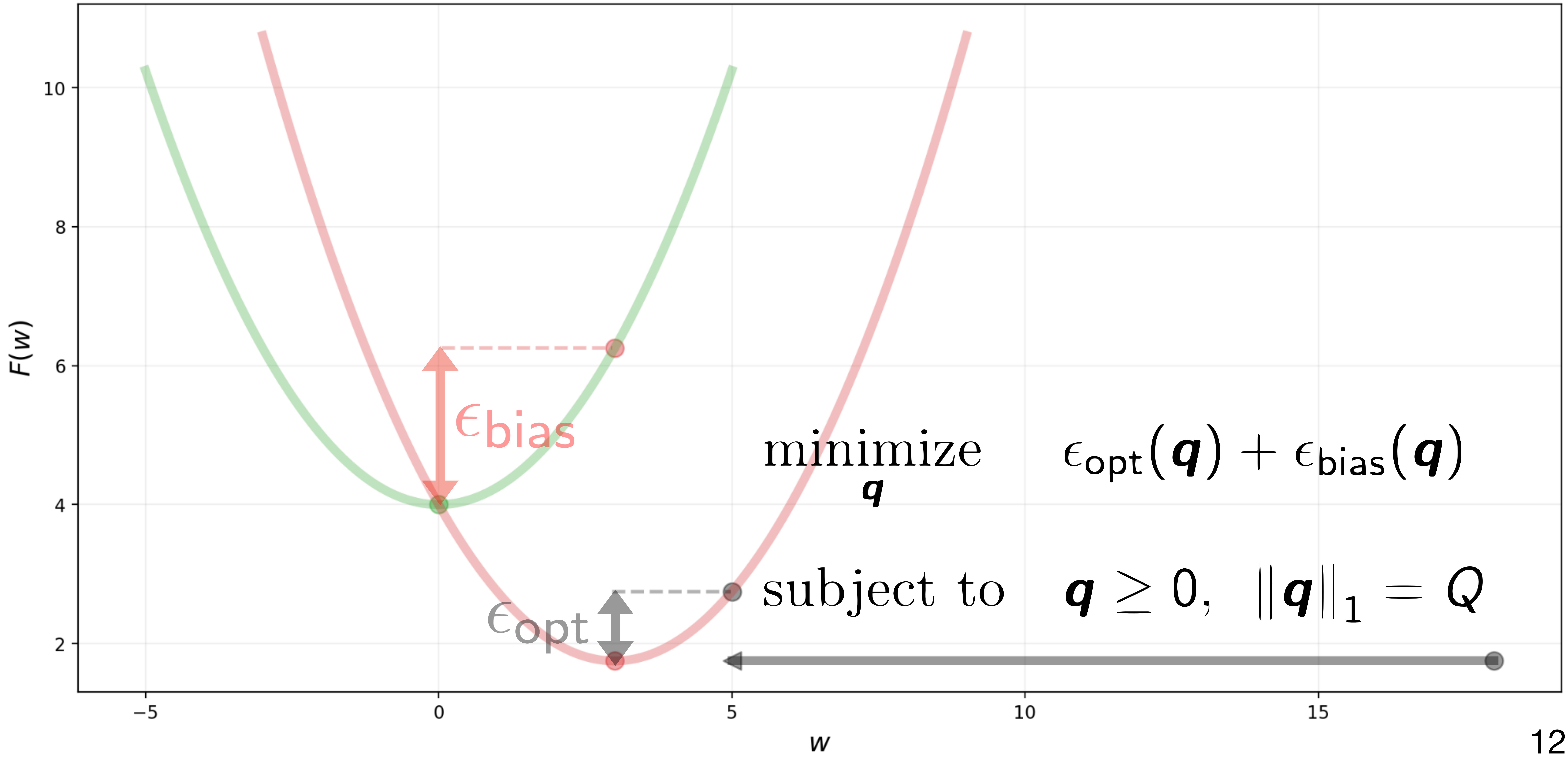


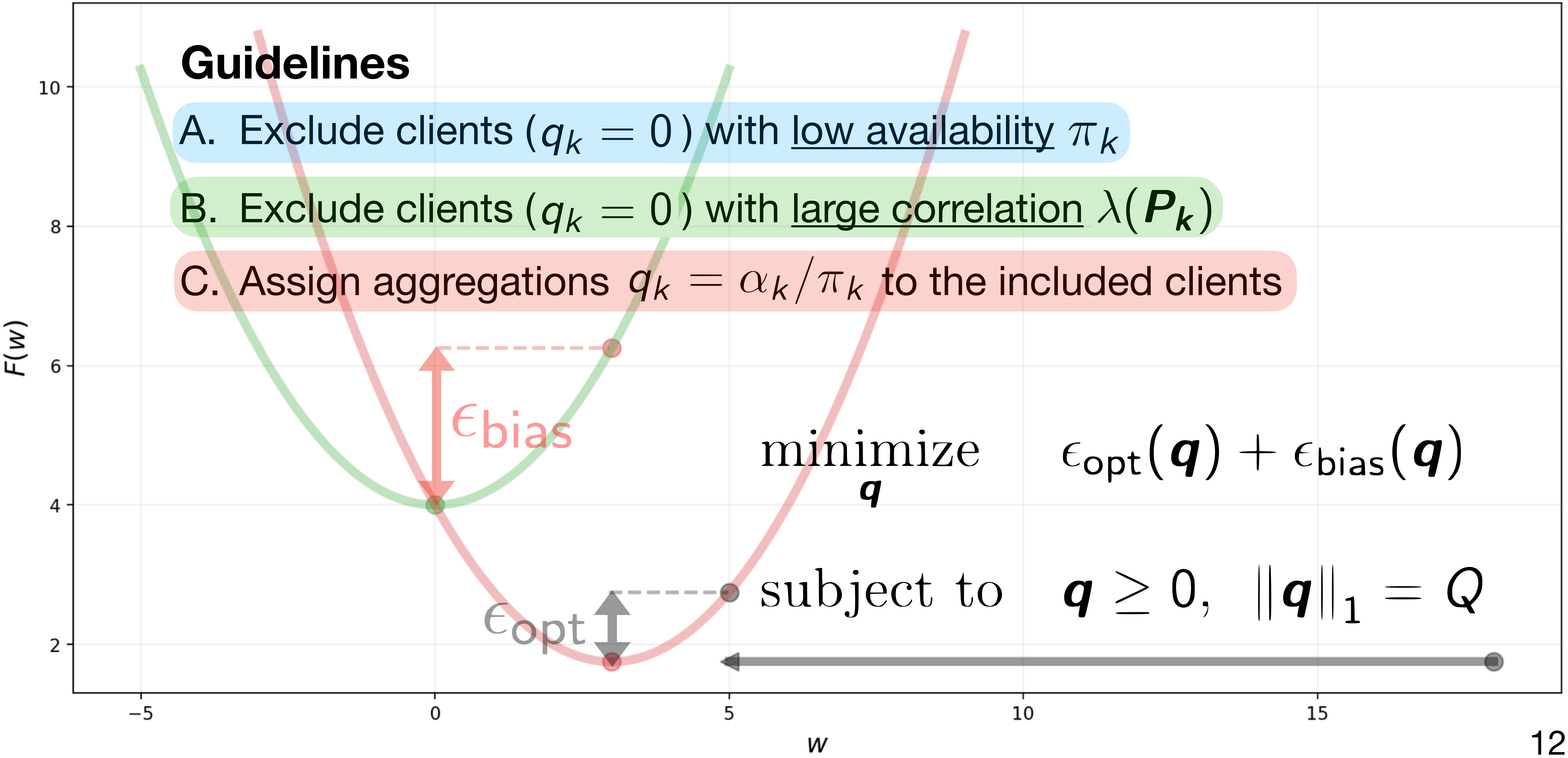
Total error : optimization error + bias error



$$\underset{\boldsymbol{q}}{\text{minimize}} \quad \epsilon_{\mathsf{opt}}(\boldsymbol{q}) + \epsilon_{\mathsf{bias}}(\boldsymbol{q})$$

$$\text{subject to} \quad \boldsymbol{q} \geq 0, \ \ \|\boldsymbol{q}\|_1 = Q$$

Total error : optimization error + bias error



Guidelines

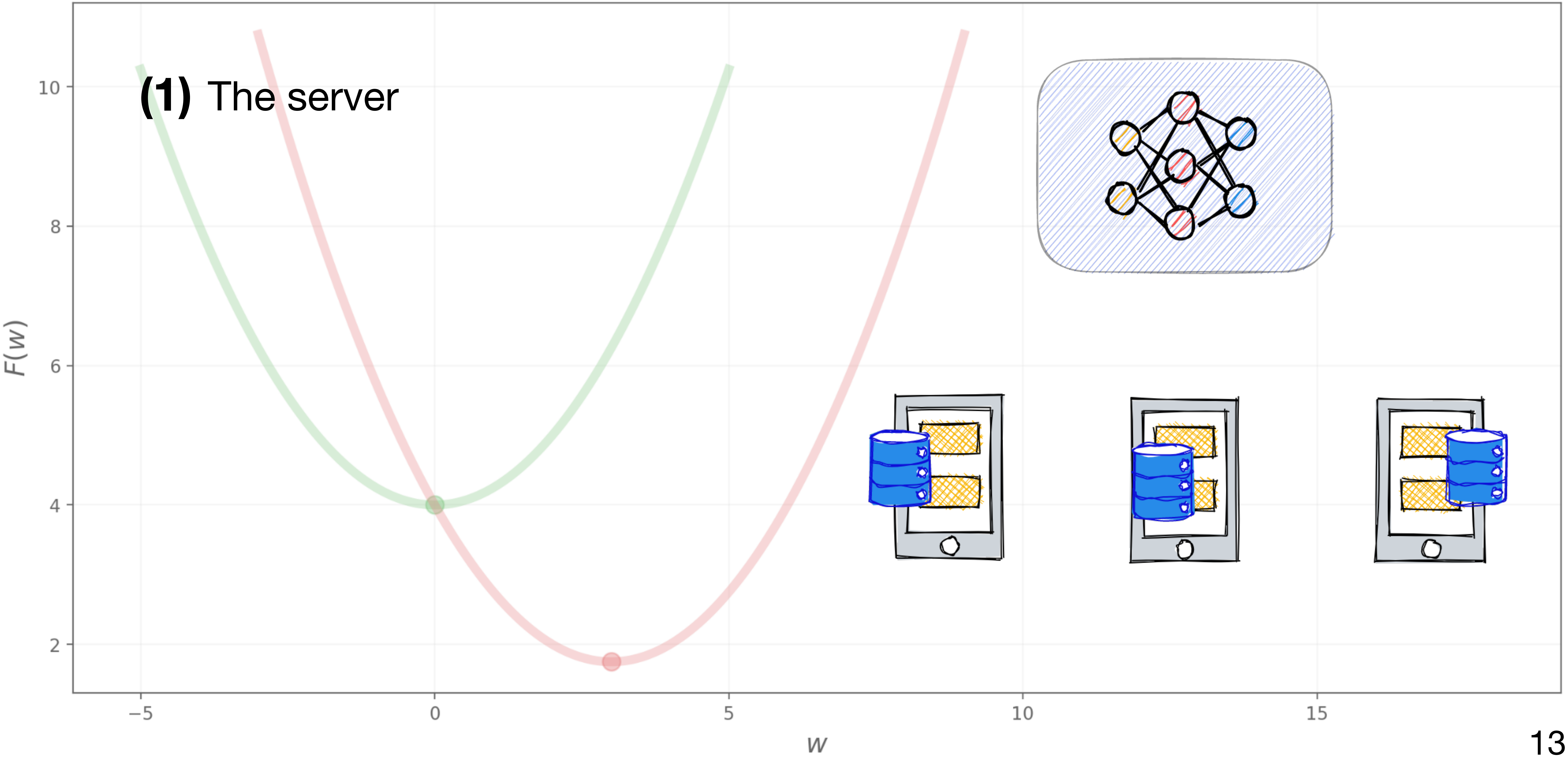
A. Exclude clients ($q_k = 0$) with low availability π_k

B. Exclude clients ($q_k = 0$) with large correlation $\lambda(\boldsymbol{P_k})$

C. Assign aggregations $q_k = \alpha_k / \pi_k$ to the included clients

$$\begin{aligned} \underset{\boldsymbol{q}}{\text{minimize}} \quad & \epsilon_{\text{opt}}(\boldsymbol{q}) + \epsilon_{\text{bias}}(\boldsymbol{q}) \\ \text{subject to} \quad & \boldsymbol{q} \geq 0, \ \|\boldsymbol{q}\|_1 = Q \end{aligned}$$

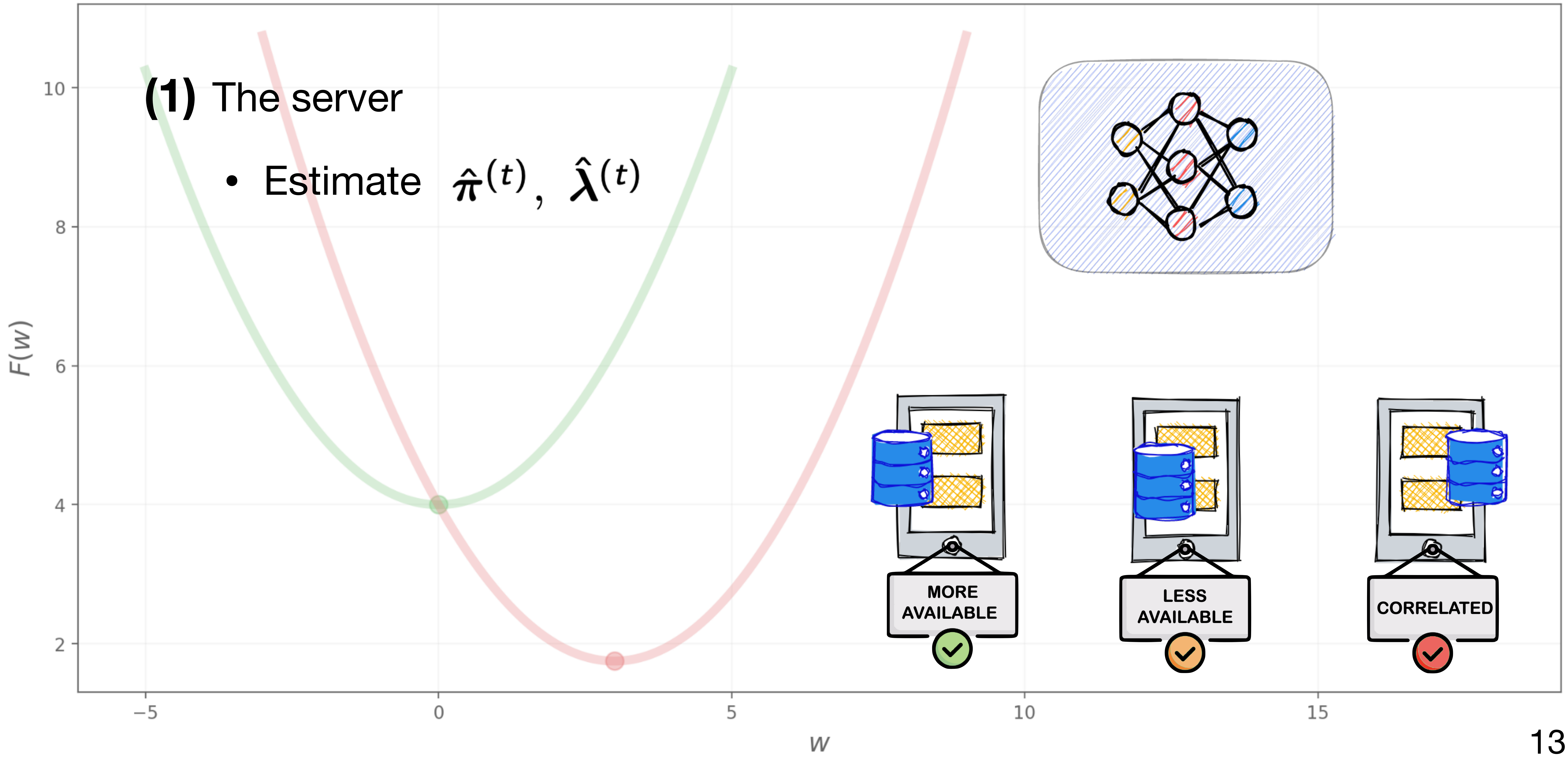
Correlation-Aware FL (CA-Fed)



Correlation-Aware FL (CA-Fed)

(1) The server

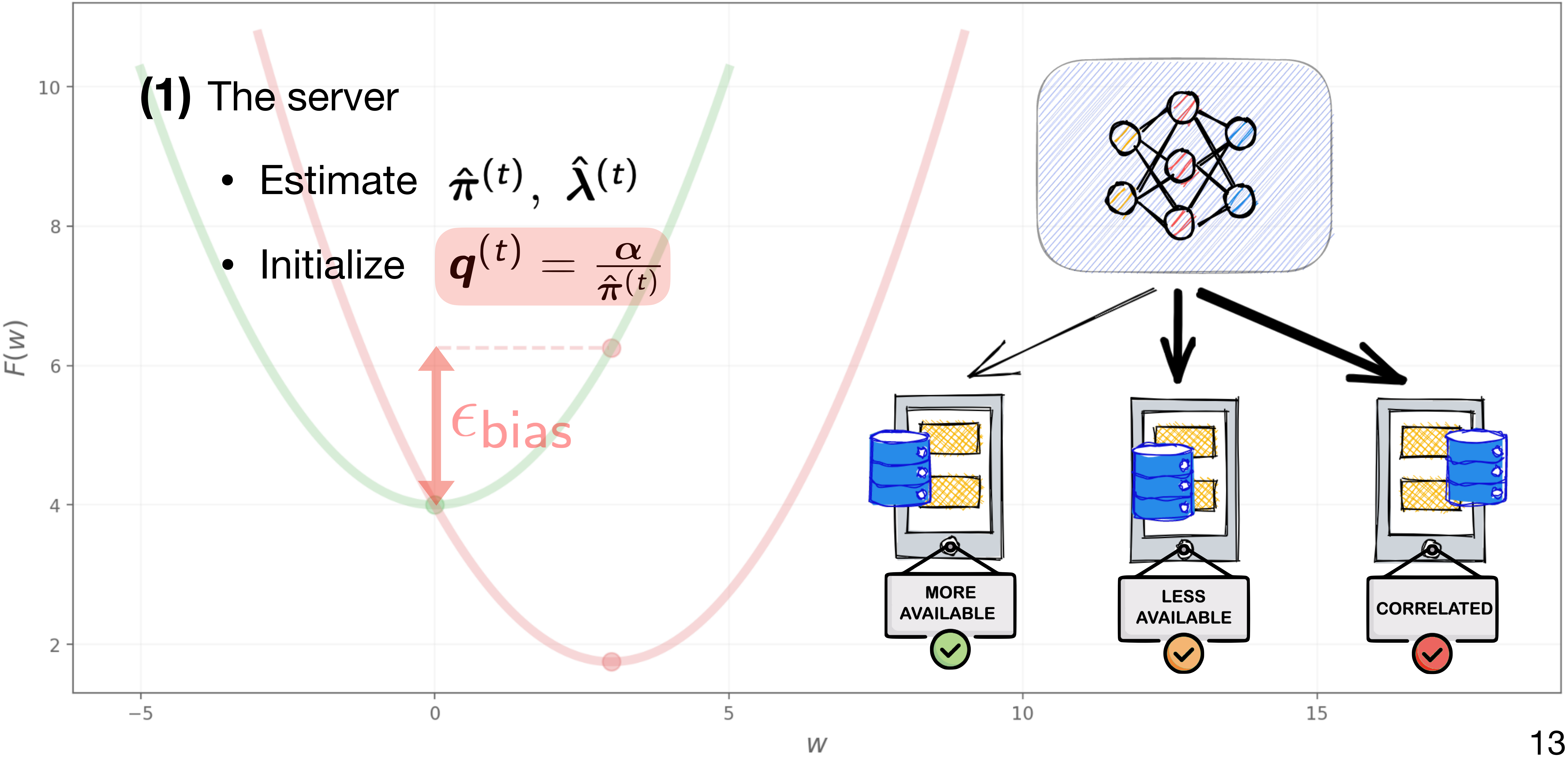
- Estimate $\hat{\pi}^{(t)}, \hat{\lambda}^{(t)}$



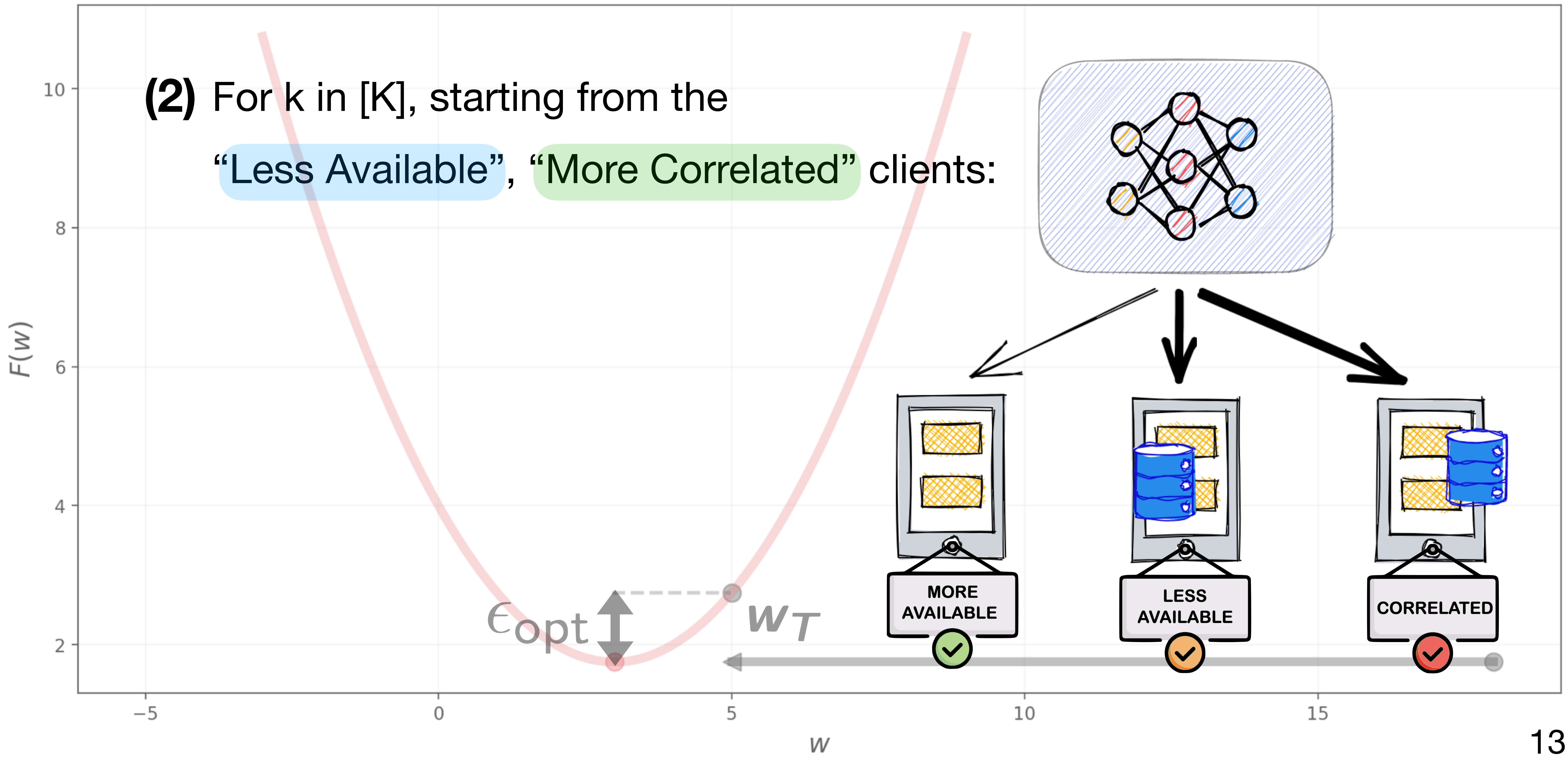
Correlation-Aware FL (CA-Fed)

(1) The server

- Estimate $\hat{\boldsymbol{\pi}}^{(t)}, \ \hat{\boldsymbol{\lambda}}^{(t)}$
- Initialize $\boldsymbol{q}^{(t)} = \frac{\boldsymbol{\alpha}}{\hat{\boldsymbol{\pi}}^{(t)}}$



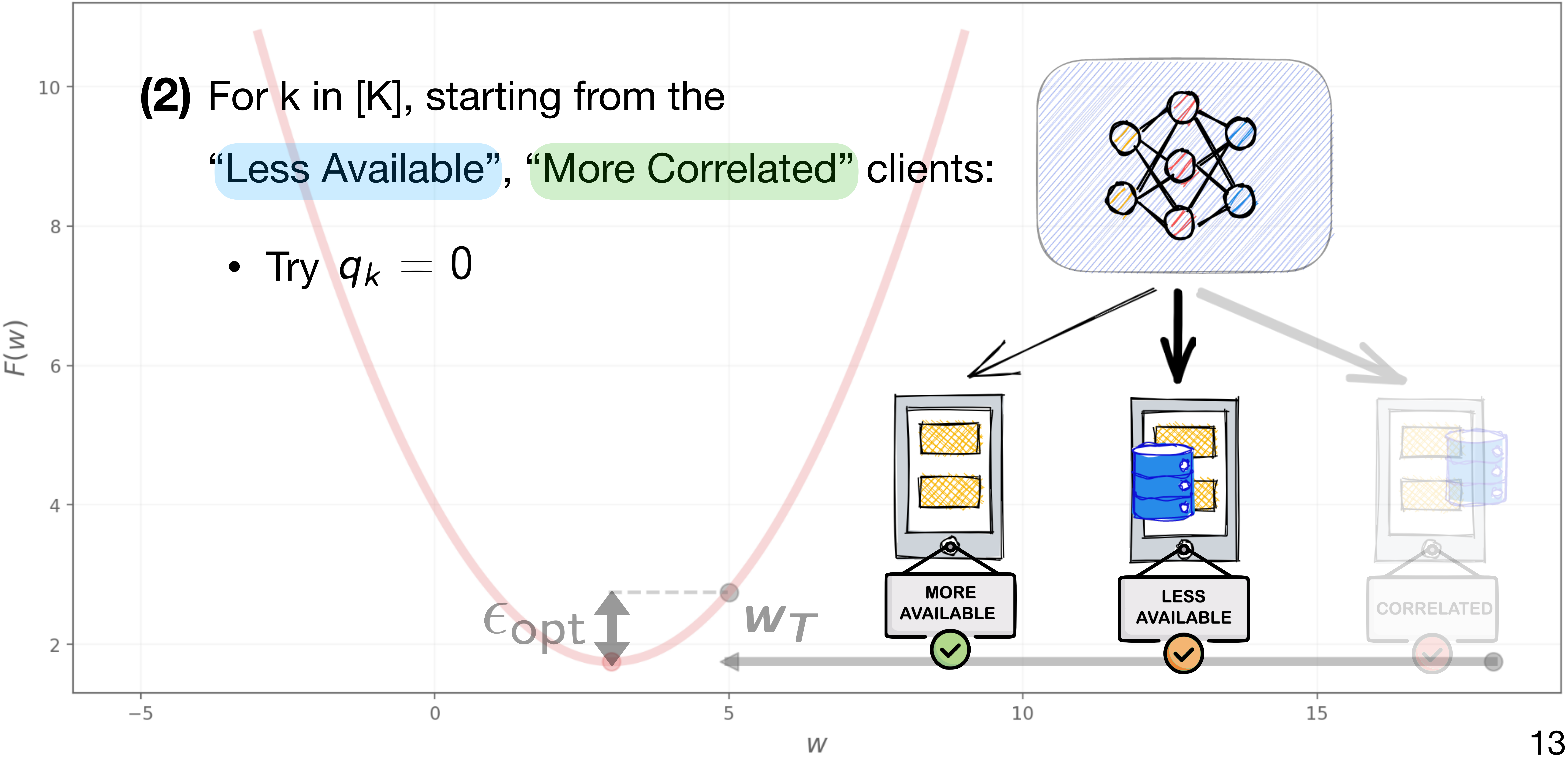
Correlation-Aware FL (CA-Fed)



Correlation-Aware FL (CA-Fed)

(2) For k in [K], starting from the "Less Available", "More Correlated" clients:

- Try $q_k = 0$

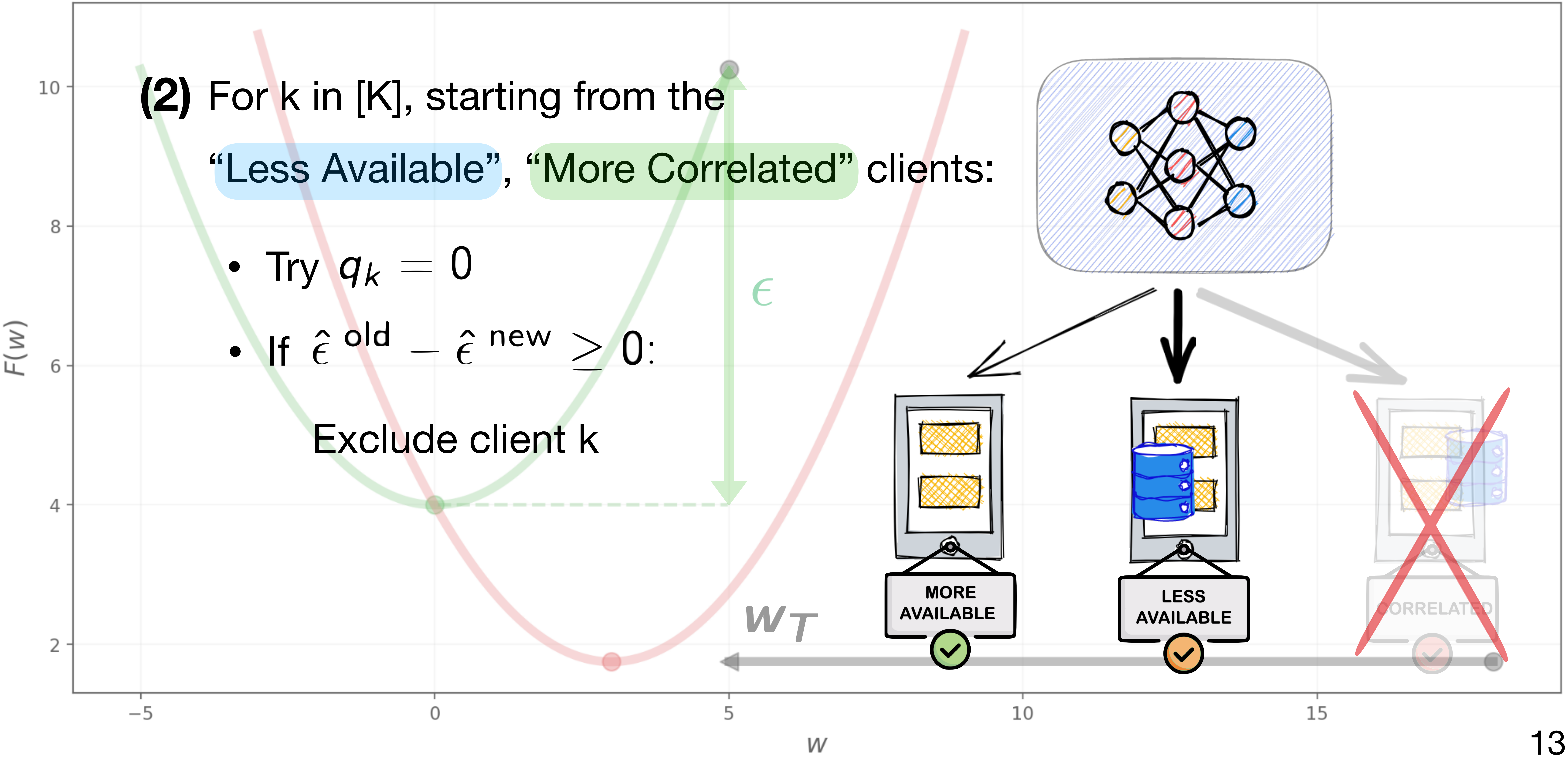


Correlation-Aware FL (CA-Fed)

(2) For k in [K], starting from the "Less Available", "More Correlated" clients:

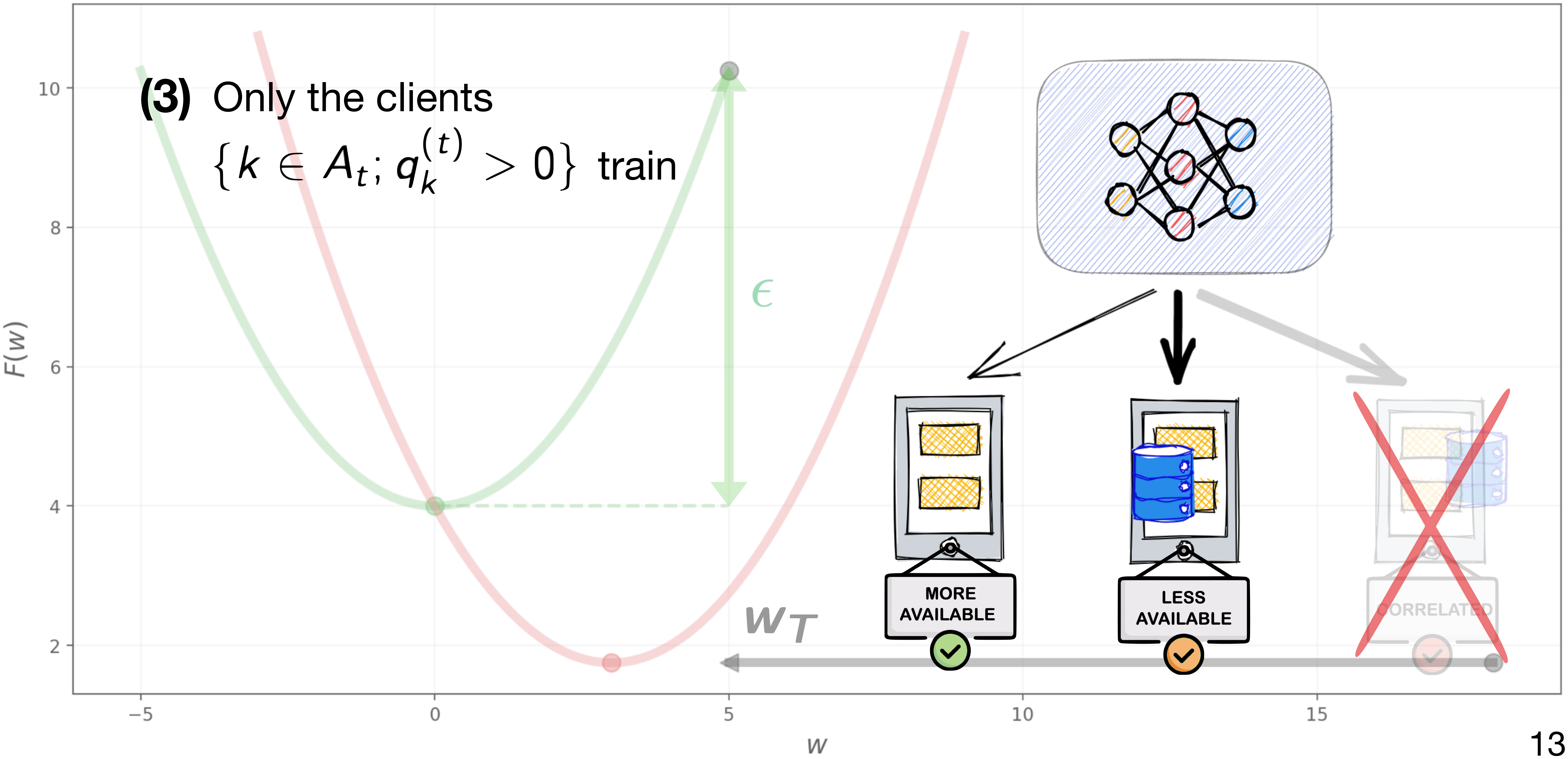
- Try $q_k = 0$
- If $\hat{\epsilon}^{\text{old}} - \hat{\epsilon}^{\text{new}} \geq 0$:

  Exclude client k



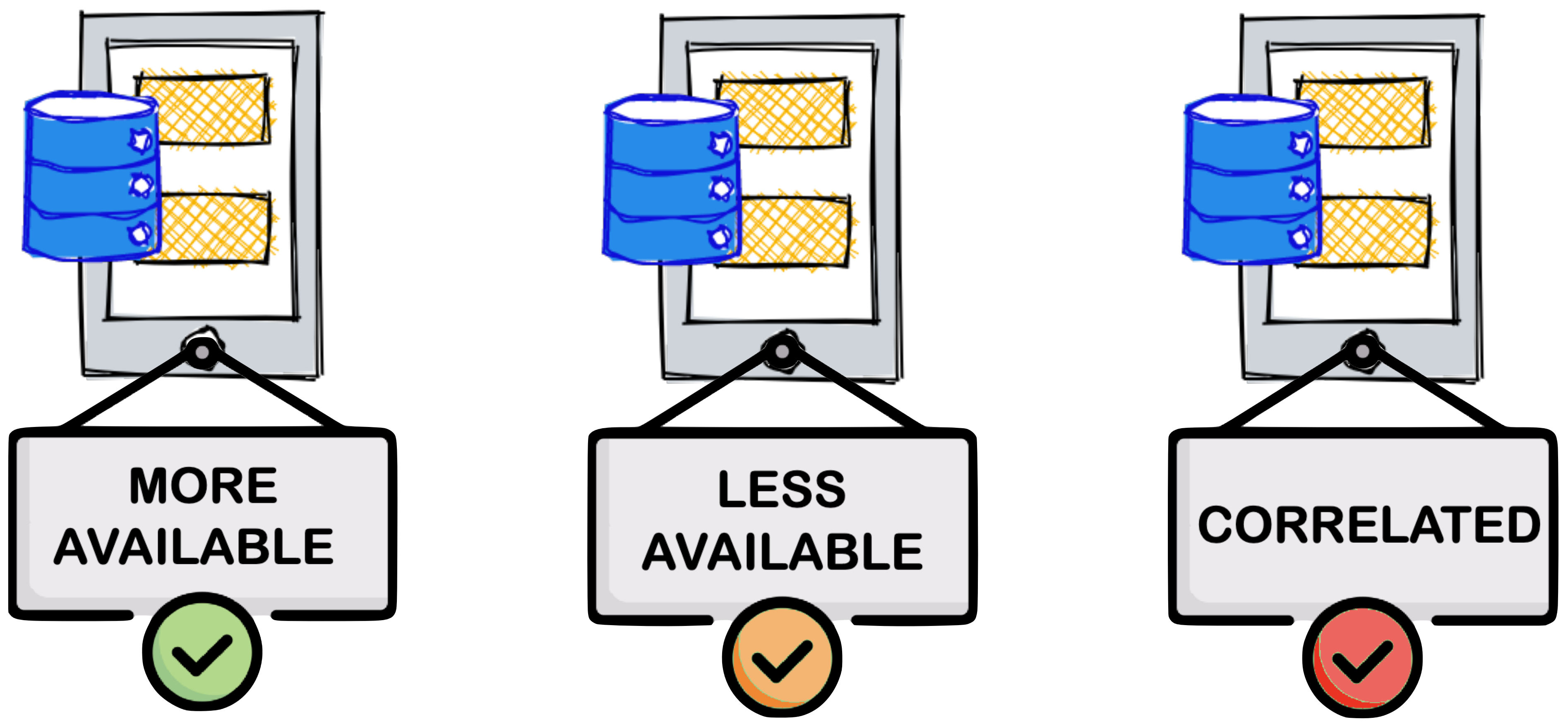
Correlation-Aware FL (CA-Fed)

(3) Only the clients $\{k \in A_t; q_k^{(t)} > 0\}$ train



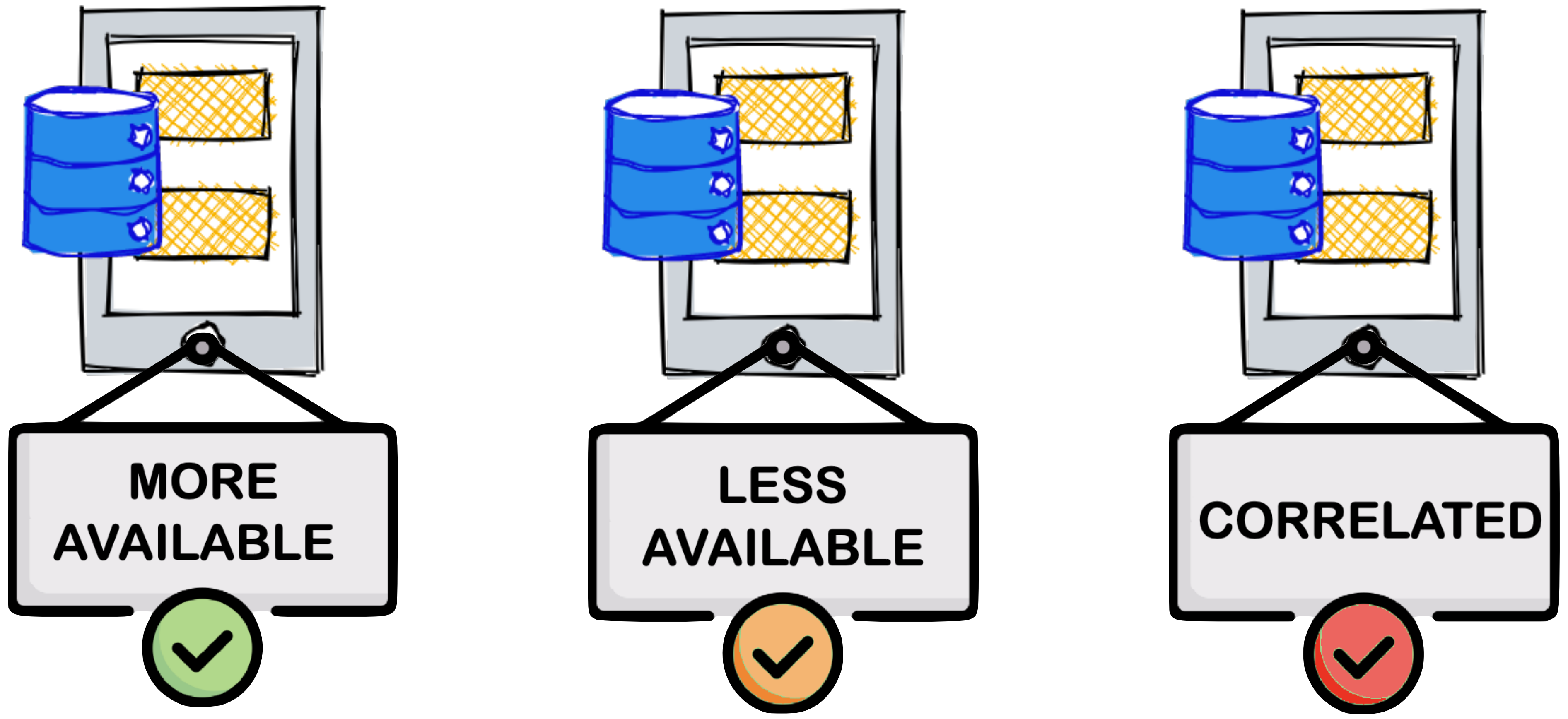
Experimental setting

- Population of K=100 clients, divided in:

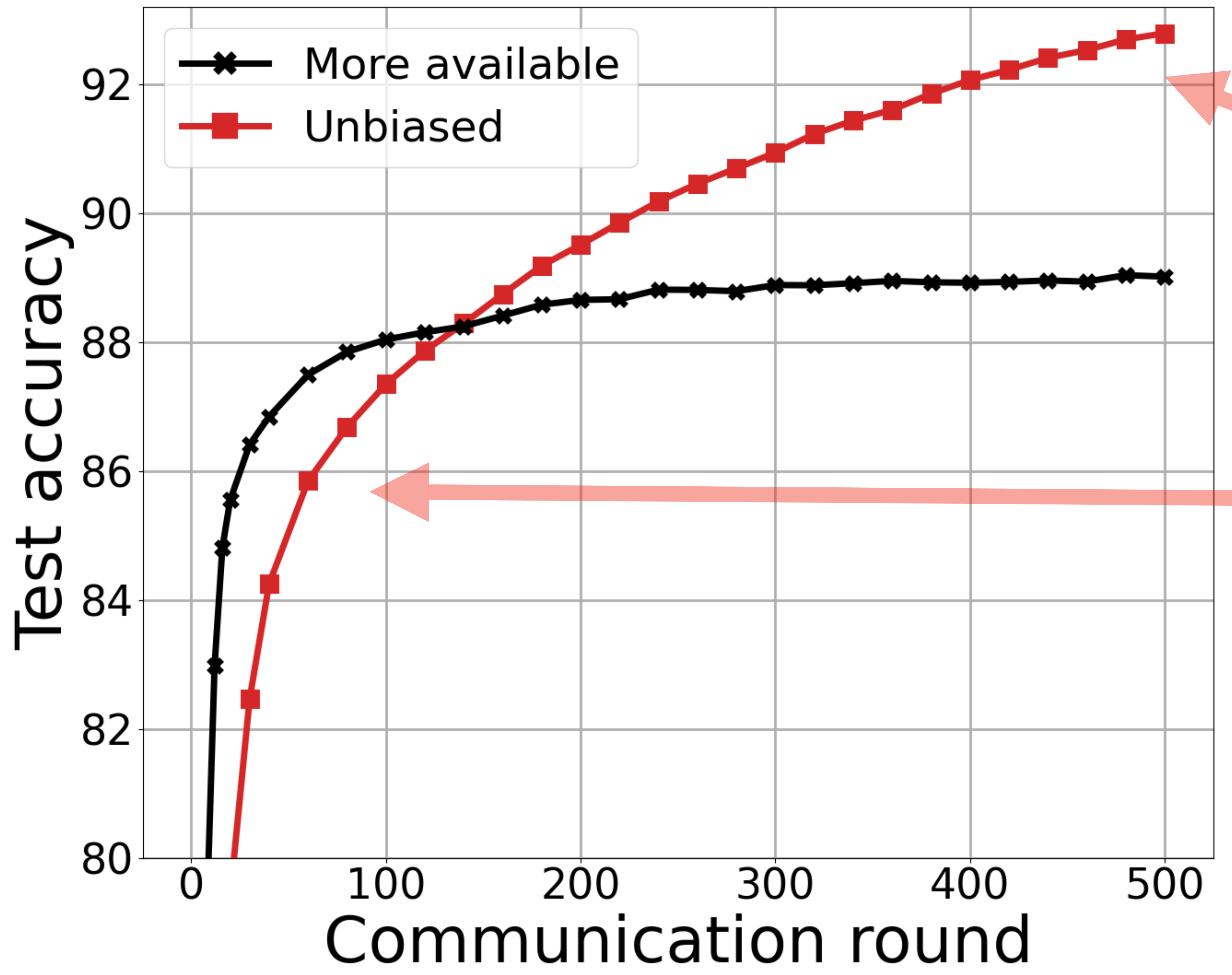


Experimental setting

- Population of K=100 clients, divided in:



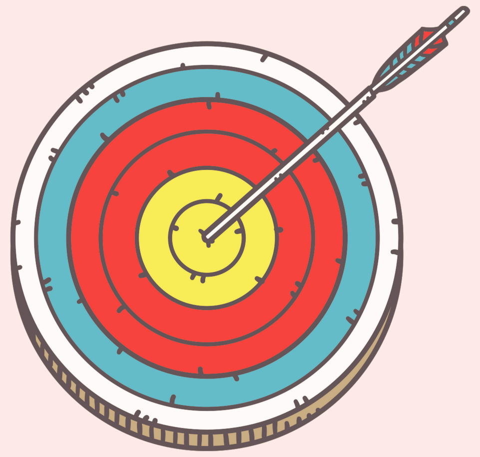
- Trade-off:



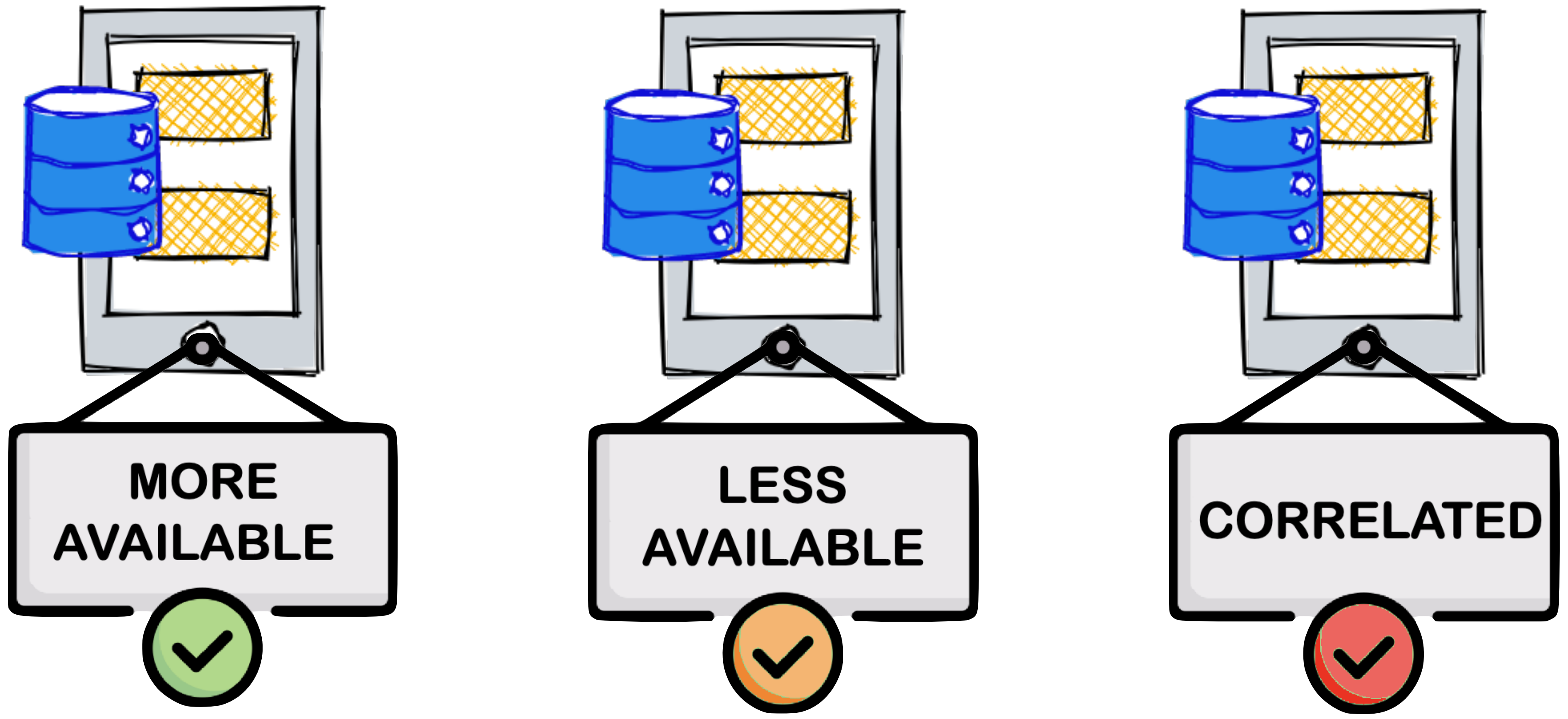
Unbiased

Minimize the bias error

Slower convergence to the target objective

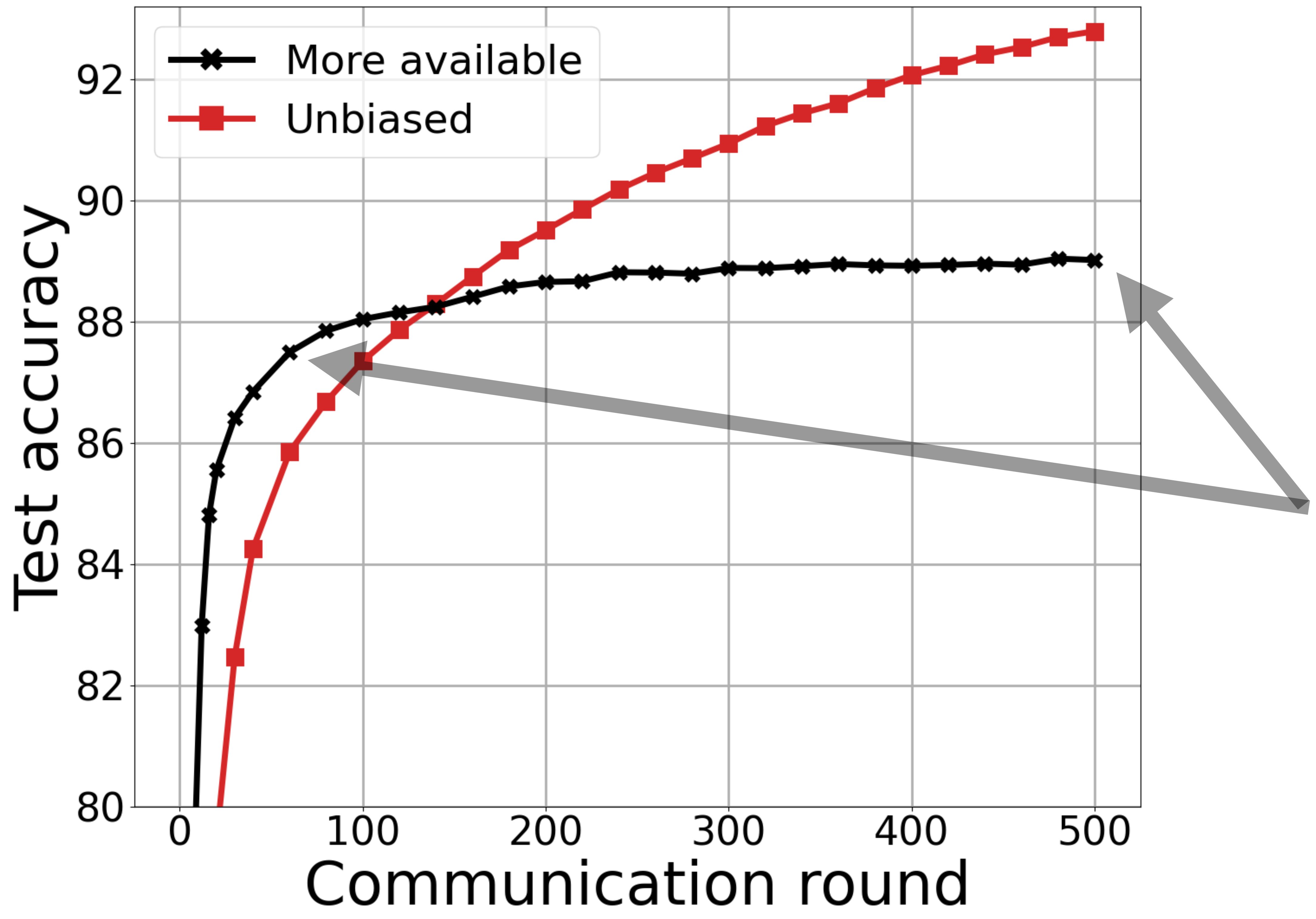


Experimental setting



- Population of K=100 clients, divided in:

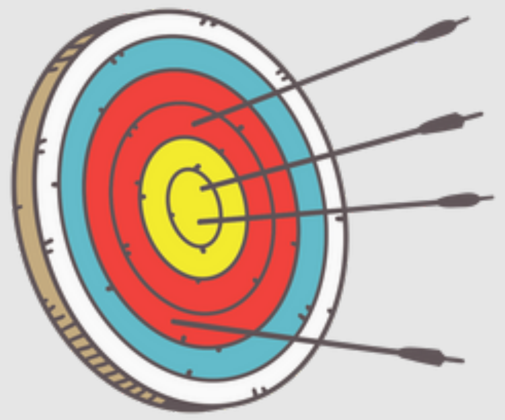
- Trade-off:



More Available

Minimize the optimization error

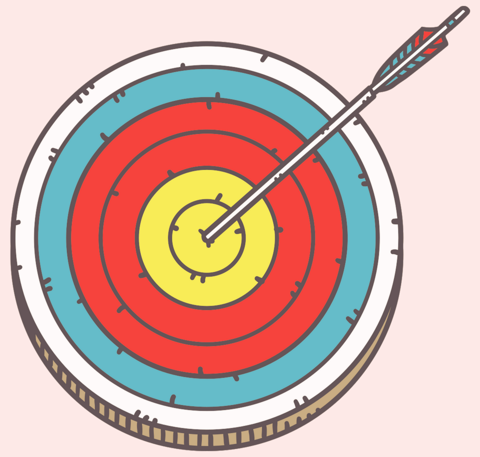
Faster convergence to a biased objective



Unbiased

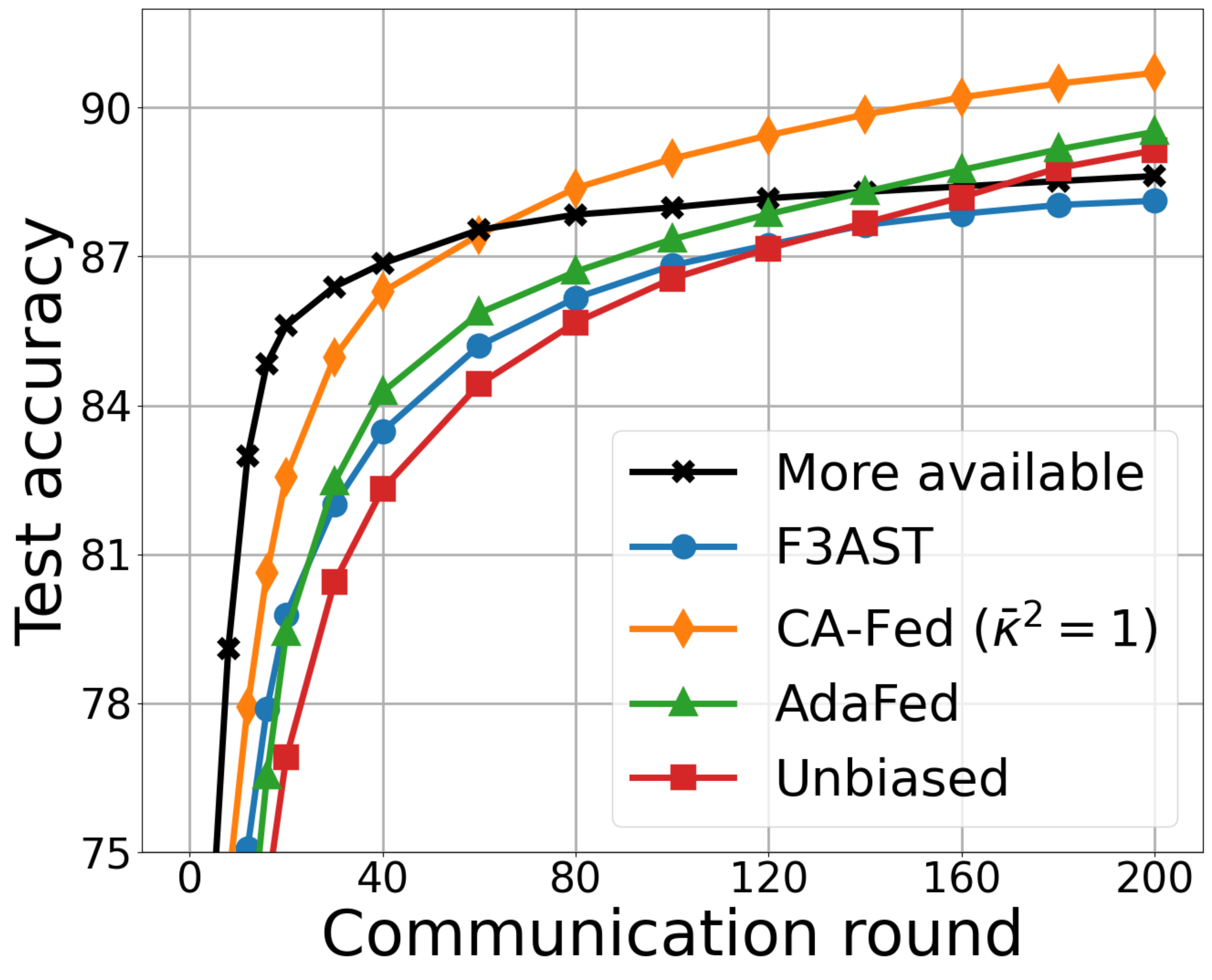
Minimize the bias error

Slower convergence to the target objective

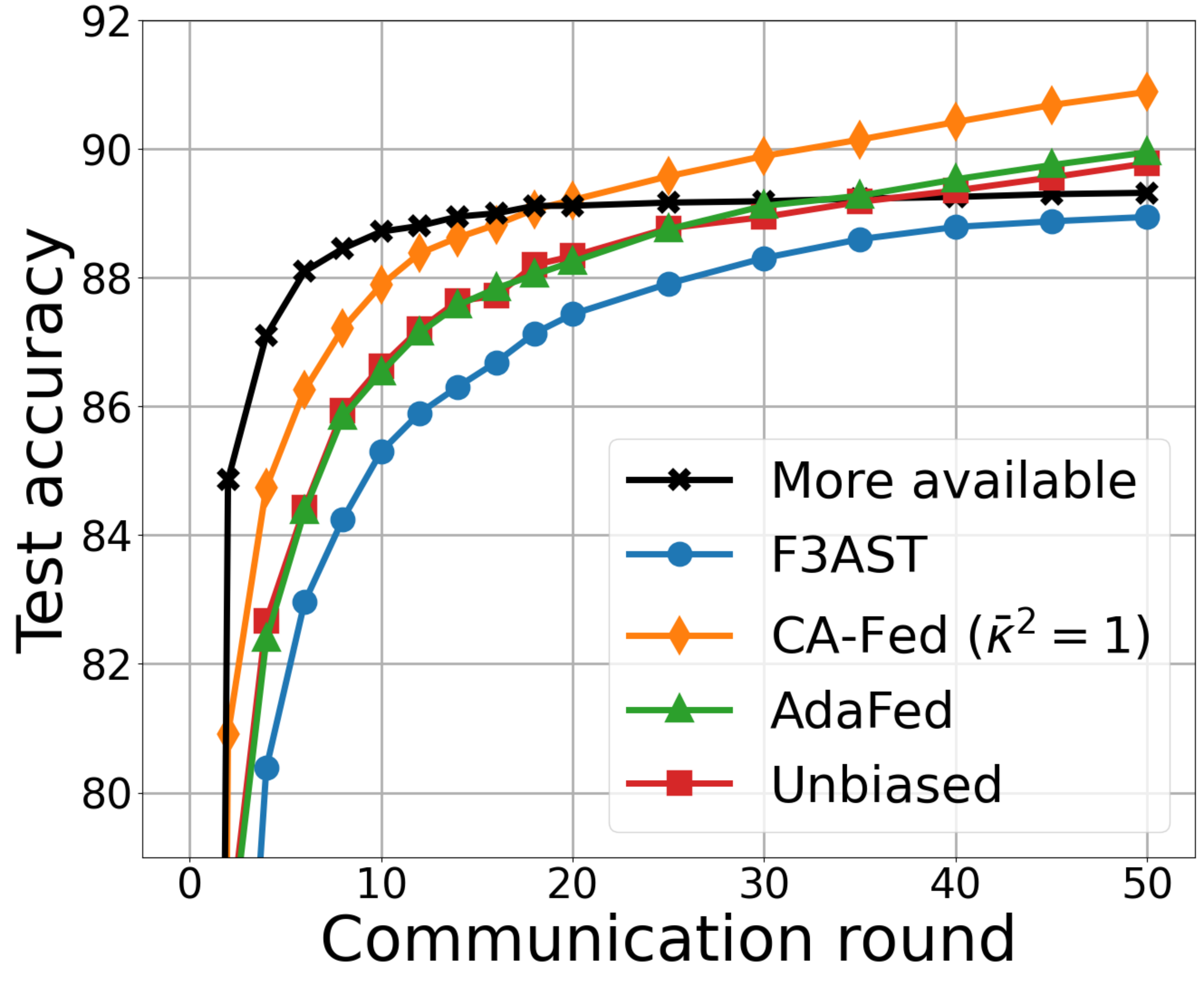


Experimental results

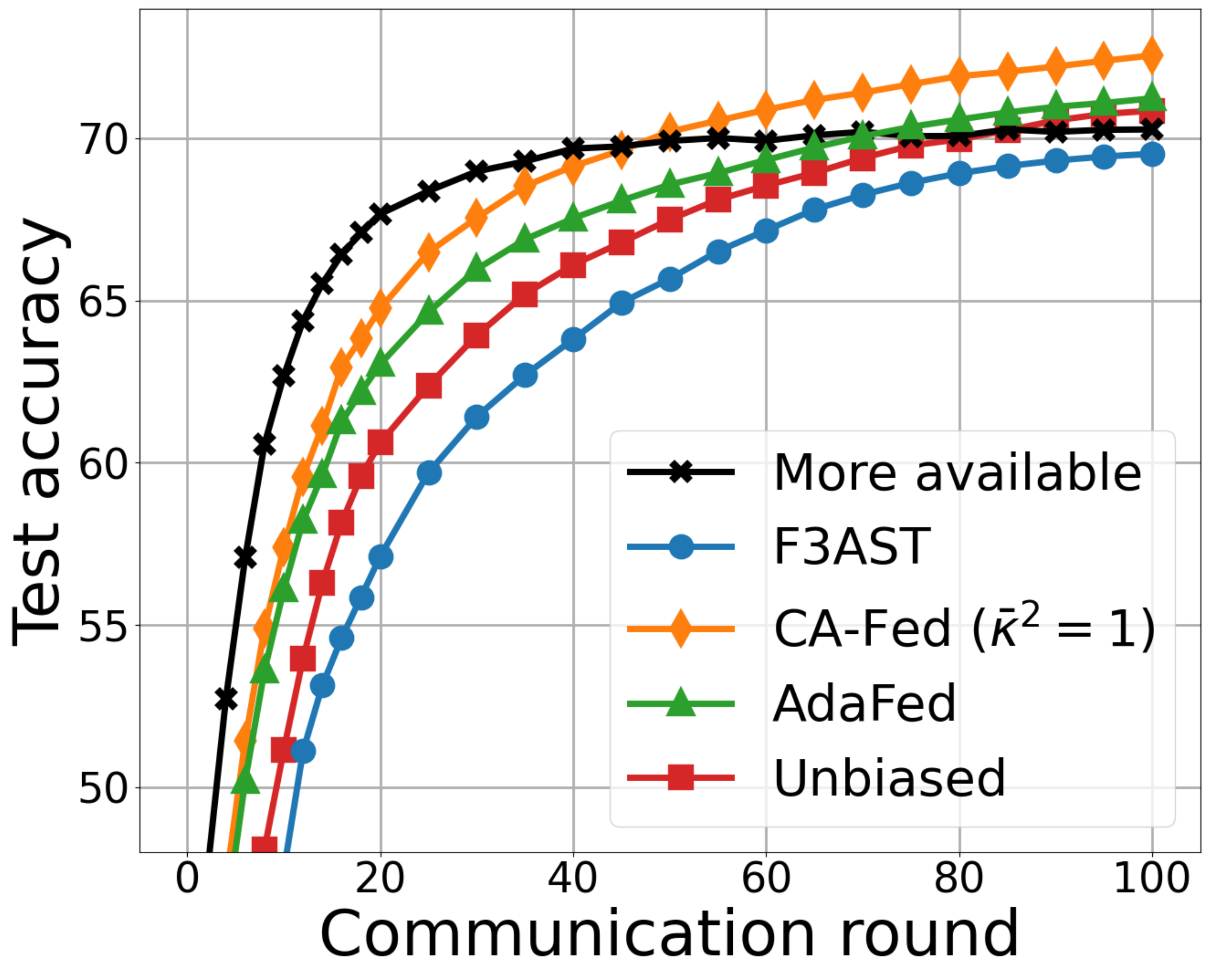
- We compare CA-Fed with More Available, Unbiased, AdaFed, and F3AST



(a) Synthetic LEAF



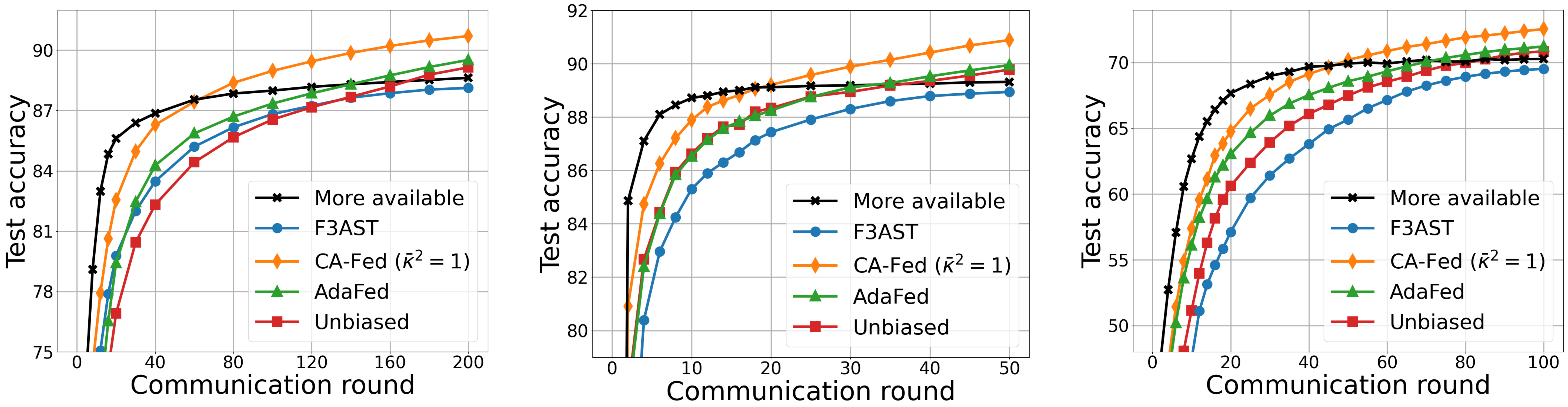
(b) MNIST



(c) CIFAR10

Experimental results

- We compare CA-Fed with More Available, Unbiased, AdaFed, and F3AST



(a) Synthetic LEAF (b) MNIST (c) CIFAR10

CA-Fed achieves higher accuracy in a shorter time

Conclusions

- First convergence analysis under heterogeneous and correlated client availability
- Adaptively excluding less available and highly correlated clients can be effective
- Further discussions and experiments in our paper!

Thank you for your attention!

Paper Code