A Time and Space Routing Game Model applied to Visibility Competition on Online Social Networks

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How to advertise on Online Social Network ?

Imagine you are a photographer who wants to become popular on Facebook. Which strategies could you use ? :

- The first strategy is to pay Facebook to do advertisment for you.
- The second one consists in posting in different News Feeds : Groups, Pages, or Users' Timeline.

A *News Feed* is a Feed where the user's friends and subscribed pages' news are published. News Feed exists in most popular social networks, like Facebook, Twitter or Pinterest.

How to advertise in Online Social Network ?

Buying Ads on Facebook

Positive effects :

- Reach a lot of people,
- easy to execute and to monitor.

Negative effects :

- You don't know where your ads appears,
- it is expensive.

Posting on Facebook News Feed

Positive effects :

- Free,
- you control all the process,
- create a real interaction with your community.

Negative effects :

- Require knowledge in Software Engineering and Statistics for an automatic system.

How to advertise on Online Social Network ?

Let's assume that our photographer wants to use the second strategy. Then he needs to find answers to the following questions :

- How to measure his posting strategy efficiency in each News Feed ?
- In which News Feed it would be more efficient to post ?
- At what time of day the posts would have more visibility ?
- Which topic of message he should choose to reach popularity ?

The goal of this study is to provide a general framework that answers to this questions.

How to be visible on a News Feed ?

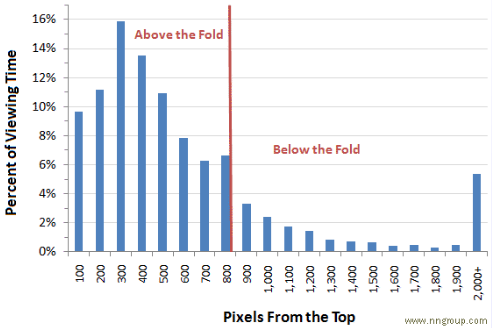


FIGURE: Distribution of user fixations [1]

Answer : Having messages always on the top of the News Feed

1. http ://www.nngroup.com/articles/scrolling-and-attention/

An intuitive definition of the Visibility Measure

Visibility Measure of source j

The probability that when a message comes in the News Feed, it comes from j.

Exemple : In this figure, the probability that a message comes from j in $[t, C+t]$ is $\frac{2}{3}$.

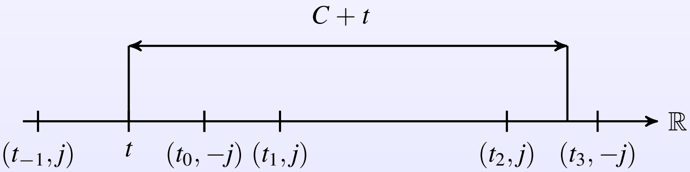


FIGURE: Arrival time of messages in a News Feed

Formalism of the Visibility Measure

We consider the following :

- The instant of arrivals of messages from a source j, $1 \leq j \leq J$, in a News Feed, is modeled by a stationary ergodic point process (SEPP). Its intensity λ_j is finite and non null.
- By using Palm probabilities theory [2] we prove that when an arrival of messages occurs, the probability that the message comes from source j is given by :

$$\frac{\lambda_j}{\sum_i \lambda_i}$$

2. Baccelli, F., Bremaud, P. (2003). Elements of queueing theory : Palm Martingale calculus and stochastic recurrences (Vol. 26). springer.

Is a unique SEPP realistic ?

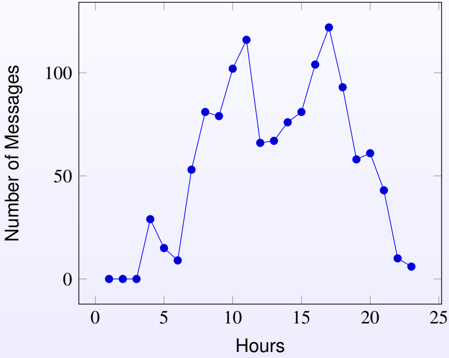


FIGURE: Number of messages per hour over seven months of a Facebook Page

Flow of messages cannot be modeled by a unique SEPP.

Visibility Measure 2.0 :

To address the issue on stationarity we define Q peak intervals.

- A peak interval q is a time interval when sources need to be visible.
- Peak intervals are disjoints.
- We define Q point processes for each peak interval and we assume that each one of these processes are independents.
- Then when an arrival of messages occurs, in a peak interval q, the probability that it is from source j is given by :

$$\frac{\lambda_{(j,q)}}{\sum_i \lambda_{(i,q)}}$$

where $\lambda_{(j,q)}$ is the intensity of the point process that model the arrival of messages from j in the peak interval q.

What about the topics of messages ?

Final Visibility Measure :

We propose that for each peak interval $[a_q, b_q]$, a message can give information about a topic c, with $1 \leq c \leq C$.

- We define C point processes for each peak interval and we assume that each one of these processes are independents.
- Then when an arrival of messages occurs, in a peak interval q, the probability that it is from source j about the topic c is given by :

$$\frac{\lambda_{(j,c,q)}}{\sum_i \sum_{c'} \lambda_{(i,c',q)}}$$

  where $\lambda_{(j,c,q)}$ is the intensity of the point process that model the arrival of messages from j in the peak interval q about the topic c.
- This measure takes into account that a source j can decrease the visibility on a topic by increasing the visibility of another topic.

Game Formulation

The questions that we will try to answer in this section are :

- How to maximize our visibility on each News Feed ?
- What happens when severeal sources try to maximize their own visibility ?

An example of News Feed game (1)

Let us consider a News Game where :

- There are two photographers 1 and 2.
- Each one of them can post pictures about sport (Only one topic in this case), with a flow of 4 pictures/hour.
- They have to choose between the two News Feeds (a, b).
- In the News Feed a there is 2 photos/hour non-controlled, and 1 photos/hour on the News Feed b.

An exemple of News Feed game (2)

We consider that a photographer send all his pictures in a unique News Feed. In order to solve the game, we have four cases to compute. We get the following payoff matrix, and we can compute all Nash Equilibria :

	News Feed a	News Feed b
News Feed a	$\frac{4}{4+4+2} = 0.4, \frac{4}{4+4+2} = 0.4$	$\frac{4}{4+2} = 0.67^*, \frac{4}{4+1} = 0.8^*$
News Feed b	$\frac{4}{4+1} = 0.8^*, \frac{4}{4+2} = 0.67^*$	$\frac{4}{4+4+1} = 0.45, \frac{4}{4+4+2} = 0.45$

Congestion Game vs Routing Game

The game described above is a congestion game, and the strategies of players are discrete.

Impossible to get a closed form expression of the Nash Equilibrium.

We proposed to study a case where strategies are continuous and the payoff is the sum of the visibility measures in each News Feed. In the case of the Photographer,

- The Photographer 1 can send $\lambda_{(1,a)}$ photos/hour in the News Feed a and $\lambda_{(1,b)}$ photos/hour in the News Feed b.
- And his payoff is :

$$\frac{\lambda_{(1,a)}}{\lambda_{(1,a)} + \lambda_{(2,a)} + 2} + \frac{\lambda_{(1,b)}}{\lambda_{(1,b)} + \lambda_{(2,b)} + 1}$$

This new Game is called a Routing Game

Strategy of a source

The strategy vector of the source j is

$$\vec{\lambda}_j = (\lambda_{(j,q,c,f)})_{q,c,f} = (\lambda_{(j,c,l)})_{c,l}$$

with $l = (c,f)$. We assume that :

- when a source chooses to send a message to a specific News Feed, it cannot send the same message to a different News Feed,
- a source can only send a limited number of messages per time unit.

Then the previous assumptions imply that for all j and for all c,

$$\sum_l \lambda_{(j,c,l)} + \lambda_{(j,c,0)} = \phi_{(j,c)},$$

where $\lambda_{(j,c,0)}$ is the message's flow about a topic c that the source j doesn't want to send to any News Feed.

An exemple of a News Feed Game (3)

Let us consider the case where the photographer 1 can also send messages about two topics c and d. Then the following picture describes the visibility measure on each News Feed, about each topic.

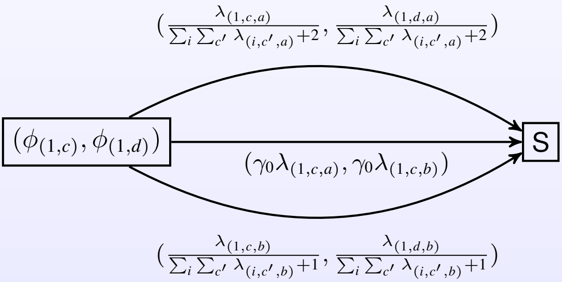


FIGURE: Visibility Measure of Photographer 1

We propose to restrict ourself to the case of **parallel link topology**.

Utility function of a source

How to define the Utility of each source, knowing that we are in a multi-objectif problem ?

Weighted sum of visibility measures

We propose to model the utility of j by

$$U_j(\vec{\lambda}_j, \vec{\lambda}_{-j}) = \sum_l \left[\frac{\sum_c \gamma_{(j,c,l)} \lambda_{(j,c,l)}}{\sum_i \sum_c \lambda_{(i,c,l)} + \sigma_l} \right] + \gamma_0 \sum_c \lambda_{(j,c,0)}, \quad (1)$$

where $\sum_{l,c} \gamma_{(j,c,l)} = 1$.

Background Nash Equilibrium

Definition : Nash Equilibrium

The decision vector $\underline{\vec{\lambda}} = (\underline{\vec{\lambda}}_1, \ldots, \underline{\vec{\lambda}}_J)$ is a Nash Equilibrium if for all $j \in \{1, \ldots, J\}$,

$$U_j(\underline{\vec{\lambda}}_j, \underline{\vec{\lambda}}_{-j}) = \max_{\vec{\lambda}_j} U_j(\vec{\lambda}_j, \underline{\vec{\lambda}}_{-j}),$$

Why should we study Nash Equilibrium in this situation ?

Nash Equilibrium

Results

- At any Nash Equilibrium, in each News Feed, each player sends messages about a unique topic,
- at any Nash Equilibrium, the sum of flow in each News Feed is the solution of a Concave programming problem,
- the Nash Equilibrium is unique,
- we can get a closed form of the Nash Equilibrium :

$$\underline{\lambda}_{(j,l)} = X_l - \frac{\gamma_0}{\gamma_{(j,l)}} X_l^2,$$

with

$$X_l = \frac{J - 1 + \sqrt{(J-1)^2 + 4(\gamma_0 \sum_i \frac{1}{\gamma_{(i,l)}} \sigma_l)}}{2\gamma_0 \sum_i \frac{1}{\gamma_{(i,l)}}}.$$

Numerical Study I

We recall that J is the number of players.

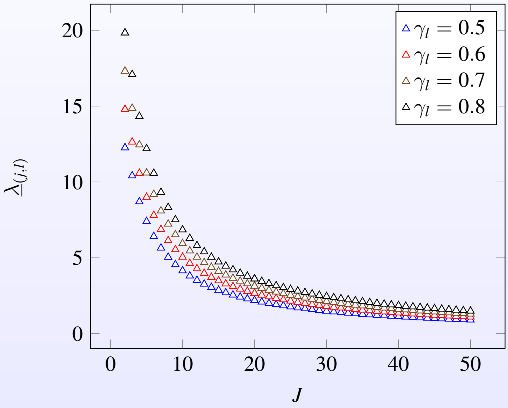


FIGURE: Evolution of $\underline{\lambda}_{(j,l)}(J)$ for $\gamma_0 = 0.01$.

Numerical Study II

We recall that σ_l is the uncontrolled flow in a News Feed l.

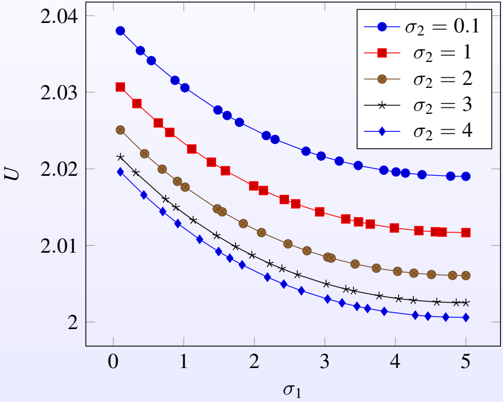


FIGURE: Evolution of $U(\sigma_l)$ with $\gamma_1 = 0.5$, $\gamma_2 = 0.5$, $J = 5$, $\gamma_0 = 0.1$ and $\phi = 20$.

Symmetric topology

How to extend the parallel topology ?

Definition of a News Feed Graph

Definition of a News Feed Graph

A News Feed Graph is a bipartite graph where the nodes of the first partition are the sources and the nodes of the second partition are the News Feeds.

A News Feed m is define by :

- its type l, i.e its uncontrolled flow σ_l
- the set $\mathcal{J}_m(l)$ of connected sources to l.

A sources j is define by :

- The set $\mathcal{M}_l(j)$ of connected News Feed of type l to j,
- a vector of weights $\vec{\gamma}_j$,
- a vector of demands $\vec{\phi}_j$.

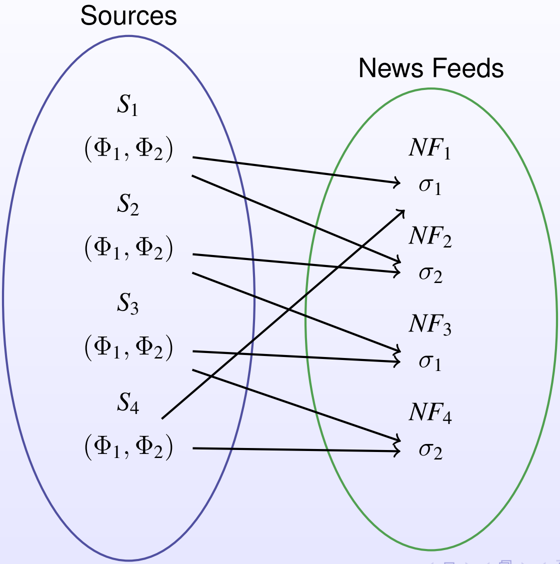
Definition of a symmetric topologies

Definition :

A symmetric News Feed Graph is a News Feed Graph where :

- For all m, m' and for all l, $card(\mathcal{J}_m(l)) = card(\mathcal{J}_{m'}(l))$.
- For all j, j' and for all l, $card(\mathcal{M}_l(j)) = card(\mathcal{M}_l(j'))$.
- For all j, j', for all l and for all c, $\gamma_{(j,c,l)} = \gamma_{(j',c,l)}$.
- For all j, j', $\vec{\phi}_j = \vec{\phi}_{j'}$.

Example of Symmetric News Feed topology :



Nash Equilibrium

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- at any Nash Equilibrium, the sum of flow in each News Feed is solution of a Concave programming problem,
- the Nash Equilibrium is unique,
- we can get a closed form of the Nash Equilibrium.

Learning Algorithm

How to find a decentralized learning algorithm converges to a PNE ?

- Our game doesn't admit a potential function,
- doesn't verify the submodularity property.

Hard to prove the convergence

Learning Algorithm

But, by using singularly perturbed Stochastic Approximation, we are able to design a two time scale algorithm where :

- On the fast time scale, each source imitates the average behavior of sources on each News Feed at each time,
- On the slow time scale, the source uses a gradient scheme to compute its optimal strategy.

$$
\begin{aligned}
\dot{\lambda}_{(j,l)}(t) &= \left[\frac{1}{J}\sum_{i}\lambda_{(i,l)}(t) - \lambda_{(j,l)}(t)\right]_{+} \\
&\quad +\epsilon\left[\frac{1}{\sum_i \lambda_{(i,l)}(t)+\sigma_l} - \frac{\lambda_{(j,l)}(t)}{(\sum_i \lambda_{(i,l)}(t)+\sigma_l)^2} - \frac{\gamma_0}{\gamma_l}\right]_{+}.
\end{aligned}
$$

Numerical study of the convergence : fast timescale

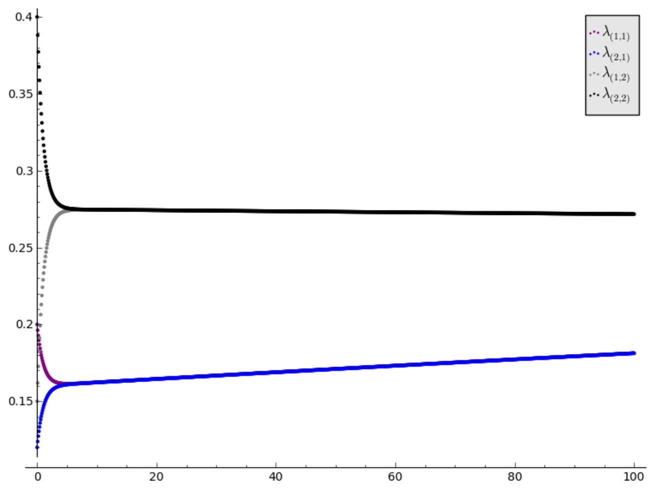

FIGURE: Evolution of $\lambda_{i,l}$ at the fast timescale

Numerical study of the convergence : slow timescale

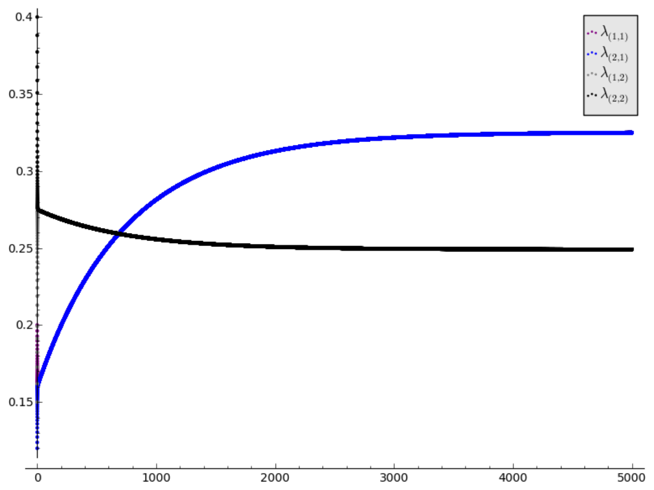

FIGURE: Evolution of $\lambda_{i,l}$ at the slow timescale

QUESTIONS ?

Slides to be posted at : www-sop.inria.fr/members/Alexandre.Reiffers/