Dissipation and dispersion control of a quadratic-reconstruction advection scheme

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The presented work is motivated by the developement of approximations for unsteady compressible flows and acoustic propagation, in which advection is dominating. Our design of advection schemes is influenced by several observations, and in particular:
- uniform meshes are most favourable to an accurate propagation,
- non-uniformity of mesh amplifies the numerical dispersion of signal to advect,
- dispersed components of a signal, when not damped, turn into oscillations,
- high-order advection schemes for cartesian meshes need much less flops to be assembled than high-order advection schemes for unstructured meshes.

For many complex geometries it has been found interesting to use meshes that are locally unstructured. As a result, a valuable strategy is to use an unstructured approximation in these regions and a cartesian one in the rest of computational domain.

In [1], the authors apply a vertex-centered MUSCL-like scheme which is second-order accurate on any region of the mesh and of "superconvergent" fifth-order accuracy on cartesian subregions. In the S-MUSCL scheme, the vertex-centering option is useful: (1) for superconvergence, since a finite-difference error analysis is directly applicable to aligned nodes, (2) for reducing the number of degrees of freedom in case of matrix-based algorithms. Due to its low dissipation combined with high tolerance to irregular meshes, this scheme is very useful for Large Eddy Simulations around complex geometries such as off-shore spars geometries involving truss, buoys, risers, as in [2]. Rather surprisingly, the same scheme performed well for mesh-adaptative calculation of compressible external flows, [3], as well as for acoustics [4].

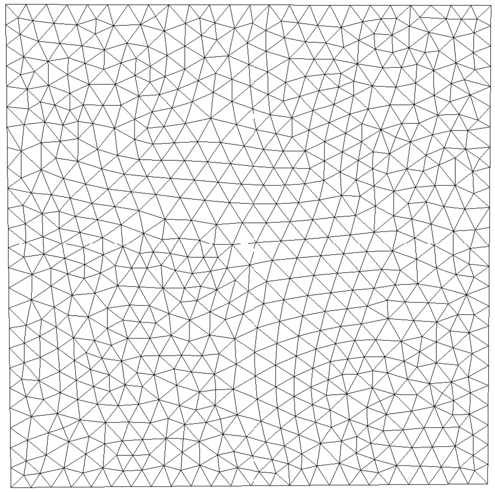
However, for both locally unstructured and strongly adaptive meshes, we are interested in studying an increase in the "unstructured accuracy order".

A second scheme relying on a quadratic reconstruction method (QRM) was then introduced and applied to nonlinear acoustics in [5]. The scheme is vertex centered. A quadratic reconstruction is derived from means on neighboring cells. Godunov-type upwinding is applied for numerical integration on cell interfaces.

In [5] the new QRM scheme was compared with the S-MUSCL for acoustic propagation test cases. These schemes were run with a sample of irregular and unstructured meshes. It appeared that:
- the S-MUSCL scheme was less costly than the QRM one.
- the S-MUSCL scheme was generally more accurate than the QRM one.
- This advantage was amplified for coarse meshes.
- This advantage disappeared only for fairly irregular meshes.

A main possible explanation of the relatively disappointing behavior of the QRM scheme is that the influence of internal dissipation dominates. Indeed, the first dissipation term in QRM is a fourth derivative with a weight proportional to the cube of local mesh size. The S-MUSCL is stabilized by sixth derivatives with a weight proportional to the power five of local mesh size.

In contrast, replacing in the QRM scheme the Riemann solver by the half-sum of right and left fluxes produces a fourth-order scheme, which, infortunately is not stable enough for our purposes. In the proposed communication, we derive a superconvergent S-QRM scheme. This is obtained by introducing in the QRM scheme two new terms in the cell-wise interpolation. The first term reduces importantly the fourth-order



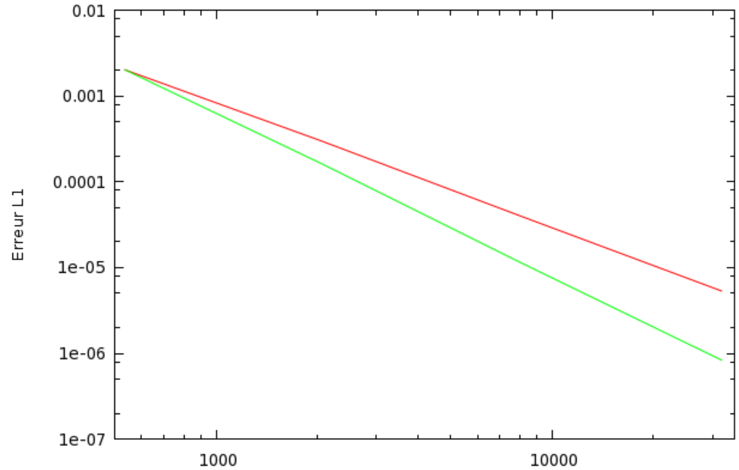


Figure 1: Convergence of and advection test case for unstructured meshes: sample of the mesh sequence, comparison of L^1 errors for the QRM scheme (third-order slope) and the S-QRM (fourth-order slope)

dispersion error. The second term equips the new scheme with a well-dimensionalised fifth-order dissipation term. These terms are easily derived from the quadratic reconstruction and increase the assembly computational cost of less than 20%. The accuracy on unstructured meshes is measured as equal to fouth-order (Fig.). Numerical experiments with different meshes for the advection of a smooth signal and of a level-set parametrized interface will illustrate our analysis.

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