Doctoral Module for the I2S Doctoral School

Advanced Markov Modeling

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Summary

This course proposes an introduction to some aspects of Markov chains which are "advanced" in the sense that they are not taught in the basic classes on the topic. The Markov chains considered here will be on discrete (and often finite) state spaces.

A particular attention will be devoted to practical aspects of the computations, related to algorithmic complexity. Some techniques presented are oriented towards the solution of Markov Chains of very large size.

This class is, in priority, for "practitioners" who use random dynamical systems: physicists, biologists, computer scientists. It will be taught in English, unless all participants are fluent in French.

The class will *not* address statistics.

Teacher: Alain Jean-Marie, with the participation of specialists for illustrations in different scientific fields. Duration: 20h. Credits: 5 ECTS.

Prerequisites

The course will use a set of mathematical concepts which will not be recalled: probabilities and stochastic processes on denumerable state spaces, linear algebra.

Students should be at ease with the basic theory of Markov chains on discrete state spaces, even if it will be recalled at the beginning of the class.

Synopsis

1. Basic theory (4h)

   Markov chains on Discrete state spaces, continuous time and discrete time.

   Classification of states, transient and stationary distributions.

   Relationship continuous/discrete: uniformization.

2. Computation of distributions (5h)

   Computation of stationary distributions; balance equations; generating function methods.

   Matrix methods; spectral decomposition; computation through uniformization.

   "Quasi Birth-Death" processes.

3. Simulation (5h)

   This chapter is devoted to the "Monte Carlo" simulation of Markov chains, in discrete and in continuous time.

   Generation of pseudo-random numbers; inverse function method; rejection method; alias/Walker method.

   "Forward" simulation: temporal discretization; uniformization; event-driven simulation; structuring and sequential selection; convergence issues; stationarity tests.

   "Backwards" simulation: coupling of trajectories; method of Propp and Wilson.

   Rare event simulation.

4. Illustrations (4h)

   Particle models in Physics and their simulation.

   Evolution models in Biology and Hidden Markov Chains.

5. Extra topics (2h): may include complements on one of the above topics, or be about *Reversibility*, *Composition of models*, *Markov-modulated Markov chains*, *Semi-Markov*, *Markov Additive or Markov Reward Processes.*