ECINADS Deliverable D30-anisotropic turbulence modelling

URANS/OES/DDES Tensorial eddy-viscosity modelling

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Abstract

IMFT has contributed in the context of URANS modelling with a directional anisotropy tensor transport modelling in the Organised Eddy Simulation (OES), (Braza, Perrin, Hoarau, (2006)), by means of URANS/DRSM. A tensorial eddy-viscosity concept for the eddy-viscosity has been derived, directionally sensitized to capture non-equilibrium turbulence physics. The turbulence anisotropy tensor in the DRSM model has been projected according to the principal directions of the strain-rate, by transporting a directional criterion of stress-strain misalignment, based on the DESIDER-EU program experiment “The IMFT circular cylinder’, Perrin et al (Exps. in Fluids 2007), and modelling development by Bourguet et al, (AIAA J 2007, J Fluids & Struct. 2008). This modelling is also useful in modifying the RANS turbulence length scale in DES.

1.1 Tensorial eddy-viscosity concept in the turbulence behaviour law

This modelling takes into account modification of the turbulence spectrum in the inertial range, due to the non-linear interaction between the coherent and random turbulence processes (Perrin et al, Flomania book, 2006). Therefore, modified turbulence scales are required for flow physics modelling of the turbulent stresses. The equations of motion in the time-domain are the phase-averaged (or ensemble – averaged) Navier-Stokes equations, (collected works in the book by Dervieux, Braza, Dussauge, (1998)-ETMA EU program, Jin & Braza, (1994), Cantwell & Coles, (1983)). It is recalled that by using Differential Reynolds Stress transport Modelling (DRSM) the eddy-diffusion coefficient C_μ used in OES two-equation modelling (Bourdet et al., 2007) was evaluated, by adopting the Boussinesq behaviour law as a first approximation. However, this tends to reinforce isotropic character especially in the near wall region. The first order turbulence behaviour law (in the sense of the strain rate expressions) has been reconsidered by means of a *directional* eddy viscosity tensor that aims at reinforcing turbulence anisotropy within non-equilibrium regions. This anisotropic constitutive law involves the elements of a spectral decomposition applied to the mean strain-rate tensor, whose respective ‘weights’ are the components of the eddy-viscosity tensor.

The IMFT’s circular cylinder test-case allowed quantification of the afore mentioned anisotropy character in the near region of a strongly detached turbulent flow, thanks to an ensemble of advanced measurements technique involving especially time-resolved 3C-PIV (Perrin et al, 2009 in *Notes on Num. Fluid Mech.* Vol. 103). These results allowed quantification of a *directional* misalignment between the turbulence anisotropy tensor and the strain rate. This has been used as a criterion for the projection of DRSM anisotropy tensor on the principal directions of the strain rate.

This approach is complementary to modifications of turbulence consitutive laws by means of scalar eddy-viscosity (Non Linear Eddy-Viscosity Models, NLEVM, (Gatski and Speziale, (1993), among other) and to the C_{as} model, (Revell et al, 2005, 2007) that suggested a scalar criterion of stress-strain misalignment derived from DRSM.

1.2 Summary of the OES anisotropic first-order model

Three transport equations have been derived from DRSM to close the anisotropic constitutive law previously described. The correponding anisotropic first-order closure scheme thus involves five equations in addition to the three momentum equations in the general three-dimensional case: two transport equations for k and ε, as well as three corresponding to each component of directional misalignment coefficient, CV_i. The scalar eddy-viscosity is replaced by the tensorial one in the whole system. The OES turbulence damping function (Jin & Braza, 1994) is used. The modelling consists therefore of solving the following set of equations:

$$-\langle u_i u_j \rangle + \frac{2}{3} k \delta_{ij} = 2 \left(\nu_{td}\right)_\alpha S_{ij}^{\alpha} \tag{1}$$

with $S_{ij}^{mi} = \lambda_m^S V_{ij}^m$ and λ_m^S are the eigenvalues of the strain rate, S. Contrary to linear EVM, projection, the coefficients CV_i are no more modelled but predicted exactly as new state variables by DRSM transport equations. The SSG (Sarkar, gatski, Speziale RSM) model has been adopted to derive the related transport equations.

with $\left(\nu_{td}\right)_i = \frac{C_{V\,i}}{2\lambda_i^S} k$ (2)

$$\frac{DC_{V\,i}}{Dt} = \left(\frac{4}{3} + c_3^* II_a^{\frac{1}{2}} - c_3\right) V_{\alpha\beta}^i S_{\alpha\beta} + \left(2 - 2c_4\right) V_{\alpha\beta}^i a_{\alpha\gamma} S_{\beta\gamma} - \frac{c_2 \varepsilon}{k} V_{\alpha\beta}^i a_{\alpha\gamma} a_{\gamma\beta} + \left(2 - 2c_5\right) V_{\alpha\beta}^i a_{\alpha\gamma} \Omega_{\beta\gamma} \tag{3}$$

$$+ \left(1 - c_1\right) \frac{\varepsilon}{k} C_{V\,i} + \left(1 + c_1^*\right) C_{V\,i} a_{\alpha\beta} S_{\alpha\beta} + \frac{c_2 II_a \varepsilon}{3k} + \frac{2\left(c_4 - 1\right)}{3} a_{\alpha\beta} S_{\alpha\beta} - a_{\alpha\beta} \frac{DV_{\alpha\beta}^i}{Dt} + D^{C_{V\,i}} \tag{4}$$

The diffusion term $D^{C_{V\,i}}$ combines viscous and turbulent diffusion contributions and is approximated by:

$$D^{C_{V\,i}} = \frac{\partial}{\partial x_\alpha}\left(\left(\nu\delta_{\alpha\beta} + \frac{(\nu_{tt})_{\alpha\beta}}{\sigma_{C_{V\,i}}}\right)\frac{\partial C_{V\,i}}{\partial x_\beta}\right) \tag{5}$$

$\sigma_{C_{V\,i}}$ coefficients are set to the value of one. Moreover the second term issued from the derivation of *CV_i*, vanishes if the anisotropy tensor is replaced by its approximation, $\hat{a}$: $-\hat{a}_{\alpha\beta} DV^{i}_{\alpha\beta}/Dt = 0$

The above equations are joined to the turbulence kinetic energy and dissipation transport equations in case of a two-equation model:

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_\alpha}\left(\left(\nu\delta_{\alpha\beta} + \frac{(\nu_{tt})_{\alpha\beta}}{\sigma_k}\right)\frac{\partial k}{\partial x_\beta}\right) + P_k - \varepsilon - \frac{2\nu k}{{y_n}^2},$$
$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_\alpha}\left(\left(\nu\delta_{\alpha\beta} + \frac{(\nu_{tt})_{\alpha\beta}}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_\beta}\right) + c_{\varepsilon_1} f_1 \frac{\varepsilon}{k} P_k - c_{\varepsilon_2} f_2 \frac{\varepsilon^2}{k} - \frac{2\nu\varepsilon}{{y_n}^2}\exp\left(-0.5y^+\right)$$

The damping functions and constants are provided in the following table:

f_μ	f_1	f_2	c_{ε_1}	c_{ε_1}	σ_k	σ_ε
$1-\exp\left(-0.0002\ y^+ - 0.000065\ y^{+2}\right)$	1	$1-0.22\exp\left(-\frac{k^2/(\varepsilon\nu)^2}{36}\right)$	1.44	1.92	1	1.3

Table 1 Damping functions and constants for the tensorial-eddy-viscosity anisotropic URANS/OES modelling

A detailed description of the above developments can be found in Bourguet et al (2007), Bourguet (2008).

Results

IMFT contribution in ECINADS mainly focuses on elaboration of efficient URANS eddy-viscosity turbulence modelling for capturing strongly detached coherent structures around lifting structures and compare with EARSM and DRSM. The objective is to capture turbulence anisotropy, thin shear-layer interfaces downstream of separation and a significant statistical content of different classes of coherent vortices generated around a lifting structure. The aim is therefore to compare the performances of advanced URANS versus DES approaches as explicated in WP3. The turbulence anisotropy modelling is achieved by means of a tensorial eddy viscosity concept within the OES, Organised Eddy Simulation approach, solving the phase-averaged Navier-Stokes equations, (Bourguet et al, 2008, chapters presented in Haase, Braza, Revell, 2009). The turbulence stresses modelling in these equations yields to transport a 3D criterion for stress-strain directional misalignment that illustrates strong non-equilibrium character. These facts were previously quantified thanks to refined physical experiments at IMFT, for high-Re flows around obstacles, by using 3C-Time-resolved-PIV, (Perrin et al, 2009). The directional misalignment equations have been derived from Galerkin projection of the DRSM modelling. Therefore, the present OES modelling includes three transport equations for the misalignment criterion that provide the three principal-direction components of the tensorial eddy-viscosity, producing a directional Cv_i coefficient. The present approach is able to capture negative production regions related to inverse turbulence cascade (Ouvrard et al, 2010, Braza et al 2010). A more simplified version of the OES modelling, yielding the same number of equations as for a two-equation modelling can be derived from DRSM by adopting a constant $C\mu$ coefficient, being of lower value than in equilibrium turbulence.

Applications: Transonic buffet over a supercritical airfoil and incompressible flow around a tandem cylinder (generic configuration of basis of a landing gear).

Turbulence Modelling		**Considered flows**
S-A DES, k-ε-OES	IMFT	OAT 15 A
k-ε-OES, k-ω-OES DES	IMFT	Tandem cylinders

Application I. Transonic buffet over a supercritical airfoil

Re=3.1 x 10 6, incidence i= 3.3°, Mach number Ma=0.73. Results by Jacquin eta la, ONERA Meudon, AIAA J. 2009.

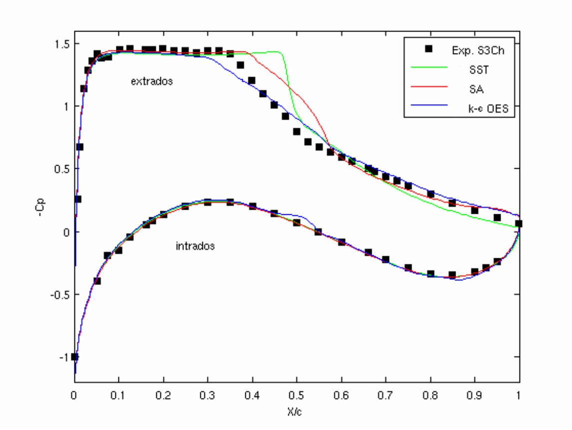

Figure 1 Averaged *Cp* coefficient according to URANS - OAT15A flow. Incidence a°=3.5, M=0.73, Re=3 x 10^6

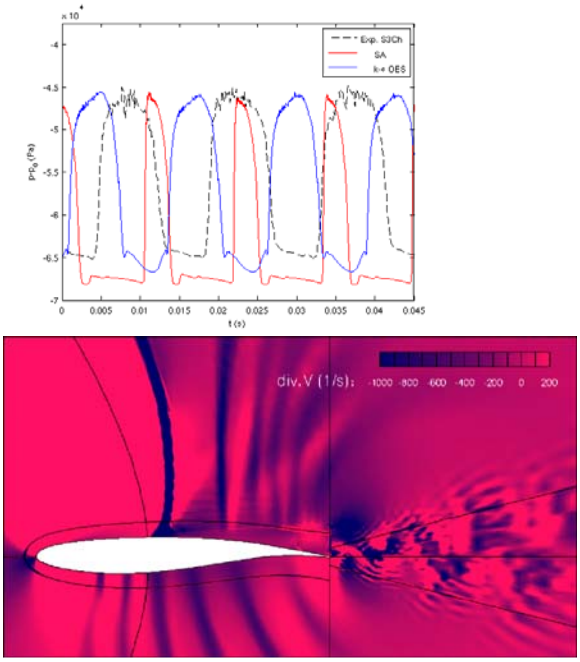

Figure 2. Comparison of the wall pressure fluctuations with the experiment (left), iso (div(V)) contours illustrating SWBLI (Shock Wave Boundary Layer Interaction) as well as shock-vortex (von Kármán) interaction, by means of URANS- k-ε-OES modelling.

In the context of ***hybrid turbulene modeling,*** IMFT has performed DES-SA, DDES-k-ω-SST, and DES-k-ω-OES approaches. DDES stands for “Delayed DES”. In this last one, the l_{RANS} turbulence length scale in the RANS part of DES is replaced by the l_{OES} turbulence length scale. This takes into account non-equilibrium turbulence effects in the near region, due to turbulence stresses anisotropy and ensures a significantly improved URANS region around the body. This approach allows a smooth passage towards the outer region subjected to the LES mode.

Application II: Tandem cylinders, Re= 160,000 - capturing Kelvin-Helmholtz instability and vortices responsible for acoustic noise. Experiments by the NASA-Langley Research centre, Jenkins et al, AIAA 2006, 2009.

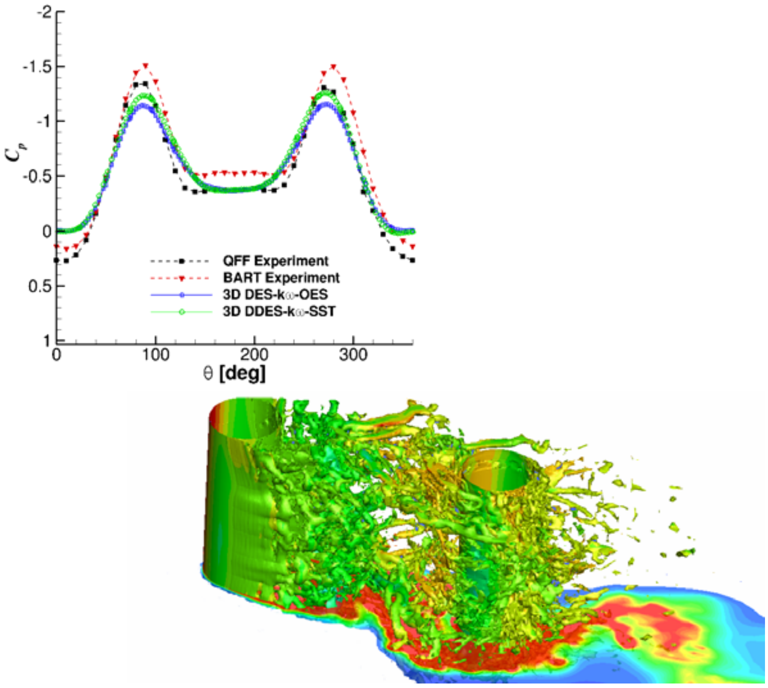

Figure: ST11-tandem of cylinders: *Cp* wall pressure coefficient of the second cylinder, (left); iso-vorticity field, DDES-*k*-*ω*-OES modeling.

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