COMPUTATION OF COMPLEX UNSTEADY FLOWS AROUND BLUFF-BODIES THROUGH VMS-LES MODELING

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Introduction (1)

As any numerical model, Large Eddy Simulation aims at providing an approximate prediction by neglecting "small" components of the exact flow. Assuming an universal behavior of small scales, they can be filtered and replaced by some modeling. Due to this assumption, LES is generally not efficient for high Reynolds number flows.

In particular boundary layers at High Reynolds number cannot be efficiently computed with LES.

Hybrid schemes like Detached Eddy Simulation have succeeded in showing that a RANS modeling of boundary layer can be combined with a LES modeling of detached eddies. See the lecture of Pr. Salvetti.

The present study restricts to flows in which we assume that we have *avoided* the problems related to boundary layers.

Introduction(2)

In a LES model, while an important assumption is that a large part of turbulent energy is accounted in simulated eddies, two sources of dissipation can affect these eddies:

- Numerical dissipation: monotony devices can affect eddied just larger than grid scale, other stabilization terms can have some damping influence.

- LES model may damp simulated eddies, even in laminar case.

The principle of Variational Multi-Scale LES methods is to use grid coarsening in order to avoid damping of simulated eddies.

Introduction(3)

- This study focusses on VMS-LES:
  - Modeling:
    - VMS-LES with different SGS models
    - VMS-LES with laminar boundary layer
    - VMS-LES with (fully) turbulent boundary layer
  - Applications:
    - Evaluation on simplified geometries
    - Application to a complex geometry

Plan:

II) Numerical model

III) Turbulence modelling:

1) VMS-LES
1) SGS Modelling
1) Boundary treatment

IV) Applications

V) Concluding remarks

II Numerical model:

$$\boxed{\text{Mixed-Element-Volume method (MEV)}}$$

II Numerical model

<u>Towards a mixed-element-volume formulation</u>:

- We are interested by *compressible flows*.

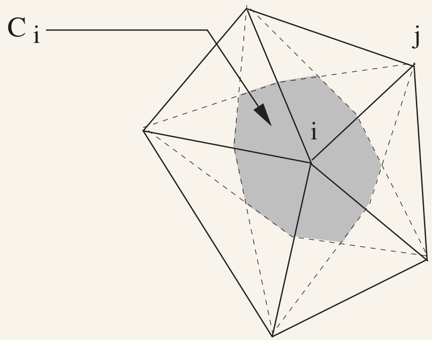
- We shall master the numerical dissipation by applying a high-derivative model : sixth-order dissipation, allowed by a finite-volume reconstruction.

- A variational finite-element formulation will permit a variational multiscale statement.

- Finite-volume coarsening by cell-agglomeration will also tranpose into a finite-element coarsening.

II Numerical model

Basic options of the MEV method:



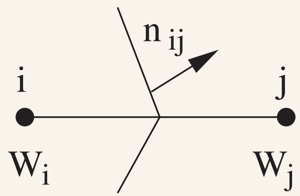
- Degrees of freedom located at nodes i
- Median cells C_i (dual mesh built from a non-structured tetrahedrization).
- Variational formulation with 2 types of test functions:
  P1 FE functions Φ_i, and characteristic functions $\mathcal{X}_i$.
- FE evaluation of the diffusive fluxes.
- FV evaluation of the convective fluxes.

II Numerical model

First order MEV:

$$W_t + \nabla.\mathcal{F}(W) = \nabla.\mathcal{R}(W)$$

$$Vol(C_i)\frac{dW_i}{dt} + \sum_{j\in N(i)} \Phi_{ij} = -\int_{T,i\in T} \mathcal{R}(W)\cdot\nabla\Phi_i$$



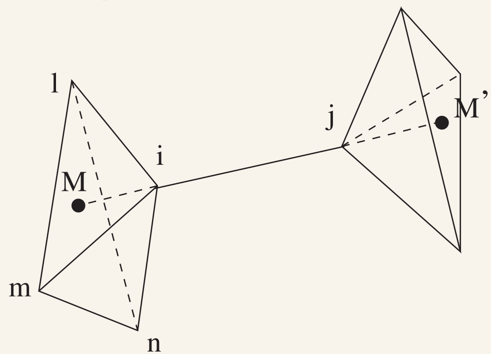
Reconstruction using 7 approximate gradients:

$$\Phi_{ij} = \frac{\mathcal{F}(W_{ij})\cdot\nu_{ij} + \mathcal{F}(W_{ji})\cdot\nu_{ij}}{2} - \frac{1}{2}\delta|\tilde{A}_{ij}|(W_{ji} - W_{ij})$$

$\delta \approx 0.01.$

II Numerical model

Extended finite-volume flux integration: W_{ij} and W_{ji} are defined from:
- variables values at the eight vertices of the two tetrahedra at ends of edge ij,
- nodal gradients at vertices, interpolated at M, M'.



$\Rightarrow$ dissipation based on 6$^{\text{th}}$ order derivatives, monitored by δ.

Time-advancing schemes: either N steps Runge-Kutta explicit scheme or second-order backward difference scheme.

III Turbulence models:

1: A variational multiscale method for the large eddy simulation (VMS-LES)

III Turbulence models (VMS-LES)

Mean features of the VMS-LES approach (Hugues et al.,CVS 2000):

- No spatial filtering of the Navier-Stokes equations as for LES but a variational projection of these equations.
- Scales separated *a priori*.
- The effects of the unresolved structures modeled only in the equations governing the “small resolved structures” (and not in the “large resolved structures” like in LES).

III Turbulence models (VMS-LES)

- Extension by Koobus and Farhat (CMAME2004) to

  - the compressible Navier-Stokes equations
  - unstructured meshes
  - a mixed element-volume framework
  - vortex shedding flows.

III Turbulence models (VMS-LES)

Compressible Navier-Stokes equations:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{u}) & = 0 \\ \\ \dfrac{\partial \rho \mathbf{u}}{\partial t} + \nabla.(\rho \mathbf{u} \otimes \mathbf{u} + P\mathbf{Id}) & = \nabla.\boldsymbol{\sigma} \\ \\ \dfrac{\partial E}{\partial t} + \nabla.[(E+P)\mathbf{u}] & = \nabla.(\boldsymbol{\sigma} u) + \nabla.(\lambda \nabla T) \end{cases}$$

Mixed element-volume spatial semi-discretization:

$$\begin{cases} A(\mathcal{X}_i, \mathbf{W}) & = \int_\Omega \dfrac{\partial \rho}{\partial t} \mathcal{X}_i \, d\Omega \; + \; \int_{\partial Sup \mathcal{X}_i} \rho \mathbf{u}.\mathbf{n} \mathcal{X}_i \, d\Gamma & = 0 \\ \\ \mathbf{B}(\mathcal{X}_i, \Phi_i, \mathbf{W}) & = \int_\Omega \dfrac{\partial \rho \mathbf{u}}{\partial t} \mathcal{X}_i \, d\Omega \; + \; \int_{\partial Sup \mathcal{X}_i} \rho \, \mathbf{u} \otimes \mathbf{u} \, \mathbf{n} \mathcal{X}_i \, d\Gamma & \\ & \; + \int_{\partial Sup \mathcal{X}_i} P \mathbf{n} \mathcal{X}_i \, d\Gamma \; + \; \int_\Omega \boldsymbol{\sigma} \nabla \Phi_i \, d\Omega & = \mathbf{0} \\ \\ C(\mathcal{X}_i, \Phi_i, \mathbf{W}) & = \int_\Omega \dfrac{\partial E}{\partial t} \mathcal{X}_i \, d\Omega \; + \; \int_{\partial Sup \mathcal{X}_i} (E+P) \mathbf{u}.\mathbf{n} \mathcal{X}_i \, d\Gamma & \\ & \; + \int_\Omega \boldsymbol{\sigma} \mathbf{u}.\nabla \Phi_i \, d\Omega \; + \; \int_\Omega \lambda \nabla T.\nabla \Phi_i \, d\Omega & = 0 \end{cases}$$

III Turbulence models (VMS-LES)

FE space decomposition:

$$\mathcal{V}_{EF} = \overline{\mathcal{V}}_{EF} \oplus \mathcal{V}'_{EF} \oplus \widehat{\mathcal{V}}_{EF}$$

FV space decomposition:

$$\mathcal{V}_{VF} = \overline{\mathcal{V}}_{VF} \oplus \mathcal{V}'_{VF} \oplus \widehat{\mathcal{V}}_{VF}$$

$$\Downarrow$$

$$\mathbf{W} = \overline{\mathbf{W}} + \mathbf{W}' + \widehat{\mathbf{W}}$$

où:

“ $-$ ” $=$ large resolved scales
“ $'$ ” $=$ small resolved scales
“ $\wedge$ ” $=$ unresolved scales

III Turbulence models (VMS-LES)

- "Large resolved scales" equations:

$$\left\{\begin{array}{ll} A(\overline{\mathcal{X}}_i, \overline{\mathbf{W}} + \mathbf{W}' + \widehat{\mathbf{W}}) & = 0 \\ \mathbf{B}(\overline{\mathcal{X}}_i, \overline{\Phi}_i, \overline{\mathbf{W}} + \mathbf{W}' + \widehat{\mathbf{W}}) & = \mathbf{0} \\ C(\overline{\mathcal{X}}_i, \overline{\Phi}_i, \overline{\mathbf{W}} + \mathbf{W}' + \widehat{\mathbf{W}}) & = 0 \end{array}\right. \Leftrightarrow \left\{\begin{array}{ll} A(\overline{\mathcal{X}}_i, \overline{\mathbf{W}} + \mathbf{W}') + A^*(\overline{\mathcal{X}}_i, \overline{\mathbf{W}}, \mathbf{W}', \widehat{\mathbf{W}}) & = 0 \\ \mathbf{B}(\overline{\mathcal{X}}_i, \overline{\Phi}_i, \overline{\mathbf{W}} + \mathbf{W}') + \mathbf{B}^*(\overline{\mathcal{X}}_i, \overline{\Phi}_i, \overline{\mathbf{W}}, \mathbf{W}', \widehat{\mathbf{W}}) & = \mathbf{0} \\ C(\overline{\mathcal{X}}_i, \overline{\Phi}_i, \overline{\mathbf{W}} + \mathbf{W}') + \mathbf{C}^*(\overline{\mathcal{X}}_i, \overline{\Phi}_i, \overline{\mathbf{W}}, \mathbf{W}', \widehat{\mathbf{W}}) & = 0 \end{array}\right.$$

- "Small resolved scales" equations:

$$\left\{\begin{array}{ll} A(\mathcal{X}'_i, \overline{\mathbf{W}} + \mathbf{W}' + \widehat{\mathbf{W}}) & = 0 \\ \mathbf{B}(\mathcal{X}'_i, \Phi'_i, \overline{\mathbf{W}} + \mathbf{W}' + \widehat{\mathbf{W}}) & = \mathbf{0} \\ C(\mathcal{X}'_i, \Phi'_i, \overline{\mathbf{W}} + \mathbf{W}' + \widehat{\mathbf{W}}) & = 0 \end{array}\right. \Leftrightarrow \left\{\begin{array}{ll} A(\mathcal{X}'_i, \overline{\mathbf{W}} + \mathbf{W}') + A^*(\mathcal{X}'_i, \overline{\mathbf{W}}, \mathbf{W}', \widehat{\mathbf{W}}) & = 0 \\ \mathbf{B}(\mathcal{X}'_i, \Phi'_i, \overline{\mathbf{W}} + \mathbf{W}') + \mathbf{B}^*(\mathcal{X}'_i, \Phi'_i, \overline{\mathbf{W}}, \mathbf{W}', \widehat{\mathbf{W}}) & = \mathbf{0} \\ C(\mathcal{X}'_i, \Phi'_i, \overline{\mathbf{W}} + \mathbf{W}') + \mathbf{C}^*(\mathcal{X}'_i, \Phi'_i, \overline{\mathbf{W}}, \mathbf{W}', \widehat{\mathbf{W}}) & = 0 \end{array}\right.$$

- "Unresolved scales" equations:

$$\left\{\begin{array}{ll} A(\widehat{\mathcal{X}}_i, \overline{\mathbf{W}} + \mathbf{W}' + \widehat{\mathbf{W}}) & = 0 \\ \mathbf{B}(\widehat{\mathcal{X}}_i, \widehat{\Phi}_i, \overline{\mathbf{W}} + \mathbf{W}' + \widehat{\mathbf{W}}) & = \mathbf{0} \\ C(\widehat{\mathcal{X}}_i, \widehat{\Phi}_i, \overline{\mathbf{W}} + \mathbf{W}' + \widehat{\mathbf{W}}) & = 0 \end{array}\right.$$

III Turbulence models (VMS-LES)

The “unresolved scales” are not captured by the numerical computation. Their effect on the “large and small resolved scales” are therefore modeled.

$$\Downarrow$$

- The effect of the “unresolved scales” on the “large resolved scales” is negligible compared to their effect on the “small resolved scales”.
- The effect (energy dissipation) of the “unresolved scales” on the “small resolved scales” is modeled by an analogy with the eddy viscosity model.

III Turbulence models (VMS-LES)

- "Large resolved scales" equations:

$$\begin{cases} A(\overline{\mathcal{X}}_{i_h}, \overline{\mathbf{W}}_h + \mathbf{W}'_h) & = 0 \\ \mathbf{B}(\overline{\mathcal{X}}_{i_h}, \overline{\Phi}_{i_h}, \overline{\mathbf{W}}_h + \mathbf{W}'_h) & = \mathbf{0} \\ C(\overline{\mathcal{X}}_{i_h}, \overline{\Phi}_{i_h}, \overline{\mathbf{W}}_h + \mathbf{W}'_h) & = 0 \end{cases}$$

- "Small resolved scales" equations:

$$\begin{cases} A(\mathcal{X}'_{i_h}, \overline{\mathbf{W}}_h + \mathbf{W}'_h) & = 0 \\ \mathbf{B}(\mathcal{X}'_{i_h}, \Phi'_{i_h}, \overline{\mathbf{W}}_h + \mathbf{W}'_h) + \int_\Omega \boldsymbol{\tau}'_h \nabla \Phi'_{i_h} \, d\Omega & = \mathbf{0} \\ C(\mathcal{X}'_{i_h}, \Phi'_{i_h}, \overline{\mathbf{W}}_h + \mathbf{W}'_h) + \int_\Omega \frac{C_p \mu'_t}{Pr_t} \nabla T'_h . \nabla \Phi'_{i_h} \, d\Omega & = 0 \end{cases}$$

where:

$\boldsymbol{\tau}'_{ij} = \mu'_t (2\boldsymbol{S}'_{ij} - \frac{2}{3}\boldsymbol{S}'_{kk}\delta_{ij})$, $\boldsymbol{S}'_{ij} = \frac{1}{2}(\frac{\partial \mathbf{u}'_i}{\partial \mathbf{x}_j} + \frac{\partial \mathbf{u}'_j}{\partial \mathbf{x}_i})$, μ'_t: small scale turbulent viscosity.

III Turbulence models (VMS-LES)

Two-level decomposition:

$$\begin{array}{l} \mathcal{V}_{EF_h} = \overline{\mathcal{V}}_{EF_h} \oplus \mathcal{V}'_{EF_h} \\ \mathcal{V}_{VF_h} = \overline{\mathcal{V}}_{VF_h} \oplus \mathcal{V}'_{VF_h} \end{array} \;\Rightarrow\; \mathbf{W}_h = \overline{\mathbf{W}}_h + \mathbf{W}'_h$$

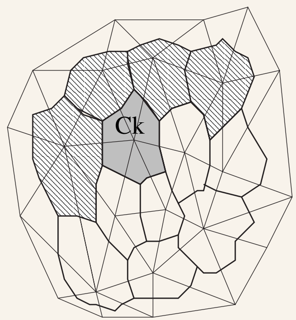
Final VMS-LES governing equations:

$$\begin{cases} A(\mathcal{X}_{i_h}, \mathbf{W}_h) & = 0 \\ \mathbf{B}(\mathcal{X}_{i_h}, \Phi_{i_h}, \mathbf{W}_h) + \int_\Omega \boldsymbol{\tau}'_h \nabla \Phi'_{i_h} \; d\Omega & = \mathbf{0} \\ C(\mathcal{X}_{i_h}, \Phi_{i_h}, \mathbf{W}_h) + \int_\Omega \dfrac{C_p \mu'_t}{Pr_t} \nabla T'_h . \nabla \Phi'_{i_h} \; d\Omega & = 0 \end{cases}$$

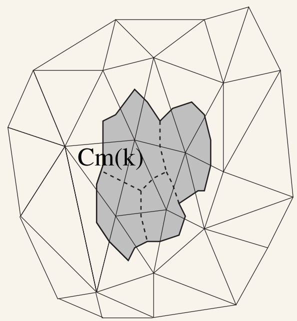
III Turbulence models (VMS-LES)

A priori scales separation: defining Φ'_{i_h} and χ'_{i_h}

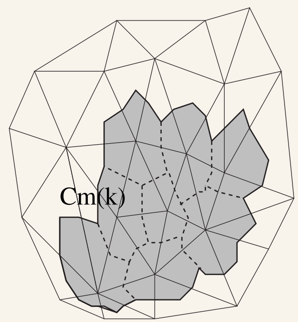
Dual mesh partioned into macro-cells:



unstructured mesh and dual mesh
(hashed cells denote already
agglomerated cells)



macro−cell Cm(k)
(1st−level 1−layer agglomeration)



macro−cell Cm(k)
(1st−level 2−layer agglomeratio

III Turbulence models:

2: Subgrid Scale models

III Turbulence models (SGS models)

$Smagorinsky$ SGS term (Smagorinsky, 1963):

$$\int_\Omega \boldsymbol{\tau}'_h \nabla \Phi'_{i_h} \, d\Omega \quad , \quad \boldsymbol{\tau}'_{ij} = \mu'_t (2\boldsymbol{S}'_{ij} - \frac{2}{3}\boldsymbol{S}'_{kk}\delta_{ij})$$
$$\mu'_t = \overline{\rho}(C'_s\Delta')^2|\boldsymbol{S}'| \tag{1}$$
$$S'_{ij} = \frac{1}{2}(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$$

where C_s is the constant model set to 0.1, and Δ measures the local mesh size:

$$\Delta^{(l)} = Vol_l^{1/3} \tag{2}$$

where Vol_l is the volume of the $l-th$ grid element.

Let us consider the two flow configurations:
Flow 1: Laminar flat boundary layer,
Flow 2: Laminar shear layer.

For both, Smagorinsky's model gives positive SGS dissipation:
Bad Laminar behavior, bad transition behavior.

III Turbulence models (SGS models)

The $Vreman$ eddy viscosity model (Vreman,POP 2003):

$$\nu_{LES}(W) = c(\frac{B_\beta}{\alpha_{ij}\alpha_{ij}})^{\frac{1}{2}} \tag{3}$$

with

$$\alpha_{ij} = \partial u_j / \partial x_i$$

$$\beta_{ij} = \Delta^2 \alpha_{mi}\alpha_{mj}$$

$$B_\beta = \beta_{11}\beta_{22} - \beta_{12}^2 + \beta_{11}\beta_{33} - \beta_{13}^2 + \beta_{22}\beta_{33} - \beta_{23}^2$$

The constant $c \approx 2.5 C_s^2$ where C_s denotes the Smagorinsky constant.

- does not create SGS dissipation on many flows including Flow1.

III Turbulence models (SGS models)

The $WALE$ eddy-viscosity model (Nicoud-Ducros, FTC1999):

$$\nu_{LES}(W) = C_w \Delta^2 \frac{(S_{ij}^d S_{ij}^d)^{\frac{3}{2}}}{(S_{ij}^d S_{ij}^d)^{\frac{5}{2}} + (S_{ij}^d S_{ij}^d)^{\frac{5}{4}}} \tag{4}$$

$$S_{ij}^d = \frac{1}{2}(g_{ij}^2 + g_{ij}^2) - \frac{1}{3}\delta_{ij}/g_{kk}^2 \quad , \quad g_{ij}^2 = g_{ik}g_{kj} \quad , \quad g_{ij} = \partial u_i / \partial x_j$$

The constant C_w is set to 0.1.

- create very small SGS dissipation on Flow1 and Flow2.

III Turbulence models:

3: Boundary layer treatment

Motivation: when increasing Rey, :

- DNS applies well to very low Rey,
- then CDNS for slightly higher Reynolds
- then MILES
- then VMS
- then LES
- then hybrid (DES,...)
- then (dep. cases) URANS,...

III Turbulence models (BL treatment)

In contrast to this hierarchy, we try to show when VMS-LES is

- better than standard LES for higher Reynolds flows
- good enough

VMS-LES + laminar BL

VMS-LES + Wall law

IV Applications:

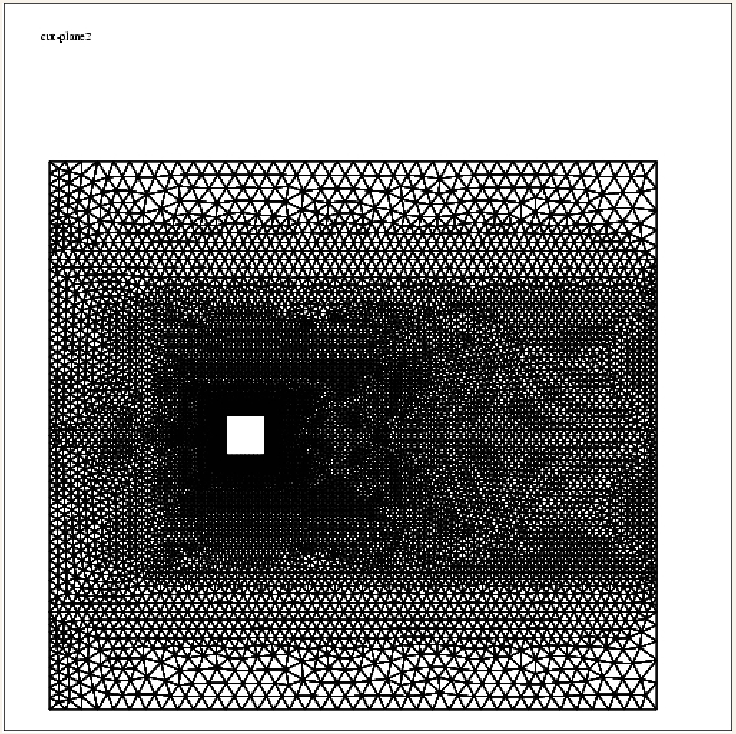
1) Low-Reynolds bluff body flow

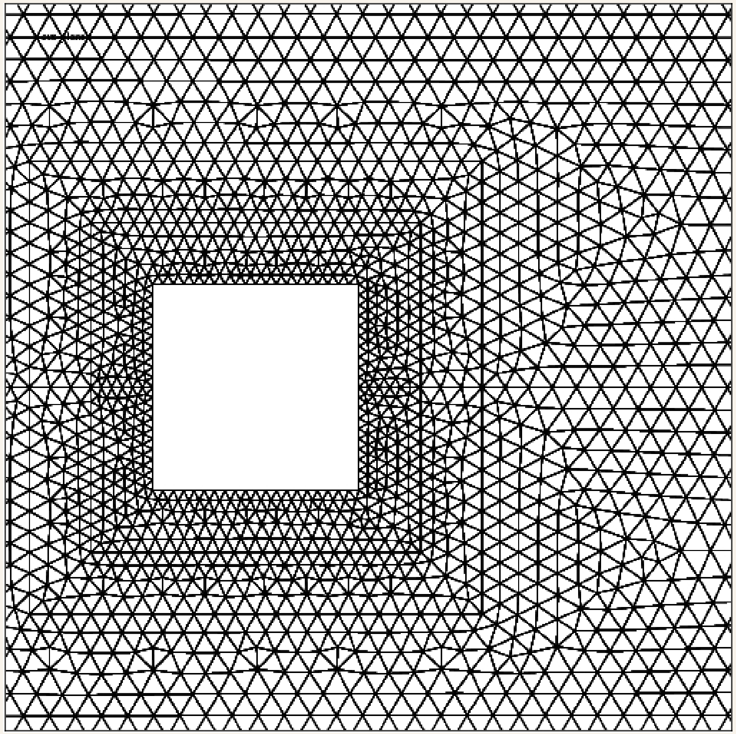
2) High-Reynolds bluff body flow

IV Applications (square cylinder)

1. Flow around a square cylinder

Flow parameters: Reynolds = 22000, Mach = 0.1, mesh: 200K nodes.





IV Applications (square cylinder)

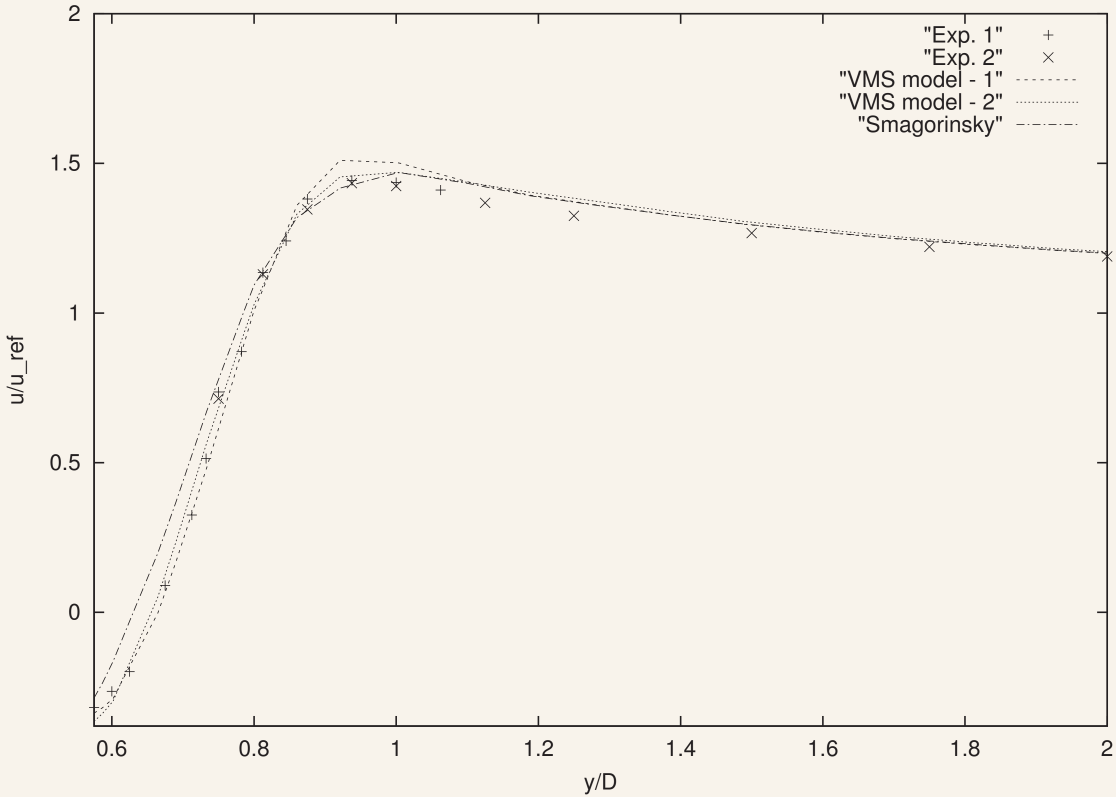
LES	$\overline{C_d}$	C'_d	C'_l	S_t	l_r	$-\overline{C}_{p_b}$
LES Smagorinsky	2.00	0.19	1.01	0.136	1.5	1.31
VMS-LES (A1)	2.10	0.17	0.98	0.134	1.4	1.48
VMS-LES (A2)	2.10	0.18	1.08	0.136	1.4	1.52
Rodi *et al.*	[1.66,2.77]	[0.10,0.27]	[0.38,1.79]	[0.07,0.15]	[0.89,2.96]	-
Sohankar and Fureby	[2.00,2.32]	[0.16,0.20]	[1.23,1.54]	[0.127,0.135]	[1.29,1.34]	[1.30-1.63]

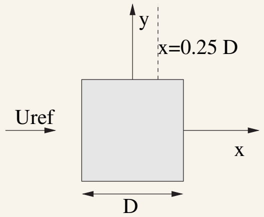
Experiences	$\overline{C_d}$	C'_d	C'_l	S_t	l_r	$-\overline{C}_{p_b}$
Lyn *et al.*	2.10	-	-	0.132	1.4	-
Luo *et al.*	2.21	0.18	1.21	0.13	-	1.52
Bearman *et al.*	2.28	-	1.20	0.13	-	1.60

$\Rightarrow$ Global improvement with VMS-LES.

IV Applications (square cylinder)

Profile of the mean streamwise velocity at $x = 0.25D$:



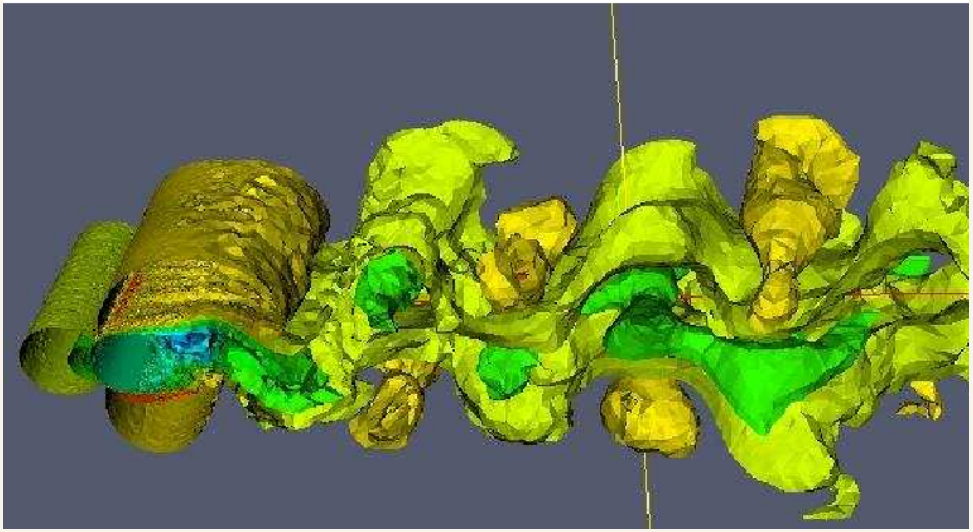


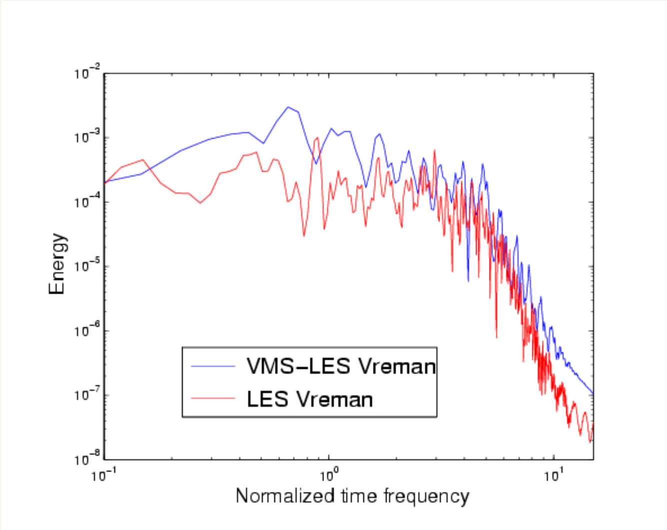
IV Applications (circular cylinder)

2: Flow around a circular cylinder

Flow parameters: Reynolds = 3900, Mach = 0.1, mesh: 290K nodes.

Instantaneous streamwise velocity, Fourier energy spectrum of spanwise velocity



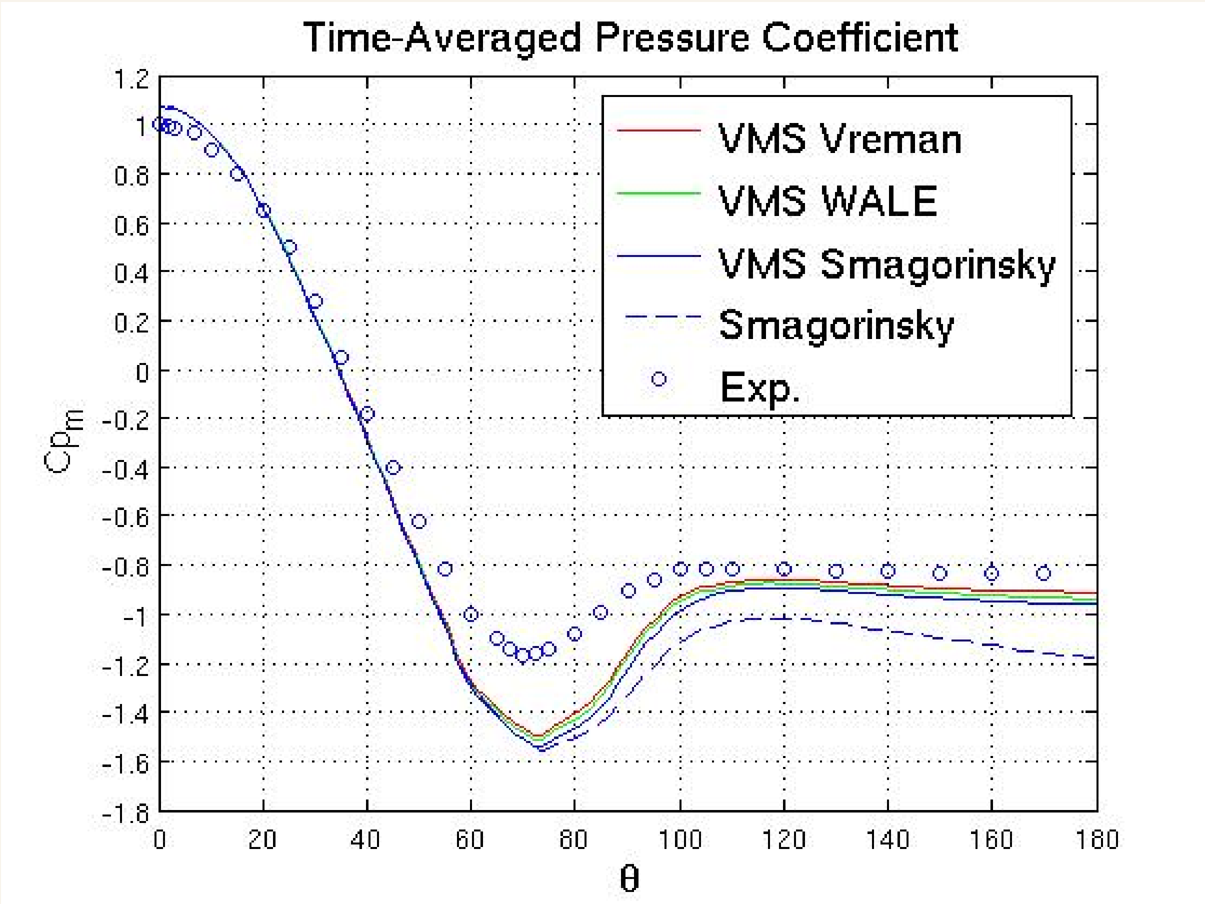


IV Applications (circular cylinder)

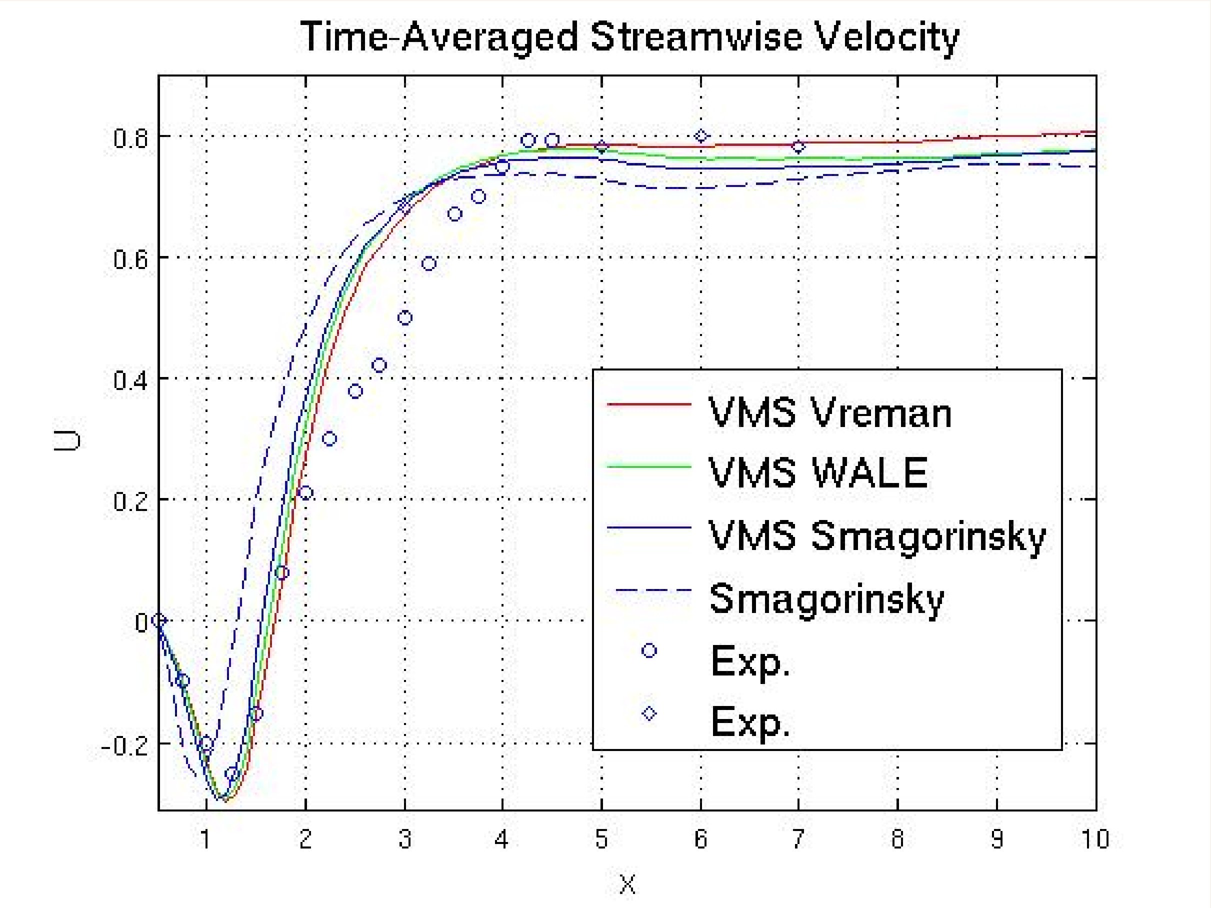
Data from:	$\overline{C_d}$	St	l_r	θ_{sep}	$\overline{C_{P_b}}$	U_{min}
LES Smagorinsky	**1.16**	**0.212**	**0.81**	**101**	**-1.17**	**-0.26**
VMS-LES Smagorinsky	**1.00**	**0.221**	**1.05**	**88**	**-0.96**	**-0.29**
VMS-LES Vreman	**0.99**	**0.221**	**1.12**	**88**	**-0.91**	**-0.30**
VMS-LES WALE	**0.97**	**0.223**	**1.19**	**89**	**-0.94**	**-0.29**
No model	**0.96**	**0.225**	**1.24**	**90**	**-0.90**	**-0.30**
Numerical data (Dyn. LES)						
Kravchenko-Moin	1.04	0.210	1.35		-0.94	-0.37
Breuer	1.07		1.197	87.7	-1.011	
Lee-Park-Lee-Choi	0.99	0.212	1.36		-0.94	-0.33
Experiments						
Norberg	0.99±0.05	0.215±0.05			-0.88±0.05	-0.24±0.1
Son-Hanratty				86 ±2		
Cardell		0.215±0.005	1.33±0.05			
Ong-Wallace		0.21±0.005	1.4±0.1			-0.24±0.1
Lourenco-Shih "Exp."			1.18±0.05			

Table 1: Circular cylinder: Bulk coefficients, comparison with experimental data and with other simulations in the literature. $\overline{C_d}$ denotes the mean drag coefficient, St the Strouhal number, l_r the mean recirculation length: the distance on the centerline direction from the surface of the cylinder tot he point where the time-averaged streamwise velocity is zero, θ_{sep} the separation angle, $\overline{C_{P_b}}$ the mean back-pressure coefficient and U_{min} the minimum centerline streamwise velocity.

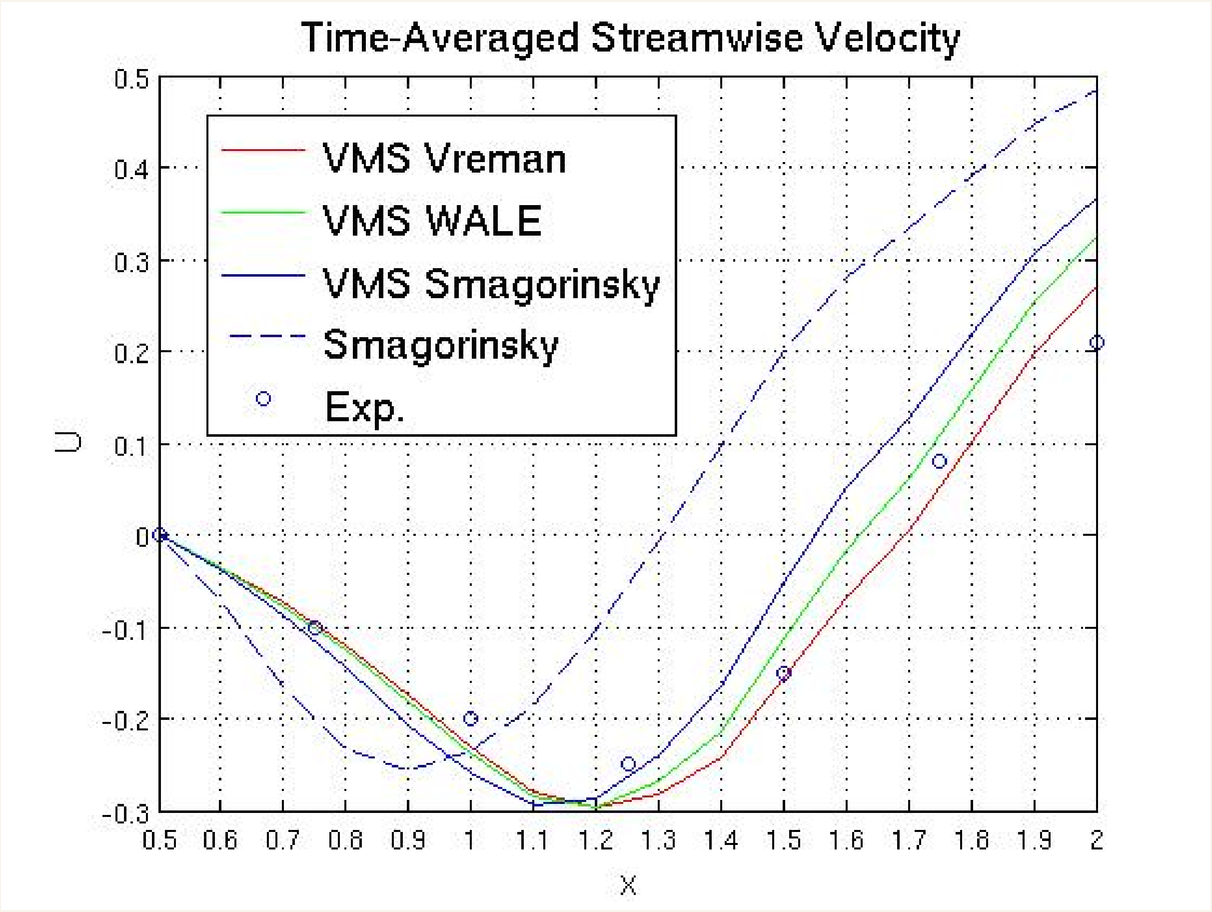
IV Applications (circular cylinder)



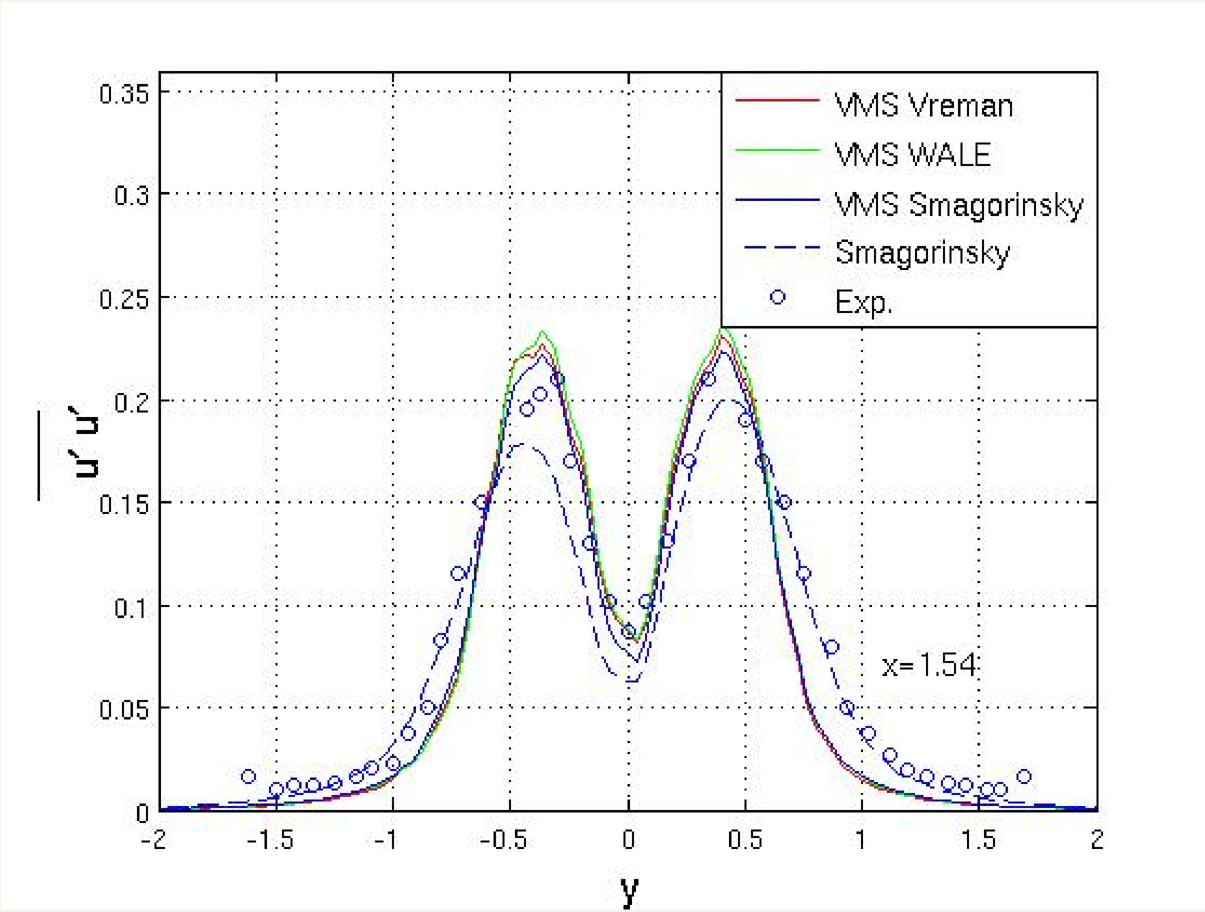
IV Applications (circular cylinder)



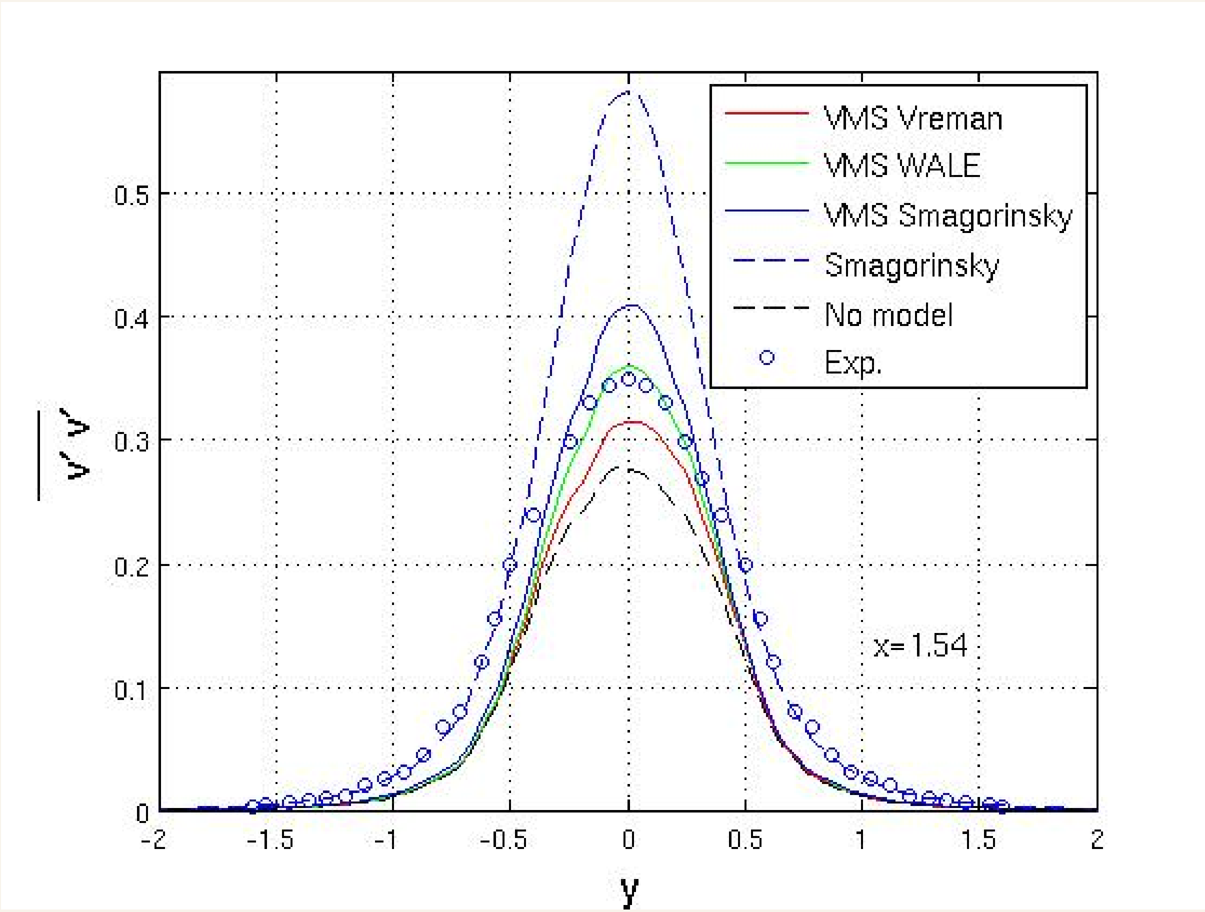
IV Applications (circular cylinder)



IV Applications (circular cylinder)



IV Applications (circular cylinder)



IV Applications

3: Flow around a spar

Spar is a moving structure:

- held by elastic moorings:

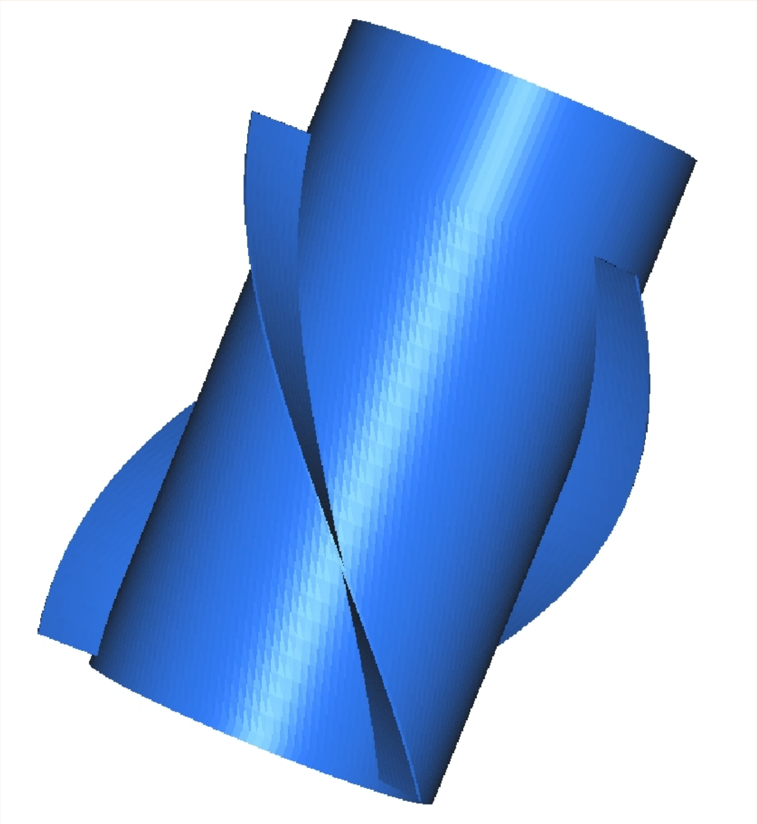
-$\Rightarrow$ Vortex induced motion.

- parameters:

incident velocity,

“raw” angle of strake.

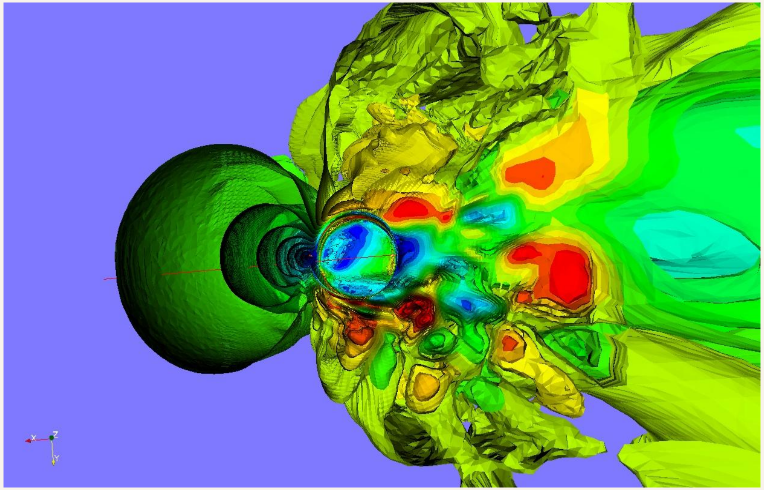
- 520 K vertices.

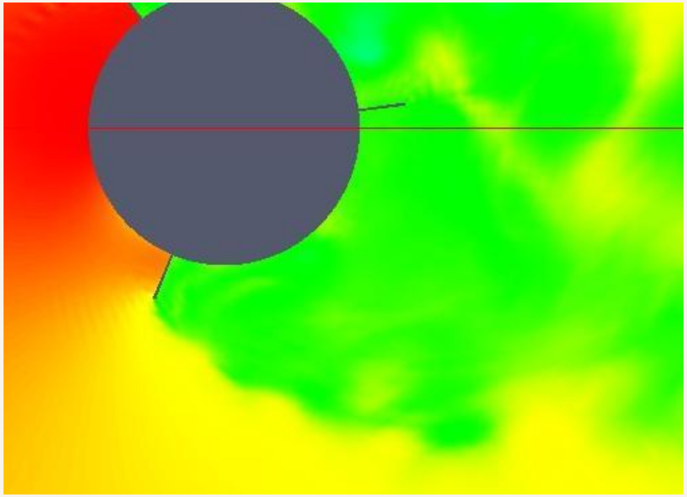


IV Applications (Flow around a spar)

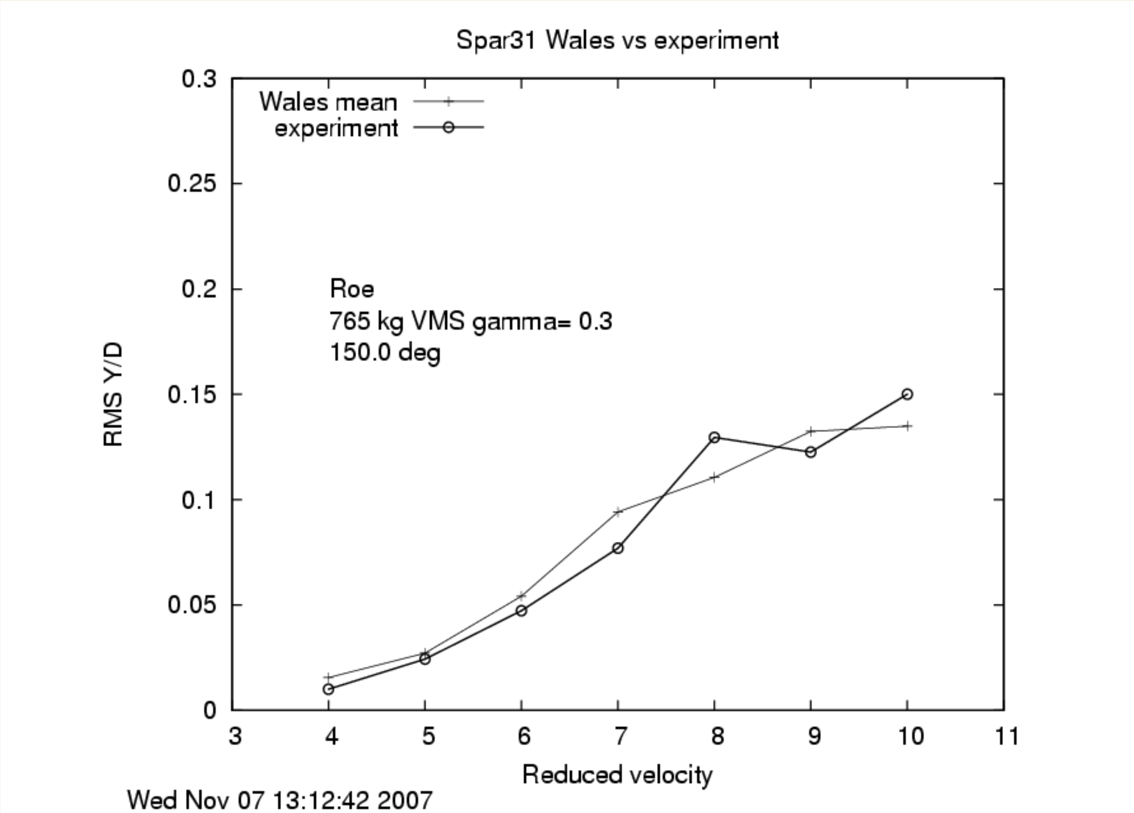
Flow past a spar, Re=300000

VMS-LES with Smagorinsky model, velocity module





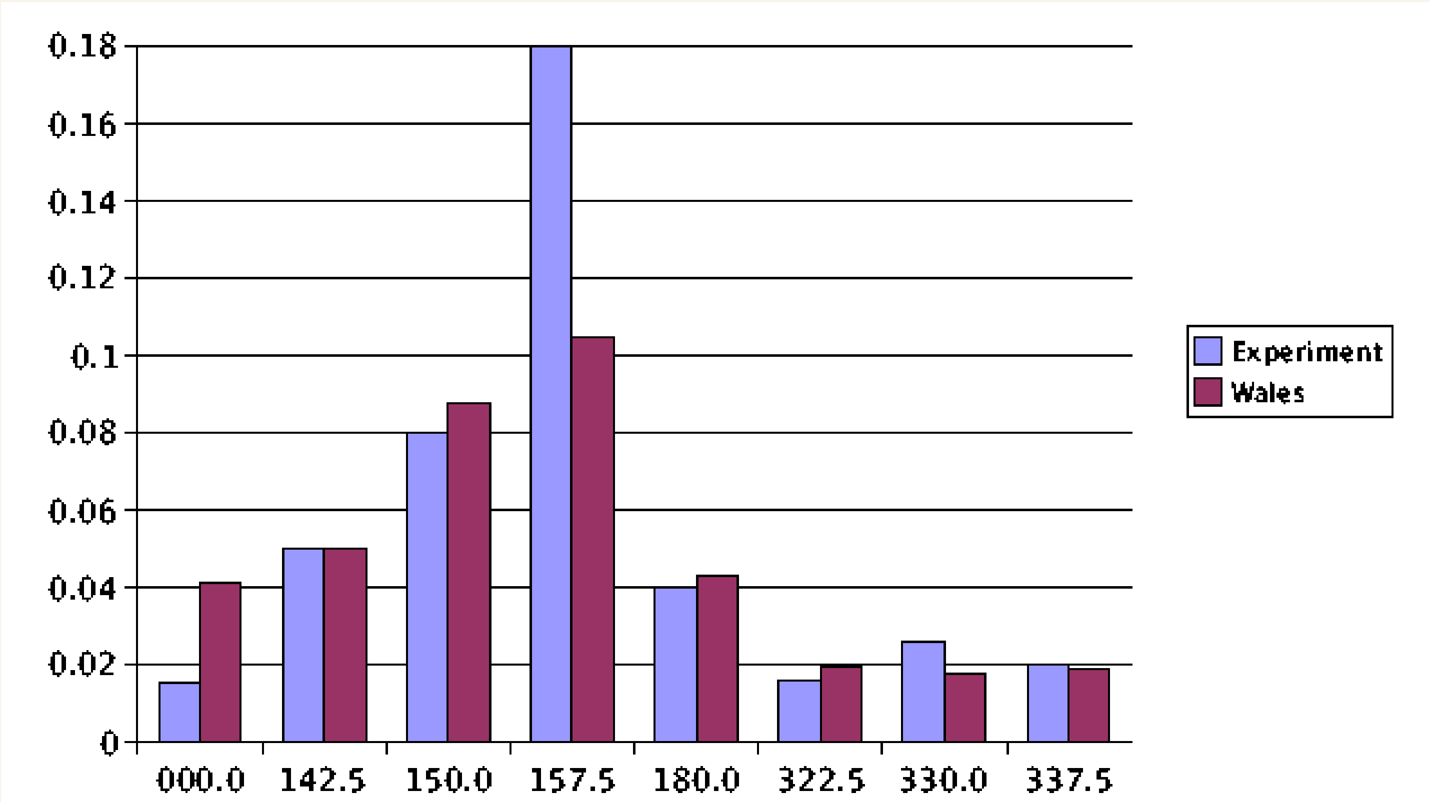
IV Applications (Flow around a spar)



Time-averaged lateral position for different velocities

IV Applications (Flow around a spar)

RMS of deviation with respect to **spar angle**, VR=7



V CONCLUDING REMARKS

The main design principle in the proposed methods is to avoid dissipation of non-subgrid eddies, neither by numerics, nor by LES models.

We have applied VMS-LES with two flow configurations:
- A flow past a circular cylinder with laminar boundary layer.
- A flow past a spar in which boundary layer has a minor impact.

Both computations show that combining VMS-LES with adapted models carries improvements.

The main output of this study is that, combined with VMS, the simplified physical models perform well. Newer models (Vreman's, WALE) take some advantages.

The natural evolution of these models is to be involved in a hybrid RANS/LES formulation, e.g. as presented in the lecture of Prof. Salvetti.

Retour plan