Mathematical Morphology
a non exhaustive overview

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Mathematical Morphology

- Shape oriented operations, that “simplify image data, preserving their essential shape characteristics and eliminating irrelevancies” [Haralick87]

Overview

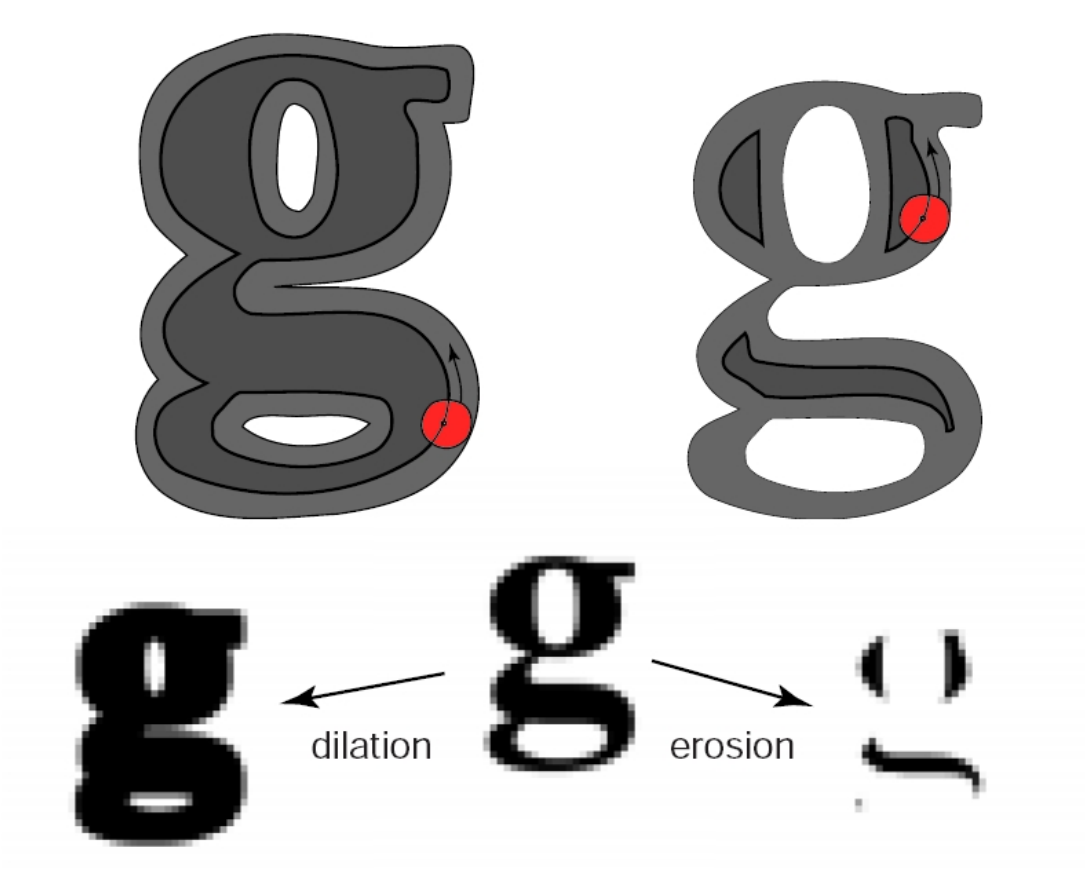
- Basic morphological operators
- More complex operations
- Conclusion and References

Overview

- **Basic morphological operators**
  - **Binary**
  - Grayscale
  - Color
  - Structuring element
- More complex operations
- Conclusion and References

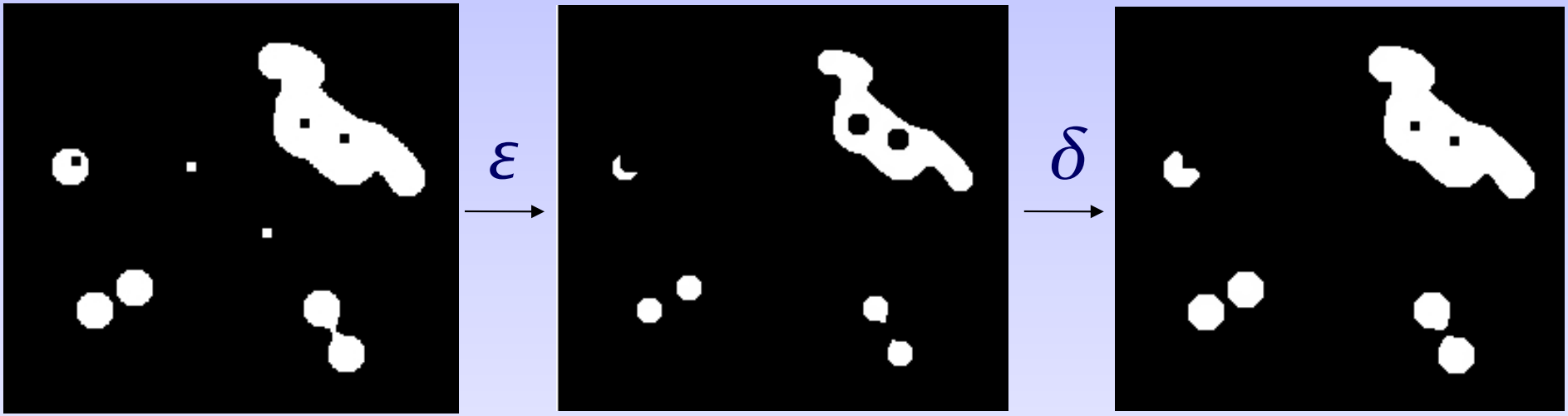
Basic operators: binary

- Dilation δ, erosion ε by a structuring element



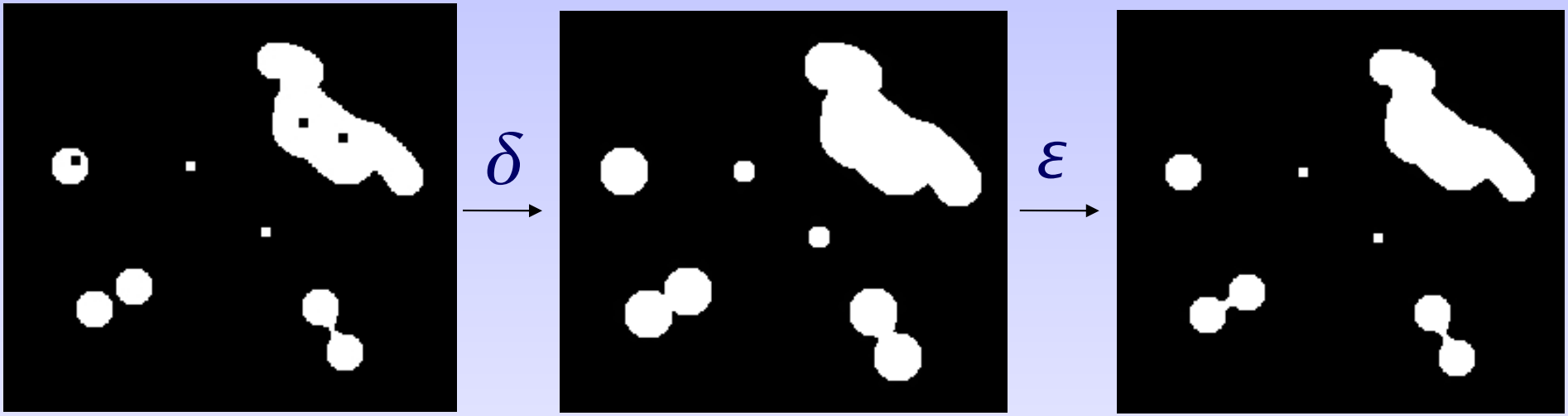
Basic operators: binary

- Opening $\delta \circ \varepsilon$: remove capes, isthmus and islands smaller than the structuring element



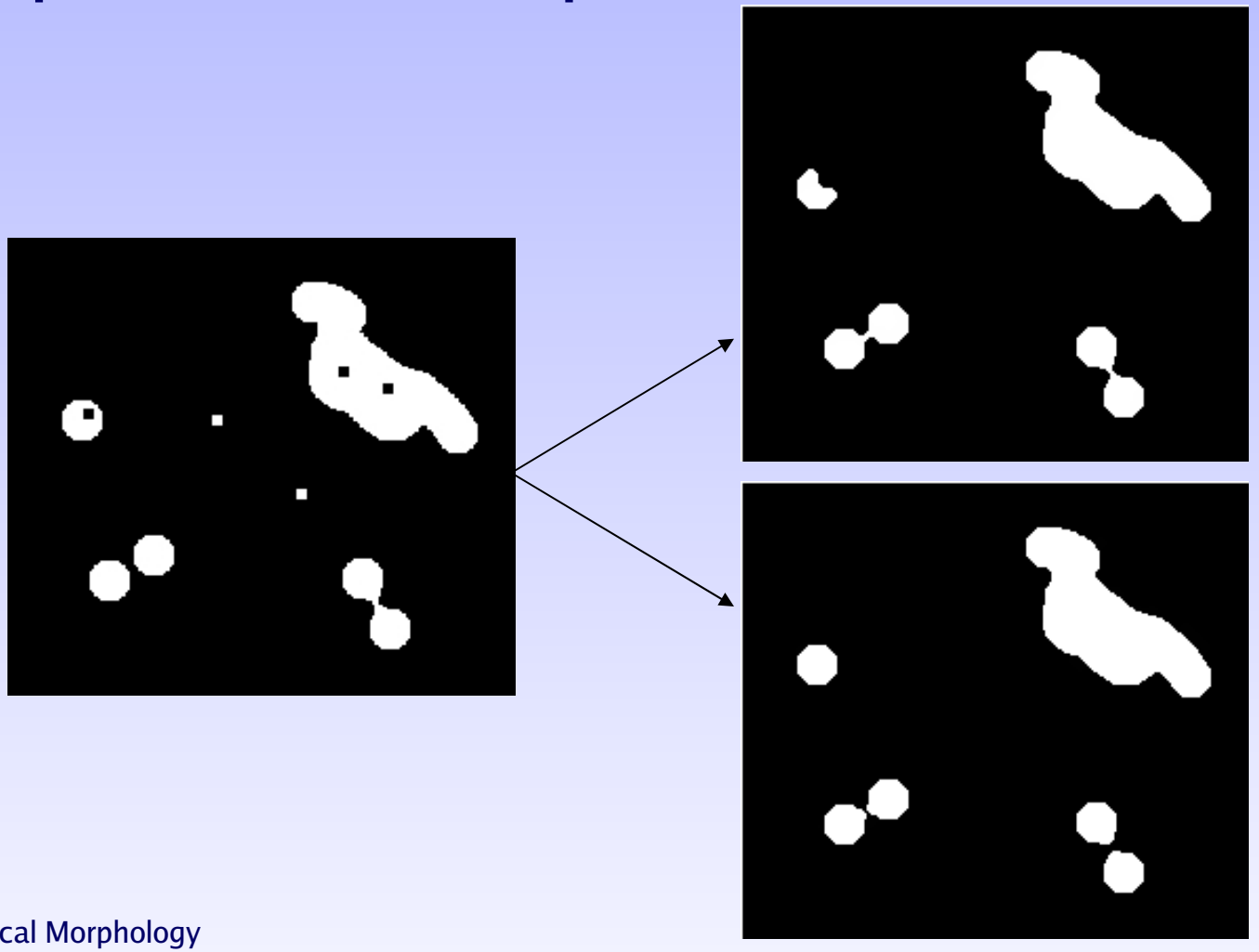
Basic operators: binary

- Closing $\varepsilon \circ \delta$: fill gulfs, channels and lakes smaller than the structuring element



Basic operators: binary

- Sequencial filter: open-close or close-open



Overview

- **Basic morphological operators**
  - Binary
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  - Color
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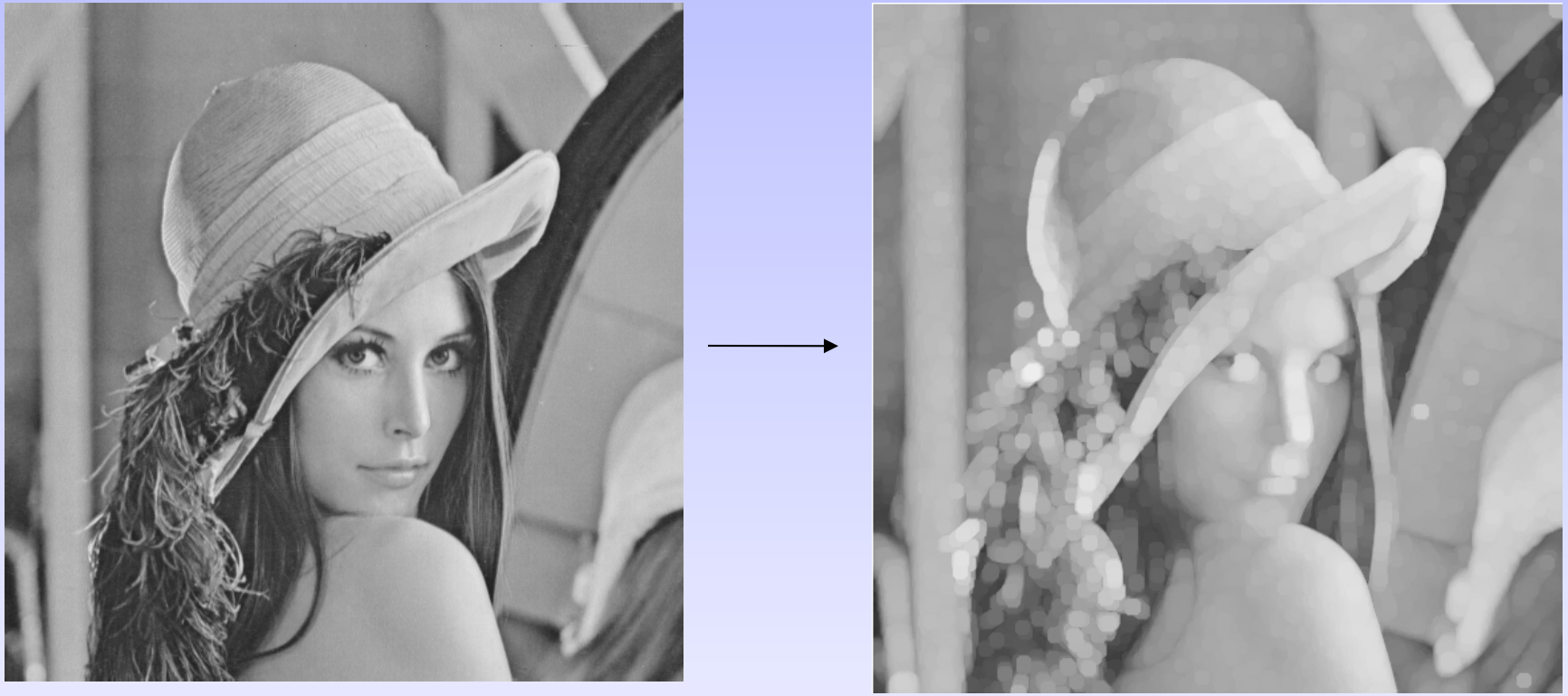
Basic operator: grayscale

- Dilation δ: max over the structuring element

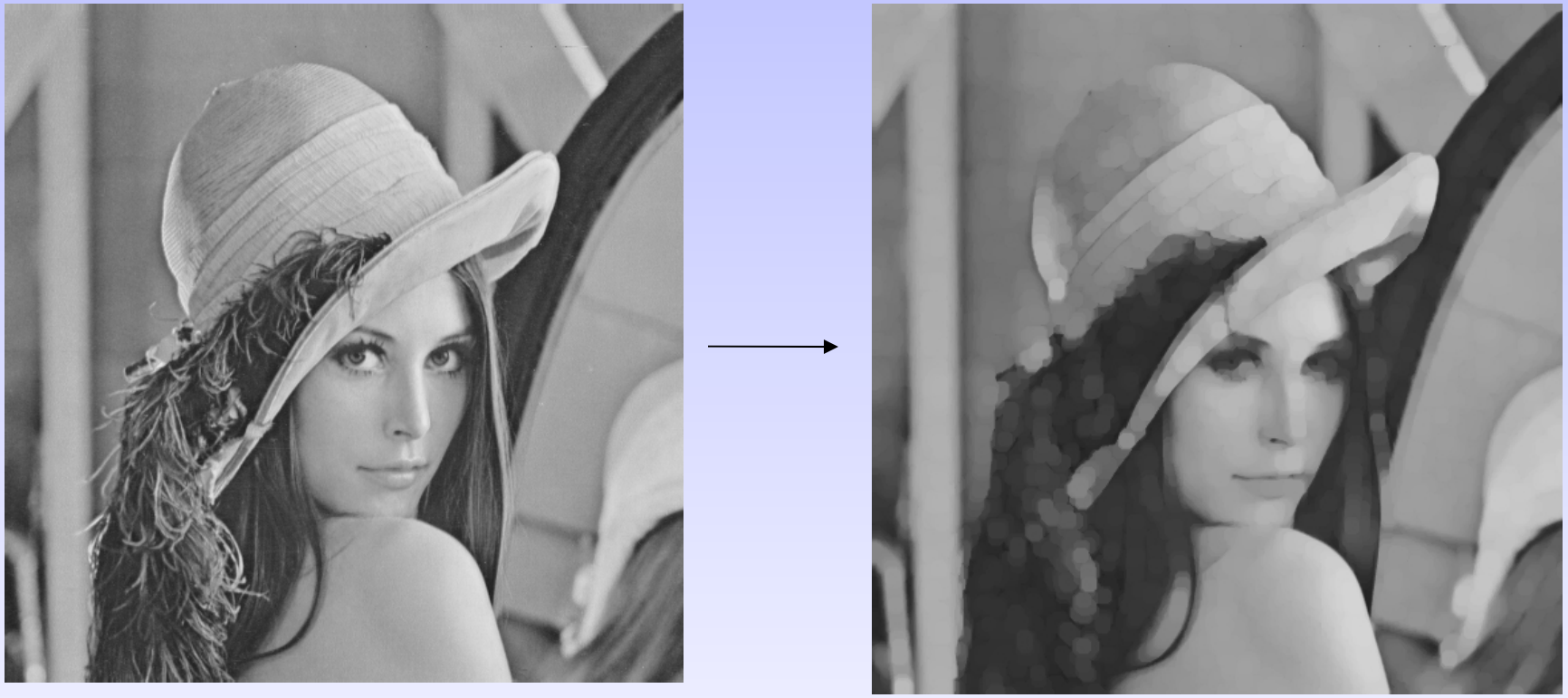


Basic operator: grayscale

- Erosion ε: min over the structuring element

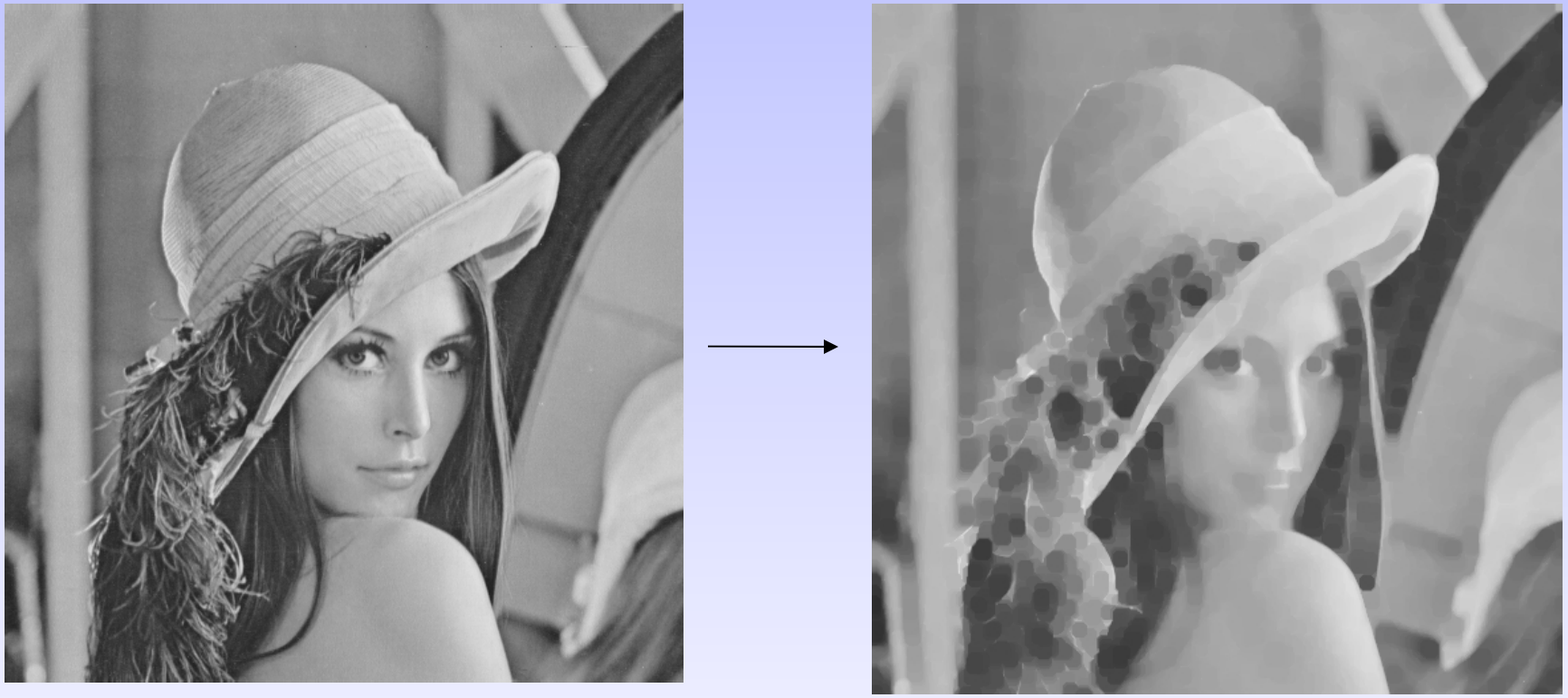
Basic operator: grayscale

- Opening $\delta \circ \varepsilon$: remove light features smaller than the structuring element



Basic operator: grayscale

- Closing $\varepsilon \circ \delta$: remove dark features smaller than the structuring element



Basic operator: grayscale

- Sequential filter (open-close or close-open): remove both light and dark features



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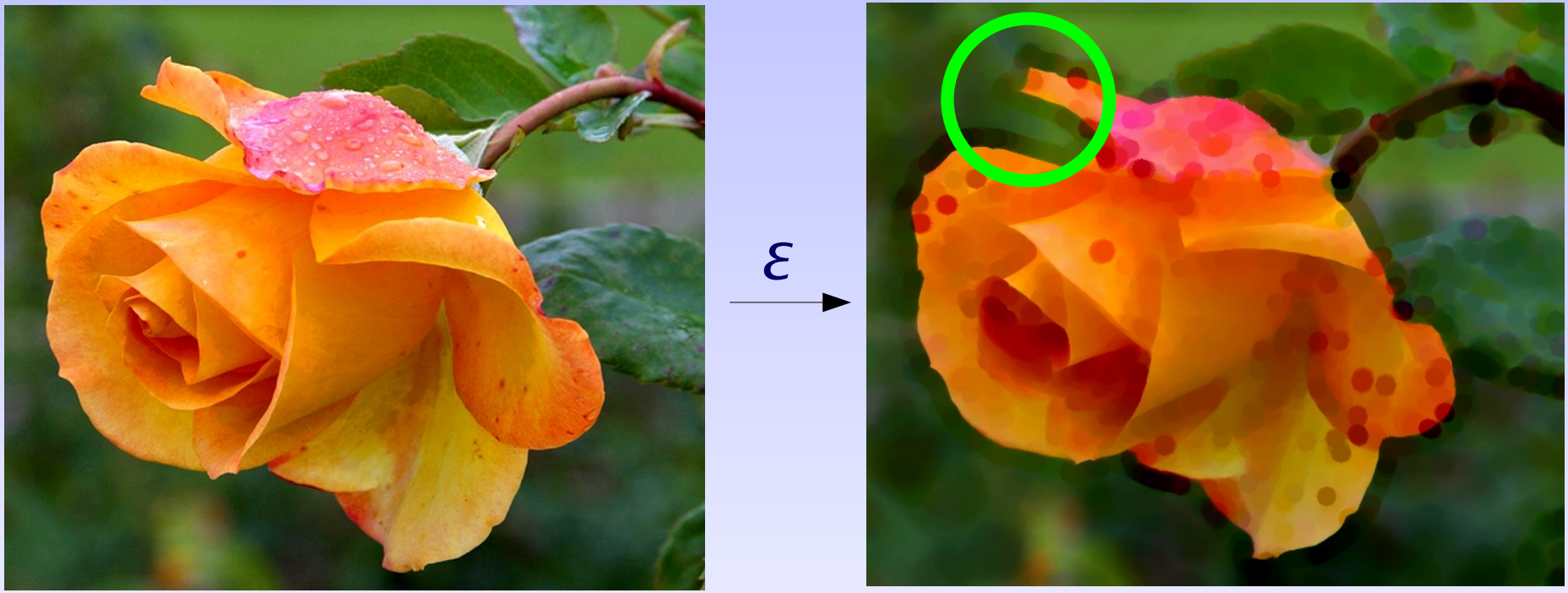
Color images

- Process each channel separately: color ghosting with basic operators



Color images

- Process each channel separately: color ghosting unnoticeable with sequential operators

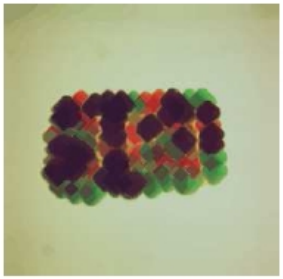
opening →

Color images

- Several ordering strategy

(a) Example colour image

(b) MSS erosion



(c) *L*-ordered erosion

(d) *S*-ordered erosion

(e) *H*-ordered erosion

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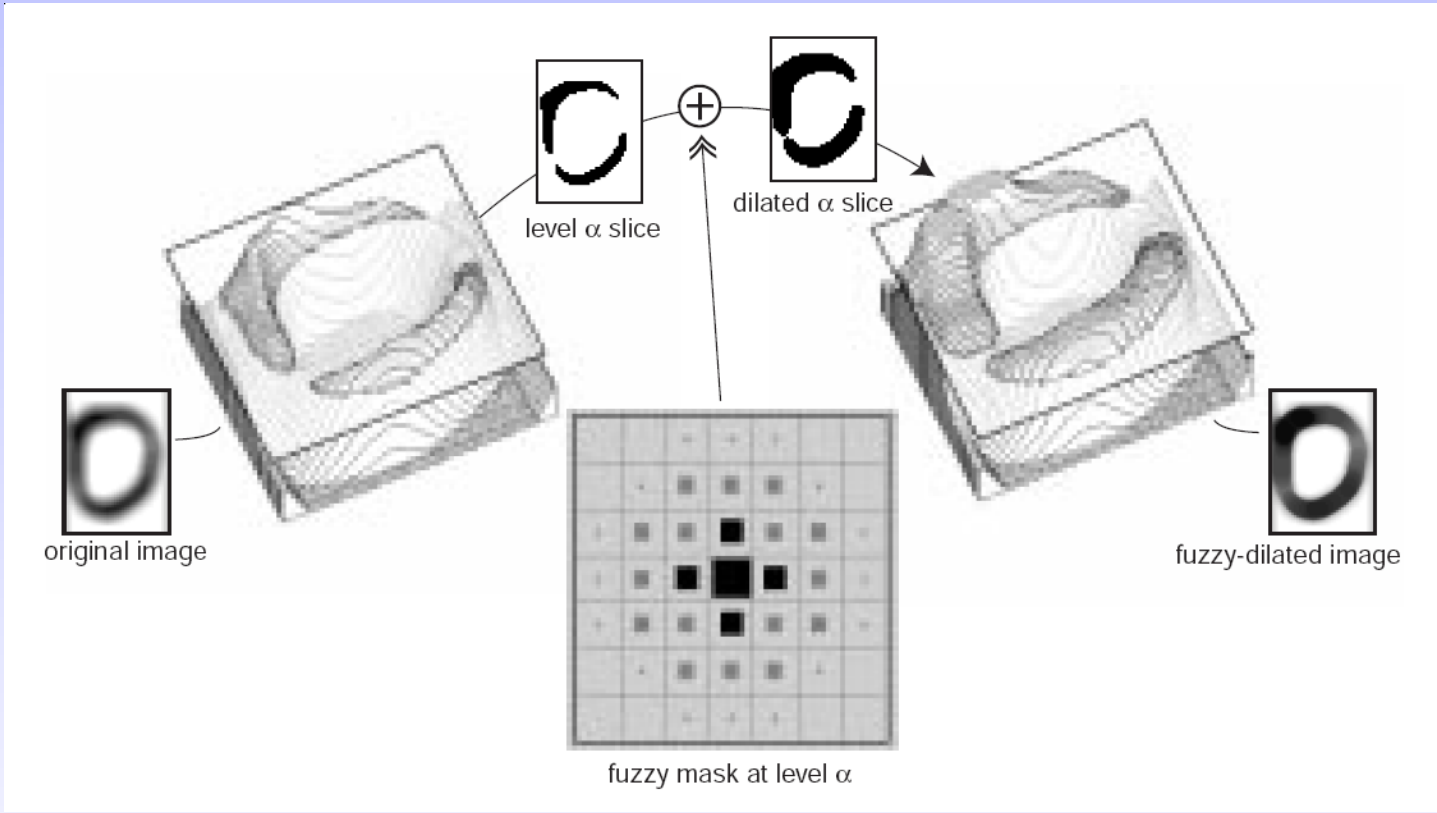
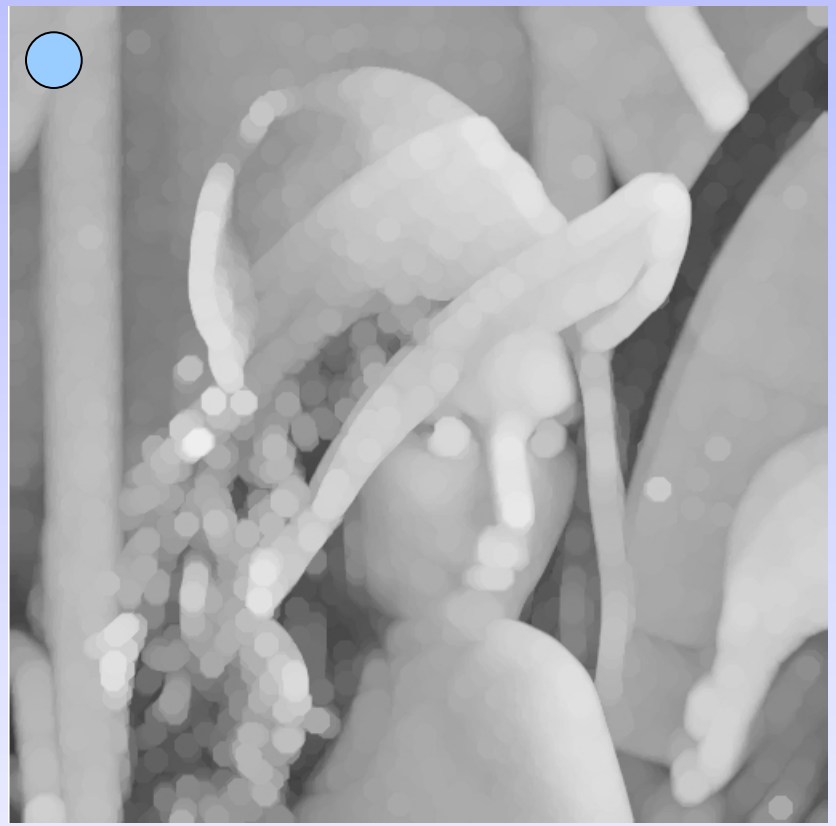
Structuring element

- Usually, flat element (binary)
- Grayscale element: fuzzy morphology



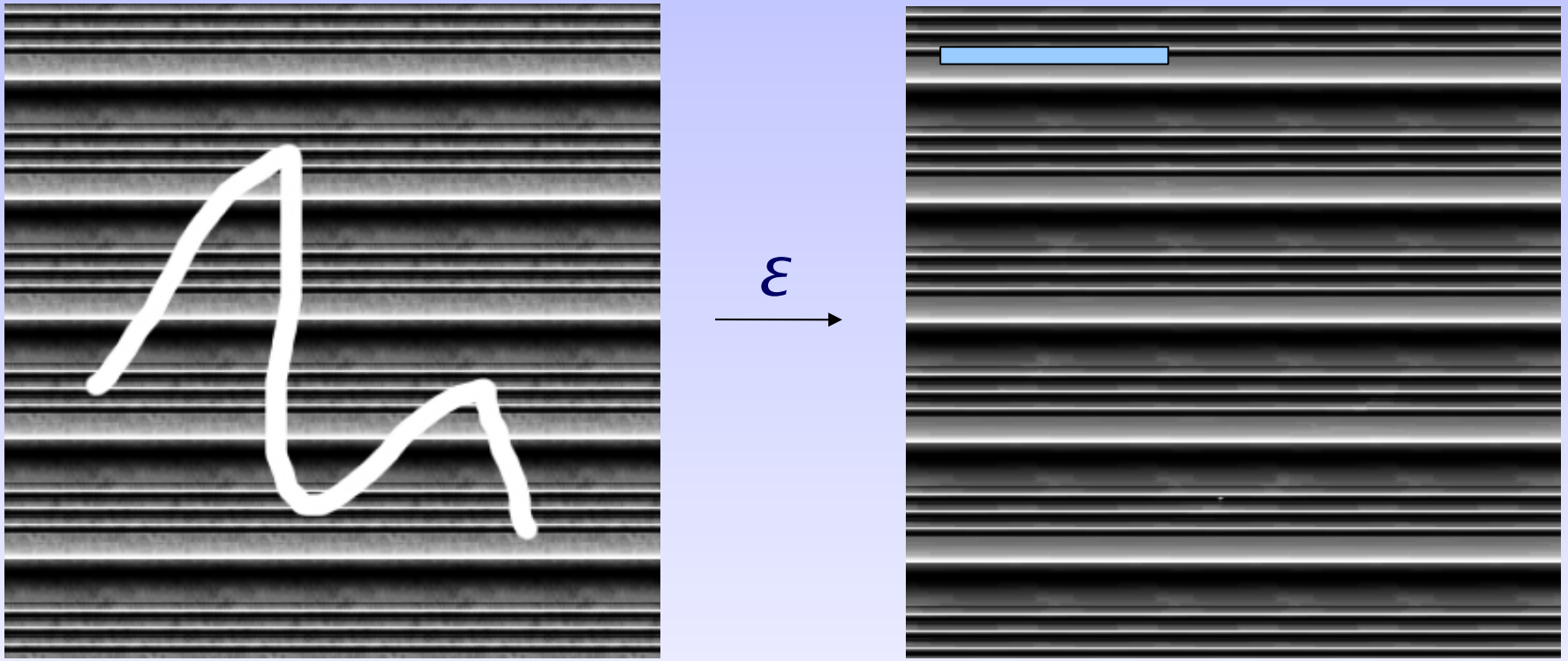
Structuring element

- Shape has an impact!



Structuring element

- Choose the structuring element according to the image structure



Structuring element

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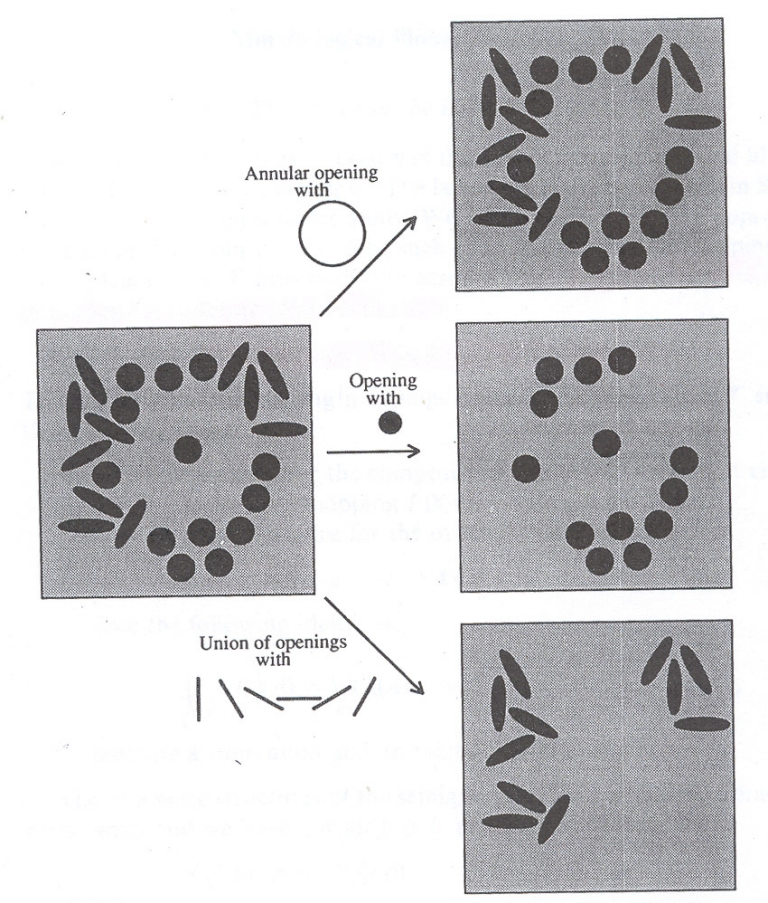


Overview

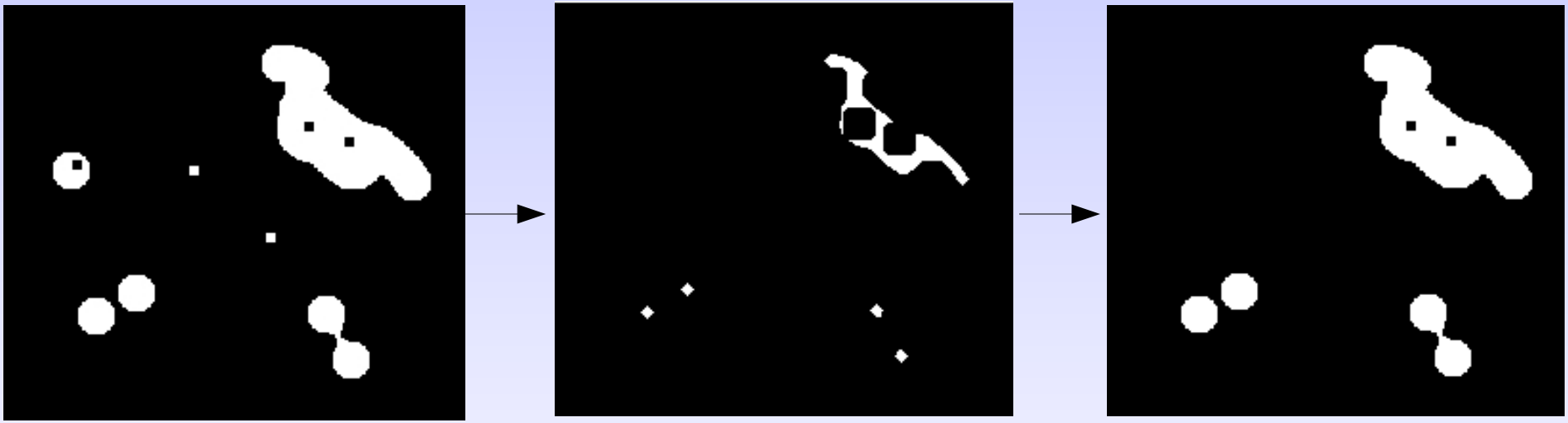
- Basic morphological operators
- **More complex operations**
  - **Reconstruction operators**
  - Top hat, sharpening, distance, thinning, segmentation...
- Conclusion and References

Reconstruction operators

- Remove features smaller than the structuring element, without altering the shape
- Reconstruct connected components from the preserved features

Reconstruction operators: binary

- Opening by reconstruction:
  - Erosion: $f'(0) = \varepsilon f$
  - Iterative reconstruction: $f'(t+1) = min(\delta f'(t), I)$ until stability



Reconstruction operators: binary

- Closing by reconstruction:
  - Dilation: $f'(0) = \delta f$
  - Iterative reconstruction: $f'(t+1) = max(\varepsilon f'(t), I)$ until stability

Reconstruction operators: grayscale

- Opening by reconstruction: remove unconnected light features

Reconstruction operators: grayscale

- Closing by reconstruction: remove unconnected dark features

Reconstruction operators: grayscale

- Sequential filter by reconstruction: open-close

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- Basic morphological operators
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  - Reconstruction operators
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Top Hat

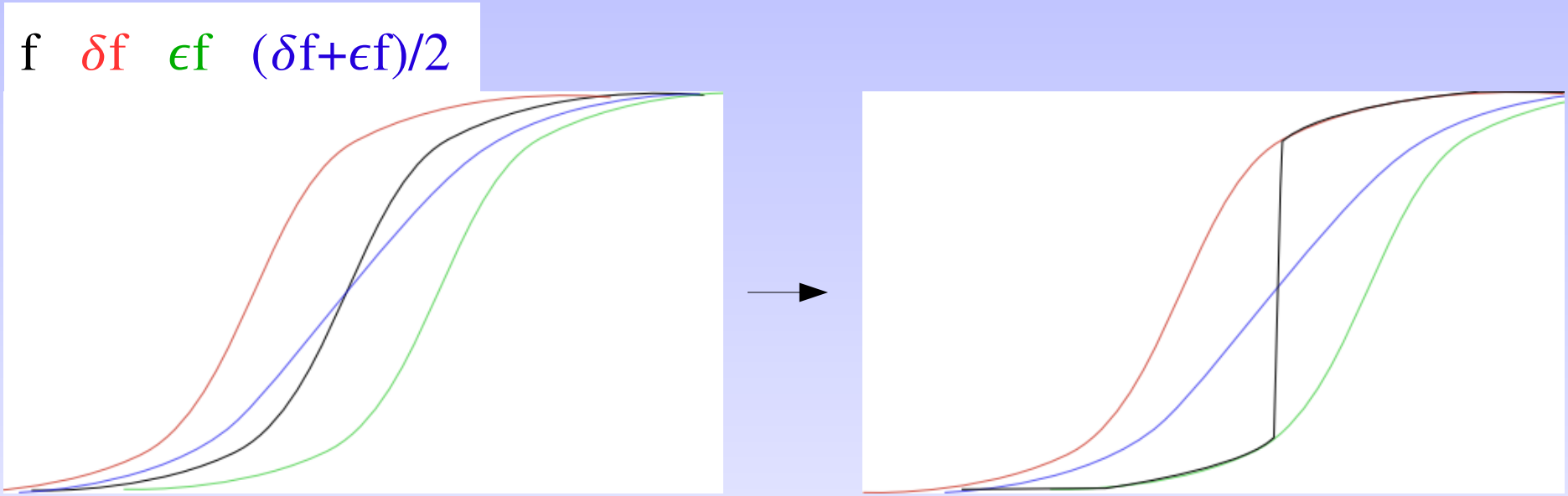
- White top-hat: *f-opening(f)*
  Extract light features

Top Hat

- Black top-hat: *closing(f)-f*
  Extract dark features

Edge sharpening

- Toggle mapping

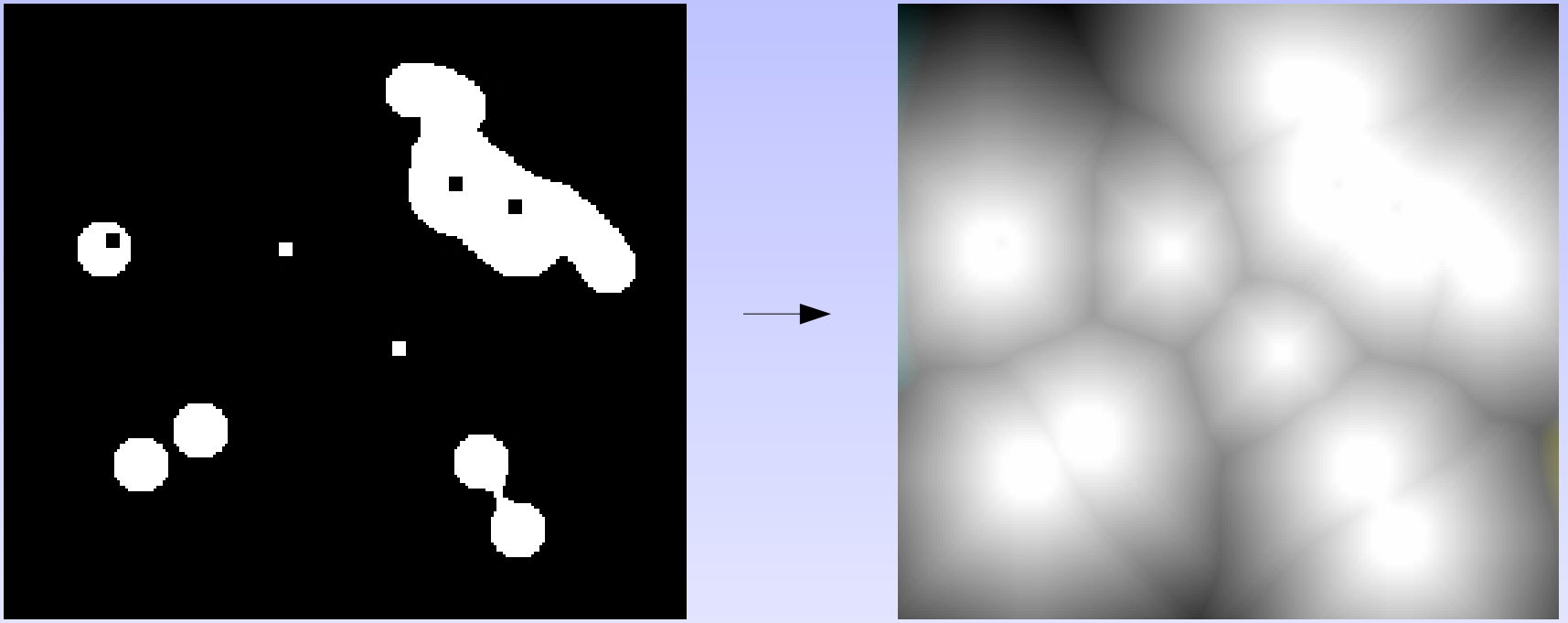


Edge sharpening

- Toggle mapping

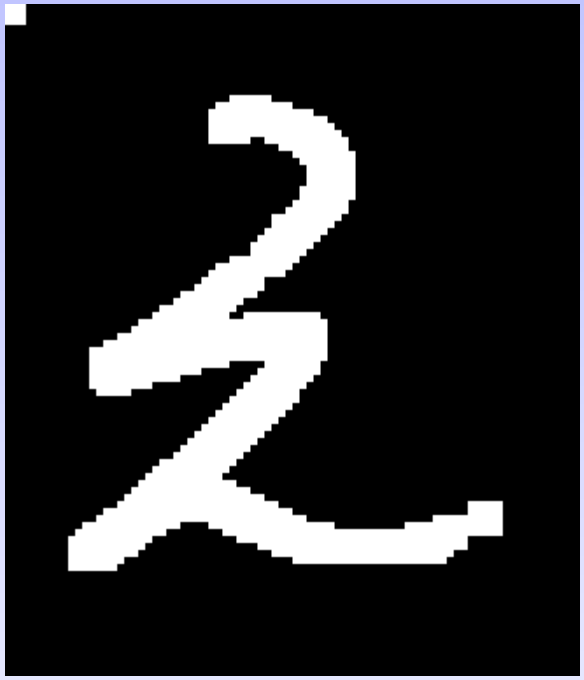
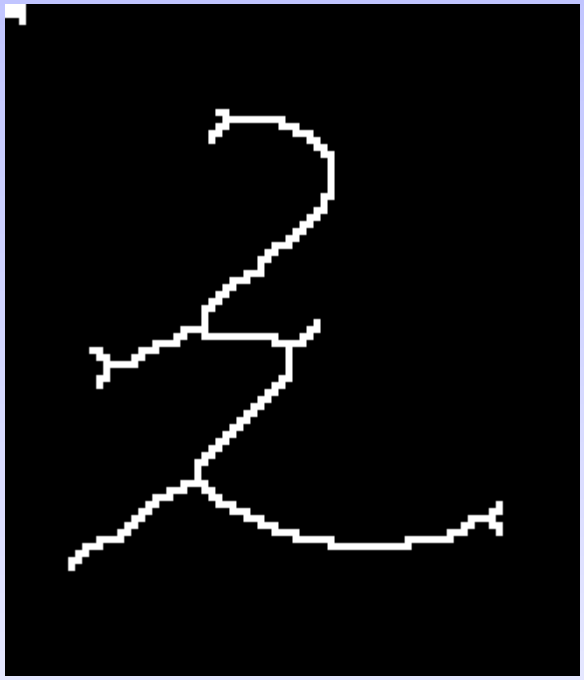
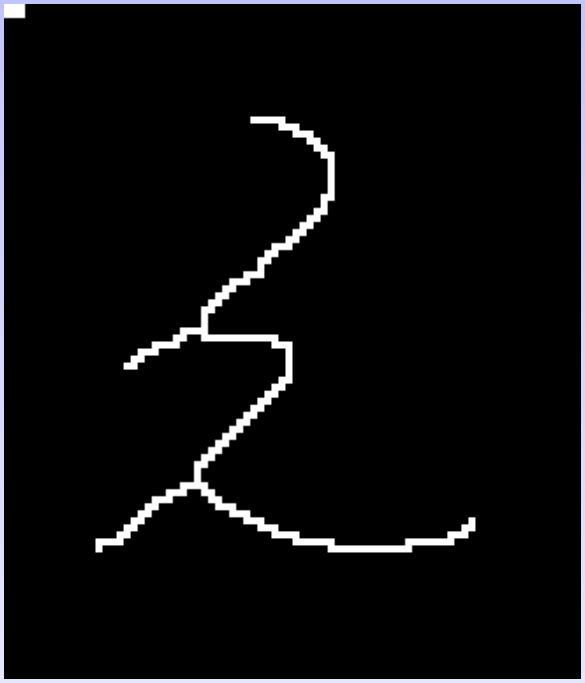
Distance function

- Distance from binary elements



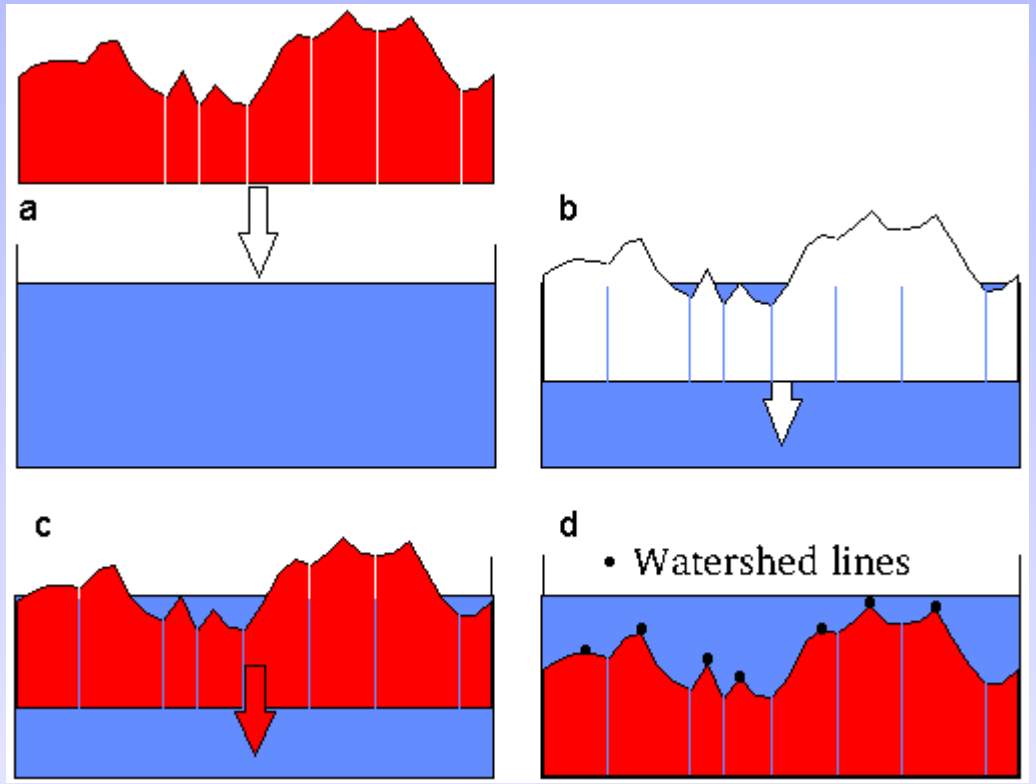
Thinning

- Binary (or grayscale ?) skeleton

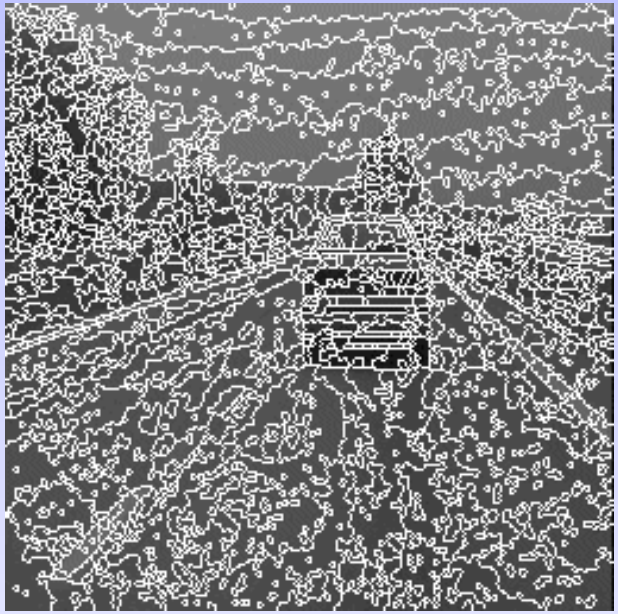
Segmentation

- Watershed:
  - Image = heightfield
  - Flood the image from its minima
  - Lake junctions give the segmentation



Segmentation

- Watershed: hierarchical results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Conclusion

- Powerful toolbox for many image analysis tasks
- Not famous because not useful?
- Not used because not famous?
- Based on a whole mathematical theory
- But can be very practical (maybe too much?)
- French!

References

- Pierre Soille, 2003: Morphological Image Analysis, Principles and Applications. (Practical approach)
- Jean Serra and Luc Vincent, 1992: An Overview of Morphological Filtering. (Mathematical approach)