Reconfiguration of the Routing in WDM Networks with Two Classes of Services

David Coudert, Florian Huc,
Dorian Mazauric, Nicolas Nisse
MASCOTTE, INRIA, I3S, CNRS, Univ. Nice Sophia
Sophia Antipolis, France
{firstname.lastname@sophia.inria.fr}

Jean-Sébastien Sereni
CNRS (LIAFA, Univ. D. Diderot), Paris and
KAM (Faculty of Math. and Physics), Charles Univ., Prague
{sereni@kam.mff.cuni.cz}

***Abstract*—In WDM backbone networks, the traffic pattern evolves constantly due to the nature of the demand itself or because of equipment failures leading to reroute affected connections. In this context, requests are routed greedily using available resources without changing the routing of pre-established connections. However, such a policy leads to a poor usage of resources and so to a higher blocking probability: new connection requests might be rejected while network resources are sufficient to serve all the traffic. Therefore, it is important to regularly reconfigure the network by rerouting established connections in order to optimize the usage of network resources.**

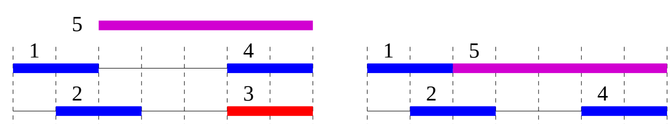
In this paper, we consider the network reconfiguration problem that consists in switching existing connections one after the other from the current routing to a new pre-computed routing. Due to cyclic dependencies between connections, some requests may have to be temporarily interrupted during this process. Yet, the number of requests simultaneously interrupted has to be minimized. Furthermore, it might be impossible for the network operator to interrupt some connections because of the contract signed with the corresponding clients. In this setting, the network reconfiguration problem consists in going from a routing to another one given that some priority connections cannot be interrupted.

The network reconfiguration problem without priority connections has previously been modeled as a cops-and-robber game [1], [2]. Here, we first extend this model to handle priority connections. Then we identify cases where no solution exists. Using a simple transformation, we prove that the reconfiguration problem with priority connections can be reduced to the problem without this constraint. Finally, we propose a new heuristic algorithm that improves upon previous proposals.

***Index Terms*—WDM networks, network reconfiguration, cops-and-robber game, process number.**

I. Introduction

Optimizing the usage of network resources is a critical issue for telecom companies operating high speed WDM backbone networks [3], [4]. The traffic demand increases rapidly (e.g. 6 millions new Internet users per month in China) and is subject to constant variation with the deployment of new services (mobile Internet, peer-to-peer, on-demand TV). Therefore, the routing of the traffic pattern has to be updated regularly to ensure an efficient usage of network resources and to preserve enough flexibility to accommodate new connection requests rapidly. For example, Fig. 1(a) represents a path network using two wavelengths where four connections are initially established. After the termination of request 3, it is not possible to accept request 5 although the routing depicted in Fig. 1(b) is possible. Hence, to reduce the blocking probability [5], [6], some requests have to be switched on other routes, and so the routing has to be reconfigured [7]. Fault tolerance is another critical concern of the networks. Indeed, failures in the backbone network may have an important impact with huge financial repercussions. Therefore, when a failure occurs, traffic restoration or protection mechanisms are used to ensure the continuation of affected connections. Such mechanisms are fast and ensure low traffic perturbation, but the resulting routing of connections requests may be inefficient. Since the repairing time of the network could be long (days), it is interesting to optimize the usage of resources and so to reconfigure the routing.



(a) Initial routing R_1 of requests $1, 2, 3, 4$. Request 3 finishes and a new request starts.

(b) The new routing R_2 satisfying requests $1, 2, 4, 5$.

Fig. 1. Example of reconfiguration.

In this paper, we concentrate on a routing reconfiguration problem. It consists in switching a set of connection requests from the current routing to another pre-determined routing, under the constraint that connections are switched one by one. Our study is independent of the destination routing and its computation is not considered here.

To switch an established connection from a lightpath to another, one has to ensure that some destination resources are available. For instance, in Fig. 1(a), connection 4 must be switched before the establishment of connection 5. To model all the dependencies between connections in the reconfiguration phase, we use the notion of dependency digraph [7]. Given the initial routing R and the new routing R' we want to achieve, the dependency digraph contains a node per connection and there is an arc from node u to node v if connection v uses resources in R that will be used by u in R'. So an arc from u to v indicates that connection v must be switched before connection u.

When the dependency digraph is a DAG (Directed Acyclic Graph), the scheduling of the switches is straightforward, starting from the leaves. However, cyclic dependencies may exist and so the dependency digraph may contains cycles. In such cases, reconfiguration is feasible only if we allow to temporarily interrupt some connections in order to break the dependency cycles. The objective is thus to minimize the number of connections that will be simultaneously interrupted. Initially, this problem has been studied with an heuristic algorithm [7]. Later, the network reconfiguration problem has been defined in terms of the *process number* [1], [2], an analogous of cops-and-robber games [8], [9]. Using this formulation, this problem has been shown NP-complete [1], [2]. More precisely, a game is defined where agents are successively placed to and removed from the vertices of the dependency digraph [1], [2]. In this setting, the minimum number of agents used during the game equals the smallest number of connections simultaneously interrupted during the reconfiguration process. In this game, an agent placed on a node models the interruption of the corresponding connection, and a connection can be switched, or *processed*, if all its outneighbors in the dependency digraph are either covered by an agent or have already been processed. In other words, a connection can be switched if all of the connections using the same resources have either been interrupted, or have already been switched. Furthermore, an agent can be reused as soon as the corresponding connection is switched. The process number is the minimum number of agents needed in the game.

The reconfiguration of the routing allows the network operator to optimize the usage of network resources. However, this process forces to interrupt temporarily some connections, and so induces some traffic perturbations that some clients may not accept. Moreover, some clients may sign a specific contract forbidding interruptions, and so the operator offers two classes of services. To cope with these two classes, we introduce a new constraint imposing that some particular connections, called the *priority connections*, cannot be interrupted. In the process number game formulation, this constraint is modeled by a particular class of nodes of the dependency digraph, called the `black` nodes, that are those nodes corresponding to the priority connections. During the game, the `black` nodes cannot host agents, i.e., the single way to process them is to deal with all their outneighbors first. It is worth-mentioning that a direct cycle of `black` nodes in the dependency digraph makes the reconfiguration impossible, thus the number of such nodes (and so clients) must be small. Furthermore, due to `black` nodes the process number of the dependency digraph may increase (see Sec. III).

Sec. II is devoted to the formal definition of the process number and the game on the dependency digraph without `black` nodes. In Sec. III we introduce the modeling with `black` nodes. We give impossibility results. Namely, we prove that reconfiguration can be performed if and only if no subset of `black` nodes induces a strongly connected component of the dependency digraph. Then, we show that both problems, with and without `black` nodes are equivalent. More precisely, when a reconfiguration is possible, we present a simple transformation of any digraph D with `black` nodes into another digraph D^* without `black` nodes with the same process number and such that it is straightforward to adapt the rerouting strategy for D^* into a strategy for D. In Sec. IV, we recall the basic principles of the heuristic designed by Coudert and Mazauric [10] and propose some improvements to solve more efficiently our problem. Finally, in Sec. V, we evaluate the respective efficiency of heuristic algorithms through simulations.

II. Modeling

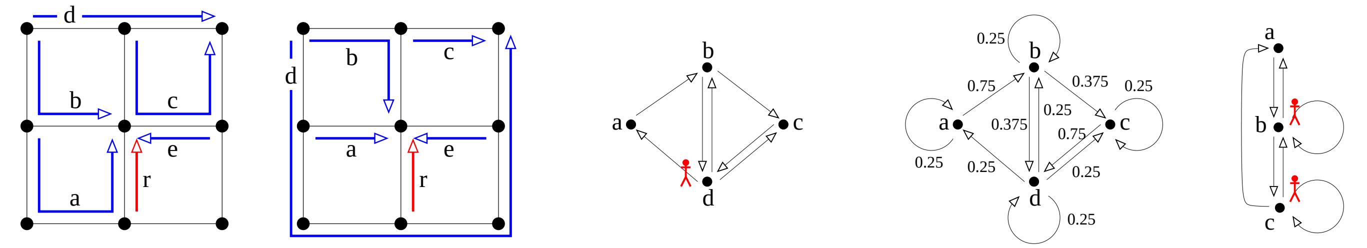
We first recall the modeling proposed in [1], [2] to solve the routing reconfiguration problem when it is allowed to interrupt any connection request during a reconfiguration phase.

The routing reconfiguration problem can be expressed as a cops-and-robber game [1], [2], similar to the game-theoretical model of the *pathwidth* of a graph [8], [11]. Hence, it can be seen as an exploration of the dependency digraph D by agents using the following rules. Interrupting a request r is represented by placing an agent on the corresponding vertex u_r in D. A vertex is *processed* when the corresponding request has been rerouted, and *process strategy* is a sequence of the three following actions allowing to reroute all requests with respect to the constraints represented by the dependency digraph D.

- R_1 Put an agent on a vertex (*interrupt a connection*).
- R_2 Remove an agent from a vertex if all its out-neighbors are either processed or occupied by an agent (*release a connection to its final route when destination resources are available*). The vertex is now processed (*the connection has been rerouted*).
- R_3 Process a vertex if all its out-neighbors are occupied by an agent (*destination resources are available, and so the connection can be rerouted*).

A *p-process strategy* is a strategy which processes the digraph using p agents and the *process number*, $\mathrm{pn(D)}$, is the smallest p such that a p-process strategy exists. For example, a star has process number 1 and the strategy consists in positioning an agent on the central vertex (R_1). Thus all the leaves can be processed (R_3) and finally the agent can be removed (R_2). A path of four vertices or more has process number 2, a cycle of size five or more has process number 3, and a $n \times n$ grid, $n \geq 3$, has process number $n + 1$.

In the example of Fig. 2, the network is a 3×3 grid where links have a single wavelength in each direction. With the routing of Fig. 2(a), the new request r cannot be accepted but the routing of Fig. 2(b) is possible. Fig. 2(c) represents the dependency digraph for switching from routing Q_1 to routing Q_2. The process strategy first places an agent at node d (R_1). The remaining digraph is a direct path a, b, c, and so we process c, b and then a (R_3). Finally, the agent can be removed to process d (R_2). So the process number of this digraph is 1 and a single connection has to be temporarily interrupted.



(a) Routing Q_1 of requests a, b, c, d, e. The new request r cannot be satisfied.

(b) Routing Q_2 satisfying requests a, b, c, d, e and r.

(c) Dependency digraph D for switching from routing Q_1 to Q_2.

(d) D_μ from D to apply *flow circulation* algorithm.

(e) $D^*(d)$ from D if d is a `black` node.

Fig. 2. Fig. 2(a) is a grid where links have a single wavelength in each direction. New request r can not be accepted, although the routing of Fig. 2(b) is possible. Figs. 2(c) and 2(d) represents dependency digraph D and corresponding D_μ to switch from routing Q_1 to routing Q_2. If d is a `black` node, digraph $D^*(d)$ of Fig. 2(e) is obtained from D by removing d and adding a complete bipartite digraph from $N^-(d)$ to $N^+(d)$.

Although digraphs with process number 1 and 2 have been characterized [2], computing $\mathrm{pn(D)}$ is NP-complete for general digraphs and the same result holds when D is a symmetric digraph, and hence for the underlying graph G. It has also been proved that $\mathrm{pw}(G) \le \mathrm{pn(G)} \le \mathrm{pw(G)} + 1$, where $\mathrm{pw}(G)$ is the pathwidth of G. Because the problem of computing the pathwidth is NP-complete in general [12], numerous works have been directed toward particular graph classes. For instance, several polynomial-time, or even linear-time, algorithms have been proposed that compute the pathwidth of trees [13], cographs [14], permutation graphs [15], circular arc graphs [16] (in particular, this class contains interval graphs). Due to the relation between the pathwidth of a graph and its process number, all aforementioned results compute the process number up to one for any of these graph classes. Bodlaender *et al.* [17] proposed a $O(\log^2 n)$-approximation for the pathwidth problem in arbitrary graph. Furthermore, the pathwidth problem is not in APX [18], meaning that there is no polynomial time algorithm that can ensure a constant approximation factor k unless $P = NP$, for any constant k. This motivates us to develop efficient heuristic algorithms.

III. Coping with multiple classes

We now extend the model to the case where some requests, the *priority* connections, cannot be interrupted during a reconfiguration phase. In our setting, the priority connections are modeled by a specific class of vertices of the dependency digraph, the `black` nodes. The constraint over the priority connections can be reformulated by the fact that a `black` node cannot host an agent during the process strategy. So rule R_1 can be applied only to other nodes. For example, suppose that node d of Fig. 2(c) is a `black` node. In this situation the process strategy consists in putting agents on both nodes b and c (R_1) which allows us to process node a (R_2), then to process node d (R_2) and finally to process nodes b and c. Such a strategy requires 2 agents while the process number of the digraph of Fig. 2(c), without `black` node, is 1. Note that the presence of a single `black` node may increase drastically the number of agents. For instance, consider a star with a large number of leaves where the central node is a `black` node.

Let $\mathrm{pn^*(D, Q)}$ be the process number of a digraph $D = (V, A)$ with a set $Q \subseteq V$ of `black` nodes. We will show in Sec. III-A that in some cases it is not possible to perform the reconfiguration and thus $\mathrm{pn^*(D, Q)}$ is not defined. We show in Sec. III-B that if the reconfiguration is possible, then $\mathrm{pn^*(D, Q)} = \mathrm{pn(D^*)}$ where D^* is a digraph without `black` nodes obtained from D by a simple transformation.

A. Impossibility results

When the dependency digraph contains a direct cycle of `black` nodes, it is impossible to break it and so no feasible process strategy exists. Let us formalize this as a lemma.

Lemma 1: Let $D = (V, A)$ be a dependency digraph and let $Q \subseteq V$ be the set of `black` nodes. If D contains a direct cycle of `black` nodes then no process strategy exists.

Proposition 1: Given a dependency digraph $D = (V, A)$ and a set of `black` nodes $Q \subseteq V$, we can decide in time $O(|V| + |A|)$ if a process strategy exists.

Proof: The existence of a process strategy relies on the existence of a direct cycle of `black` nodes. So, it suffices to test if the digraph D_Q, obtained by removing all nodes of $V - Q$ from D, has a strongly connected component. ■

As a consequence of Lemma. 1, it is always possible to compute a process strategy when $|Q| \le 1$. Now, when $|Q| \ge 2$ an interesting question is to determine the probability that the dependency digraph contains a strongly connected component of `black` nodes. This probability increases with $|Q|$, and so we have to determine the size of Q for which this probability is $o(1)$.

Indeed, there is a trade off to determine for operators between the number of clients for which no traffic interruption is allowed and the blocking probability. When the number of such client is large, then it is almost impossible to reconfigure the routing and so the blocking probability will be high.

B. Transformation

To compute the number of agents needed to process a digraph D containing `black` nodes, we can construct a digraph D^* from D without `black` nodes such that the process number of D^* equals the number of agents needed to process the initial digraph D with black nodes. Moreover, the process strategy for D directly follows the process strategy of D^*.

Definition 1: Let $D = (V, A)$ be a digraph and let $v \in V$. The digraph $D^*(v)$ is obtained from D by removing v and adding an arc from every vertex of $N^-(v)$ to every vertex of $N^+(v)$. Note that the preceding transformation can yield loops if $N^-(v) \cap N^+(v) \neq \emptyset$.

For example, let D be the digraph depicted in Fig. 2(c) and let d be a `black` node. Using above transformation, we obtain the digraph D^* represented in Fig. 2(e). Since $N^-(d) \cap N^+(d) = \{b, c\}$, nodes b and c have loops in D^*. We have $\mathrm{pn(D^*)} = 2$ and from a process strategy for D^*, we can deduce a process strategy for $(D, \{d\})$, as proved in Proposition 2.

Proposition 2: For every digraph $D = (V, A)$ and set $Q := \{v\}$ for some vertex $v \in V$ which has no loop, we have $\mathrm{pn^*(D, Q) = pn(D^*(v))}$.

Proof: Let P^* be a processing strategy for (D, Q). We apply it to $D^*(v)$, ignoring the step that processes the vertex v. This processes the whole graph $D^*(v)$: there can only be a problem when we process a vertex $u \in N_D^-(v)$. But when any such step is reached, the vertex v has already been processed earlier since P^* is a valid strategy. Consequently, all the vertices of $N_D^+(v)$ are already processed or covered by an agent, and the vertex u can safely be processed. Thus, $\mathrm{pn(D^*(v)) \leq pn^*(D, Q)}$.

Conversely, let P be a processing strategy for $D^*(v)$. We obtain P^* from P by adding the step "process v" just before the first vertex of $N^-(v)$ is processed. By the definition of $D^*(v)$, all the vertices of $N^+(v)$ must be already processed or covered by an agent. Consequently, we obtain a valid processing strategy for (D, Q), and thus $\mathrm{pn^*(D, Q) \leq pn(D^*(v))}$. ■

More generally, by recursively applying Proposition 2, given a dependency digraph $D = (V, A)$ and a set $Q \subseteq V$ of `black` nodes, we can construct digraph D^* without `black` nodes and deduce the process strategy of (D, Q) from the process strategy for D^*.

Corollary 1: Let $D = (V, A)$ be a digraph and let $Q := \{v_1, v_2, \ldots, v_f\} \subseteq V$. We set $D_1 := D^*(v_1)$ and, for each $i \in \{2, 3, \ldots, f\}$, we set $D(v_{i+1}) := D_i^*(v_{i+1})$ if v_{i+1} has no loop in V_i. Then, $p^*(D, Q)$ is finite if and only if D_f is defined, and then $\mathrm{pn^*(D, Q) = pn(D_f)}$.

Notice that if v_{i+1} has a loop in V_i, then D contains a direct cycle of `black` nodes and so the digraph cannot be processed.

To conclude this section, we prove an upper bound on the amount of extra agents needed to process a dependency digraph $D = (V, A)$ containing `black` nodes compared to the process number of D without `black` nodes. For $Q \subseteq V$, the set of vertices that are outneighbors of some vertex in Q is $N^+(Q)$.

Proposition 3: For every digraph $D = (V, A)$ and every set $Q \subseteq V$ of `black` nodes, $\mathrm{pn^*(D, Q) \leq pn(D) + |N^+(Q)|}$. Moreover, this bound is asymptotically tight.

Proof: Let P be a processing strategy for D using $\mathrm{pn(D)}$ agents. Let us apply P to (D, Q) with the following adaptations. At every step of P that consists of placing an agent at some vertex $v \in Q$, we replace this operation by the sequence of operations that consists of placing an agent at every node in $N^+(v)$. If a vertex in $N^+(v)$ is a `black` node, we also place an agent at every vertex of its outneighborhood and so on, recursively. Then, all the `black` nodes considered at this step are processed. Now, if at some step of the process strategy, P places an agent at an already occupied vertex (this may happen if this vertex is the outneighbor of a `black` node), then we simply skip this step. Finally, if P processes a vertex that is not occupied (in P) but that is occupied by an agent according to the modified strategy, then this agent is removed. The obtained strategy is a process strategy for (D, Q), because when a vertex v of D is occupied by an agent in P, either it is also the case in our strategy, or v is a `black` node and has been processed. Moreover, the modified strategy uses at most $\mathrm{pn(D) + |N^+(Q)|}$ agents.

To prove that this bound is tight, let $k \geq 1$. Consider the digraph D composed of k symmetric cliques $C_1, C_2, \ldots, C_k$ of size k, and of the $k + 1$ vertices $v_0, \cdots, v_k$. For any $i \in \{1, 2, \ldots, k-1\}$, add the arcs from v_0 to v_i and from v_i to all the vertices of C_i, and add all the arcs from C_i to C_{i+1} and from C_i to v_0. We first prove that $\mathrm{pn(D) = k}$: the strategy consists in placing an agent at v_0, and, for i from k down to 1, processing C_i (using additional $k - 1$ agents), and then v_i. To conclude, let $Q = \{v_0, \cdots, v_k\}$. We prove that $\mathrm{pn^*(D, Q) = \sum_{i \leq k} |C_i| = k^2}$. By applying the transformation described above at the vertices $v_1, \cdots, v_k$, we obtain a digraph in which the `black` node v_0 is an in-neighbor of all the vertices in $\bigcup_{i \leq k} C_i$. The result follows from Proposition 2. ■

IV. Heuristic Algorithms

In this section we present an heuristic for the reconfiguration problem. This heuristic is based on an improvement of a previous heuristic [10]. We first recall it and then we further study its behavior and possible improvements. Then, we explain how this heuristic can be used when the network contains some `black` nodes.

A. Principles of the heuristic of Coudert and Mazauric [10]

Before going details, let us notice that the process number of a digraph is simply the maximum over the process numbers of its strongly connected components. Therefore, at each step of the execution of the heuristic of Coudert and Mazauric [10], a similar process will be executed to any strongly connected components of the digraph.

The heuristic of Coudert and Mazauric [10] proceeds as follows. At each step, an unprocessed and unoccupied vertex is chosen. Then, an agent is placed at this vertex and the protocol processes all possible vertices. Moreover, the agents occupying a processed vertex are removed. The main part of the heuristic consists in choosing the vertex where an agent will be placed. For this purpose, the *flow circulation* method is used. Intuitively, every vertex of the network wants to be processed as soon as possible without being interrupted. In that regards, the better choice for a vertex v would be to place an agent at one of its outneighbors. The flow circulation method somehow models a voting process at the end of which the vertex that maximizes the happiness of all the vertices is chosen to receive an agent.

Let us describe the flow circulation method more formally. Consider a strongly connected component C of the digraph induced by the vertices that are neither occupied nor processed yet. A weight is assigned to any vertex of C, then a flow circulates in the following way. At each round, any vertex u in C sends a uniform fraction of its weight to its out-neighbors. Simultaneously, u receives a fraction of the weight of its in-neighbors and updates its new weight as the sum of the flows it has received. The same process is executed during $|V|$ rounds and the vertex with maximum final weight is chosen. By modeling this process by a discrete Markov chain, Coudert and Mazauric [10] proved that the vector of weights always admits a unique stationary vector and that convergence is achieved after $|V|$ rounds. In Fig. 2(d), we can show the digraph of Fig. 2(c) from a Markov point of view to apply the flow circulation method.

Lemma 2 ([10]): The worst case time complexity of the proposed heuristic is $O(n^2(n+m))$.

B. Improvements

In this section, we further study the behavior of the heuristic described above and propose a modification allowing to improve its performances.

To illustrate the behavior of the previous heuristic, let us consider the graph defined by a path of cliques. More precisely, the (symmetric) graph G is composed of $r > 0$ cliques $C_1, \cdots, C_r$ each of size 6. Moreover, C_i intersects C_{i-1} and C_{i+1} in 2 vertices, and no other cliques intersect C_i for $0 < i < r$. The optimal process strategy for G consists in placing agents at the vertices in $C_1 \cap C_2$. Then, more agents are placed at the vertices of C_1 but in one vertex in $C_1 \setminus C_2$, that is then processed. Then, all agents at $C_1 \setminus C_2$ are removed and placed at $C_2 \cap C_3$. Again, other agents are placed at the yet unoccupied vertices of $C_2 \setminus C_3$ but one, that is then processed. And so on. Such a strategy uses exactly 5 agents.

In this particular class of graph, we can observe a faulty behavior of the heuristic. Indeed, while the first steps executed by the algorithm consists in placing agents at the vertices of C_1, once the vertices in $C_1 \setminus C_2$ have been processed, the algorithm places agents at the vertices in $C_3 \setminus C_4$. Finally, some agents are placed in the vertices in $C_1 \setminus C_2$ and the vertices in C_2 are occupied and processed. Therefore, at least 7 agents are used. It is worth to notice that during the processing of the vertices in C_2, the agents occupying $C_3 \setminus C_4$ are useless and should not be taken into account. The new heuristic that we propose takes advantage of this remark.

The above remark leads to the following improvement of the heuristic of [10]. At every step of the execution of the previous algorithm, after having chosen a vertex at which an agent will be placed, the vertices on which rule R_2 applies are processed. In our algorithm, before processing those nodes (and only if at least one such node exists), we remove all agents that are occupying a node v not in-neighbors of which have been processed previously. Indeed, such a vertex v has not been used before in contrast with another kind of vertices occupied by agents that cannot be removed: those occupied nodes belonging to the in-neighborhood of some processed node u that have been used for the processing of u. The agents that have been removed at this stage are not taken into account in the count of the number of agents.

It is straightforward to adapt the heuristic to digraphs containing `black` nodes. Indeed, it is not necessary to implement the transformation described in Sec. III. We rather proceed in the following way: when choosing a candidate vertex to place an agent, `black` nodes are not considered. Thus, the complexity of the algorithm is unchanged.

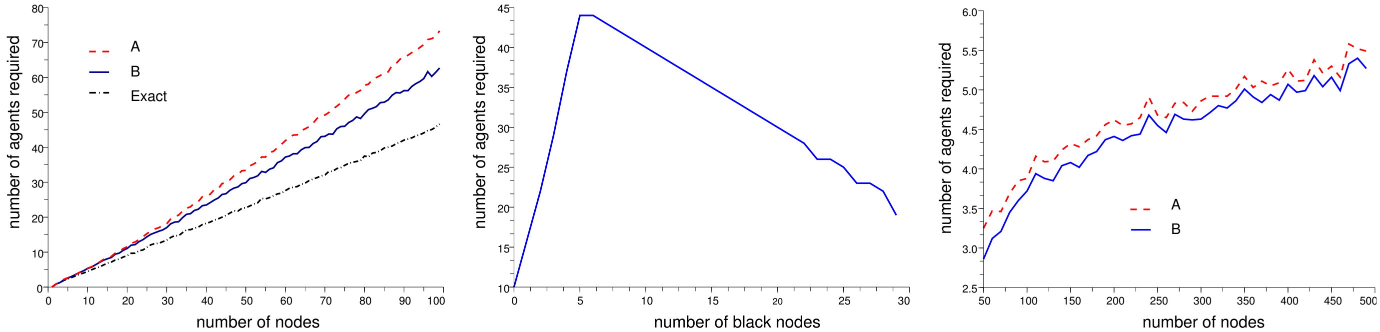
V. Simulations

To validate the heuristic algorithm proposed above, we have done experiments on different topologies. Recall that computing the process number is NP-complete in general. Therefore, and to make fair comparisons, we have restricted ourself to digraph classes for which efficient exact algorithms (or good approximations) computing the process number have been designed. For each type of topology, described below, and for a range of network sizes, we create 100 random instances of networks belonging to this type. Then we run on each of them the two heuristics described, and an algorithm computing the pathwidth (and so the process number up to one). The results are presented in Fig. 3.

Circular arc graphs are particularly interesting because they corresponds to the case where the physical topology is a ring. A graph is a *circular arc graph* if (i) its vertices can be represented by a subinterval over the circular interval $[1, n]$ and (ii) two vertices are adjacent if the corresponding interval intersect. When the network is a ring, each communication between processors represents an interval. Therefore, when proceeding to rerouting, two paths interfer if the corresponding intervals intersect. This yields to a dependency digraph that is precisely a circular arc graph. To make comparisons, we have also implemented an algorithm to compute optimally the pathwidth of circular arc graphs [16]. Fig. 3(a) shows that our heuristic algorithm (B) is better than (A) [10], and that the deviation from the optimal value is small.

Digraphs with process number 2 have been characterized, and a polynomial time algorithm to decide whether a digraph has process number 2 has been given [2]. On such digraphs, the improvement of the new heuristic algorithm (B) is small compared to heuristic algorithm (A), as reported in Fig. 3(c). However, it shows that both algorithms are relatively fast, since it took us only one minute to perform all the simulations depicted in Fig. 3(c) on a standard laptop.

We now consider *circulant* digraphs with n vertices $(v_1, \ldots, v_n)$ and arcs from v_i to $\{v_{i+1}, v_{i+2}, \ldots, v_{i+k}\}$, for $i = 1 \ldots n$ and for some fixed k (indexes are taken modulo n). In particular, the directed cycle $(v_1, \ldots, v_n)$ is a subgraph of D. Simulation has been done in order to illustrate the impact of the `black` nodes in terms of the process number (see Proposition 3). In absence of `black` nodes, the optimal



(a) Circular arc graphs

(b) Circulant digraphs with 50 nodes and arcs from node i to nodes $\{i+1, i+2, \ldots, i+10\}$.

(c) 2-digraphs

Fig. 3. Simulation results on circular arc graphs, circulant digraphs and 2-digraphs. The heuristic algorithm of Coudert and Mazauric [10] corresponds to A, and the heuristic algorithm proposed in this paper corresponds to B. For each size of digraph, 100 digraphs are randomly generated for our experiments.

process strategy uses k agents and consists of the following: place agents at $(v_1, \ldots, v_k)$ and then iteratively processes v_n, v_{n-1} to v_{k+1}. Without any `black` node, our heuristic computes this optimal process number for any pair n, k. In Fig. 3(b), we consider a circulant graph with 50 nodes and outdegree $k = 10$. We gradually increase the number $|Q|$ of `black` nodes. For each value of $|Q|$, we run 100 tests with random positions of the `black` nodes and plot the maximum value found. While $|Q| \leq 5$, with high probability, the `black` nodes create a configuration similar to the one of the graph defined in the proof of Proposition 3, increasing the number of agents up to $\sum_{v\ \mathtt{black}_{node}} |N^+(v)|$. When $|Q|$ increases too much, with high probability, strongly connected components consisting of `black` nodes are created leading to many impossible reconfigurations. Simultaneously, when $|Q|$ is large enough, the only valid configurations are very specific and forbid situations like in Proposition 3.

VI. Conclusion

In this paper, we have investigated the reconfiguration problem: switching connections routed in a WDM backbone network from the current routing to a pre-determined destination routing under the constraints that (1) connections are switched one after the other, and (2) some connections may refuse any temporary interruption. This last constraint, modeled using `black` nodes in the dependency digraph, may cause impossibility situation for the reconfiguration and so the number of `black` nodes must remain small. Then, we have shown how to handle `black` nodes in the process strategy by constructing a new dependency digraph without `black` nodes. Finally, we have proposed a new heuristic algorithm that performs well according to our experiments.

An interesting open question is now to evaluate the number of priority clients that an operator may accept in the network in order to ensure the feasibility of the reconfiguration with high probability. Another important issue is also to validate our heuristic by considering realistic network environments.

Acknowledgments

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