Fine partitions of planar graphs

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Fix a non-negative integer k. Let $G = (V, E)$ be a (connected) graph. We want to partition the vertex-set V into two parts V_1 and V_2 such that the subgraph $G[V_i]$ of G induced by V_i has maximum degree at most k, for each $i \in \{1, 2\}$. Such a partition is *fine*.

Knowing whether a fine partition exists is NP-complete in general, and stays NP-complete even when restricted to the class of planar graphs [2]. On the other hand, if G has maximum degree at most $2k + 1$, then a fine partition always exists: take one corresponding to a maximum cut.

What if we consider only planar graphs of maximum degree at most $2k + 2$? If $k = 0$, then we can find one if and only if G is not an odd cycle.

It was shown [1] that, if $k \in \{1, 2\}$, then the corresponding decision problem is NP-complete. Yet, the existence of an integer $K \geq 3$ such that for all $k \geq K$, every planar graph of maximum degree at most $2k+2$ admits a fine partition is conjectured [1].

Any progress on this conjecture would be nice. We can also consider non-symmetric versions of the problem.

References

[1] R. Corrêa, F. Havet, J.-S. Sereni: About a Brooks-type theorem for improper colouring. *Australasian Journal of Combinatorics*, 43:219–230, 2009.

[2] L. Cowen, W. Goddard, C. E. Jesurum: Defective coloring revisited. *J. Graph Theory* 24(3):205–219, 1997.