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Workpackage 4: Algorithmic Solutions and Technical Recommendations for Optical Networks I

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Algorithmic Solutions and Technical Recommendations for Optical Networks I

Responsible Partner: Centre National de la Recherche Scientific (F) & Universite de Nice-Sophia Antipolis (F)

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Partners:
Computer Technology Institute (GR)
University of Geneva (CH)
Centre National de la Recherche Scientific (F)
Universite de Nice-Sophia Antipolis (F)
Christian-Albrechts-Universitaet zu Kiel (D)
Universita degli studi di Salerno (IT)
Universita degli studi di Roma “Tor Vergata” (IT)

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In the state-of-the-art survey for WP4 [1] we have presented some important algorithmic challenges for the design and planning of networks. In this report we stress the problems and solutions we have investigated during the first year of the project.

This includes providing effective wavelength routing and traffic grooming algorithms, but also some theoretical results for the resources allocation problem. After a short introduction on networks we will develop these three aspects of the problem along with the solutions we have proposed. In the recommendations given at the end of each section we summarize what should be addressed in the next coming research work.

Then we conclude with the description of future works and expected results.

1 WDM Networks

The Wavelength Division Multiplexing () technology, a variation of Frequency Division Multiplexing for fiber optic channels, makes it possible to optimally use the optical fibers - often already installed - by better utilizing their available capacities. This is achieved through multiplexing several wavelength channels onto the same fiber. In networks, the huge bandwidth available on an optical fiber is divided into multiple channels. Each channel carries bandwidth up to several gigabits per second. A minimum unit of resource allocation is an optical channel, which consists of a route and a wavelength assigned on each link along the route and is called a *lightpath*. If wavelength translation is performed in optical switching, then each channel may be assigned different wavelengths on each link along the route; otherwise the wavelength continuity constraint must be satisfied on all links along the route. Of course, two lightpaths sharing a link must use different wavelengths on that link.

In the following sections we will give more definitions when necessary.

2 Optical routing

One of the key objectives of optical routing is to allow true *all-optical* networking. The new optical backbones will allow the transport of both SDH (Synchronous Digital Hierarchy) circuits and wavelengths in the optical domain without electronical/optical conversion as it is often the case today. In this section we present new results for wavelength assignment covering two important parameters that are usually omitted in the classical Wavelength Assignment Problem: links of the network are made of several fibers and a limited number of wavelength translators may be used.

2.1 Design of Multifiber WDM Networks [11, 12]

In this papers, we study multifiber optical networks with Wavelength Division Multiplexing (WDM). Assuming that the lightpaths use the same wavelength from source to destination, we extend the definition of the well-known Wavelength Assignment Problem (WAP) to the case where there are k fibers per link, and w wavelengths per fiber are available. This generalization is called the (k, w)-WAP. We develop a new model for the

(k,w)-WAP, based on *conflict hypergraphs*. Conflict hypergraphs more accurately capture the lightpath interdependencies, generalizing the conflict graphs used for single-fiber networks. By relating the (k,w)-WAP with the hypergraph coloring problem, we prove that the former is $\mathcal{NP}$-complete, and present further results with respect to the complexity of that problem. We consider the two natural optimization problems that arise from the (k,w)-WAP : the problem of minimizing k given w, and that of minimizing w given k. We develop and analyze the practical performances of two methodologies based on hypergraph coloring, one for each of the two optimization problems, on existing backbone networks in Europe and in the USA. The first methodology relies on two heuristics based on a randomized approximation algorithm and the second consists on an integer programming formulation.

2.2 Lightpath Assignment for Multifibers WDM Optical Networks with Wavelength Translators [10, 8, 9]

We consider the problem of finding a lightpath assignment for a given set of communication requests on a multifiber WDM optical network with wavelength translators. Given such a network, and w the number of wavelengths available on each fiber, k the number of fiber per link and c the number of partial wavelength translation available on each node, our problem stands for deciding whether it is possible to find a *w-lightpath* for each request in the set such that there is no link carrying more that k lightpaths using the same wavelength nor node where more than c wavelength translations take place. Our main theoretical result is the writing of this problem as a particular instance of integral multicommodity flow, hence integrating routing and wavelength assignment in the same model. We then provide three heuristics mainly based upon randomized rounding of fractional multicommodity flow and enhancements that are three different answers to the trade-off between efficiency and tightness of approximation and discuss their practical performances on both theoretical and real-world instances.

2.3 Wavelength Assignment in Trees and Rings [2, 7, 6]

In the context of the standard wavelength assignment problem we consider two important network topologies: bidirected trees and undirected rings.

For trees, one direction of research [2] was to define the class of greedy wavelength assignment algorithms that use randomization, extending in this way the class of deterministic greedy algorithms which have been studied extensively in the last years. We studied the limitations of these algorithms proving several lower bounds while we developed a wavelength assignment algorithm that assigns at most $7L/5+o(L)$ wavelengths on any set of paths of load L on a binary tree network of depth $o(L^{1/3})$, with high probability. For the analysis of the upper and lower bounds, new tail inequalities for random variables following hypergeometrical probability distributions were developed.

The randomized wavelength assignment algorithm can be thought of as applying some kind of randomized rounding on fractional path colorings. These are equivalent to solutions to the natural relaxation of the integer linear program corresponding to the wavelength assignment instance. In [7], we show that each symmetric set of paths of load L on a binary tree has a fractional path coloring of cost $1.367L$. By applying similar techniques with those in [2], we obtain an $1.367 + o(1)$ approximation algorithm for wavelength assignment of symmetric sets of paths on binary trees (again, with some restrictions on the depth of the tree). Symmetric sets of paths are important since many services that will be supported by all-optical networks in the future will require bidirectional reservation of optical bandwidth.

Another direction of research was to try to approximate the optimal solution of the wavelength assignment problem using almost optimal solutions to the corresponding fractional path coloring problem as a guide. In [6], by simplifying and extending previous work, we present polynomial time algorithms that compute almost optimal fractional path colorings in bounded-degree trees and in rings. Our methods cover the case of multiple fibers as well. By applying a novel randomized rounding technique and using known wavelength assignment algorithms as subroutines, we obtain approximation algorithms with improved approximation ratios in bounded-degree trees and in rings with one or multiple fibers. The randomized rounding technique can be applied to any network. Our approach gives improved existential upper bounds for the cost of the optimal wavelength assignment as a function of the cost of the corresponding optimal fractional path coloring solution. For the analysis, we use new tail inequalities for generalizations of occupancy problems.

Recommendations

The multifiber model gives a better modelization of networks but the complexity of the coloring problem still dramatically increase with the number of available wavelengths per fiber. Then, Integer Linear Programming formulation of the problem cannot be computed in reasonable time and space. Randomized rounding of the multicommodity flow LP-relaxation and heuristics have to be used, and we should provide efficient algorithmic solutions using these mixed strategies for real size networks.

3 Traffic Grooming

Efficient optical routing aims to minimize the number of different wavelengths used in the network but also the number of electronical/optical conversions (hops for lightpaths). Another way for reducing the cost of the network is to group the traffic in such a way that some units of traffic may share some optical channels.

3.1 Traffic Grooming in Unidirectional WDM Ring Networks [3, 4]

In [3, 4] we address the problem of traffic grooming in WDM rings with all-to-all uniform unitary traffic. The goal is to minimize the total number of SONET add-drop multiplexers (ADMs) required. We have shown that this problem corresponds to a partition of the edges of the complete graph into subgraphs, where each subgraph has at most C edges (where

C is the grooming ratio) and where the total number of vertices has to be minimized. Using tools of graph and design theory, we optimally solve the problem for practical values and infinite congruence classes of values for a given C. Among others, we give optimal constructions when $C \geq N(N-1)/6$ and results when $C = 12$. We also show how to improve lower bounds by using refined counting techniques, and how to use efficiently an ILP program by restricting the search space.

3.2 Traffic Grooming in WDM Networks with Multi-Layer Switches [14]

We develop traffic grooming algorithms for WDM networks with multi-layer switches. We consider a node as an N-layer switch, in which a given layer k is an aggregated set of elements of layer $k-1$. Typical examples of layers are wavelengths, bands and fibers. The cost of a given node depends on the number of input and output ports of each layer. Assuming this model and a traffic matrix - with unity elements in layer 0 - minimizing the cost of the network will consist in grooming traffic in such a way that as much traffic as possible will be switched in the higher possible layer (fibers in our example).

When some traffic is switched along a path in the network within the same layer, we represent it with a pipe. Each pipe has an associated linear cost depending on the current layer and on the number of nodes crossed in that pipe.

We present an integer linear programming formulation for this model that aims to minimize the overall cost of the network for a given input traffic matrix. We ran experiments using the CPLEX optimization package on various topologies such as actual networks like the Pan-European all optical network as well as rings and meshes of various sizes.

Recommendations

Traffic grooming is another hard problem. In the specific case of rings, one may found optimal solutions based on design theory but in the case of mesh networks, or when the traffic pattern is irregular, no practical good solution is known. One may want to design mesh networks built up with rings and study how to take advantage of the underlying topology.

Actually, the traffic grooming problem occurs at different levels in the network, starting from the virtual connections of the traffic demand matrix down to the physical fibers of the optical backbone. Each upper level has to be groomed into a lower level, corresponding to the embedding of a virtual guest graph into another virtual host graph. The precise model for this process has to be study first. We have introduced a model of pipes that we believe general enough to cover various type of networks such as MPLS, ATM, SDH and WDM. Now we should develop new algorithms using this model.

Then after, we intend to study the impact of routing over grooming.

4 Resource Allocation

Many resource allocation problems can be formalized by specific problems that are called packing or covering problems. Hence, in order to solve real size instances it is essential to rely on a fast algorithm solving, even approximatively, the relaxed problem (real numbers). Hence we propose a fast approximation algorithm for the packing problem in the following section.

The list coloring problem is another way to model many allocation problems such as frequency allocation or .

4.1 Approximation algorithms for the packing problem [16, 15]

We present an approximation algorithm based on Lagrangian decomposition via a logarithmic potential reduction to solve a general packing or min-max resource sharing problem with M nonnegative convex constraints on a convex set B. We generalize a method by Grigoriadis and Khachiyan to the case with weak approximate block solvers (i.e. with only constant, logarithmic or even worse approximation ratios). We show that the algorithm needs at most $O(M(\epsilon^{-2} \ln \epsilon^{-1} + \log M))$ calls to the block solver, a bound independent on the data and the approximation ratio of the block solver. For small approximation ratios the algorithm needs at most $O(M(\epsilon^{-2} + \ln M))$ calls to the block solver.

As an application of our the method above, we study the problem of minimizing the maximum edge congestion in a multicast communication network. We are given a graph $G = (V, E)$ to represent a communication network where $|V| = n$ and $|E| = m$ and a set of multicast requests $S_1, \ldots, S_k \subseteq V$. A solution is a set of k trees $T_1, \ldots, T_k$ where T_i connects the vertices of S_i. The goal of the *multicast congestion* problem is to find a solution of k trees minimizing the maximum number of times an edge is used. Interestingly the underlying block problem here is the classical m minimum Steiner tree problem that can be solved only approximately. We show how to use approximation algorithms for the minimum Steiner tree problem to solve the edge congestion problem approximately.

4.2 Choosability of bipartite graphs with maximum degree Δ [5]

The list coloring problem is a variation and generalization of the well-known problem of coloring the vertices of a graph with as few colors as possible so that adjacent vertices get distinct colors. The additional requirement in this concept is that every vertex v has to be colored with a color from a set $L(v)$ of allowed colors which is assigned to every vertex of the graph.

Let $G = (V(G), E(G))$ be a graph. A *list assignment* is an assignment of a set $L(v)$ of integers to every vertex v of G. An L-coloring is an application C from $V(G)$ into the set of integers such that $C(v) \in L(v)$ for all $v \in V(G)$ and $C(u) \neq C(v)$ if u and v are joined by an edge. A (k, k')-*list assignment* of a bipartite graph G with bipartition (A, B) is a list assignment L such that $|L(v)| = k$ if $v \in A$ and $|L(v)| = k'$ if $v \in B$. A bipartite graph is (k, k')-*choosable* if it admits an L-coloring for every $(k.k')$-list assignment L. In this paper, we study the (k,k')-choosability of graphs. Alon and Tarsi (92) proved in an algebraic and non-constructive way, that every bipartite graph with maximum degree Δ is $(\lceil \Delta/2 \rceil + 1, \lfloor \Delta/2 \rfloor + 1)$-choosable. In this paper, we give an alternative and constructive proof to this result. We conjecture that this result is sharp (i.e. there is a bipartite graph with maximum degree Δ that is not $(\lceil \Delta/2 \rceil, \lfloor \Delta/2 \rfloor + 1)$-choosable) and prove it for $\Delta \leq 5$.

Moreover, for a fixed $\Delta \in \{4,5\}$, we show that given a bipartite graph with maximum degree Δ and a $(\lceil \Delta/2 \rceil, \lfloor \Delta/2) \rfloor + 1)$-list assignment L, it is NP-complete to decide if G is L-colorable. At last, we give upper bounds for the minimum size $n_3(\Delta)$ of a non $(3,3)$-choosable bipartite graph with maximum degree Δ: $n_3(5) \leq 846$ and $n_3(6) \leq 128$.

4.3 The Station Placement Problem [13]

The paper introduces a new resource location problem which we call the Station Placement (SP) problem. Let $G = (V, E)$ be a graph, s be a distiguished vertex, l be a function that associates to each edge in the graph a positive integer, k an integer less than or equal to $|V|$ and D be a subset of V. A solution to the k-SP problem is a family $(S_1, ..., S_k, S_{k+1} = D)$ along with a set of functions $(\phi_1, \ldots, \phi_k)$ such that ϕ_i maps S_{i+1} to $S_i \cup \{s\}$ that minimizes the cost function $\sum_{i=0}^{k} \sum_{v \in S_{i+1}} w(\phi_i(v), v)$.

Although formulated as a resource location problem, the SP problem has application in multicasting in communication networks. Assume that the source s wants to send a message to the vertices in D. A possible algorithm for the multicasting could be as follows: identify a set S_1 of intermediate stations and assign each vertex of D to a vertex of S_1. The communication takes places in two steps. First, s establishes a virtual circuit with each of the vertices of S_1 and transfers the message. In the second phase, each vertex of S_1 establishes circuits with its assigned destinations so that the message is finally delivered to the destination vertices. In general, one can have k sets $S_1, \cdots, S_k, S_{k+1} = D$ of intermediate stations. Notice that the cost function defined above is exactly the cost of the multicast operation.

In the paper we proved that the k-SP problem is NP-Complete on general graphs by reducing its decision version to Set Cover. We also give a dynamic programming algorithm to solve the 1-SP problem on directed trees. We also give a $O(\log |D|)$ approximation algorithm for the k-SP on general graphs. We also study two variants of the SP problem: the Restricted Station Placement and the Generalized Restricted Station Placement problem.

Recommendations

The practical use of these algorithms is not straightforward for networks and we will focus on applying these tools to routing and grooming algorithms.

5 Future Work

Part of the future work was described in the recommendations, but other issues have to be addressed in this project.

5.1 Survivability

The all-optical routing cited in section 2 really makes sense when combined with protection and restoration, as it is today one important value of the SDH service. This involves provisioning of services over disjoint lightpaths and this increases dramatically the complexity of routing. As we did for grooming, this problem will be studied for rings and meshes.

5.2 Virtual topologies

As already stated, the wavelength assignment problem and the grooming problem are not independents. We intend to improve our model of virtual network in order to be able to gather and grooming and also be able to deal with the IP/ATM/SDH/WDM multilayer scenario.

5.3 Dynamic traffic

The static traffic scenario has to be studied first and is already hard to solve. However we should investigate the dynamic scenario, when some connection may change along the time. This seems to be an important issue in the study of MPLS networks.

5.4 Experimentation

Implementations of all the algorithms proposed here will be tested against existing networks. We already have developed a first software able to compute routing and grooming for a simple network model.

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