Crescco	CRESCCO IST-2001-33135 Critical Resource Sharing for Cooperation in Complex Systems

Deliverable D4.1
Optical Networks: State-of-the-art Survey

Responsible Partner: Centre National de la Recherche Scientific (F) & Universite de Nice-Sophia Antipolis (F)

Report Version: 1.0
Report Preparation Date: 02/08/02
Classification: PUB
Contract Start Date: 01/01/02 Duration: 36 months
Project Co-ordinator: University of Patras

Partners:
Computer Technology Institute (GR)
University of Geneva (CH)
Centre National de la Recherche Scientific (F)
Universite de Nice-Sophia Antipolis (F)
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information society technologies

Project funded by the European Community under the "Information Society Technologies" Programme (1998-2002)

Efficient Access to Optical Bandwidth Routing and Grooming in WDM Networks: state-of-the-art survey

Abstract

In this survey we present some important algorithmic challenges for the design and planning of WDM networks. This includes wavelength routing and traffic grooming. In both cases the problem is to compute lightpaths in order to carry a traffic demand over the network, but the objectives are different. In the case of wavelength routing and assignment (WRA for short) the problem is to minimize the number of different wavelengths used in the network or the maximum congestion over one link. While in the case of the grooming problem the objective is to minimize the cost of the end to end equipment into the nodes by grouping the traffic requirements. These two objectives may be contradictory and should be balanced into a global processing of the design of the network.

When the network topology is fixed (e.g. rings, grids, or trees) and when the traffic pattern is also fixed (e.g. unitary *all-to-all*) some closed formulaes are known about the maximum number of wavelengths required for routing or the optimal number of OADMs required in the nodes.

In the case of mesh networks (arbitrary topology), no exact solutions are known and the detailed integer linear formulation are not efficient as they lead to excessive computation time. Then one has to describe simpler models or heuristics to give practical solutions.

1 Introduction

The emergence of wavelength division multiplexing (WDM) networking components allows carriers to build metro networks provided with access and transport functionalities as well as wide area backbone networks. The WDM technology, a variation of Frequency Division Multiplexing for fiber optic channels, makes it possible to optimally use the optical fibers - often already installed - by better utilizing their available capacities. This is achieved through multiplexing several wavelength channels onto the same fiber. In WDM networks, the huge bandwidth available on an optical fiber is divided into multiple channels. Each channel carries bandwidth up to several gigabits per second. A minimum unit of resource allocation is an optical channel, which consists of a route and a wavelength assigned on each link along the route and is called a *lightpath*. If wavelength translation is performed in optical switching, then each channel may be assigned different wavelengths on each link along the route; otherwise the wavelength continuity constraint must be satisfied on all links along the route. Of course, two lightpaths sharing a link must use different wavelengths on that link.

A lot of undergoing projects plan to construct networks with hundred of WDM nodes (see [1] for instance). Planning such networks is a complex task and a lot of undergoing research is currently done with emphasis on routing and grooming the traffic. Such networks carry wavelengths paths - or lightpaths - multiplexed onto fibers. An example of a WDM network is provided in Fig 1 with the European backbone defined in the COST239 action [40].

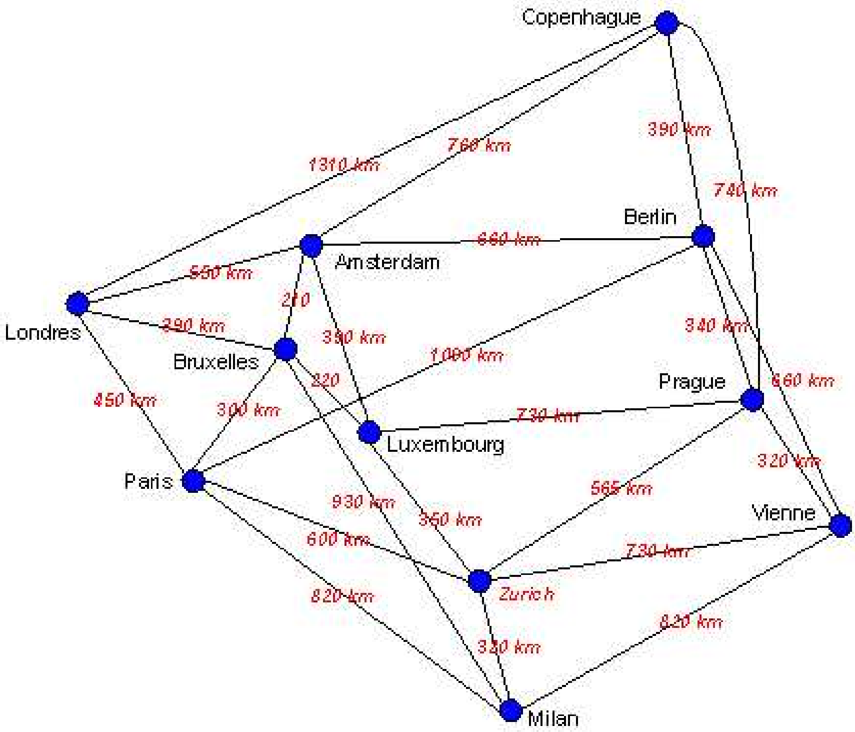

Figure 1: European Project COST239 WDM Network

The equipment at the nodes consists of optical add/drop multiplexers (OADM) and fiber or wavelength optical crossconnects (W-OXC/F-OXC). The OADMs equipments realize the

[1] `http://www.velocita.com`

interface with the electrical layer, and the OXCs equipments realize the switching in the optical layer.

The objective of the planning process is to minimize the size (number of input/output ports) of W-OXC switch matrices. Indeed, each time a fiber has to be demultiplexed at a node (extracting wavelengths from the input fiber and multiplexing them again onto the output fibers of the F-OXC), a W-OXC of size equal to the number of wavelengths included in the fiber has to be used. Current WDM technology multiplexes fibers with more than a hundred wavelengths channels. Hence, the size of the W-OXC used should be (order of) a hundred times the number of fiber ports attached to the F-OXCs. In the last generation of equipments, the use of wavebands is also taken into consideration. The concept of wavebands was introduced in WDM ring networks in [52, 94]. A waveband (band for short) is made of a set of wavelengths and is routed across the nodes as a single channel, using a single port of a waveband optical crossconnect (B-OXC) (see Fig 2 which shows an Optical Cross Connect (OXC) for two-stage multiplexing).

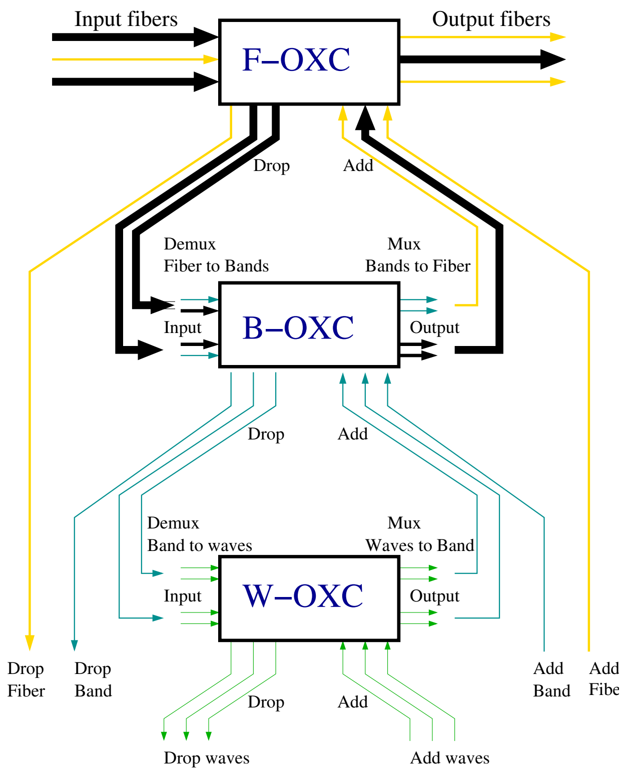

Figure 2: A 3-layer switch

Thus, if it is possible to divide the set of wavelengths into groups or wavebands (of say 8 wavelengths for instance), then, a ratio of the size of a group (8 for instance) might be saved in the number of ports used to switch the traffic at the B-OXC layer. Of course, this is not possible when all the wavelengths use different lightpaths and different fibers across the node. Therefore, the objective is to compute paths such that groups of lightpaths use common sub-paths as far as possible, and then to map these groups onto wavebands. This is what is called grooming, and in fact this grouping applies at different layers: wavelengths are groomed into wavebands and wavebands are groomed into fibers. Note that this grooming problem is also studied in the case of grouping OC-3 or OC-12 onto wavelengths in SONET rings. Instead of minimizing the size of W-OXCs or B-OXCs the problem is to minimize the number of add/drop multiplexers and optimal or near optimal solutions were proposed for some specific problems like WDM ring networks with *all-to-all* uniform traffic (see section 3.4.1).

In the following sections we will first present the wavelength routing problem and the solution proposed in the literature. Then we will present the traffic grooming problem and some survivability issues. Then we will conclude with the research perspectives in this domain. We have restricted ourselves to these problems but optical routing with converters and dynamic traffic (e.g. on-line traffic requests) are also important problems in WDM networks.

2 Optical routing

2.1 Generalities

Our definitions and general properties are valid for graphs and digraphs. We will always use the terms graph, path and edge, even if the right terms for digraphs should be digraph, dipath and arc (or directed edge). If a result is valid only for graphs or digraphs, it will be mentioned.

A traffic demand I is a multiset of ordered pairs $(x, y) \in V \times V$. All demands are assumed to be unitary. Actually, a demand with bandwidth b corresponds to b unit demands. This is equivalent to say that demands are *splittable*.

The optical routing problem is usually considered as the combination of two classical combinatorial problems:

- Routing : each traffic demand has to be assigned a path. The set of such paths is called a *routing* R for (G, I);

- Coloring : each path in R has to be assigned a wavelength, in such a way that any two conflicting paths (i.e. sharing an edge) are assigned different wavelengths;

- The goal is to minimize the total number of wavelengths used. We denote this minimum possible number by $\mathtt{w}(G, I)$.

The *conflict graph* of (G, I, R) is defined as the graph $C(G, I, R)$ whose vertices are the paths in R and where there is an edge connecting two paths *iff* they share an edge. We thus have to find a vertex coloring for $C(G, I, R)$. Somehow, $\mathtt{w}(G, I)$ is the minimum chromatic number among all possible conflict graphs: $\mathtt{w}(G, I) = \min_R \chi(C(G, I, R))$. Due to the relation with vertex coloring, we use the term color instead of wavelengths, and we say that we color the paths.

2.1.1 Path coloring problem

The *Path Coloring Problem*[2] consists in coloring the paths once a routing R is fixed. We denote $\mathtt{w}(G, I, R) = \chi(C(G, I, R))$ the minimum number of colors needed to color the paths of R in G. The Path Coloring Problem corresponds to the usual vertex coloring problem, restricted however to a specific class of graphs: namely the path intersection graphs. Those have been studied for long in the literature. For cycles they are called circular arc graphs and their coloring has been studied extensively [104, 65, 70]. Golumbic and Jamison [56] have also studied the case of undirected trees. They proved that this problem is equivalent to multigraph edge coloring. At last, Chlamtac, Ganz and Karmi [22] showed that, whenever G contains a large mesh, it is possible to define a set of paths such that the resulting conflict graph is simply any given graph. In the general case, the Path Coloring Problem is thus exactly the vertex coloring one.

Note that the Path Coloring Problem has some practical issues. First, due to technological constraints, the routing may be fixed in advance. Second, most of the existing optical routing algorithms are made of two steps: one that computes a routing and one that color then the paths. Note finally that in the case of tree networks, the optical routing reduces to the Path Coloring Problem, since there is only one "good" routing.

2.1.2 Routing

For two steps algorithms, designing a "good" routing is quite relevant. As it is not easy to figure out what a good routing is (no simple property ensures that a routing R minimizes $\chi(C(G, I, R))$, the following criteria is used : the maximum edge load.

The load of a routing R is the maximum number of paths sharing some edge, denoted by $\pi(G, I, R)$. As $\mathtt{w}(G, I, R) \geq \pi(G, I, R)$ for any routing R, solving the minimum maximum edge load problem can be of first importance for optical routing. We call it the *Minimum Load Problem*: finding a routing R such that $\pi(G, I, R)$ is minimum. This optimum value is denoted by $\pi(G, I)$.

The classical routing problem consists in deciding if a set of requests in some capacitated network is feasible or not. In the case of uniform capacity, the question is what is the

[2] also called Wavelength Assignment Problem (WAP)

minimum capacity value such that the requests can be satisfied, which corresponds actually to finding the minimum load of a routing. As we will see, the minimum load problem is NP-hard, but it can be approximated often quite well. Indeed, whenever $\pi(G, I)$ sufficiently large compare to $|E|$ by rounding randomly a fractional solution with the routine of Raghavan and Thomson [89].

Let us mention finally that finding an optical routing using k colors is equivalent to solving a routing problem for some induced graph mainly made of k copies of the initial graph. This equivalence is of little practical importance in general, since the induced routing problem is an edge-disjoint path problem, which is very hard to solve. Nevertheless, it is quite useful for multicasting instances of requests, or in the case of multifiber networks.

2.2 General hardness results

Most of the hardness results in this area rely on the hardness of the two sub-problems that we have defined: the Path Coloring Problem and the Minimum Load Problem.

2.2.1 Exact computation

The oldest hardness result is due to Fortune, Hopcroft and Willie, and in [44] they proved that deciding if two given requests can be routed in a directed network along edge-disjoint paths is NP-hard. In other words, on directed network the feasibility of the integral multicommodity flow problem is NP-hard, even with two unit commodities. This means that the optical routing problem is NP-hard even for directed networks with $|I| = 2$.

For undirected networks, an analogous result, proven by Even, Itai and Shamir [41], states that finding k edge-disjoint paths for k given requests in some undirected network is NP-hard. Therefore the optical routing problem is NP-hard for undirected networks, even if $\mathtt{w}(G, I) = 1$. Note that this last result is somehow optimal: from the results of Robertson and Seymour [93], k edge-disjoint paths can be found in time $f(k).|E|$ in any undirected graph with $|E|$ edges ($f(k)$ being extremely large). Note also that Jarry [62] proved that for symmetric directed graphs the situation is similar, that is: the multicommodity flow with bounded traffic k can be solved in time $f(k).|E|$.

Since optical routing is a specific edge-disjoint path problem, it follows that optical routing on undirected or symmetric networks is solvable in polynomial time when the traffic is bounded.

The case of specific topologies was first discussed by Erlebach and Jansen [36]. They proved that the optical routing problem in directed trees and cycles is NP-hard, by reducing it to the circular arc graph coloring. Note that this result does not hold when $\mathtt{w}(G, I)$ is bounded, since then a usual dynamic programming method allows to compute $\mathtt{w}(G, I)$ in time $n^{O(d\mathtt{w})}$ for bounded degree trees and for cycles.

In [37] they extended their hardness results to directed symmetric trees, cycles and grids. The hardness holds even when $\mathtt{w}(G, I, R) = 3$: they proved for trees that finding an optical routing using at most 3 colors is NP-hard. Their reduction relies on the fact that the optical routing problem reduces to multigraph edge coloring.

More generally, the optical routing problem is polynomial whenever the associated edge-disjoint path problem is so. For more results on the complexity of the edge-disjoint path problems on specific topologies, we refer to the book edited by Korte, Lovász, Prömel and Schrijver [67], and to the chapter of A. Frank [46].

2.2.2 Hardness of Approximation

Most of the hardness proofs for specific topologies do not extend to any hardness result about the approximability of the optical routing problem. For directed trees and cycles, the question of approximating $\mathtt{w}(G, I)$ is tightly related to approximating circular arc graph coloring, and this question remains open.

For undirected trees, the question reduces exactly to multigraph edge coloring.

For a general directed network, Jarry [62] proved that deciding if there exists an optical routing using only one color in some given (G, I) where G has $|V| = \mathcal{O}(k^2)$ vertices, or if at least k colors are necessary, is NP-hard. It follows that $\mathtt{w}(G, I)$ cannot be approximated within a factor less than $|V|^{1/2}$. A similar result for undirected networks is still to be found.

2.3 Relation between $\mathtt{w}$ and $\pi(G, I)$

The relation between the two parameters $\mathtt{w}$ and π is hard to capture. For some topologies the ratio $\mathtt{w}/\pi$ is bounded, while for others we may have $\pi >> \mathtt{w}$. Some trivial bounds exist still.

First, if a routing uses some paths of length at most L, then the maximum degree of $C(G, I, R)$ is less than $L.(\pi(G, I, R) - 1)$. From Vizing's theorem [105], $\mathtt{w}(G, I, R) \leq L.(\pi(G, I, R) - 1) + 1$. Hence $\mathtt{w}(G, I) \leq L_0.(\pi(G, I) - 1) + 1$, where L_0 is the minimum length such that there exists a minimum routing using paths of length at most L_0.

A refined analysis due to Aggarwal et al. [1, 2] proves that $\mathtt{w}(G, I) \leq 2\pi(G, I)\sqrt{|E|}$. Indeed, as for any routing it holds that $\sum \text{Load}(e) \leq |E|.\pi(G, I, R) = \sum \text{Length}(P)$, the corresponding conflict graph contains at most $\sqrt{|E|}$ paths with length greater than $\sqrt{E}.\pi(G, I)$ and each other path has at most $\sqrt{|E|}.\pi(G, I)$ conflicts. Such high degree vertices in the conflict graph are colored each with one new color, and the coloring is completed by applying the above method with $L_0 = \sqrt{|E|}$.

The above upper bound on $\mathtt{w}(G, I)$ is somehow the best possible since the authors also provided a pathological example (G, I) for which $\mathtt{w} = \Omega(\pi \cdot \min\{L, \sqrt{|E|}\})$. Many other pathological examples are based on this construction, we shortly describe it.

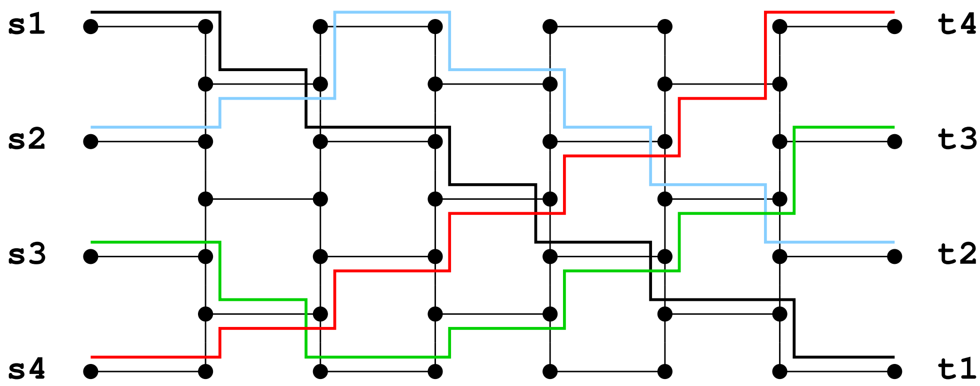

Figure 3: Pathological instance in the mesh-like network.

In [7] Beauquier proposed an extension of this pathological case to directed networks. Such an example is still to be found for symmetric directed networks. He also used the large gap between π and $\mathtt{w}$ in the mesh-like network to prove another counter-intuitive result: minimizing $\mathtt{w}$ and π may be opposite goals. His network has $\mathcal{O}(k^2)$ nodes. Then, on one hand any optimal routing requires at least $k.\mathtt{w}$ colors and on the other hand any optimal coloring induces a load of at least $k.\pi(G, I)$.

2.4 Links with connectivity

The usual measure of the network connectivity allows to derive some (weak) bounds on $\mathtt{w}$. We denote $\lambda(G)$ the edge connectivity and we refer to [28, 10] for basics on connectivity and its relations with single-commodity flow and edge-disjoint routing.

Recall that Shiloach and Tarjan [97] proved that any instance I of requests can be routed along edge-disjoint paths if $\lambda(G) \geq k, k = |I|$. Note that this theorem follows quite simply from Edmons theorem [32], which states that if the connectivity of a vertex r is k, then k edge-disjoint directed spanning trees rooted in r can be found in polynomial time.

An immediate consequence of those theorems is the following: for any directed network G and any instance I, an optical routing using less than $\lceil |I|/\lambda(G) \rceil$ colors can be found in polynomial time.

2.5 Links with edge expansion and distance parameters

Some general results were derived by considering any instance as a subset of a k-relation. In the directed case a *k-relation* is an instance such that a node is the source of at most k requests and the destination of at most k requests. In the undirected case a k-relation is such that every node is involved in at most k requests. Any instance is a *k-relation* for a suitable k (namely the maximum vertex degree in the traffic graph). In the case of directed networks, a *k-relation* can be considered as the union of k permutations (using the Kœnig-Hall theorem on matching decomposition of bipartite graphs).

In his thesis [87] and in [88], Pankaj proved that some bad permutations may need many colors. The idea is to construct an instance I such that the average distance between a source and a destination is large. This average is computed according to a Moore-like estimation of the size of a ball of radius d centered at some vertex: the number of vertices at distance less than d is at most $\sum_{i=0}^{d-1} \Delta^i = \frac{\Delta^d - 1}{\Delta - 1} \leq \Delta^d$. This leads to the existence of a permutation instance such that $\mathtt{w}(G, I_1) \geq \pi(G, I_1) \geq \frac{\lfloor \log_\Delta N/2 \rfloor}{2\delta}$.

In the case of vertex-transitive graphs, Pankaj [87] derived a stronger result: applying the automorphism group allows to find a permutation instance in which each vertex is connected to an antipodal vertex. It follows that there exists a permutation instance I_1 such that $\mathtt{w}(G, I_1) \geq \pi(G, I_1) \geq \frac{D}{\Delta}$. Note that this bound is somehow optimal since the antipodal permutation $x \rightarrow x + 111\ldots111$ in the hypercube can be routed with only one color and has some average edge load equal to 1.

Most of the general upper bounds on $\mathtt{w}(G, i)$ rely on the fact that $\mathtt{w}(G, I) \leq L(R)\pi(G, I, R)$ for any routing using paths of length at most $L(R)$, so we present results dealing about routing with small load and short paths.

For instance, a general relation was proven by Raghavan and Upfal, providing some estimation of $\mathtt{w}$ from the edge expansion of the graph. Their work was inspired by another on edge-disjoint paths by Broder et al. (see the survey [48] or [16]). They constructed a routing by connecting each source to its destination via a random walk of appropriate length L. The transition matrix of the random walk is Q where $q_{ij} = 1/d_i$ for $(i, j) \in E$, $q_{ij} = 0$ otherwise, and $q_{ii} = 1/2$. The limit distribution of this chain is $P(x = x_0) = \frac{d(x)}{2|E|}$ on the vertices, and it is uniform on the edges. The mixing time of the random walk is driven by λ the largest absolute eigenvalue of Q different from 1. By choosing $L \sim \log(kn)/\log(\lambda)$, they ensure two properties: first, there exists a walk of length L connecting any source to any sink; second, for any edge the actual load is almost the expected load, that is $Lk|V|/|E| \leq Lk$. We refer to Jerrum and Sinclair works on rapidly mixing Markov chains [98].

They obtained thus a routing along paths of length L forcing a load kL, and an optical routing using $O(kL^2)$ colors.

Results from spectral graph theory (see the book of Chung [23]) put in relation λ and the edge expansion of a graph. The edge expansion is defined as $\beta = max_{S \subset V, |S| \leq |V|/2} |S|/\Gamma(S)$. It can be remarked that $n\beta$ approximates very well the maximum load l of a cut for the *all-to-all* instance: $l = max_{S \subset V} |S||\overline{S}|/|\Gamma(S)|$. It is known that λ provides an estimation for $\beta(G)$. Indeed, $1 - O(\beta^{-1}) \leq \lambda \leq 1 - O(\beta^2)$.

By applying this relation, Aggarwal et al. obtained $\mathtt{w} \leq L^2 k \leq k(\log^2 n)\beta^2$. Using a construction similar to that of Figure 3, it was also proved that their bound was quite tight since for any $\beta \leq 1$ and any $1 \leq k \leq N$, there exists a planar directed graph and a k-relation I_k such that $\mathtt{w}(G, I_k) = \Omega(k/\beta^2)$.

Leigthon and Rao [73] proved a similar result for directed networks: for any bounded degree symmetric directed graph and any permutation, there is a polynomial algorithm to find a routing with load $O(\log N/\beta)$ using path of length $O(\log N/\beta)$. Aumann and Rabani [4] used this result to prove that any *k-relation* can be routed with $k \log^2 n/\beta^2$ colors.

Note that the upper bound for undirected graphs, $\mathtt{w}(G, I_k) \leq k \log^2 n\beta^2$, relies strongly on a property of the Laplacian of undirected graphs, and on the semi-definite property of this matrix. No similar result is known for directed graphs. Still pathological cases can be constructed, proving that there exist instances such that $\mathtt{w}(G, I_k) = \Omega(k/\beta^2))$. Similarly the symmetry hypothesis is essential in the Leigthon and Rao's work. Similar results are still to be found for general directed networks.

2.6 Broadcasting (one-to-all) and multicast (one-to-many) instances

A set of request is called a *broadcasting* in $G = (V, A)$ if $I = \{(u_0, v)\}_{v \in V(G)}$ for some u_0. The following immediate bounds on $\mathtt{w}(OTA(u_0), G)$ where given by Bermond et al. [15]: first $\mathtt{w}(OTA(u_0), G) \geq \frac{N-1}{d(u_0)}$ (where $d(u_0)$ denote the degree or the outdegree of u_0) moreover $\mathtt{w}(OTA(u_0), G) \leq \lceil \frac{N-1}{\lambda(G)} \rceil$. Note that for superconnected regular graphs (such that $\lambda(G) = d$) we have $\mathtt{w}(OTA(u_0), G) = d$.

From Mader and Hamidoune work [79], edge-transitive graphs are superconnected, and many other Cayley graphs belong to that class.

The generalization to the case of any graph due to Beauquier et al [8], states that optical routing is solvable in polynomial time for any multicast instance. Moreover $\mathtt{w}(G, I_{\mathtt{M}}) = \pi(G, I_{\mathtt{M}})$ since the proof shows that $\mathtt{w}(G, I_{\mathtt{M}})$ is the maximum load of a cut.

2.7 Gossiping (*all-to-all*) Instances

We recall that the so called *Gossiping* or *all-to-all* is $I_{\mathtt{A}} = V(G) \times V(G)$. First, the complexity of optical routing for the *all-to-all* instance is still open. Note that the only related problem for which there exists a hardness result is the the minimum load problem: Saad proved it to be NP-hard via a quite complex reduction in the case of vertex load. The case of edge load is still open. The load of a *all-to-all* routing had been largely studied.

First Chung [24] studied the minimization of the vertex load for a *all-to-all* instance, and named this parameter the *vertex forwarding index*. The edge load in directed and undirected version was studied by Heydemann et al [58, 57]. More recently $\pi(G, I_{\mathtt{A}})$ had been proven to be closely related to the edge expansion of G, namely $\pi(G, I_{\mathtt{A}}) = O(\log(|V|)n/\beta)$.

Note that most of the results linking routing with the edge expansion can be interpreted from Linial et al. [77] work on graph embedding into low dimensional spaces, which implies that for an instance I, $\pi(G, I)$ is a most $\log(|I|)$ the load of the most weighted cut.

The good news are that $\pi(G, I_{\mathtt{A}})$ can be quite well approximated by randomized rounding, and that for Cayley graph it is equal to the average load $\sum_{x,y} d(x,y)/|E|$.

Many papers proved that $\mathtt{w}(G, I_{\mathtt{A}}) = \pi(G, I_{\mathtt{A}})$ for specific networks, but no general result, neither positive or negative, had been derived. The question whether or not $\mathtt{w}(G, I_{\mathtt{A}}) = \pi(G, I_{\mathtt{A}})$ for the *all-to-all* instance is still open. Does the equality hold for any graph? For a sub-class of networks (like edge transitive graph, or edge transitive Cayley graphs)? The only evidence for $\mathtt{w} \neq \pi$ comes from *weighted graphs*. In a weighted graph, each vertex is given a positive integer weight $\mathtt{w}(x)$ and *all-to-all* is then defined as establishing $\mathtt{w}(x)\mathtt{w}(y)$ paths between the vertices x and y. Any weighted graph *all-to-all* instance can be easily transformed into an equivalent instance on a graph when the weights are > 0. There exist weighted graphs for which $\mathtt{w}(G, I_{\mathtt{A}}) \neq \pi(G, I_{\mathtt{A}})$ but they use vertices with weight 0.

The equality $\mathtt{w}(G, I_{\mathtt{A}}) = \pi(G, I_{\mathtt{A}})$ had been proven when G is

- an undirected Cycle (Bermond et al. [15] and independently Wilfong [108]); a symmetric directed cycle [15]

$$\mathtt{w}(C_N, I_{\mathtt{A}}) = \pi(C_N, I_{\mathtt{A}}) = \lceil \lfloor N^2/4 \rfloor / 2 \rceil$$

- the n dimensional undirected hypercube (Pankaj [87] or Bermond et al. [15])

- The n dimensional symmetric directed hypercube H_n

$$\mathtt{w}(H_n, I_{\mathtt{A}}) = \pi(H_n, I_{\mathtt{A}}) = 2^{n-1}.$$

- the Cartesian sum $K(n_1, n_2, \ldots, n_d)$ of complete graphs (Beauquier **??** or see [103, 102] for some slightly weaker results) $\mathtt{w}(K(n_1, n_2, \ldots, n_d), I_{\mathtt{A}}) = \pi(K(n_1, n_2, \ldots, n_d), I_{\mathtt{A}}) = \prod_{i=1}^{d} n_i$

- the Cartesian product of directed symmetric cycles (tori)

$$\mathtt{w}(C_n^d, I_{\mathtt{A}}) = \vec{\pi}(C_n^d, I_{\mathtt{A}}) = n^{d+1}/8, \text{ if } n \text{ is even}$$

$$\vec{\pi}(C_n^d, I_{\mathtt{A}}) = (n^2-1)n^{d-1}/8 \leq \mathtt{w}(C_n^d, I_{\mathtt{A}}) \leq (n+1)^{d+1}/8 = \mathtt{w}(C_{n+1}^d, I_{\mathtt{A}}) \text{if } n \text{ is odd}$$

(Beauquier [6])

- the Cartesian product of symmetric directed paths (grids)

$$\mathtt{w}(P_n^d, I_{\mathtt{A}}) = \vec{\pi}(P_n^d, I_{\mathtt{A}}) = n^{d+1}/4, \text{ if } n \text{ is even}$$

$$\vec{\pi}(P_n^d, I_{\mathtt{A}}) = (n^2-1)n^{d-1}/4 \leq \mathtt{w}(P_n^d, I_{\mathtt{A}}) \leq (n+1)^{d+1}/4 = \mathtt{w}(P_{n+1}^d, I_{\mathtt{A}})\text{if } n \text{ is odd}$$

- a directed tree, indeed the result holds for weighted trees (Gargano et al [49])
- a tree of rings [3] (Beauquier et al. [9] generalize the case of trees).

Note that in the undirected case $\pi(G, I_{\mathtt{A}}) \neq \mathtt{w}(G, I_{\mathtt{A}})$ since a simple subdivided star provide an example with $\pi = 20$ and $\mathtt{w} = 24$. Indeed no scheme computing an optical routing for *all-to-all* instances in undirected trees is known.

2.8 Specific Topologies

Since the optical routing problem is hard to solve on a general network, a lot of work had been devoted in order to provide efficient (approximation) algorithm or negative results for specific topologies. Most of the work focused on very simple networks like cycles, trees or grids. Since these networks are the blocks commonly used to build real life networks those results have strong practical impact.

2.8.1 Trees

As already claimed, the optical routing problem in trees reduces to the Path Coloring Problem, or the vertex coloring problem of path intersection graph on a tree. The simplest tree is a path, note that in that case we have to color an interval graph. Such a problem is polynomial and can be solved by a greedy algorithm (which could be seen as a trivial dynamic programming algorithm) that would proceed from left to right. Another simple tree class is the star. In that case, the situation differs in the directed and undirected case.

In the case of undirected trees, Golumbic et Jamison [55, 56] proved that coloring a path intersection graph based on a tree is equivalent to coloring the edges of a multigraph. Indeed coloring some paths of a star is equivalent to multigraph edge coloring (since any path is adjacent two edges). Then coloring path of a tree can be reduced to solving several multigraph edge coloring. Note that the equivalence proven is extremely strong, optical routing in undirected trees is solving a sequence of multigraph edge coloring on auxiliary graphs. In some sense any result on edge coloring can be applied immediately. It follows that the problem is NP-hard even on a star, then existing results on multigraph edge

[3] A `tree of rings` is any graph made up by a union of cycles such that two cycles intersect in at most one vertex and such that two distinct vertices can be connected by exactly two edge-disjoint paths

coloring can be applied: Tarjan [100], and later Raghavan and Upfal [90] used Shannon theorem [95] (see [43] for a nice presentation) which provides a $\frac{3}{2}$ approximation for multigraph edge coloring, to derive a 3/2 approximation for optical routing in trees. Then it was observed in [82] that one could get an asymptotic 9/8 approximation from Goldberg edge coloring algorithm [53, 54], last from Nishizeki and Kashiwagi [84] algorithm, Erlebach et Jansen [36] provided a 1.1 approximation scheme.

It is not known if the asymptotic approximation ratio can be 1 or not, since the only negative result is that deciding if $\mathtt{w} = 3$ or 4 is NP-complete.

In the directed case the situation is quite different. One can prove that the optical routing problem is equivalent to decompose the edges of a bipartite graph into a minimum number of matchings (see for example [7]). From König-Hall theorem, we know that the edge set can be decomposed into exactly Δ matchings where Δ is the maximum degree of a vertex. Since $\Delta = \pi(S, I)$ we conclude that the problem is polynomial and that $\mathtt{w}(S, I) = \pi(S, I)$. The idea can be generalized to star subdivision (which are tree having only one vertex with degree ≥ 3).

For general trees the problem is NP-hard:

- even if $\mathtt{w}(T, I) = 3$, (tree with unbounded degree) (from Erlebach and Jansen [36] or Kumar et al. [69]);
- in binary trees ([37, 69] or tree with depth 2 [69] (in both cases the load is unbounded)

Using dynamic programming one can prove that the problem is polynomial if the instance is bounded, or if the set of request class (two requests connecting the same vertices being considered as equivalent) is bounded. The particular case of a bounded tree being addressed in [69].

Remark: Note that in the undirected case the problem do not depend on the tree depth since one can color each substar and combine the solutions without adding any new color. In that case, the hard part of the problem consist in being able to color locally the paths crossing a star. In the directed case, coloring a star is easy, but the hard part is to combine the local coloring obtained.

Several papers considered the problem of providing an approximated solution to the optical routing problem on directed trees. Mihail, Kaklamanis et Rao [82] first derived a $15\pi(\mathcal{T}, I)/8$ approximation scheme. Kaklamanis et Persiano improved it to a 7/4 approximation [63] (independently found by Kumar et Schwabe in [71]), then this factor was improved to 5/3 by Erlebach et al. in [39, 38].

All those papers use variations of the same "greedy" technique: the idea is to operate from top to bottom coloring the paths intersecting a star and propagating the constraints downward. This leads to a *constrained edge coloring problem* in a bipartite graph (see for example [64]). The results proven were indeed stronger since it was actually proven that $\mathtt{w} \leq \rho\pi$.

For binary trees, a simplified version of the algorithms was given again with an approximation ratio of 5/3 ([19] and [61]). Furthermore 5/3 ratio is believed to be the best asymptotic ratio since there exists an instance such that $\pi = 3$ et $\mathtt{w} = 5$. Moreover it was proven by the authors that for any "greedy" algorithm, there exists an instance such that the algorithm will return a solution with cost a least $(5/3 - \varepsilon)(\pi)$.

Later two different approaches improved those ratios. The first is due to Caragiannis et al. [18] and is based on fractional coloring. Fractional coloring consists in covering the paths with set of independent paths, the goal being to minimize the total weight used (number of colors). It is tightly related to the *maximum independent set problem*, which in the case of path intersection graph reduces to find maximum number of requests that can be routed via edge disjoint paths. This problem named Maximum Routing problem was proven by Jansen et al. to be polynomial on bounded degree trees, but NP-hard on general trees.

The authors proved that fractional coloring in bounded degree trees is polynomial. They also use a randomized rounding of a fractional solution to derive a $1+5/3e$ approximation. They showed that for path intersection graph on the binary tree the fractional chromatic number is at most $\frac{7}{5}\pi$.

The second approach due to Auletta et al. [3] used a randomized algorithm. It still operates from top to bottom but instead of fixing completely the coloring. It determines a probability distribution on the colorings. It provides a $\frac{7}{5}$ randomized approximation scheme for optical routing in binary trees.

One open question is to understand better those two approaches since one can probably be considered as a randomized rounding version of the other.

The question about the worst ratio between π and $\mathtt{w}$ is still open. An easy example based on a weighted C_5 proves that we can have $\mathtt{w} \geq 5/4\pi$ even for $\mathtt{w} \to \infty$, on the other hand we know that $\mathtt{w} \leq 5/3\pi$.

2.8.2 Cycles

For cycles, when the routing is fixed (Wavelength Assignment Problem) the problem reduces to the classical circular coloring problem which is NP ([56]) and for which no negative result hold concerning its approximation. The first approximation algorithm was designed by Tucker (and it's relation with a specific multi commodity flow presented). Then Karapetyan [65] derived a $\frac{3}{2}$ approximation scheme (note that this result remained almost unknown for at least 10 years) since the number of color used is at most $\lfloor 3\varphi/2 \rfloor$, where φ is the size of the maximum clique. Until Karapetyan result was known the best algorithm was due to Shih et Hsu [96] providing a $5/3$ approximation. Note also that if the path set is *proper* (no path is a strict subpath of another) the problem becomes polynomial (Bonuccelli and Bovet [86]).

Recently, Kumar proposed a randomized algorithm [70] based on the relaxation of Tucker multicommodity flow modelisation, and using a randomized rounding to obtain an integral solution. The approximation ratio is $1 + 1/e \simeq 1.37$, with high probability when $\log|I| = o(\pi(G, I))$. Note that relaxing Tucker multicommodity flow consists indeed in solving the fractional coloring problem for the set of paths. This approximation ratio seems quite pessimistic since branch and bound strategies based on solving the relaxed fractional problem converge very fast toward the optimal even for large values of $\mathtt{w}$. After solving many instances we still have to find one for which $\mathtt{w}$ is greater than the optimum of the relaxed problem + 1. One can find a related result in [20].

When the routing is not fixed, we have some additional freedom since we can choose between different paths to connect x and y. Erlebach et Jansen designed some instance so that the routing becomes fixed, reducing the problem to the standard arc circular coloring problem [36], [109].

A trivial 2-approximation is obtained by coloring all the paths crossing an edge with $l(e)$ colors and all the other with π colors since the problem is then on a path, see [90] for the undirected case and [82] for the directed one).

Since the routing problem and the optical routing problem are quite related, let us mention a nice min-max result of Wilfong and Winkler [109]. They proved that one can route requests on a cycle with an optimal load $\pi(C_n, I)$ that is almost equal to the maximum load of a cut. The corresponding result in the undirected case is due to Frank et al. [45, 47], and follows from a more general result of Okamura and Seymour [85] that states that multi commodity flow in planar graph can be solved in polynomial time when the sources and sinks are located on the outer face of the graph.

Note that, even if the routing problem can be solved exactly, this do not yield an immediate approximation algorithm with a ratio better than 2. Indeed there exists a simple (see /citeBeau00) instance such that $\mathtt{w} = 2\pi$. So π is not an accurate indicator for $\mathtt{w}$.

2.8.3 Grid

From a result of Kramer et van Leeuwen [68] dealing about the complexity of VLSI design one can deduce that the routing problem is NP-hard for grids, and that finding a

2-approximation is also NP-Hard.

Aumann and Rabani [4] proposed a $O(\log N \log |I| \mathtt{w}(\mathcal{G}, I))$ approximation scheme. It is based on an oracle that find an approximated maximum set of requests that can be routed using edge disjoint paths (the technique is quite analogous to the one which colors a graph by greedily picking maximum independent sets). Using Kleinberg and Tardos result [66] that provides for a class of graphs including the grids a constant approximation for the maximum routing problem Rabani improved that result to $O(\log N)$.

2.9 Summary of the results on Optical routing

In this section we summarize the results presented in this survey.

Relation between $\mathtt{w}$ and π

Instance	Topology	Relation
Any	Any	$\mathtt{w} \leq 2\pi\sqrt{\lvert E\rvert}$ (tight)
Directed pathological network	Pathological	$\mathtt{w} = O(2\pi\sqrt{\lvert E\rvert})$
One-to-all	Any	$\mathtt{w} = \pi$
All-to-all	Hypercubes, Tori, Trees of rings, Grids	$\mathtt{w} = \pi$
Any	Directed trees	$\mathtt{w} \leq \frac{5}{3}\pi$
Pathological	Directed trees	$\mathtt{w} \geq \frac{5}{4}\pi$
Pathological	Directed trees	$\mathtt{w} = 5, \pi = 5$
Any	Sub divided stars	$\mathtt{w} = \pi$
Undirected or symmetric cycle	Pathological	$\mathtt{w} = 2\pi$

Relation with connectivity parameters

Instance	Topology	
Any	λ connected graph	$\lvert I\rvert/\lambda(G)$
k-relation	Undirected Graph with edge expansion β	$\mathtt{w} \leq k(\log^2 n/\beta^2)$
k-relation	Undirected Graph with edge expansion β	open

Complexity of Optical Routing

Instance	Topology	
Any	Any	$\rho \geq n^{1/2}$
One-to-all	Any	Polynomial
All-to-all	Any	Unknown
All-to-all	Undirected trees	Unknown
All-to-all	Cayley Graphs	Unknown
Any	Directed trees	$\rho \leq 1 + 5/3e, \rho \neq 1$
Any	Cycles	$\rho \leq 1 + \frac{1}{1/e}\rho \neq 1$
Any	Cycles Routing fixed (WAP)	$\rho \leq 1 + \frac{1}{1/e}\rho \neq 1$
Any	Undirected trees	Multigraph edge coloring
Bounded Traffic $\|I\| \leq k$	Directed and symmetric or undirected	Polynomial
Bounded Traffic $\|I\| \leq k$	Undirected	$\rho \neq 1$

Complexity of Fractional Optical Routing

Instance	Topology	
Any	Any	$\rho \geq n^{\epsilon}$
Any	Any	Polynomial
Any	Bounded degree trees & Bounded treewidth	Polynomial

2.10 k-fiber networks

Recently a special case of the optical routing problem where each edge is replicated k times as been addressed. This problem was introduced in order to model the fact that in telecommunication networks a fiber is usually carried by a cable containing several fibers, so practically the network is a multigraph in which each edge is replicated several time.

In the k-fiber model it is assumed that each edge is replicated k times. If we call kG the multigraph obtained from G by replicating each edge k times, the k fiber optical routing problem can be considered as a classical optical routing problem on kG. But this way of doing is not providing interesting results since as k grows the k-fiber problem becomes much easier to solve than the general optical routing problem. So some work has been directed toward proving stronger results for the k-fiber model.

Note that, Ferreira et al. [42] proved that the problem of deciding if an optical routing using $\mathtt{w}$ color exists in a k fiber network remains NP-complete for any value of k and $\mathtt{w}$.

2.10.1 Relation with π

Since the k-fiber optical routing problem is more constrained than routing in a network with capacity k it follows that $\mathtt{w}_k(G,I) \geq \lceil \frac{\pi(G,I)}{k} \rceil$. Note also that $\mathtt{w}_k(G,I) \leq \lceil \frac{w(G,I)}{k} \rceil$ since any set of paths that can be colored with k colors in the 1-fiber model can be colored with a single color in the k-fiber model. So we have $\lceil \frac{\pi(G,I)}{k} \rceil \leq \mathtt{w}_k(G,I) \leq \lceil \frac{\mathtt{w}(G,I)}{k}$. We will focus on the ratio $\frac{\pi(G,I)}{kw_k}$ which measures the efficiency of the optical routing (kw_k is the maximum capacity provided by a link, while $\pi(G,I)$ is the minimum capacity needed to carry the traffic). Intuitively $\mathtt{w}_k(G,I)$ should be upper bounded by $\frac{\pi(G,I)}{k}(1+g(k))$ with $g(k) \to 0$ when $k \to \infty$.

Note that the efficiency can grow dramatically, as for example there exist instances such that $\mathtt{w}_k(G,I) = n$ and $\mathtt{w}_{k+1}(G,I) = 1$ (see [81]).

Li and Simha [75] considered the case of 2-fiber stars and proved that $\mathtt{w}_2(P) = \lceil \pi(P)/2 \rceil$. They used a result from Ramaswami et al. [91] which shows that paths can be directed so that the load induced on the associated symmetric directed star is at most $\lceil \pi(P)/2 \rceil$, as for directed stars $\mathtt{w} = \pi$ the directed path can be colored with $\lceil \pi(P)/2 \rceil$ colors, then on any edge at most 2 paths are assigned the same color (one in each direction). They also considered the 2-fiber ring, they proved the optical routing problem to be NP-hard, and using Seymour et al. result [85] (which allows to find a routing with minimum load), and the fact that any set of path can be colored with at most $\frac{3\pi(P)}{4}$ colors they provided a 3/2 approximation.

Janos and Simons studied this property for cycles and trees [81], they proved that for any undirected star S and any $k \geq 2$ the equality $\mathtt{w}_k(S,I) = \lceil \pi(S,I)/k \rceil$ holds, and that for ring networks $w_k(S,I) = (1+\frac{1}{k})\lceil \pi(S,I)/k \rceil$ (note that this result can be seen as consequence of Tucker work [104] and was also proven by Li and Simha [75]). They also constructed specific instance on tree for which $\mathtt{w}_k(T,I) \geq (1+1/k)\pi(G,I)/k)$, and on cycles $w_k(C,I) \geq (1+1/(k-1))\frac{\pi(C)}{k}$. Note that the results on the cycle are assuming a fixed routing, so they deal about paths coloring.

2.10.2 Reduction to routing in a network with capacity c

Recall that deciding if there exist an optical routing with p colors can be reduced to the feasibility of an edge disjoint path routing problem in an associated network, the associated network being made of p copies of G and each copy representing one color.

For the k-fiber problem this construction can be adapted but the associated routing problem has then to be solved in a network with capacity k, when k grows this problem tends to be much easier to solve than an edge disjoint path problem. As example Baskiotis et al [5], using Raghavan and Thomson randomized rounding technique proved that $\mathtt{w}_k(G,I) \leq \pi(G,I)(1+O(\sqrt{\frac{log|E|}{k}}))$ when $k > \log^{2/3}|E|$, moreover the algorithm can be de-randomized easily. So in some sense as soon as k is relatively large ($k > log|E|$) there exist a 2-approximation scheme, and the approximation gets better when k grows. Rivano and Coudert [29] also studied the practical efficiency of this multi commodity flow relaxation and considered how wavelength conversion could be integrated in the relaxation.

The question on how large k must be so that good approximation schemes exists is open.

2.10.3 Modelizing the conflicts

One can model the path conflicts in a k-fiber network using a conflict graph, but in that case the routing of an instance must be given in the multigraph kG even if it is quite clear that only the routing in G is relevant. In order to be able to express the optical routing constraint from a routing in G one need to model the fact that on an edge at most k paths can be assigned the same color.

Given a set of paths (a routing) R to be colored under the k-fiber hypothesis we define an *hypergraph* coloring problem on an associated hypergraph denoted $H(G,R)$. $H(G,R)$ has as vertex set the set of paths, and to each edge of G we associate an hyperedge in $H(G,R)$ which contains all the paths crossing that edge. Note that if we replace each hyperedge by a clique we obtain the conflict graph $C(G,R)$ defined for optical routing. The path coloring reduces then to the following *hypergraph coloring* problem : find a vertex coloring so that in any hyperedge each color appear at most k times (this approach was introduced in [42] by Ferreira et al.).

Note that when $k = 1$ all the vertices appearing in the same hyperedge are given different colors, which means that in that special case the hypergraph coloring reduces to color $C(G,H,I)$.

2.10.4 Path coloring problem

We consider now the path coloring variant, in which the routing is fixed. For a set of path P we denote $\mathtt{w}_k(P)$ the minimum number of color needed to color the paths under the k fiber hypothesis.

As in the non routed case we have $\lceil \pi(P)/k \rceil \leq \mathtt{w}_k(P) \leq \lceil \mathtt{w}(P)/k \rceil$.

Margara and Simon [80] proved that for any fixed network G there exist a value of $k_0(G)$ such that for $limit_{\pi(P)\to\infty} w_k(P)/\lceil \frac{\pi(P)}{k} \rceil = 1$.

The result is coming from the finiteness of G, indeed since G is finite one can show that any subset of path P with high enough load can be partitioned into two subsets P_1 and P_2 such that the $\pi(P) = \pi(P_1) + \pi(P_2)$. It follows that any set of path P can be decomposed using a finite ground set $\{P_1, P_2, \ldots P_p\}$ called *prime sets of paths*. That is $P = \alpha_1 P_1 \cup \alpha P_2 \cup \alpha_p P_p$ with $\sum \alpha_i \pi(P_i) = \pi(P)$, where $\alpha_i P_i$ means taking α_i times the set P_i. Then choosing k_0 as the smallest common divisor of the numbers $\pi(P_i)$ we obtain that the set $\alpha_i P_i$ can be colored with $\lceil \alpha_1 \pi(P_1)/k_0 \rceil$ colors. It follows that $\mathtt{w}_{k_0}(P) \leq \sum_{i=1,2,\ldots p} \lceil \alpha_i \pi(P_i)/k_0 \rceil \leq p + \frac{\sum \alpha_i \pi(P_i)}{k_0} = p + \frac{\pi(P)}{k}$.

Note that this result if of little practical use since k_0 can be extremely large (in the case of a cycle with n nodes, on can show that prime set have load at most $n-1$, so k_0 can be taken as $(n-1)!/(n/2)!$), moreover even when $k \geq k_0$ the result only states that $\mathtt{w}_k(G,P) \leq \pi(G)/k + p$, and the number of prime sets can be extremely large.

Rivano [92] also studied the problem and used hypergraph coloring results (based on (hypergraph coloring algorithm by Liu [78], and on variation of lovatz local lemma due to Srinivasan [74]) to derive approximation schemes, they proved if L_{max} is the maximum length of a path then for $k \geq \frac{\log L_{max}}{2}$ there exist a randomized scheme which colors the paths with $\pi(P)/k(1+o(1))$ colors.

2.10.5 Fixing $\mathtt{w}$ and minimizing k

Note that the k fiber problem open some new perspectives, especially when the routing is fixed, since then one need to color a set of paths so that at most k paths share a given edge, this can be seen as a covering problem with set of path of load k. Studying the relation between $\mathtt{w}_k$ and π/k consist then in finding how well can the load be balanced if we partition the paths into c subsets. Which lead to the next approximation problem : given c partition a set of path P into c sets $P_1, P_2, \ldots P_c$ such that $max_{I=1,2,\ldots c} \pi(P_i)$ (or $\sum_{I=1,2,\ldots c} \pi(P_i)$) is minimized.

3 Traffic Grooming

Traffic grooming refers to techniques used to combine low speed traffic streams onto high speed wavelengths in order to minimize the network-wide cost in terms of line terminating equipment and/or electronic switching. Such techniques become increasingly important for emerging network technologies, including SONET/WDM rings and MPLS/MPλS backbones [99], for which traffic grooming is essential.

The general traffic grooming problem being NP-complete [21], recent works focus on specific issues. Most of the algorithms aim at grooming traffic in such a way that all the traffic between any given pair of nodes is carried on a minimum number of wavelengths (efficient use of the fiber, see section 2). However, a large part of the cost depends on the size of the multiplexing equipment required at each node. Hence, in order to minimize the overall network cost, algorithms have to take into account a tradeoff between the number of wavelengths used and the number of required (optical) Add-Drop Multiplexers (ADMs) [51, 114].

In the following subsections a general definition of the problem as well as proposed solutions are presented.

3.1 The General Traffic Grooming Problem

A comprehensive survey of the literature concerning the general traffic grooming problem is presented in [30]. Given the traffic demand matrix between pairs of nodes in a network represented by a directed graph and a fixed capacity per wavelength, the traffic grooming problem involves the following conceptual subproblems [30]:

1. *virtual topology:* find a set of lightpaths requests for the traffic demands (Recall a lightpath is a path associated with one wavelength). A traffic demand may have to use several consecutive lightpaths to cross the network (multihop);

2. *lightpath routing and wavelength assignment:* solve the Routing and Wavelength Assignment problem on the virtual topology of lightpaths requests found in step 1;

3. *traffic routing:* route each traffic stream through the lightpaths.

Actually, these three steps are not independent and a global optimization should be processed.

The authors then attempt to give an Integer Linear Program (ILP) formulation of the grooming problem for illustrative purpose. *No wavelength conversion is assumed, i. e. the same wavelength is assigned to a lightpath throughout the path it follows.* Three relevant objectives for traffic grooming have also been identified by the authors:

1. *Total number of lightpaths.* The cost to be minimized corresponds to the expensive electronic equipment required at the extremities of each lightpath, i. e, the number of SONET add-drop multiplexers (ADMs) for ring networks, or the number of OXC ports for a mesh topology.

2. *Network wide amount of electronic switching.* Instead of counting a unit of cost each time a lightpath is terminated, this objective takes into account the number of traffic streams carried by the lightpath.

3. *Maximum number of lightpaths terminating/originating at a node.* This model aims to minimize the electronic switching at the node where it is maximum.

A mathematical ILP formulation for the Traffic Grooming problem for optical mesh (actually networks with arbitrary topology) networks is also given in [115], where the authors seek to maximize the total successfully-routed traffic. Thus, even if the network resources are insufficient to allow all the traffic demands to be successfully routed the ILP gives a solution maximizing the network throughput.

3.2 Traffic Grooming with Multi-Layer Switches

This model proposed in [60] assumes a network with arbitrary topology in which the data is transmitted in a set of hierarchical optical containers, for example, wavelengths are included in bands and the bands, in turn, in fibers. Each level in the hierarchy, also called a layer, is mapped to a specific switching cost. This hierarchical network model has been defined in the French RNRT project PORTO with inputs from its industrial partners. The use of wavebands [72] allows to reduce the size and complexity of optical crossconnects at the nodes.

The multi-layered approach introduces the concept of a *Pipe* which is used to represent traffic switched within the same layer throughout the path it follows. The model in the paper is restricted to the two layers sub-problem: pipes of layer l are considered as traffic demands that must be carried by pipes of layer $l+1$ which are to be designed.

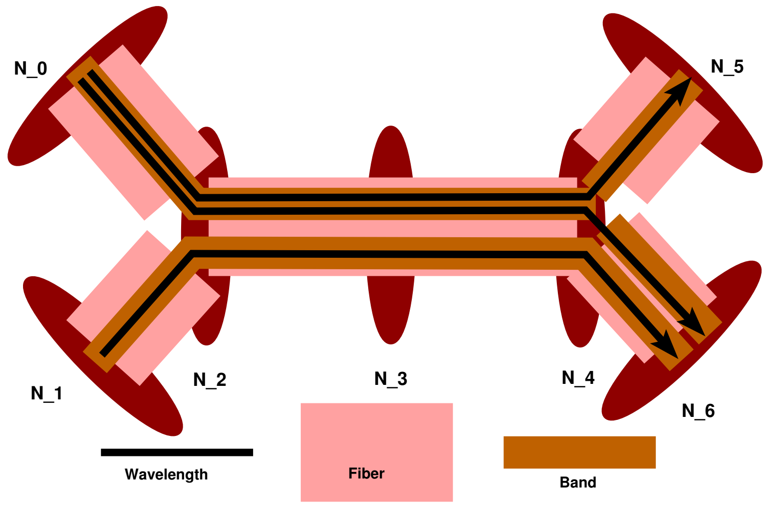

Figure 4: A grooming example

To illustrate what a pipe is, let us consider Fig. 4. We observe that some traffic is carried on the network within the same layer on certain sub-paths. This is the case of demand $N_1 \rightarrow N_6$ that crosses the network in the band layer and never has to be processed by a W-OXC. The sequence of bands from N_1 to N_6 forms a band-pipe. Another band-pipe exists from N_0 to N_4. Note that there is also a fiber-pipe running from N_2 to N_4. The grooming algorithm decides the way pipes of lower layers are included in pipes of upper layers. A different grooming could have been done by multiplexing $N_0 \rightarrow N_6$ and $N_1 \rightarrow N_6$ at node N_2 and this would have led to a different set of pipes. Usually one considers that F-OXCs are much less expensive than B-OXCs and that B-OXCs are in turn, much less expensive than W-OXCs. Hence the grooming algorithm has a strong influence on the cost of nodes. Note that the wavelength-continuity constraint along a pipe has not been considered since it is assumed that a good grooming algorithm leads to a simpler wavelength assignment problem. However it is easy to add this constraint to the model. Roughly speaking a pipe is a lightpath in a WDM model where conversion is allowed, but the optical channel is not necessary a wavelength but could be a band, a fiber or any kind of container defined in the network nodes.

3.3 Traffic Grooming Heuristics

As stated earlier the traffic grooming problem has been shown to be $\mathcal{NP}$-complete and the ILPs proposed in [30, 115, 60, 72] become extremely difficult to solve for large scale networks due to the unbounded increase in their size and complexity. Hence several heuristics have also been proposed in the literature to groom traffic efficiently.

1. *Maximizing Single-Hop Traffic (MST)* [115]: This simple heuristic seeks to establish single lightpath hops for the traffic demands as far as possible and the demands are considered in the order of their magnitudes. The lightpaths are routed along the shortest path between source and destination, subject to wavelength availability and node equipment constraints. If no more single hop lightpaths can be established, the remaining demands are routed based on the virtual topology network state created so far.

2. *Maximizing Resource Utilization (MRU)* [115]: This heuristic seeks to maximize the average traffic per wavelength link. The ratio of the traffic demand size and the physical hop distance between the source and the destination is used to determine the priority of the demand for routing. The procedure is the same as for MST except for the manner of assigning priorities to traffic demands.

3. *Grouping lightpaths with the same destination* [72]: This is applied to networks using wavebands. To construct the wavebands, the lightpaths having the same destination are grouped together since then there would be no need to ungroup them before the destination. The lightpaths are first routed. This is followed by grouping lightpaths in the same class (corresponding to one destination) into wavebands.

4. *Pipe filtering*[60]: taking all the subpaths of existing demands as the set of potential pipes is leading to hard integral linear program as this set is too large. The main goal is to discard any long pipe that would in any case be almost empty. In order to eliminate such pathological pipes one use an evaluation function f that given a pipe, its length l and the maximum amount of traffic t that it can use, assigns a pipe grade $f(l, t)$. All pipes receiving a grade lower than *mingrade* are discarded. Note

that by tuning the value *mingrade* the selectivity of this process can be adjusted (as an example we can keep 10 percents of the best pipes).

Another efficient heuristic is to limit the number of split allowed per demand (i.e. the number of different pipes that will carry that demand from source to destination).

The first two heuristics have been compared to the optimal solution (in terms of the ILP formulated) [115]. However one may like to compare the performance of the last to the optimal grooming solution for a given routing. The multi-layered ILP gives the optimal solution for a given routing (considering 2 layers at a time) in a hierarchically layered network and this can be used to test the efficiency of grooming the routed lightpaths by common destination.

Apart from these solutions proposed for grooming traffic, a novel approach to this problem is of improving the routing of the traffic demands itself so that the grooming can be done efficiently.

3.4 Traffic Grooming in Unidirectional Rings

Because much of previous physical layer network infrastructure was built around Synchronous Optical NETwork (SONET) rings, a lot of research has been done for that specific case of topology (see the surveys [83, 31]). Let recall that by using traffic grooming, one can bypass the nodes electronics for which there is no traffic sourced at this node or destined to it. Instead of having one SONET Add Drop Multiplexer (shortly ADM) on every wavelength at every node, it may be possible to have ADMs only for the wavelength used at that node (the other wavelengths being optically routed without electronic switching).

It is well known that even for this simple unidirectional ring network the number of wavelengths and the number of ADMs cannot be simultaneously minimized (see [50], or [21] for uniform traffic). Furthermore, given a traffic matrix expressed in some units of a bandwidth (for example OC-3) where $r_{i,j}$ units have to be transmitted from i to j, the solution will depend on the routing used and how connections are assigned to wavelengths. So, the general problem is even more difficult.

3.4.1 Unidirectional Ring and *all-to-all* traffic

In the following, we consider the particular case of unidirectional rings (the routing is unique) with static uniform symmetric *all-to-all* traffic (that is $r_{i,j} = 1$ for all couples (i, j)) and with no possible wavelength conversion.

In that case, for each pair $\{i, j\}$, we associate a circle (or circuit) which contains both the request from i to j and from j to i. If each circle requires only $\frac{1}{C}$ of the bandwidth of a wavelength, we can "groom" C circles on the same wavelength. C is called the *grooming* *ratio* (or grooming factor). For example, if the request from i to j (and from j to i) is one OC-12 and a wavelength can carry an OC-48, the grooming factor is 4 as 4 OC-12 can be multiplexed into an OC-48. The objective is, given the grooming ratio C and the size N of the ring, to minimize the total number of (SONET) ADMs used, denoted $A(C, N)$, and so reducing the network cost by eliminating as many ADMs as possible from the "no grooming case". For example, let $N = 4$; we have 6 circles corresponding to the 6 pairs $\{0,1\}, \{0,2\}, \{0,3\}, \{1,2\}, \{1,3\}, \{2,3\}$. Without grooming (one wavelength per request) we will have to use 2 ADMs per circle, and thus a total of 12 ADMs. Suppose now that $C = 4$. One way can be to groom on wavelength 1 the circuits associated to $\{0,1\}, \{1,2\}, \{2,3\}, \{3,0\}$ requiring 4 ADMs and on wavelength 2 the circuits associated to $\{0,2\}$ and $\{1,3\}$ requiring 4 ADMs. So a total of 8 ADMs. A better way is to groom the circuits associated to $\{0,1\}, \{0,2\}, \{0,3\}$ using 4 ADMs and those associated with $\{1,2\}, \{1,3\}, \{2,3\}$ using 3 ADMs and a total of 7 ADMs.

This case might appear very particular but it has been considered by many authors [21, 31, 50, 51, 59, 106, 107, 112, 113, 114] and numerical results, heuristics and tables have been given (see for example that in [107]). It presents the advantage of concentrating on the grooming phase (excluding the routing). It can also be applied to groom components of connections more general than two opposite pairs into wavelengths or more general classes. These components are called circles [21, 114] or circuits [107] or primitive rings [26, 27].

Ideas of *design theory* can be used to obtain optimal or quasi-optimal results improving some of the preceding results of the literature and unifying them [13]. Indeed it is possible to use the vast effort and the numerical results obtained in the last century in design theory [25], without reinventing them. Note that design theory was also used in [26, 27] for $C = 8$.

3.4.2 Other studies for Rings

In the case of unidirectional rings with arbitrary asymmetric traffic requirements the design theory proposed above does not applies. Integer Linear Programs mixed with heuristics [107] or genetic algorithms are proposed [111].

In the case of bidirectional rings, a first step is to determine the routing (clokwise or counterwise) but this goes against the protection scheme of SONET rings: in case of link failure the protocol uses the opposite direction. We introduce some survivability issues in the next following section.

4 Network survivability

Network survivability [110] (i.e. the ability to recover traffic affected by failures) is becoming a key issue in the design of ultra-high capacity networks based on WDM technology.

The survivability against equipment or link failure consists in computing new routes for the demands affected by a failure; thus the optical layer must be over-dimensioned.

Two survivability schemes can be implemented : protection or restoration. Protection can be done by using a pre-assigned capacity between nodes in order to replace the failed or degraded transport entities.

On the other hand, restoration can be realized by using any capacity available between nodes in order to find a transport entity that can replace the failed one. Furthermore, restoration is based on re-routing algorithms to find a new path to recover failed network entities, at the time the failure occurs.

Dividing the network into independent sub-networks provides an intermediate solution for survivability. Indeed it allows resource sharing within the limits of a given sub-network, and uses of fast automatic protection in case of failure [101]. The ring topology is often chosen as sub-network since it minimizes the complexity of the routing problem with full survivability for any *single failure*. Indeed we use on the cycle half of the capacity for the demands and in case of failure we reroute the traffic going through the failed link via the remaining part of the cycle using the other half of capacity. It will be interesting to get very small cycles as subnetworks as they are more easy to manage and less costly to reroute. Also, we will associate a wavelength to each cycle (in fact two: one for the normal traffic and one for the spare one). Furthermore this cycle should satisfy the disjoint routing cycle (DRC) property, implying that it is embedded in an elementary cycle of the physical graph.

The general problem can be summarized as follows: *Find a covering of the edges of a logical graph H by subgraphs H_k, such that, for each H_k, there exists in the physical graph G a disjoint routing of the edges of H_k and such that the cost of the network is minimized.*

The aim is to minimize the cost of the network; that is a very complex function depending on the size of the ADMs put in each node, the number of wavelengths (associated to the subnetworks) in transit in each optical node and a cost of regeneration and amplification of the signal. In a first approximation, some authors reduce it to minimize the number of cycles of the covering ; other minimize the sum of the number of vertices of the rings ; other insist on using very small cycles in the covering [76, 35],[17, 33, 34] and [50].

When the physical graph is a ring this corresponds *to minimize the number of subgraphs I_k in the covering* (as there is a unique physical path associated to a request). In [12, 11, 14] the problem is fully solved for the case where the physical graph is a ring and the logical graph is the complete graph (corresponding to an *all-to-all* communication pattern). Furthermore it is shown that the cycles can be chosen to be of length at most 4.

5 Future Challenges

One of the main practical issue would be to refine the modelization of WDM networks in order to take into account the diversity of the problems: coloring, grooming, routing and survivability. More experimentations should be done in order to validate the proposed models and the efficiency of the algorithms.

Regarding theoretical aspects, a lot of problems remain unsolved.

In the case of the grooming problem, most of the results presented here assume a static traffic input of the problem, except in very specific hypothesis like in [51, 114]. The case for dynamic instance of traffic should be considered, especially in adequation with GMPLS internet backbones. However, even in the static case, the grooming problem in mesh network is open. In particular, we want to study the impact of routing over grooming, and simple instances of traffic such as multicast or traffic with limited number of sources. Moreover, even for simple models like [13, 60] no approximation algorithm is known.

In the case of optical routing, many open problems are mentioned in section 2, among them let recall:

- generalization of results only known for undirected network to directed network;
- Good bounds for the *all-to-all* instance are to be found. Indeed for many usual networks the optimum number of colors is simply the load parameter, which is easy to approximate. However no relation between the number of colors and the load has been proved in that case;
- for simple networks like trees, rings, or grids finding practical algorithms computing an almost optimal solution ($1 + \varepsilon$ approximation ratio for trees and rings, constant approximation ratio for grids) is an important question. Obviously if such algorithms do not exist it remains to be proved.

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