Cops and robber games in graphs

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Cops and robber game is a combinatorical problem that has been studied for providing new structural tools and characterization of graphs (see the survey of B.Alspach [Alspach04]). In this game, one player first places a set of tokens (the cops) on the vertices of a graph G, then the second player places its token (the robber) on a vertex of G. Alternatively (starting with the cops), the players moves their token from their current location to an adjacent vertex. The goal of the cops is to capture the robber (a cop must be placed on the same vertex as the robber), while the robber wants to permanently escape. Given a graph, a question is to determine the smallest number of cops that ensures the existance of a strategy (sequence of moves) that capture the robber whatever it does. For instance, if the number of cops equals the number of vertices of G, the robber will immediatly be captured. On a grid, two cops are sufficient (you can try). A corresponding problem is to design efficient algorithms (strategies) allowing either to capture the robber using the optimal number of cops, or the evasion of the robber facing few cops. One of the main (difficult) remaining challenge in this area consists in determining whether square-root(n) cops are always sufficient to capture a robber in any n-node graph.

During recent years, some variants of this game has been considered [CCNV,FGK08,NS08]: for instance, the study of the game when the robber is faster than the cops, when the cops can ”shoot” the robber (if they can capture it when occupying a vertex at some distance $k > 0$ from the location of the robber). The case of random graphs has also been widely studied recently [BPW09,Pralat10,LP]. Many problems remains open as the characterization of the cop-win graphs (graphs in which one cop is always sufficient to capture the robber) in these different variants. In particular, few is known concerning these variants in random graphs or in some particular graph classes (the number of cops required to capture a fast robber in a grid is unknown [NS08]).

Main objectives of the PFE:

- The first goal of this project is to provide a state of the art of the cops and robber games. This study will focus on the classes of graphs in which the cop-number has been proved to be less than square-root(n) and on the techniques used for this purpose. This requires some mathematical skills and basic knowledges in probability and graph theory.
- Then, the different variants may be investigated in random graphs, or in some particular (deterministic) graph classes (e.g., grid, bounded genus graphs). Classical questions are of interest: upper and lower bounds on the cop number, characterization of graphs with a given cop number, deterministic or random algorithms allowing to compute efficient strategies for the cops/ the robber.

Required background: good knowledge in probability theory, graph theory, algorithmic, optimization, computational complexity

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