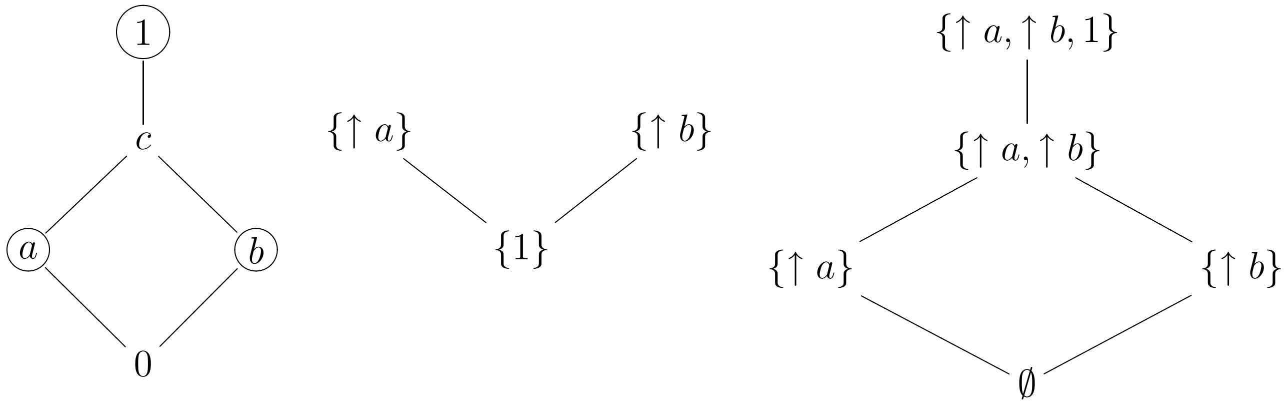
Annotated many-valued resolution: from $(\{\uparrow v_1\} : L) \vee C_1$ and $(\overline{\{\uparrow v_2\}} : L) \vee C_2$ derive $C_1 \vee C_2$ provided that $v_1 \geqslant v_2$.

Theorem 1 (Priestley representation theorem, 1970). *Let A be a bounded distributive lattice and $D(A)$ be a set of all prime filters of A ordered by inclusion. Then A is isomorphic to the lattice $O(D(A))$ of all closed and open prime filters of $D(A)$.*

Example 1. $A, D(A)$ and $O(D(A))$.



Theorem 2 (Sofronie-Stokkermans, 2000). *Let A be a bounded distributive lattice. Then maps and relations on $D(A)$ can be canonically defined.*

Theorem 3 (Sofronie-Stokkermans, 2000). *Let $X = D(A)$ be a finite partially ordered set. Then operators on $O(X)$ can be canonically defined.*

Resolution based on the Priestley duality: from $(\{\beta\} : L^f) \vee C_1$ and $(\{\alpha\} : L^t) \vee C_2$ derive $C_1 \vee C_2$ provided that $\alpha, \beta \in D(A)$ and $\alpha \leqslant \beta$.

Let $\mathbb{Z}^+$ and $\mathbb{Z}^-$ be the non-negative and non-positive integers, respectively. *The Chang algebra* is $\mathbf{C} = (C, \oplus, \neg, 0)$, where C is the lattice

$$C = \{(0, a) : a \in \mathbb{Z}^+\} \cup \{(1, b) : b \in \mathbb{Z}^-\}.$$

The zero element is $(0, 0)$ and the unit element is $(1, 0)$. The order is lexicographical. The addition is given by

$$(i, a) \oplus (j, b) = \begin{cases} (0, a + b) & \text{if } i + j = 0 \\ (1, 0 \wedge (a + b)) & \text{if } i + j = 1 \\ (1, 0) & \text{if } i + j = 2, \end{cases}$$

and the negation is given by $\neg(i, a) = (i +_2 1, -a)$.

Let f be an algebraic operation. We define f^σ to be the lower limit of f, and f^π to be the higher limit of f. *The canonical extension* of an algebra A is an algebra resulted from A after the application of σ or π to all its operations and after the embedding of A into a complete lattice.

The lattice for the canonical extension of $\mathbf{C}$ is obtained as follows:

$$(0,0) \text{ --- } (0,1) \text{ --- } (0,2) \text{ - - - } \textcircled{y} \sim\sim \textcircled{x} \text{ - - - } (1,-2) \text{ --- } (1,-1) \text{ --- } (1,0)$$

Lemma 4 (Gehrke&Priestley, 2001). *Let* $\mathbf{C} = (C, \oplus, \neg, 0)$ *be a Chang algebra and let* $f = \oplus$*. Then* $f^\sigma \neq f^\pi$*.*

Proof. From the definition of f^σ,

$$f^\sigma(x, y) = \bigvee\{f^\sigma(x, (0, a)) : a \in \mathbb{Z}^+\} = \bigvee\{\bigwedge\{(1, 0 \wedge (a+b) : b \in \mathbb{Z}^-\} : a \in \mathbb{Z}^+\} = x.$$

Likewise, from the definition of f^π,

$$f^\pi(x, y) = \bigwedge\{f^\pi((1, a), y) : a \in \mathbb{Z}^-\} = \bigwedge\{\bigvee\{(1, 0 \wedge (a+b)) : b \in \mathbb{Z}^+\} : a \in \mathbb{Z}^-\} = (1, 0).$$

Consequently, $f^\sigma(x, y) = x \neq (1, 0) = f^\pi(x, y)$. □

Let us prove that the following axiom of MV-algebra is non-canonical:

$$\neg(\neg a \oplus b) \oplus b = \neg(\neg b \oplus a) \oplus a. \qquad \text{(MV6)}$$

Lemma 5 (Gehrke&Priestley, 2001). *The equation (MV6) fails in* $\mathbf{C}^\sigma = (C^\sigma, \oplus^\sigma, \neg^\sigma, 0)$.

Proof.

$$\neg^\sigma(\neg^\sigma u \oplus^\sigma (1, 0)) \oplus^\sigma (1, 0) = (1, 0).$$

Also,

$$\neg^\sigma(\neg^\sigma(1,0) \oplus^\sigma y) \oplus^\sigma y = \neg^\sigma((0,0) \oplus^\sigma y) \oplus^\sigma y = \neg^\sigma y \oplus^\sigma y = x \oplus^\sigma y = x \neq (1,0).$$

□

Lemma 6 (Gehrke&Priestley, 2001). *The equation (MV6) fails in* $\mathbf{C}^\pi = (C^\pi, \oplus^\pi, \neg^\pi, 0)$.

Proof.

$$\neg^\pi(\neg^\pi x \oplus^\pi y) \oplus^\pi y = \neg^\pi(y \oplus^\pi y) \oplus^\pi y = \neg^\pi y \oplus^\pi y = x \oplus^\pi y = (1,0).$$

The right side of the equation yields

$$\neg^\pi(\neg^\pi y \oplus^\pi x) \oplus^\pi x = \neg^\pi(x \oplus^\pi x) \oplus^\pi x = \neg^\pi(1,0) \oplus^\pi x = (0,0) \oplus^\pi x = x.$$

Since $x \neq (1,0)$, the equation (MV6) fails. □

Theorem 7. *Let L be a Łukasiewicz logic, then the resolution method based on the Priestley duality is sound with respect to L if and only if L is finite valued.*

Proof. (⇒) As we have already shown, no non-finitely generated variety of MV-algebras is canonical. This proves the sufficiency by contraposition.
(⇐) By theorems 2 and 3 we can define canonical operators on $O(D(A))$. By lemmata 4, 5 and 6 these operators will preserve axioms of MV-algebra. This proves the soundness of the resolution rule. □

Problem 1. What are the nesessary and sufficient conditions for a given logic whose set of truth-values is a bounded distributive lattice to have a method of automated theorem proving based on the Priestley duality?