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THEORY OF INFERENCE IN MANY-VALUED LOGICS

ABSTRACT OF THE DISSERTATION
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The aim of the dissertation research is to present a uniform approach to the theory of inference in many-valued logics by means of studying their semantices and calculi.

The objects of the research are logical semantices of different kinds, logical calculi for many-valued logics, theories, where the concept of many-valuedness is formalised, and models of such theories.

Obtained state of investigation. As a result of the development of the theory of many-valued logics, many original ideas appeared. Among them we can especially point out the following ones: a computation of all precomplete classes of functions of three-valued Post logic (Yablonsky, 1954); a proof of existence of k-valued closed classes of functions which have no finite bases, for $k > 2$ (Yanov and Muchnik, 1959); a determination of functional properties of Łukasiewicz many-valued logics (Finn, 1970); a method of axiomatisation for classically extending finite-valued logics (Anshakov and Rychkov [2], 1982); a proof of completeness of infinite-valued Łukasiewicz logic with respect to ternary relevant Kripke-style semantics (Vasyukov [16], 1993); a discovery of the connection between Łukasiewicz implication and classes of prime numbers (Karpenko [5], 1996); a construction of sequential calculi for *classes* of finite-valued logics (Kosovsky and Tishkov [14], 2000). Nowadays many-valued logic is a complex ramified branch of science. Many general theoretic works are published not by independent authors, but by groups of authors, e.g., a paper [3] on signed sequential calculi for finite-valued logics, or a monograph [4] on algebraic foundations of many-valued logics.

Actuality of the dissertation theme. Today for infinite-valued logics there is no well-founded uniform proof theory, though for finite-valued logics cut-free tabular calculi were built by Rousseau in 1967. However, several calculi for infinite-valued logic were defined in literature, e.g., Aguzzoli and Ciabattoni [1] in 2000 built a sequential calculus for infinite-valued Łukasiewicz logic. This calculus was based on the Mundici's idea of a reduction of the satisfiability problem in infinite-valued Łukasiewicz logic to the satisfiability problem in an appropriate class of finite-valued logics. So, this calculus was based upon calculi for finite-valued Łukasiewicz logics. In 2001 Sofronie-Stokkermans [15] introduced an effective method of theorem proving based on a representation theorem for algebras of truth-values for a wide class of many-valued logics. In addition to its effectiveness this method is attractive

for proof theorists, as it uses Kripke structures and so provides a correspondence between many-valued and modal logics. Unfortunately, the method can't be applied to a number of frequently used many-valued logics, such as Łukasiewicz infinite-valued logic. This method is investigated in the dissertation, and there is made a conclusion about the possibility to generalise it to the class of infinite-valued logics if we take a suitable representation theorem for algebras of truth values.

Research methods. Traditional logical techniques were used in the dissertation, e.g., sequential logical calculi, as well as non-traditional ones: signed calculi, group-theoretic classification of closed classes of many-valued logic functions, Kripke structures which are dual to some many-valued algebraic semantices. The author of the dissertation introduces an original resolution calculus for theorem proving in many-valued logics which have linearly ordered sets of truth values.

Main results of the dissertation:

- The specificity of the theory of inference in many-valued logics is determined. More precisely, it is determined that the crucial point is the connection of logical calculi with some concrete truth-functional semantices, and the essence of efficient inferences in many-valued logics lies in the finding of simplified analogues of initial semantic constructions connected with corresponding logical calculi.
- A sound and complete resolution calculus for the mixed Post logic (a many-sorted logic with finite-valued predicates which was introduced by Kosovsky and Tishkov in 2000), that is for the whole class of finite-valued Post logics, is obtained.
- On the basis of resolution calculus for mixed Post logic there is obtained a common scheme of construction of analogous calculi for other mixed logics of Kosovsky and Tishkov.
- The bounds of applicability of resolution method based on the Priestley representation theorem to the theorem proving in first-order many-valued logics are determined.
- A criterion of applicability of resolution method based on the Priestley duality to an arbitrary many-valued logic is formulated and proved.

All the above listed results are well-grounded.

Theoretical importance. The theoretical importance of the dissertation is stipulated by the generalised appearance of its results. E.g., the suggested resolution method for mixed Post logic generates a scheme of construction of analogous calculi for a wide class of many-valued systems. Also, the results of the dissertation let us formulate a list of open problems in the field of theory of inference in many-valued logics. It makes possible to measure the present state of research in this logical discipline with respect to the others.

Practical importance. From the applied character of the research field it follows that the results of the dissertation could have important applications. Since their creation many-valued logics have had clear semantices. Because it is easy to find an interpretation for a many-valued logic, and because there are a lot of various interpretations exist, many-valued logics have been used not only in academic practice, but also in industry and production. They have plenty of engineering applications, and as well they are applicable to programming of tasks in different areas of computer science. Theoretical basis laid in the dissertation is a necessary condition for creation of efficient applications based on many-valued logics, and for making their industrial implementations. From the point of a practical importance a special attention should be paid to the results on infinite-valued Łukasiewicz logic. Many so called *fuzzy* logics are based on this logic. Fuzzy logics have their applications in various control tasks, e.g., control tasks for very complex processes having no simple mathematical models, such as highly non-linear processes, or the processing of linguistically formulated expert knowledge. To be concrete, let us point out some industrial realisations of fuzzy control systems: optimised planning of bus time-tables (Toshiba), archiving system for documents (Mitsubishi Electric), prediction system for early recognition of earthquakes (Institute of Seismology, Bureau of Metrology, Japan), medicine technology: cancer diagnosis (Kawasaki Medical School), recognition of handwritten symbols with pocket computers (Sony), single button control for washing-machines (Matsushita, Hitachi), improved safety for nuclear reactors (Hitachi, Bernard), simulation for legal proceedings (Meiji Gakuin University, Nagoya University).

Approbation of the dissertation results. The results of the dissertation were presented by the author at meetings of the Research Logical Seminar of Institute of Philosophy RAS, and also at the following scientific conferences:

- Smirnov Readings 3, Department of Logic, Institute of Philosophy RAS, Moscow, May 2001;
- Contemporary Logic: problems of its theory, its history, and its applications in science, Saint-Petersburg State University, June 2002;
- 14th European Summer School for Logic, Language and Information, University of Trento, Italy, August 2002;
- Smirnov Readings 4, Department of Logic, Institute of Philosophy RAS, Moscow, May 2003;
- Summer School and Workshop on Proof Theory, Computation and Complexity, Technische Universität Dresden, Germany, June–July 2003.

Contents of the dissertation

The original Russian version of the dissertation consists of 155 pages; its bibliography contains 89 entries.

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