On game semantics for intuitionistic linear logic

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In our paper [3] we study Hyland–Schalk games [2] and sequential data structures [1] as affine intensional models for propositional intuitionistic linear logic (ILL). We relate sequential data structures to games using clear categorical methods thus contributing to the study of a question formulated in [2] about connections between these.

Simple games, also known as Hyland–Schalk games, are two-player games (bipartite trees) forming a category **Gam** with morphisms from A to B being partial deterministic strategies in the game $A \multimap B$. This category is known to be a model for affine ILL. Another model for ILL is the category **Algos** of sequential data structures (sds) and affine algorithms. In **Algos** the interpretation of all the connectives, informally speaking, is essentially the same as in Hyland–Schalk games, except for the exponential ! which is interpreted in a form of Curien exponential: the game $!A$ is obtained by allowing Opponent to play many strands of the game A. That is, at any point in $!A$, Opponent may return to an earlier Player move in $!A$ and play a new response to it; Player must always respond to the last Opponent move, and therefore cannot change strands. Therefore, the relationship between the above two models is essentially the relationship between the two versions of the exponential !.

The linear exponential is precisely the connective which is responsible for the coalgebraic structure of a linear logic model. We recall that a categorical model for ILL is given by a category which (1) is symmetric monoidal closed, (2) has finite products, and (3) is equipped with a linear exponential comonad, i.e., it has a monoidal comonad such that the category of Eilenberg–Moore coalgebras for this comonad, with the induced tensor product, is a category of commutative comonoids. The property (3) is needed to model ! as a functor, in fact, a functor giving rise to free coalgebras in a natural way. A proof of whether the category **Algos** has the latter property

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is interesting in that it can be obtained by making use of the full embedding of the Kleisli category of free coalgebras for the linear exponential comonad into the Kleisli category of Eilenberg–Moore coalgebras for the comonad.

The conceptual analogy between games and sequential data structures can be shown as follows:

game semantics	sds & affine algorithms
game	sds
strategy	strategy
Opponent	cell
Player	value

In Section 2 we outline the sequent calculus for propositional intuitionistic linear logic and define what is meant by a categorical model for intuitionistic linear logic. In Section 3 we describe Hyland–Schalk games and give some definitions important for demonstrating the categorical techniques. Section 4 is concerned with the category of sds and affine algorithms. We pursue the question about connections between simple games and sequential data structures posed in [2] and prove that the category **Algos** has a linear exponential comonad. This allows us to obtain yet another proof for the theorem stating that the category in question is a model for intuitionistic linear logic. Moreover, we prove the functor $F : \mathbf{Algos} \to \mathbf{Gam}$ being faithful and linearly distributive (i.e., F is monoidal and is equipped with a distributive law $\lambda : !F \to F!$ respecting the comonoid structure). In Section 5 we outline some future directions.

References

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