Neural fields models of visual areas: principles, successes, and caveats

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Organization of the primary visual cortex

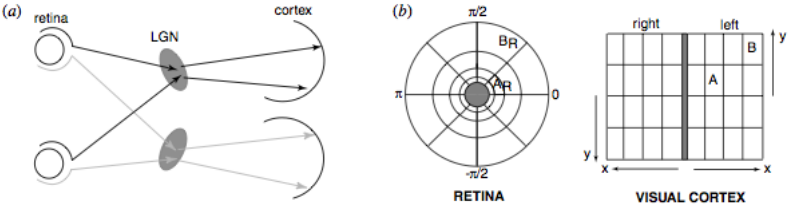

Figure 25. (*a*) Visual pathways from the retina through the lateral geniculate nucleus (LGN) of the thalamus to the primary visual cortex (V1). (*b*) Schematic illustration of the complex logarithmic mapping from retina to V1. Foveal region in retina is indicated by grey disc. Regions A_R and B_R in the visual field are mapped to regions A and B in cortex.

From Bressloff, J. Phys. A: Math. Theor., 2012

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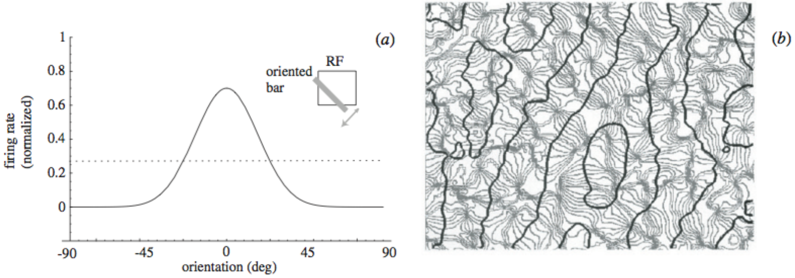

Figure 26. (a) Schematic illustration of an orientation tuning curve of a V1 neuron. Average firing rate is plotted as a function of the orientation of a bar stimulus that is moved back and forth within the receptive field (RF) of the neuron. The peak of the orientation tuning curve corresponds to the orientation preference of the cell. (b) Iso-orientation (light) and ocular dominance (dark) contours in a region of primate V1. A cortical hypercolumn consists of two orientation singularities or pinwheels per ocular dominance column. Reproduced with permission from figure 5A of [168].

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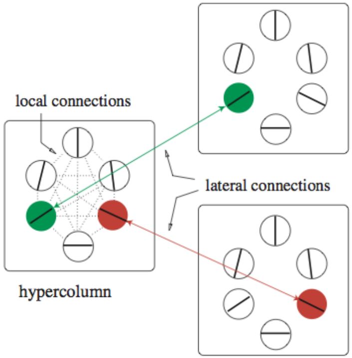

Figure 27. Schematic illustration of anisotropic horizontal connections. Orientation selective cells within a hypercolumn tend to connect to all neighbors in a roughly isotropic fashion. On the other hand, longer range horizontal connections link cells between hypercolumns with similar orientation preferences along a particular visuotopic axis.

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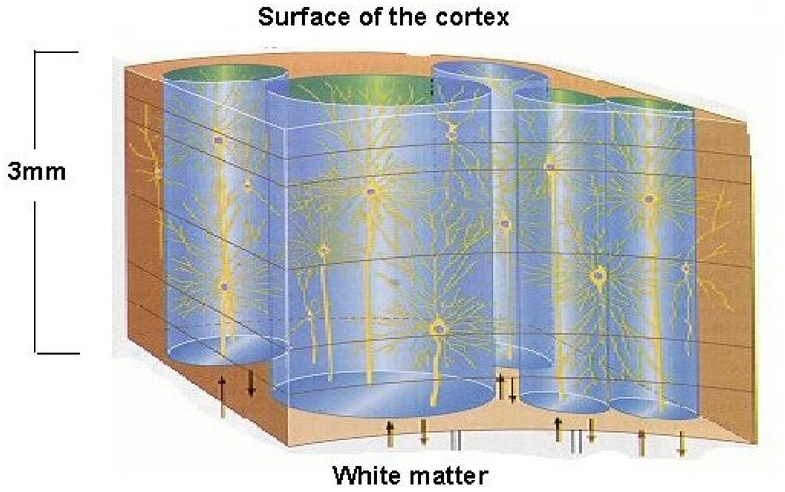

Putting things in equations

Average membrane potential:

$$V(x, t)$$

Putting things in equations

Intrinsic dynamic:

$$\tau \frac{\partial V(x,t)}{\partial t} = -V(x,t)$$

Putting things in equations

An external current (projection from the Thalamus, other cortical areas):

$$\tau \frac{\partial V(x,t)}{\partial t} = -V(x,t) + I_{\text{ext}}(x,t)$$

Putting things in equations

Local interaction:

$$\tau \frac{\partial V(x,t)}{\partial t} = -V(x,t) + I_{\text{ext}}(x,t) + \int_{\Omega} w_{\text{loc}}(x,y) S(V(y,t))\, dy$$

S is a sigmoidal function.

Putting things in equations

Lateral interaction:

$$\tau \frac{\partial V(x,t)}{\partial t} = -V(x,t) + I_{\mathrm{ext}}(x,t) + \int_\Omega w_{\mathrm{loc}}(x,y) S(V(y,t))\, dy + \int_\Omega w_{\mathrm{lat}}(x,y) S(V(y, t - |x-y|/v))\, dy$$

Note the propagation velocity v: propagation delays.

Putting things in equations

Synaptic depression:

$$
\begin{cases}
\tau \frac{\partial V(x,t)}{\partial t} & = -V(x,t) + I_{\text{ext}}(x,t) \\
& \quad + \int_\Omega w_{\text{loc}}(x,y) q(y,t) S(V(y,t))\, dy + \\
& \quad \int_\Omega w_{\text{lat}}(x,y) q(y,t) S(V(y,t-|x-y|/v))\, dy \\
\frac{\partial q(x,t)}{\partial t} & = \frac{1-q(x,t)}{\tau_q} - \beta q(x,t) S(u(x,t))
\end{cases}
$$

Is this different from using PDEs?

- PDEs have been found very useful in C.V.

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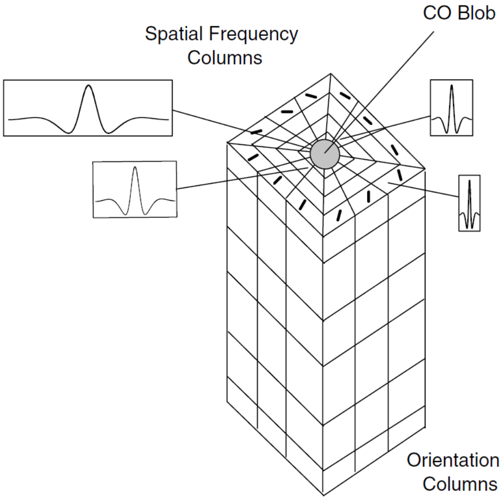
- Hence they are more general and can "implement" non-local algorithms such as A. Buades, B. Coll, and J.-M. Morel, Multiscale Model. Simul., SIAM 2005.

Orientation HyperColumns



From [Bressloff, Cowan, Golubitsky et al., Phil. Trans. R. Soc. Lond. B, 2001], drawn by Jack Cowan.

Tuning curves

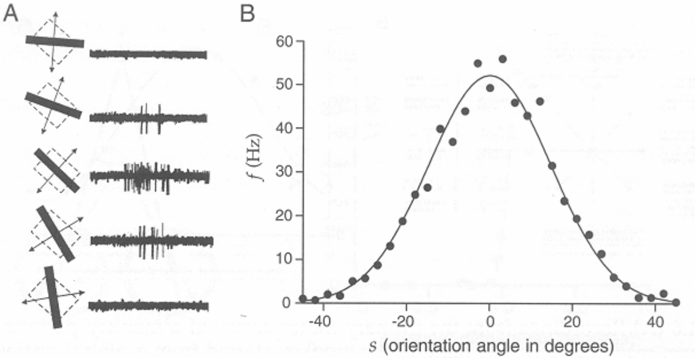
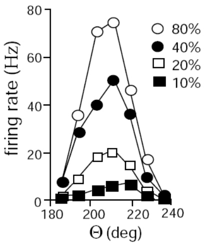
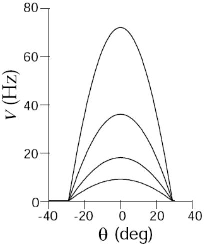
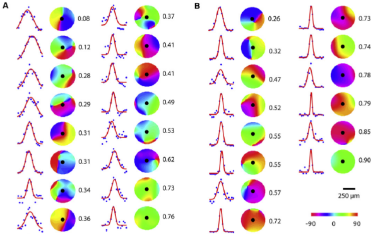


Figure 1.5 (A) Recordings from a neuron in the primary visual cortex of a monkey. A bar of light was moved across the receptive field of the cell at different angles. The diagrams to the left of each trace show the receptive field as a dashed square and the light source as a black bar. The bidirectional motion of the light bar is indicated by the arrows. The angle of the bar indicates the orientation of the light bar for the corresponding trace. (B) Average firing rate of a cat V1 neuron plotted as

Tuning curves







The equation

$$\tau \dot{V}(\theta, t) = -V(\theta, t) + \int_{-\pi/2}^{\pi/2} J(\theta - \bar{\theta}) S(\sigma V(\bar{\theta}, t)) \frac{d\bar{\theta}}{\pi} + \varepsilon I_{ext}(\theta, t) \quad t > 0$$

- Ring model of visual orientations selectivity

Some history

- Selective response of neurons to orientations [Hubel and Wiesel 1962]: balance between cortical computation and LGN feedforward input?
- Cortical models [Somers et al. 1995, Ben-Yishai 1995, Hansel and Sompolinsky 1997]
- Simplifications of spiking models [Somers et al. 1995, Douglas et al. 1995, Carandini and Ringach 1997]
- Meant to reproduce interactions between contrast and orientation selectivity [Dean 1981, Sclar and Freeman 1982, Skottun et al. 1987, Alitto and Usrey 2004]
- Computation studies of the ring model of orientation tuning: [Ermentrout 1998, Bressloff et al. 2000, Bressloff et al. 2001, Shriki et al. 2003]

Questions

- Can we predict the existence of tuning curves?
- Can we understand the dynamics of the model?
- Can we relate its parameters to known biological evidence?
- Can we come up with interesting predictions?

Answer

- YES, thanks to qualitative mathematics.

Key ingredients

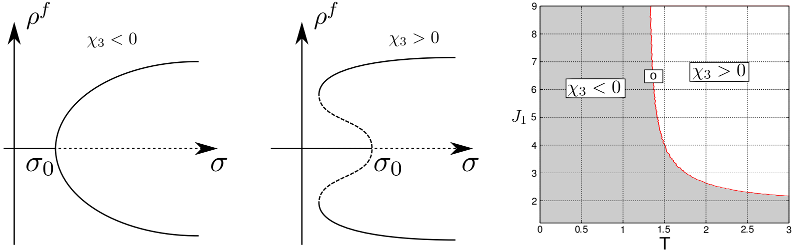
1. Analysis of the symmetries of the model (rotational).
2. Reduction to finite dimensions
3. Bifurcation analysis (with respect to the slope of the sigmoid)

Finding the tuning curves (no stimulus)

- For small values of σ there is a unique, untuned, solution [Faugeras et al., SIAM Journal of Applied Mathematics, 2008].
- A tuned solution arises when the Jacobian of the fixed point equations is singular.
- We obtain a pitchfork bifurcation:

$$0 = \frac{\sigma - \sigma_0}{\sigma_0} \rho^f + \chi_3 \left(\rho^f\right)^3$$

Finding the tuning curves (no stimulus)



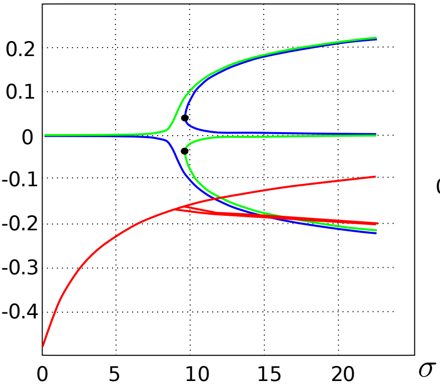
- Because of the $O(2)$ symmetry we have a continuum of tuning curves:

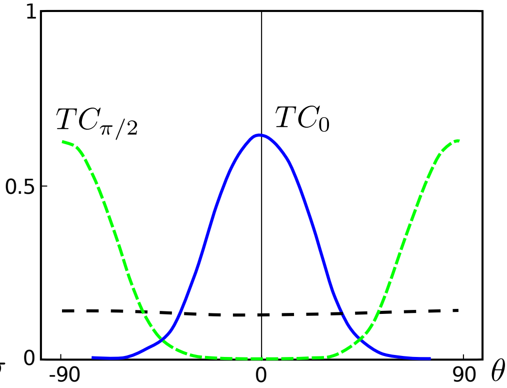
$$TC_\varphi(\theta) = S\left[\sigma\left(v_0^f + \sigma\sqrt{J_1}\rho^f \cos_2(\theta - \varphi)\right)\right].$$

Finding the tuning curves (stimulus on)

- How many tuning curves remain solutions?
- What is their stability?

Finding the tuning curves (stimulus on)

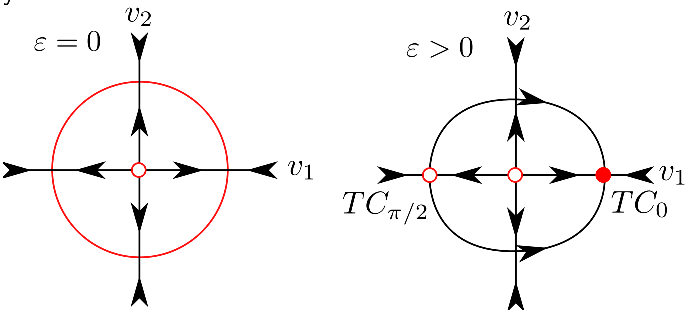




- Opening up of the pitchfork
- 3 tuning curves: 1 stable, two unstable

Finding the tuning curves (stimulus on)

The dynamics:

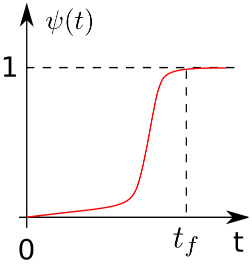


- $TC_{\pi/2}$ is unstable.
- unstable eigenvalue: $\frac{\varepsilon\beta}{\rho^f\sqrt{J_1}}$.

Biological/psychophysical prediction

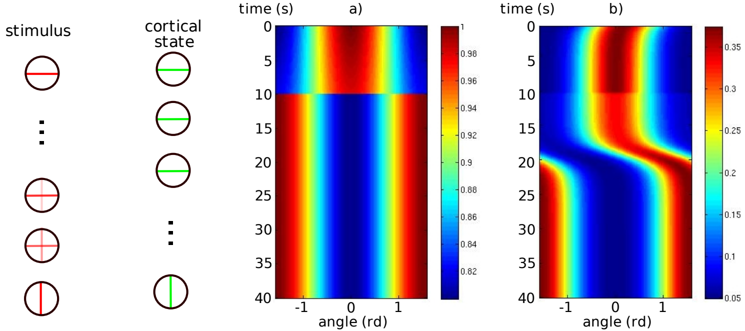
- Dynamical "mixture" stimulus:

$$I_{ext}(t) = (1-\psi(t))I_0^{DG} + \psi(t)I_{\pi/2}^{DG}$$

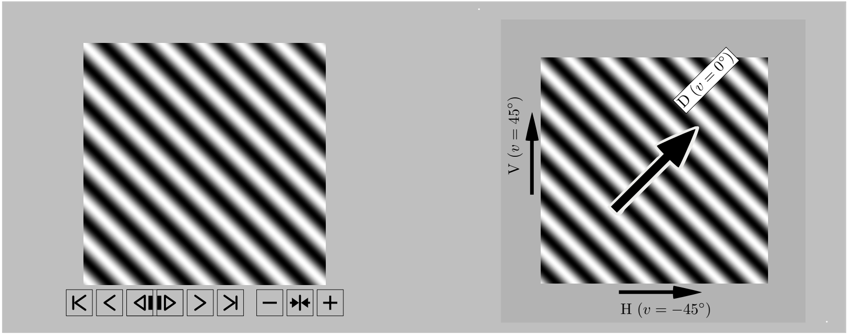


Biological/psychophysical prediction

- $\frac{\pi}{2}$ illusion:



Perceptual switching dynamics of a multistable barber pole stimulus: the phenomenon



- Initially percept is D followed by a transition to either H or V.
- Later we see regular switches between H and V.

The model

- Interaction between V1 and MT.
- V1 is represented as a ring model of directions of motion

$$\dot{A}(\theta, t) = -A(\theta, t) + S\left(\lambda\left(\int_{-\pi/2}^{\pi/2} J(\theta - \bar{\theta})A(\bar{\theta}, t)\frac{d\bar{\theta}}{\pi} + \varepsilon I_{MT}(\theta, t)\right)\right)$$

The model

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Add adaptation:

$$\begin{cases} \dot{A}(\theta, t) & = & -A(\theta, t) + S\left(\lambda\left(\int\limits_{-\pi/2}^{\pi/2} J(\theta - \bar{\theta})A(\bar{\theta}, t)\frac{d\bar{\theta}}{\pi} - \right.\right. \\ & & \left.\left. k_\alpha \alpha(\theta, t) + \varepsilon I_{MT}(\theta, t)\right)\right) \\ \varepsilon\dot{\alpha}(\theta, t) & = & -\alpha(\theta, t) + A(\theta, t), \end{cases}$$

The model

- Interaction between V1 and MT.
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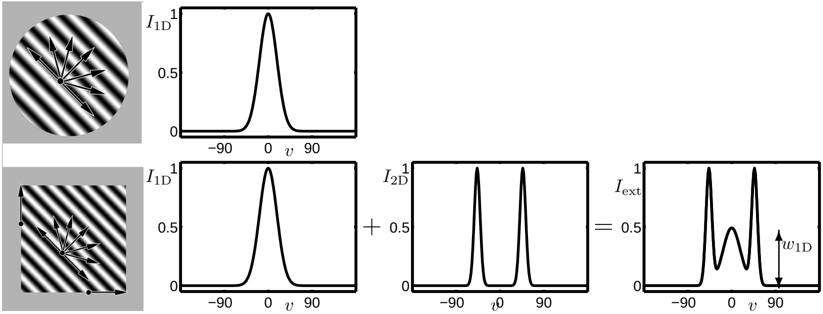
and noise

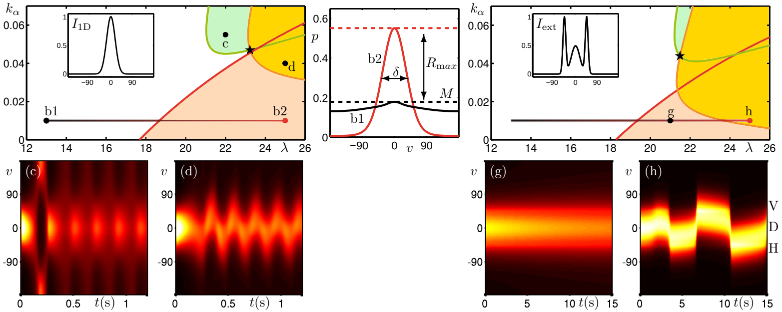
$$\begin{cases} \dot{A}(\theta,t) & = & -A(\theta,t) + S\left(\lambda\left(\int\limits_{-\pi/2}^{\pi/2} J(\theta-\bar{\theta})A(\bar{\theta},t)\frac{d\bar{\theta}}{\pi} - \right.\right. \\ & & \left.\left. k_\alpha \alpha(\theta,t) + \beta X(\theta,t) + \varepsilon I_{MT}(\theta,t)\right)\right) \\ \varepsilon\dot{\alpha}(\theta,t) & = & -\alpha(\theta,t) + A(\theta,t) \\ dX(\theta,t) & = & -\varepsilon X(\theta,t) + \sigma dW(\theta,t) \end{cases}$$

Representation of the stimulus in direction space



We assume that in MT 2D cues play a more important role than 1D cues and set $w_{1D} = 0.5$.

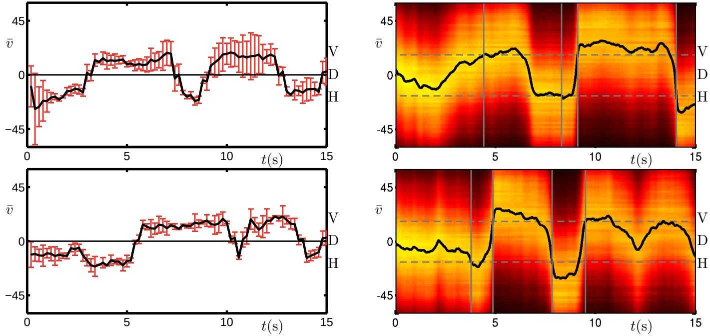
Organisation of solutions in the parameter-plane (no noise)



- Model response in terms of max firing rate matched to data from Sclar et al (1990).

Experimental recordings and characteristic behaviour

Eye movement recording data from individual 15s presentations; carried out by Andrew Meso.

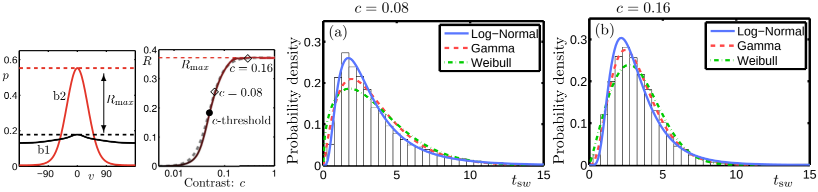


Characteristic behaviour:

- Directions H or V are held for extended durations that vary.
- The direction D is seen briefly in transitions between H and V.

Distributions of switching times

- Are the switches driven by noise or adaptation?
  Rubin and Hupé (2005), Shpiro et al (2009), Theodoni et al (2011).

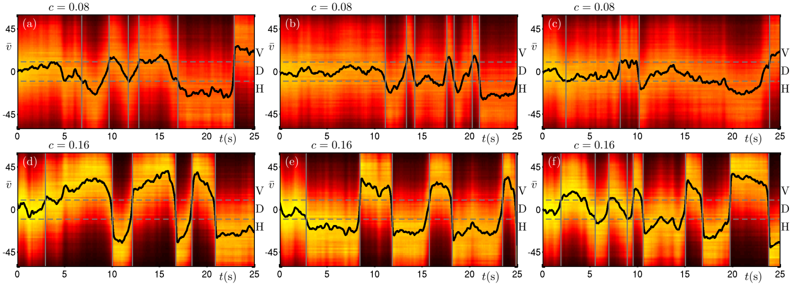


- The log-normal distribution cannot be rejected for $c = 0.08$.
- The gamma distribution cannot be rejected for $c = 0.16$.

Prediction: It is possible to move smoothly from noise-driven to adaptation-driven perceptual switching by varying the contrast.

Examples of switching behaviour with noise

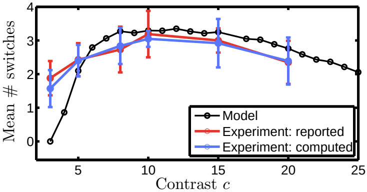
With small $\beta \neq 0$ the time between switches varies.



- Close to contrast threshold the switching appears to be primarily noise driven.
- Above contrast threshold the switching appears to be more adaptation driven.

Contrast and switching rate: all subjects together

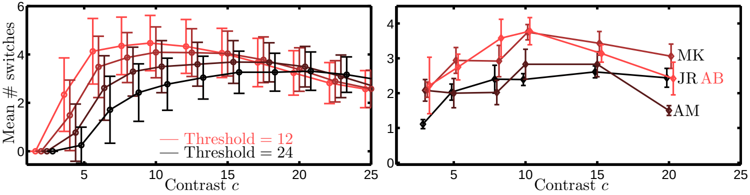
- We show mean and SD of switching time distributions over a range of contrast values.
- Error bars from experimental data come from the averaging across trials and subjects.



Common characteristics:
With increasing contrast, both the model and the experimental results show an early rise, peak and fall in the number of switches.

- Initial rise and saturation due to changing signal to noise ratio.
- Drop off above contrast threshold can be captured by assuming that $w_{1\mathrm{D}}$ decreases at high contrast.

Perception threshold: individual subject data



With increasing threshold:

- The optimal number of switches occurs at a higher contrast.
- The optimal number of switches is lower.
- The curve becomes flatter around the optimal contrast value.

Conclusion about neural fields

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4. Caveats
   - More work needs to be done on the learning side (add a dynamics on the interaction kernels)
   - They have not been applied to many computer vision problems (but see Tobi Delbruck's and Heiko Neumann's talks)

Co-workers

- Ring model of orientations: Romain Veltz
- Publications:
  - Local/Global Analysis of the Stationary Solutions of Some Neural Field Equations, Romain Veltz, Olivier Faugeras, SIAM Journal on Applied Dynamical Systems, (2010).
  - Illusions in the Ring Model of visual orientation selectivity, Romain Veltz, Olivier Faugeras, ArXiv, (2010).
  - Stability of the stationary solutions of neural field equations with propagation delays, Romain Veltz, Olivier Faugeras, The Journal of Mathematical Neuroscience, (2011).

Co-workers

- Multistable barber pole illusion:
  1. INRIA: Pierre Kornprobst, **James Rankin**
  2. INT (Marseille): Guillaume Masson, Andrew Meso
  3. Publications:
     - Bifurcation analysis applied to a model of motion integration with a multistable stimulus, J. Rankin, E. Tlapale, R. Veltz, O. Faugeras and P. Kornprobst, Journal of Computational Neuroscience, (2012).
     - Perceptual transition dynamics of a multi-stable visual motion stimulus I: experiments, A.I. Meso, J. Rankin, P. Kornprobst, O. Faugeras, G.S. Masson, Vision Sciences Society 12th Annual Meeting, (2012)
     - Motion direction integration following the onset of multistable stimuli (II): stability properties explain dynamic shifts in the dominant perceived direction, J. Rankin, A. Meso, G.S. Masson, O. Faugeras, P. Kornprobst, European Conference on Visual Perception, (2012).

The sponsor

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