Méthodes de Champ Moyenne et Multi-Échelles pour l'analyse de Populations de Neurones

CIRM October 3rd-7th 2011 - Marseille

Bi-Stability and Spatio-Temporal Irregularity in Neuronal Networks with Non-linear Synaptic Transmission

Gianluigi Mongillo

Laboratory of Neurophysics and Physiology
Paris Descartes University – CNRS UMR 8119

Joint work with:

David Hansel and Carl van Vreeswijk

Laboratory of Neurophysics and Physiology
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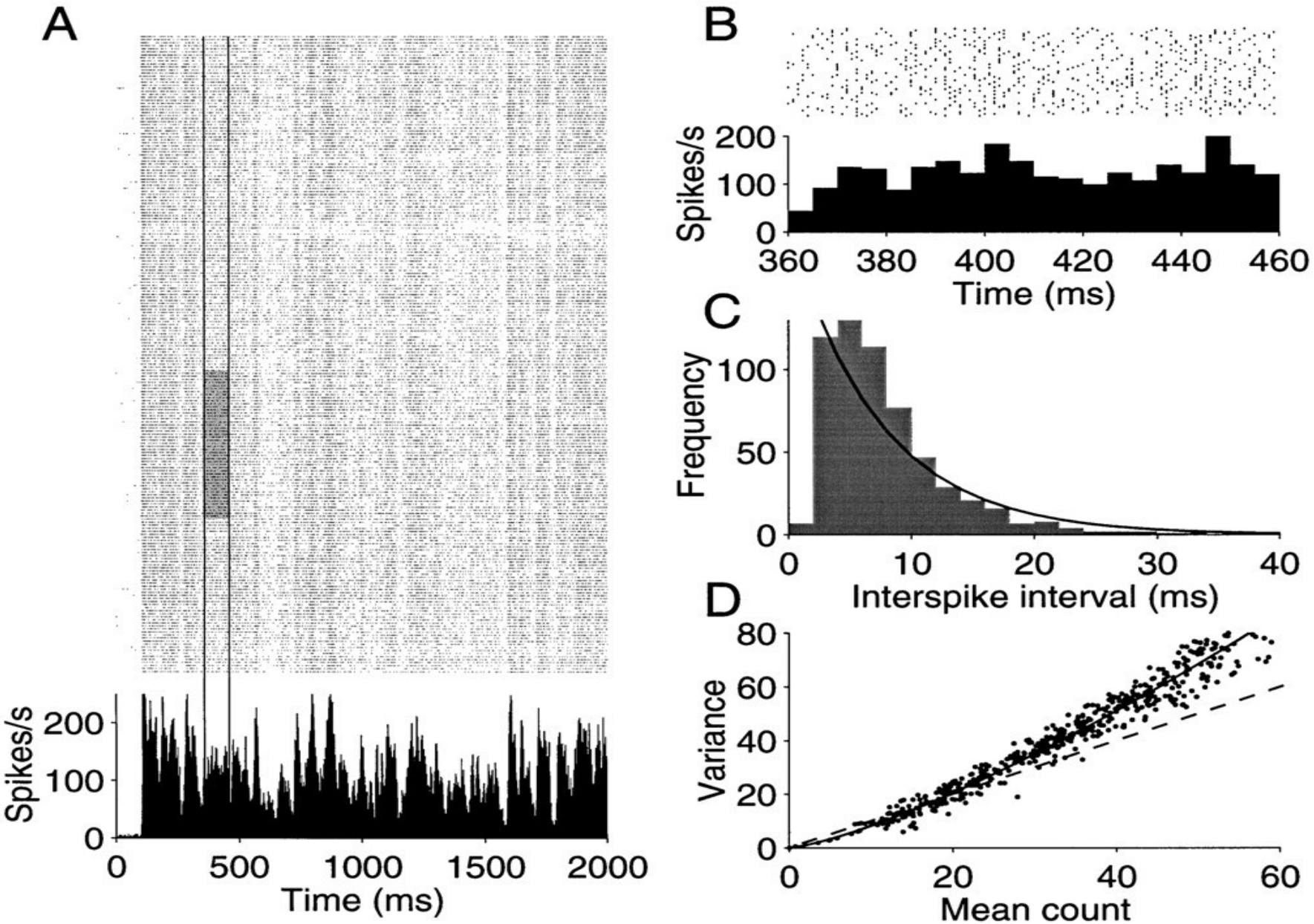
Outline

- **Spatio-temporal irregularity in patterns of neuronal activity supports the notion that cortical networks work in a regime of balanced excitation-inhibition.**

- **Networks working in the balanced regime respond linearly to external inputs, when neurons interact linearly.**

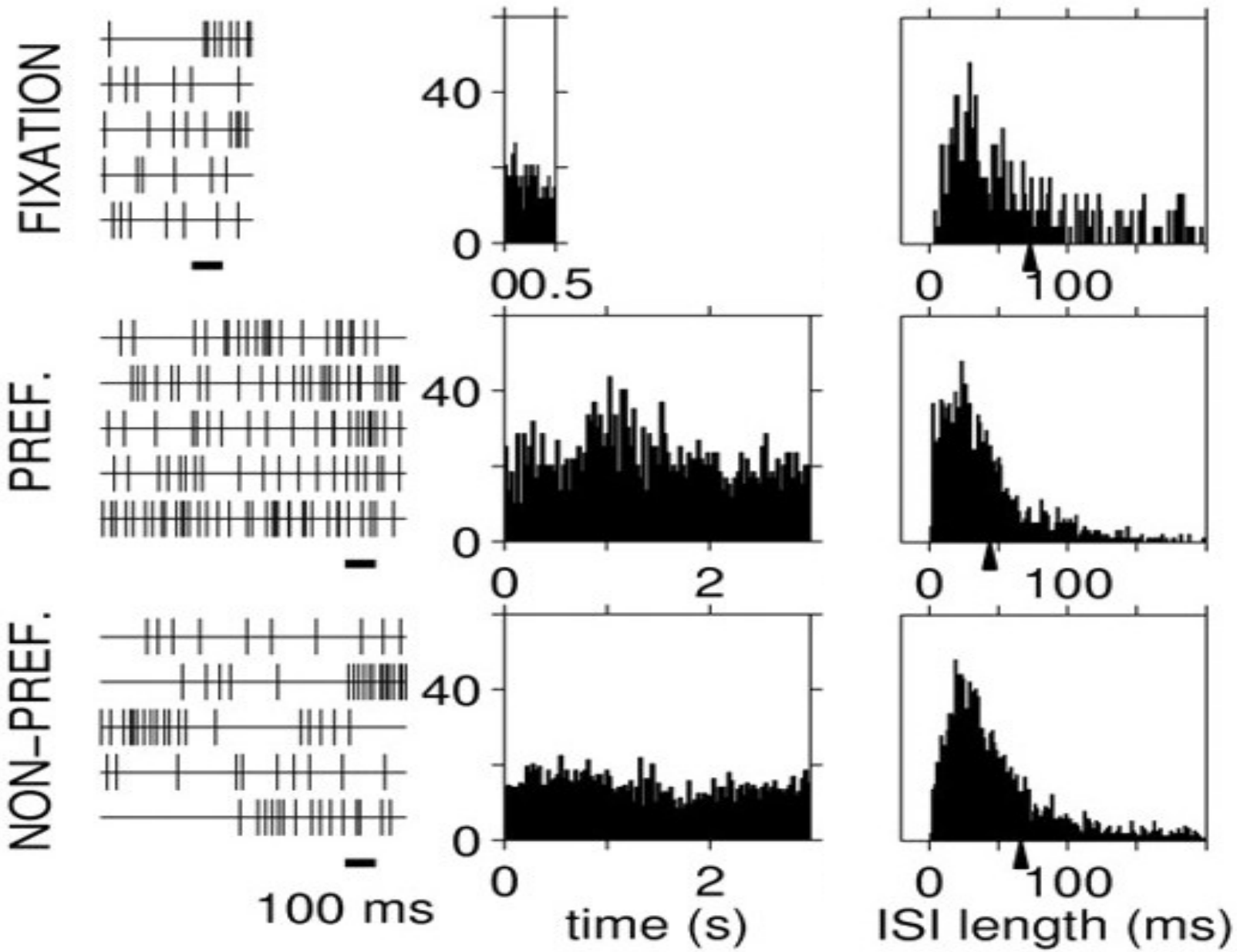
- **Short-term synaptic plasticity (STP) restores non-linearity at the network level in the balanced regime.**

In Vivo Discharge Variability - I



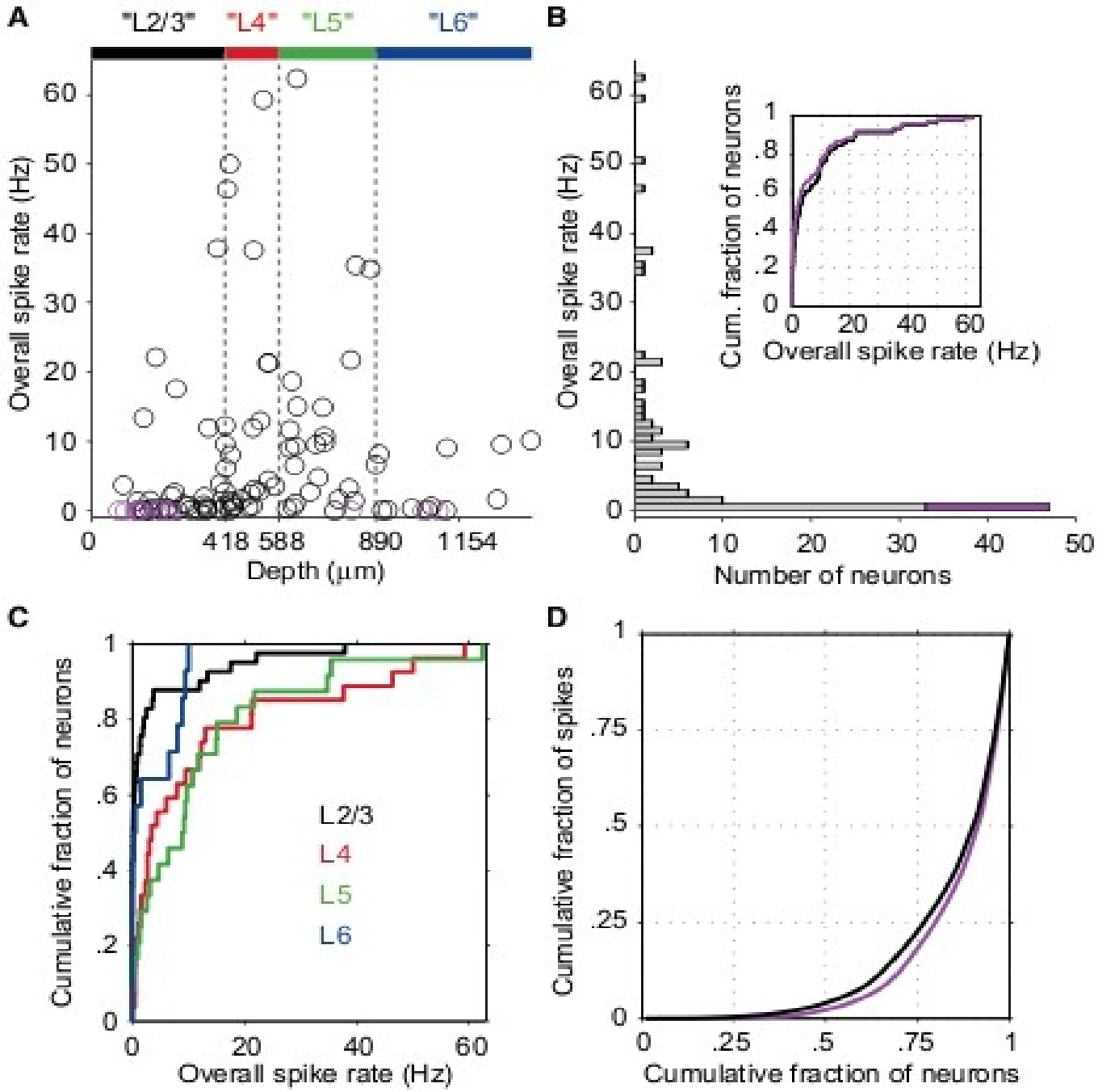
(adapted from Shadlen & Newsome, 1998)

In Vivo Discharge Variability - II



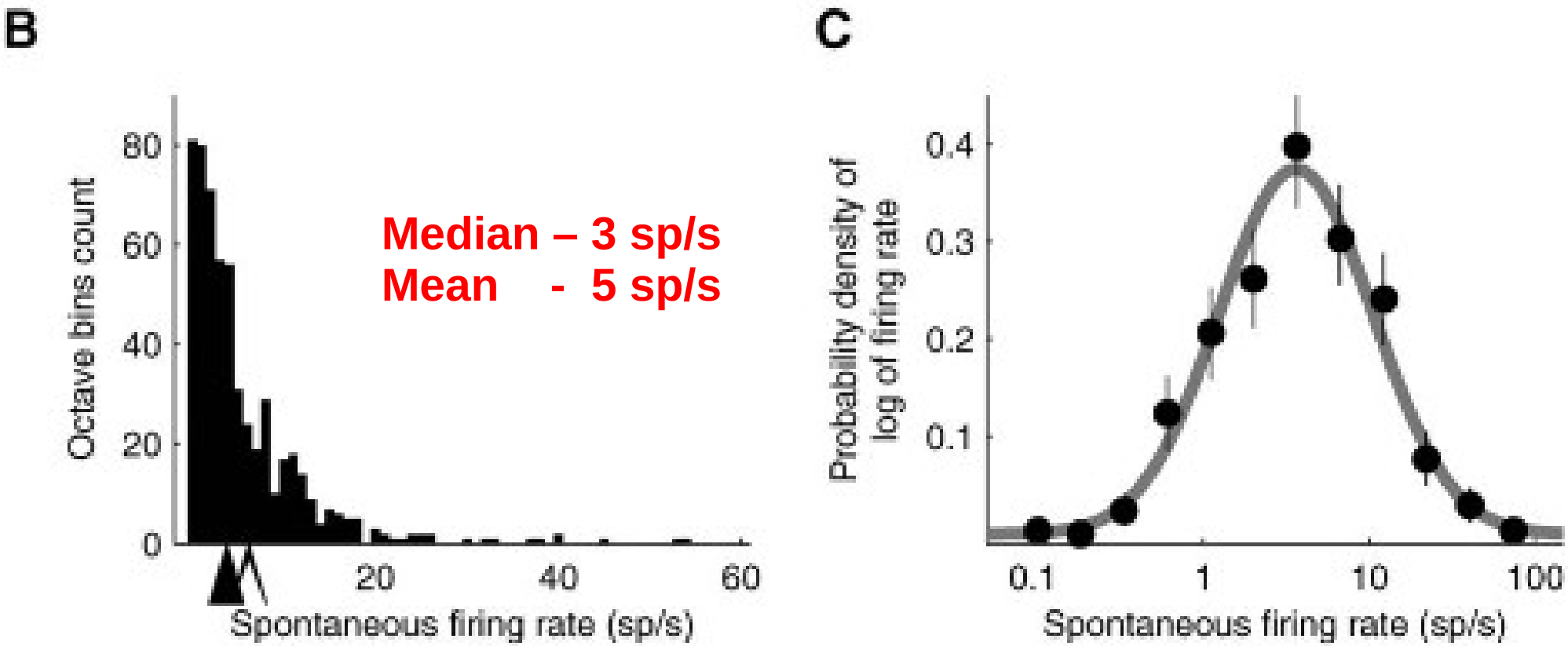
(adapted from Compte *et al.*, 2003)

In Vivo Rate Inhomogeneity - I



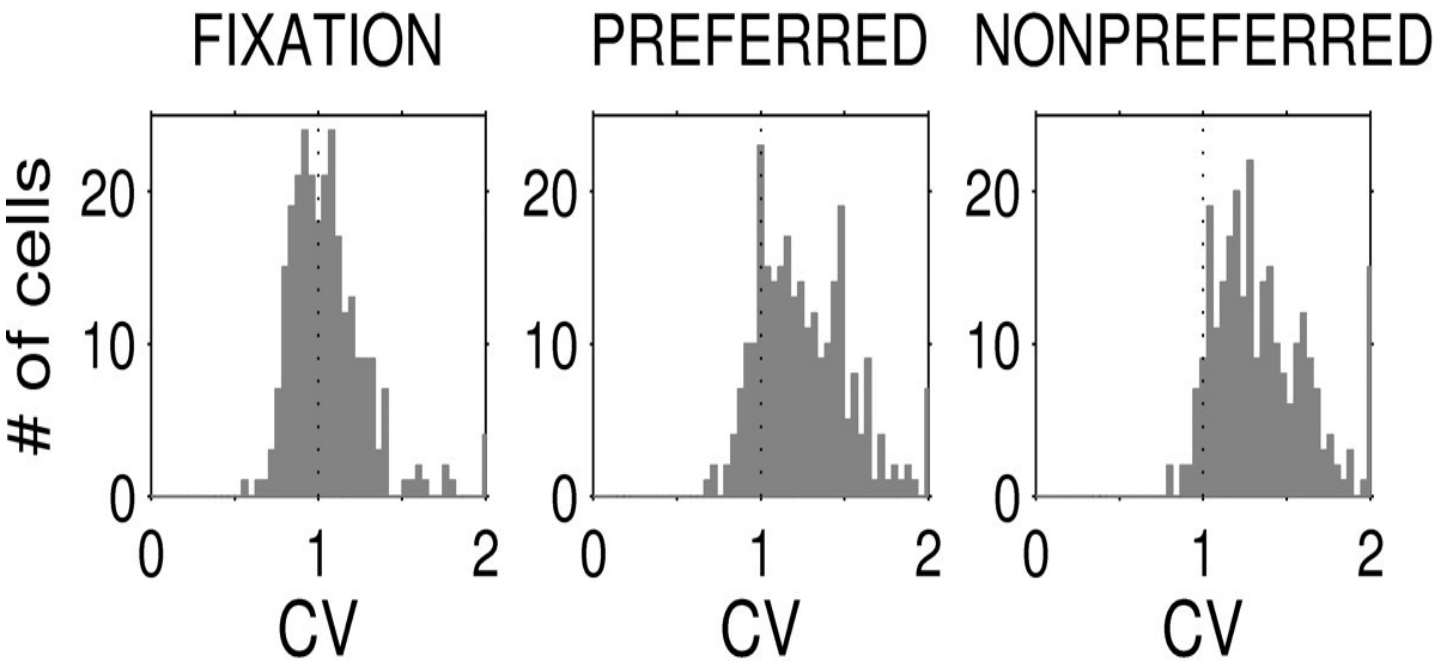
(adapted from O'Connor *et al.*, 2010)

In Vivo Rate Inhomogeneity - II

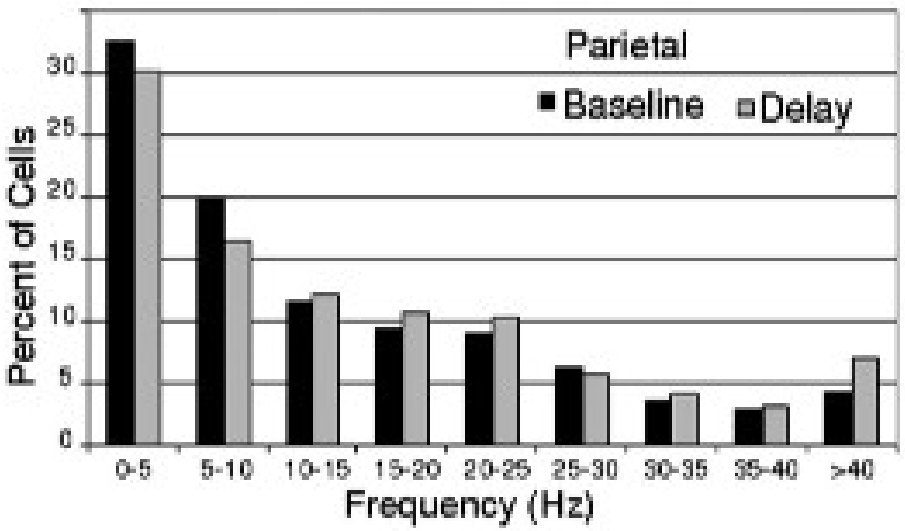


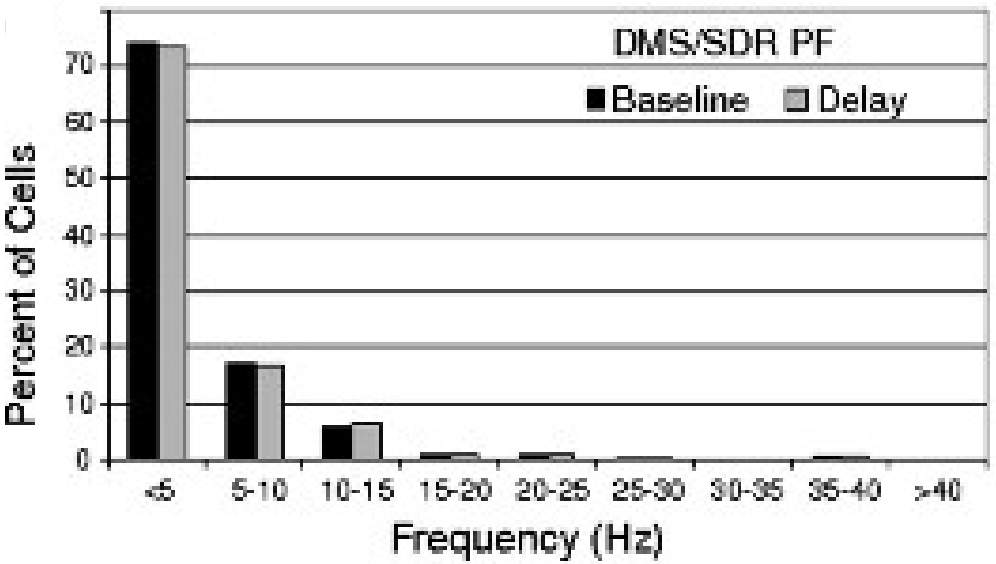
(adapted from Hromàdka *et al.*, 2008)

Spatio-temporal Irregularity During WM



(adapted from Compte *et al.*, 2003)

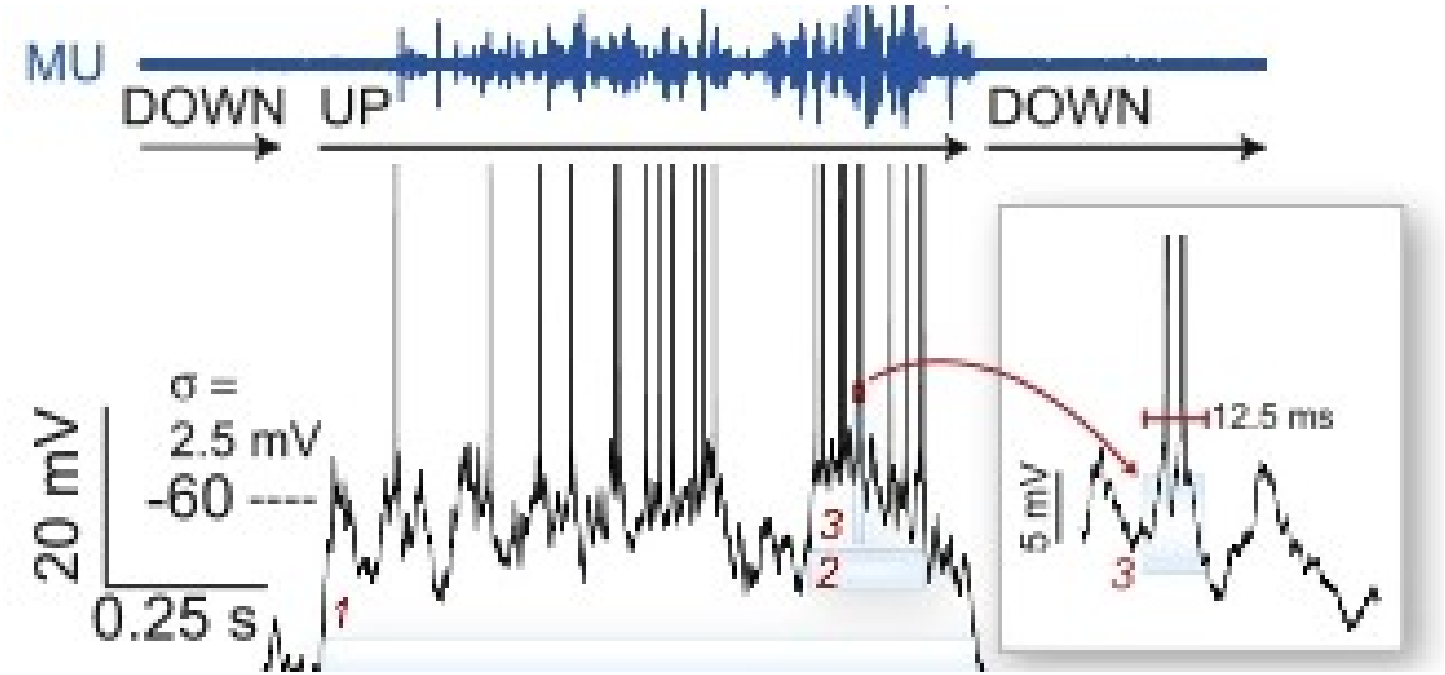




(adapted from Shafi *et al.*, 2007)

In Vitro Noisy States

(adapted from Shu *et al.*, 2003)



(adapted from Haider & McCormick, 2009)

Summary

Patterns of cortical activity exhibit some very generic features:

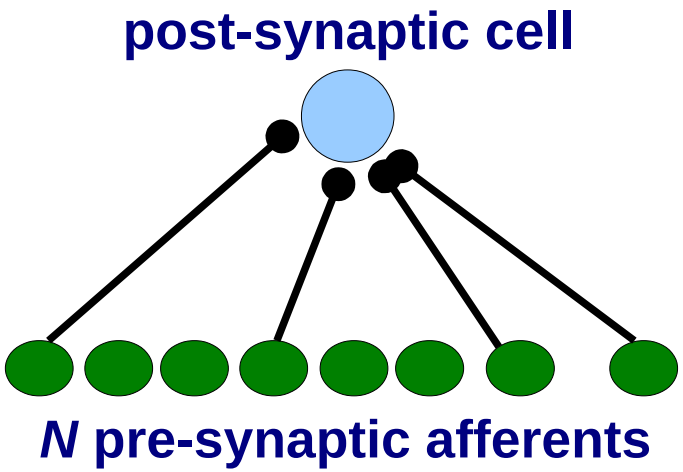
- **Low firing rates.**
- **Temporally irregular spiking resembling Poisson process.**
- **Right-skewed, long-tailed distributions of average rates.**

These features do not depend on:

- **The cortical region.**
- **The functional state of the circuits, *i.e.*, spontaneous activity, stimulus-driven or stimulus-elicited activity**

They are also observed in *in vitro* preparations (*e.g.*, slices)

Generating Spatio-Temporal Irregularity



prob. of contact: C/N with C>>1 and C/N<<1

$$\begin{cases} \mu = C \cdot J \cdot \langle \nu \rangle \\ s^2 = C \cdot J^2 \cdot \langle \nu^2 \rangle \\ \sigma^2 = C \cdot J^2 \cdot \langle \nu \rangle \end{cases} \Rightarrow \boxed{\frac{s}{\mu} \sim \frac{\sigma}{\mu} \sim \frac{1}{\sqrt{C}}}$$

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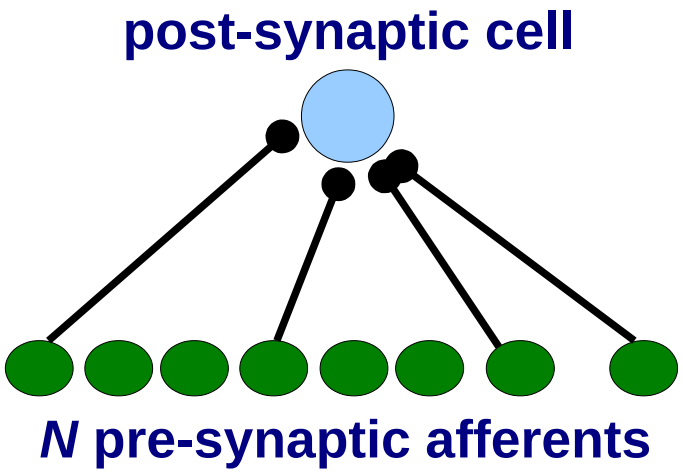

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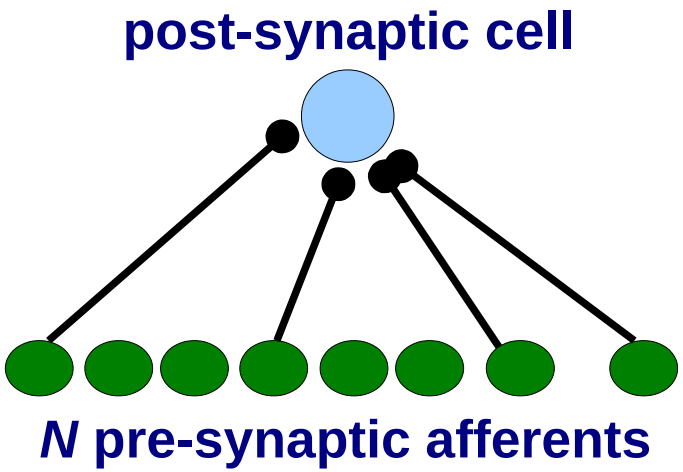
$$J \sim 1/C \qquad \begin{cases} \mu = O(1) \\ s^2 = \sigma^2 = O(1/C) \end{cases}$$

Generating Spatio-Temporal Irregularity



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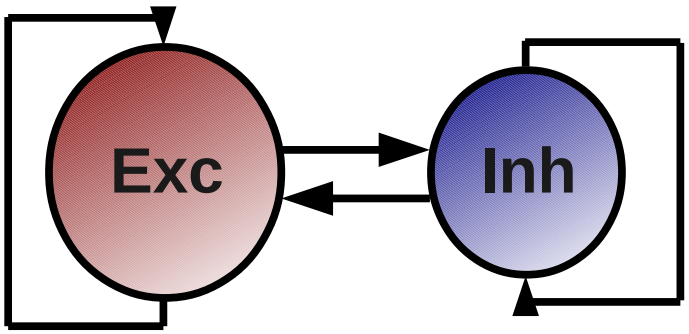
$$J \sim 1/\sqrt{C} \qquad \begin{cases} \mu = O(\sqrt{C}) \\ s^2 = \sigma^2 = O(1) \end{cases}$$

The *Balanced* Regime



$$\begin{cases} \mu_E = \sqrt{C} \cdot \left[I_E^{(ext)} + I_{EE}(\nu_E) - I_{EI}(\nu_I) \right] \\ \mu_I = \sqrt{C} \cdot \left[I_I^{(ext)} + I_{IE}(\nu_E) - I_{II}(\nu_I) \right] \\ s_{E,I}^2 = O(1); \quad \sigma_{E,I}^2 = O(1) \end{cases}$$

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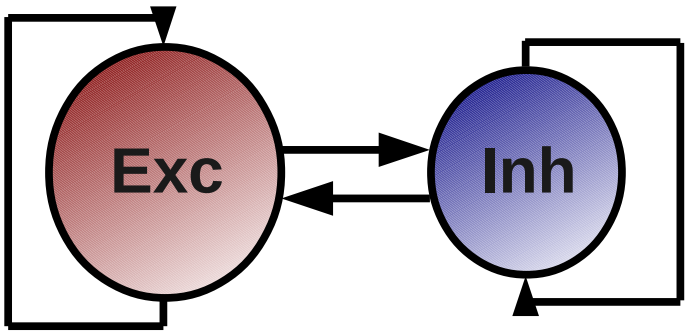

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Excitation and Inhibition should cancel to leading order to produce net inputs the same order as the fluctuations

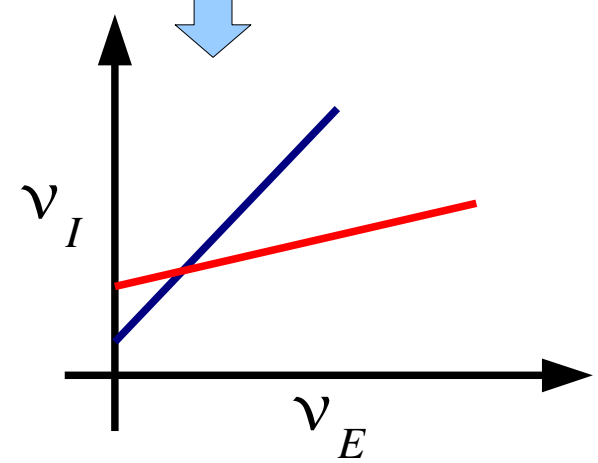
$$\begin{cases} I_E^{(ext)} + I_{EE}(\nu_E) - I_{EI}(\nu_I) = 0 \\ I_I^{(ext)} + I_{IE}(\nu_E) - I_{II}(\nu_I) = 0 \end{cases}$$

(van Vreeswijk & Sompolinsky, 1996, 1998)

Linear Synaptic Transmission

$$I_{\alpha\beta}(\nu_\beta) \sim \nu_\beta$$

$$\begin{cases} J_E^{(ext)}\nu_{ext} + J_{EE}\nu_E - J_{EI}\nu_I = 0 \\ J_I^{(ext)}\nu_{ext} + J_{IE}\nu_E - J_{II}\nu_I = 0 \end{cases} \Rightarrow \begin{cases} \nu_E = A_E \cdot \nu_{ext} \\ \nu_I = A_I \cdot \nu_{ext} \end{cases}$$



- **The network responds linearly to external inputs**
- **A single balanced state exists**

Neuronal non-linearities are effectively *washed out*

'Non-linear' Balance Conditions

Balance conditions

$$\begin{cases} I_E^{(ext)} + I_{EE}(\nu_E) - I_{EI}(\nu_I) = 0 \\ I_I^{(ext)} + I_{IE}(\nu_E) - I_{II}(\nu_I) = 0 \end{cases}$$

If currents are non-linear functions of the rates, the network responds non-linearly to external inputs and, in principle, multiple balanced solutions can exist.

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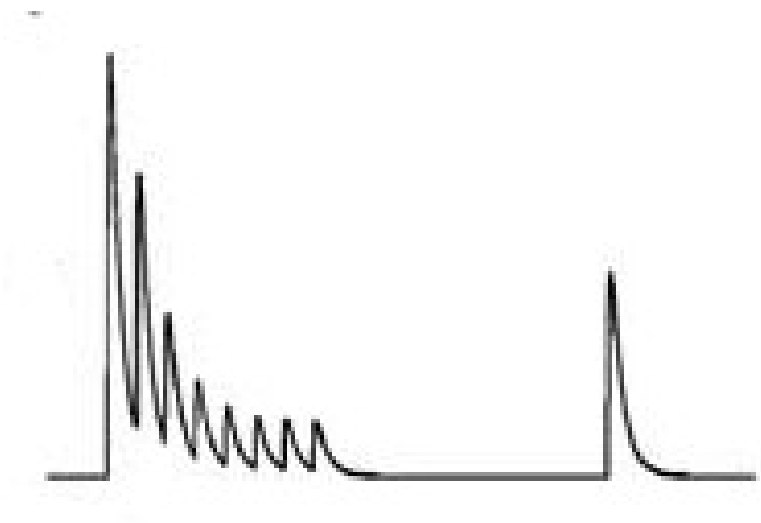
If currents are non-linear functions of the rates, the network responds non-linearly to external inputs and, in principle, multiple balanced solutions can exist.

Short-term synaptic plasticity is a potential source of such a current-rate non-linear relationship (Hansel & Mato, 2008)

Short-Term Synaptic Plasticity (STP)

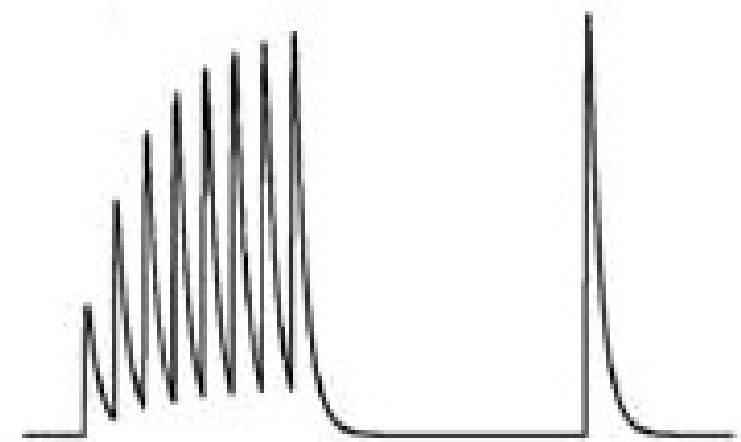
STP refers to temporary modifications of the synaptic efficacy depending on the presynaptic activity, whose time scales are on the order of ~100ms

depression



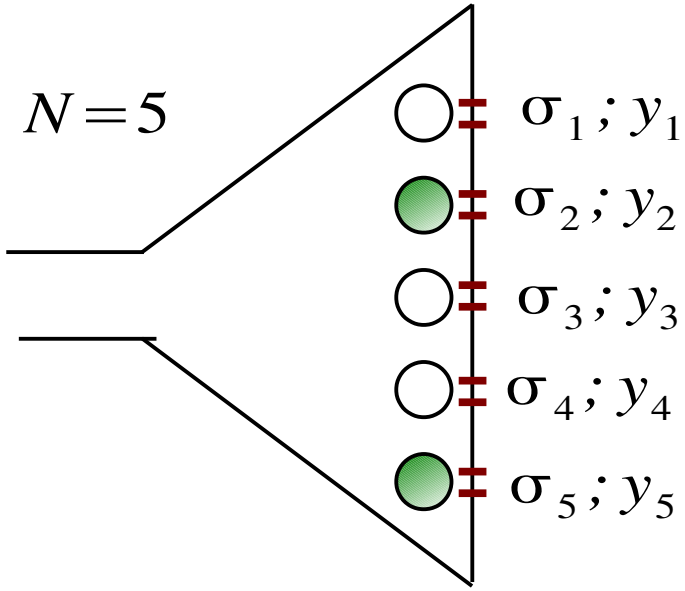
Post-synaptic responses decline over consecutive pre-synaptic spikes

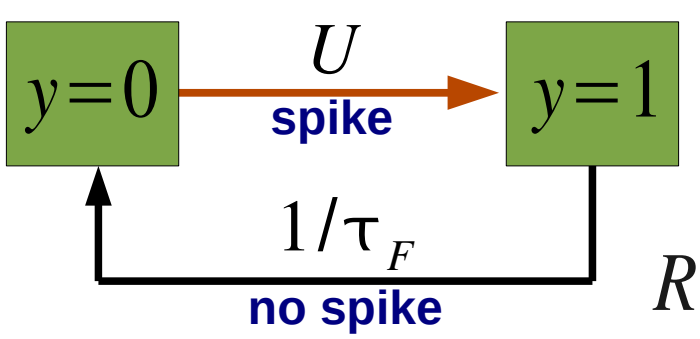
facilitation



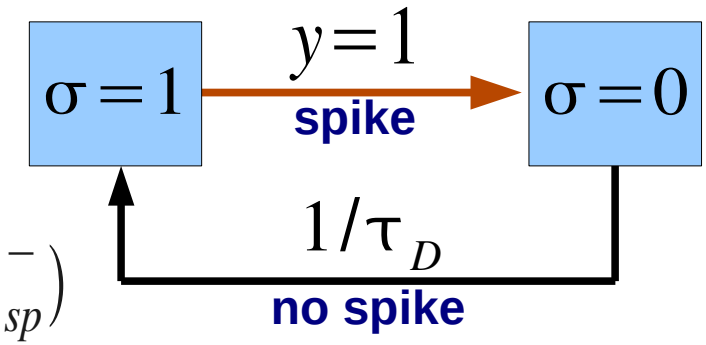
Post-synaptic responses increase over consecutive pre-synaptic spikes

A *Stochastic* STP Model





$$R = y(t_{sp}^{+}) \cdot \sigma(t_{sp}^{-})$$

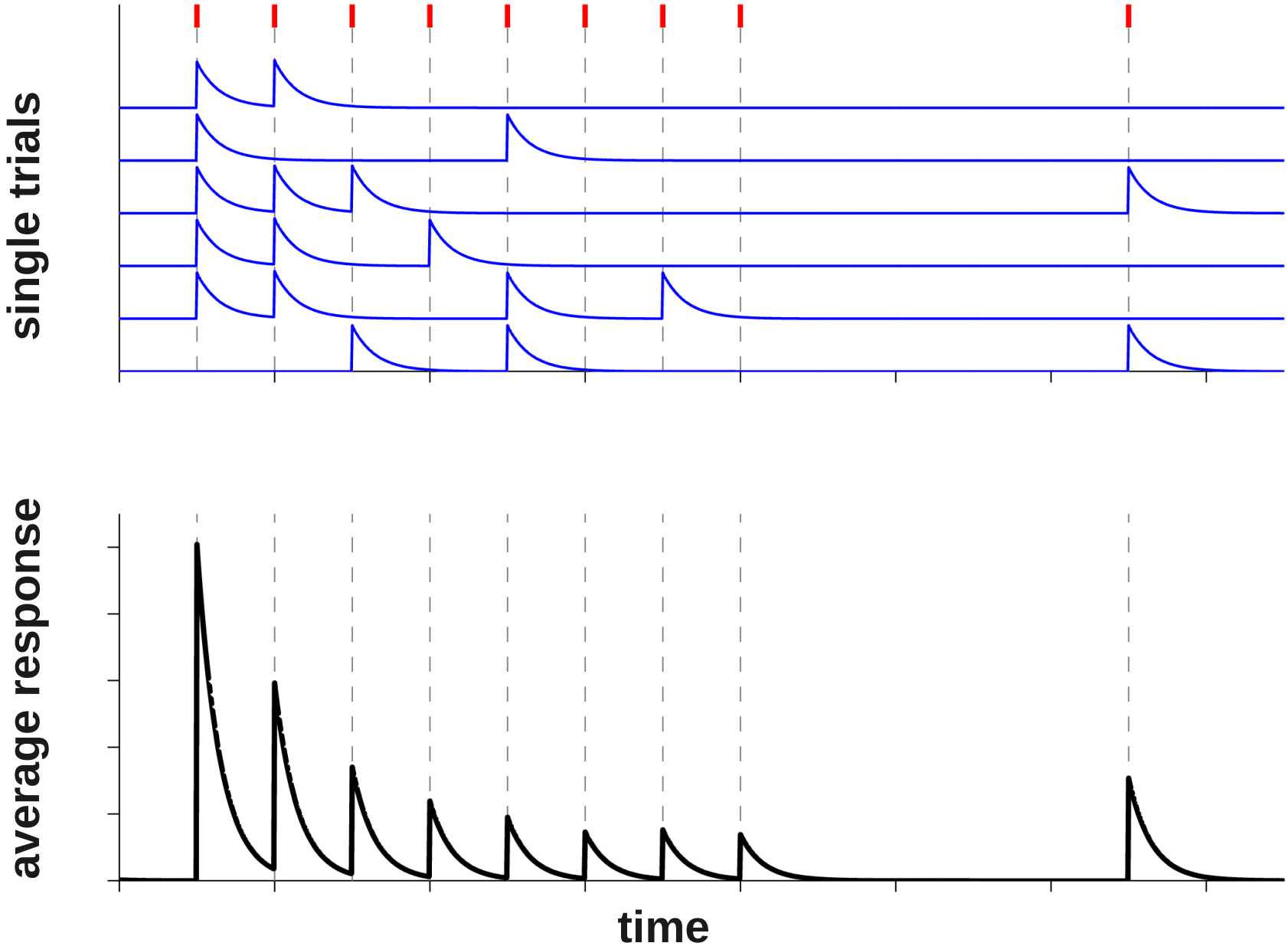


Simplified Calcium kinetics for stochastic release/facilitation

Stochastic depletion model for the depressing component

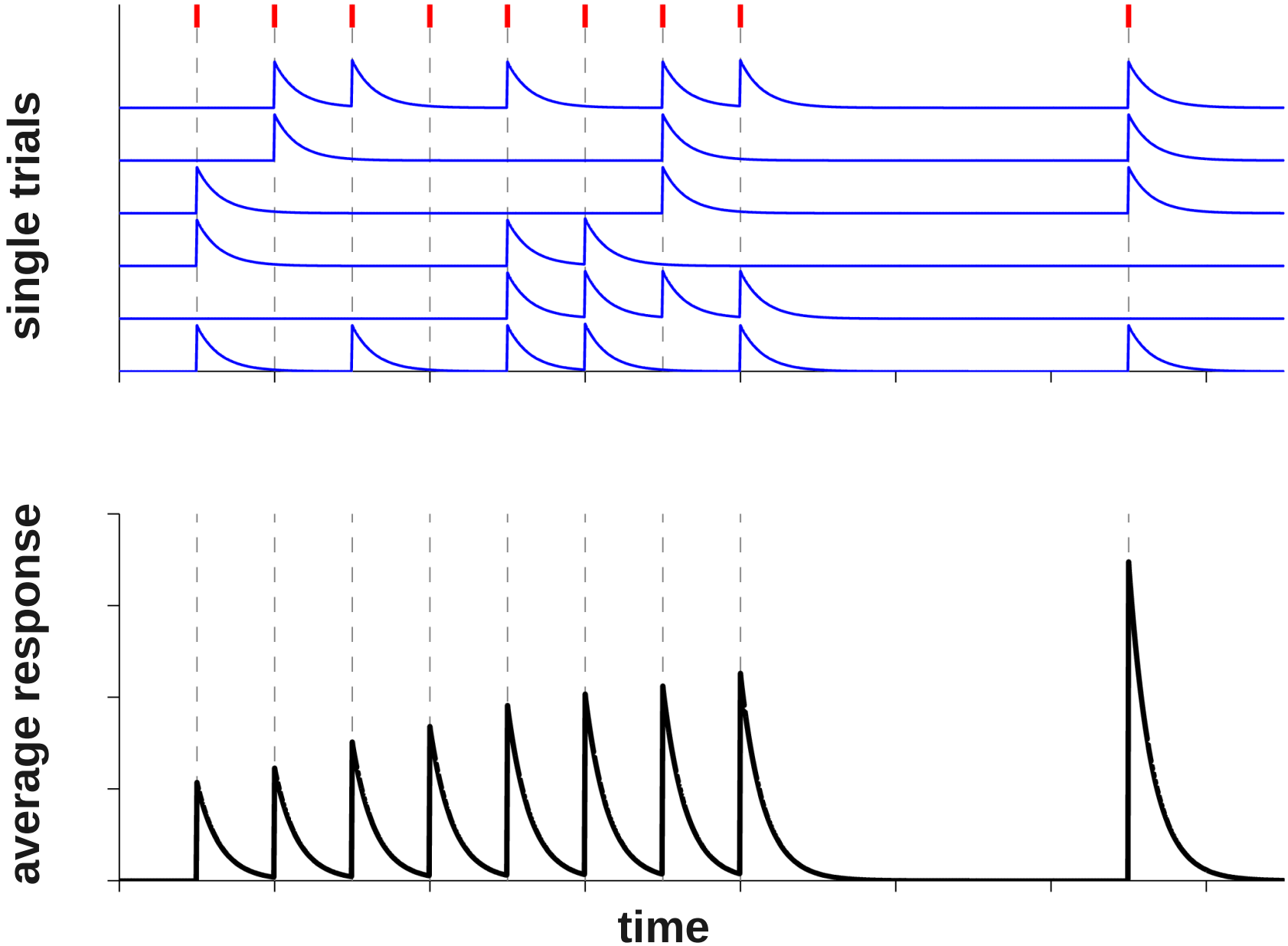
Depressing Synapse

$$U = 0.5\,;\tau_F = 20\,ms\,;\tau_D = 750\,ms$$

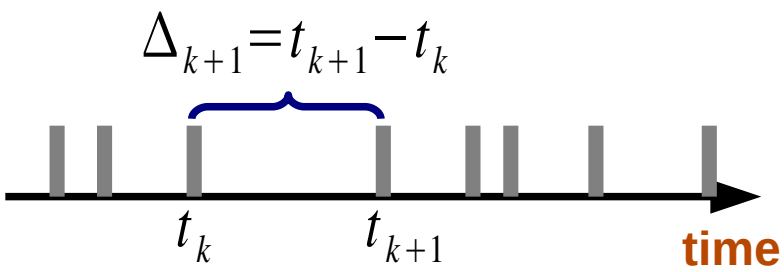


Facilitating Synapse

$$U = 0.1\,;\tau_F = 650\,ms\,;\tau_D = 100\,ms$$



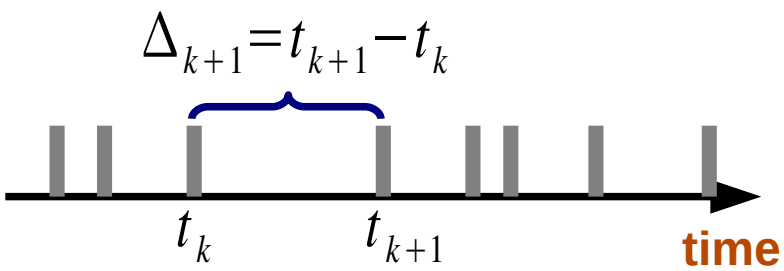
Solving the STP model



$$\begin{cases} \langle y \rangle_{k+1} = [U + (1-U)\langle y \rangle_k] \cdot e^{-\Delta_{k+1}/\tau_F} \\ \langle \sigma \rangle_{k+1} = 1 - [1 - (1-U)(\langle \sigma \rangle_k - \langle \sigma\, y \rangle_k)] \cdot e^{-\Delta_{k+1}/\tau_D} \\ \langle \sigma\, y \rangle_{k+1} = (1 - e^{-\Delta_{k+1}/\tau_D}) \cdot [U + (1-U)\langle y \rangle_k] \cdot e^{-\Delta_{k+1}/\tau_F} \end{cases}$$

$$p_R^{(k+1)} = U \cdot \langle \sigma \rangle_{k+1} + (1-U) \cdot \langle \sigma\, y \rangle_{k+1}$$

Solving the STP model



$$\int_0^\infty d\Delta\, \psi(\Delta) \begin{cases} \langle y \rangle_{k+1} = [U + (1-U)\langle y \rangle_k] \cdot e^{-\Delta_{k+1}/\tau_F} \\ \langle \sigma \rangle_{k+1} = 1 - [1 - (1-U)(\langle \sigma \rangle_k - \langle \sigma\, y \rangle_k)] \cdot e^{-\Delta_{k+1}/\tau_D} \\ \langle \sigma\, y \rangle_{k+1} = (1 - e^{-\Delta_{k+1}/\tau_D}) \cdot [U + (1-U)\langle y \rangle_k] \cdot e^{-\Delta_{k+1}/\tau_F} \end{cases}$$

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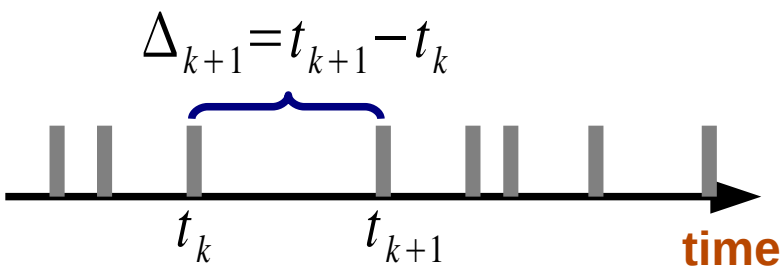

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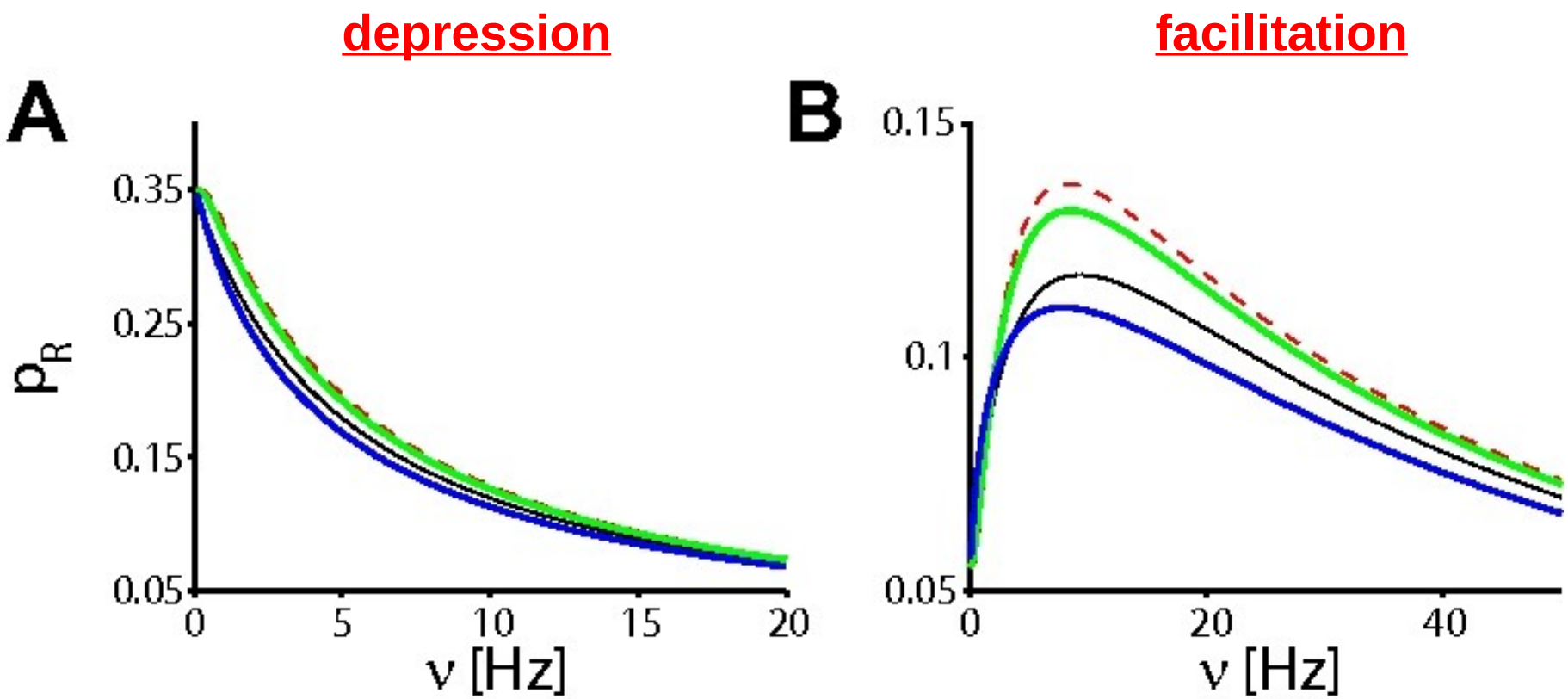
$$p_R = U \cdot \langle \sigma \rangle + (1-U) \cdot \langle \sigma\, y \rangle$$

$$q = p_R \cdot \nu$$

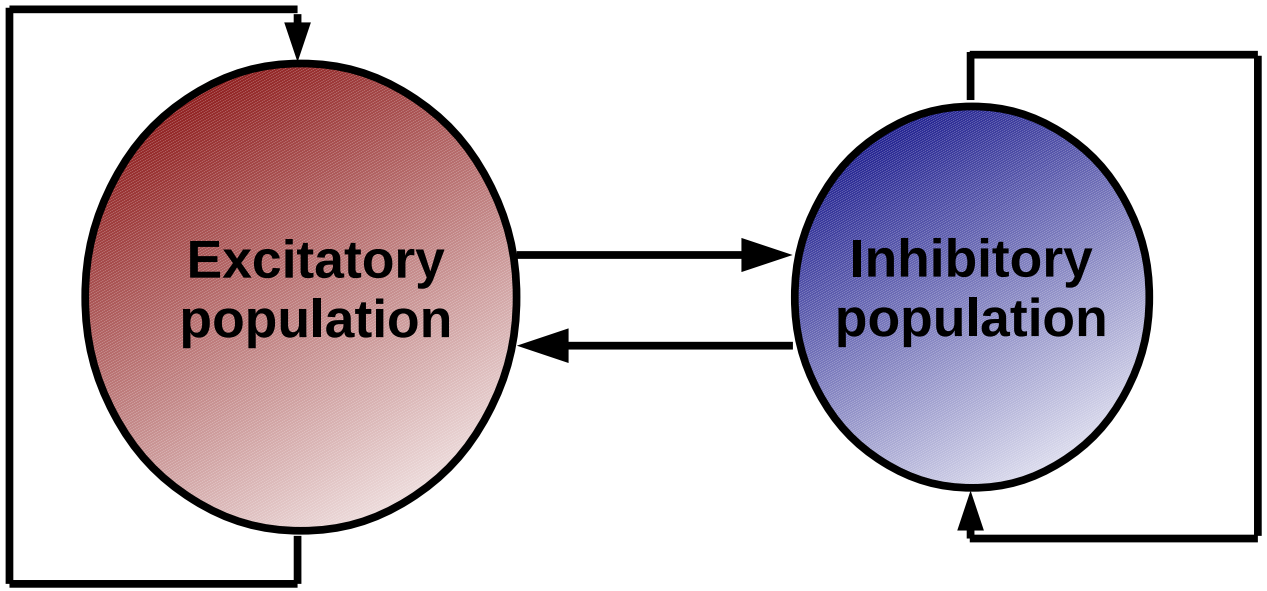
Synaptic Non-Linearities



$U = 0.35;\quad \tau_F = 150\text{ms};\quad \tau_D = 700\text{ms}$

$U = 0.055;\quad \tau_F = 650\text{ms};\quad \tau_D = 200\text{ms}$

The Network Model



- Leaky-Integrate-and-Fire neurons
- Balancing conditions
- STP described by the stochastic model

Mean-Field Analysis

For 1<< C << N

$$\mu_\alpha^i(t) = \mu_\alpha + s_\alpha \cdot \xi_i + \sigma_\alpha \cdot \eta_i(t) \quad \Rightarrow \quad \begin{cases} \mu_\alpha = I_\alpha^{(ext)} + \sum_\beta C_\beta J_{\alpha\beta} q_{\alpha\beta} \\ \sigma_\alpha^2 = \sum_\beta C_\beta J_{\alpha\beta}^2 q_{\alpha\beta} \\ s_\alpha^2 = \sum_\beta C_\beta J_{\alpha\beta}^2 Q_{\alpha\beta} \end{cases}$$

ξ_i **- Gaussian with mean 0 and variance 1**

$\eta_i(t)$ **- White Noise**

$$q_{\alpha\beta} \equiv \frac{1}{N_\alpha \cdot N_\beta} \sum_{i,j} q_{\alpha\beta}^{ij}$$

$$Q_{\alpha\beta} \equiv \frac{1}{N_\alpha \cdot N_\beta} \sum_{i,j} \left(q_{\alpha\beta}^{ij}\right)^2$$

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The C–>∞ limit

$$J_{\alpha\beta} \to J_{\alpha\beta}/\sqrt{C}\,; \quad I_{\alpha}^{(ext)} \to \sqrt{C}\cdot I_{\alpha}^{(ext)}$$

- **STP is restricted to the E→E synapses**
- **All neurons receive the same number of connections**

$$\begin{cases} I_E^{(ext)} + J_{EE}\, q_{EE} - J_{EI}\, \nu_I = 0 \\ I_I^{(ext)} + J_{IE}\, \nu_E - J_{II}\, \nu_I = 0 \end{cases}$$

$$\sigma_{\alpha}^2 = \sum_{\beta} J_{\alpha\beta}\, q_{\alpha\beta}$$

$$q_{EE}(\mu_E, \sigma_E) = K \cdot (\nu_E - \nu_0)$$

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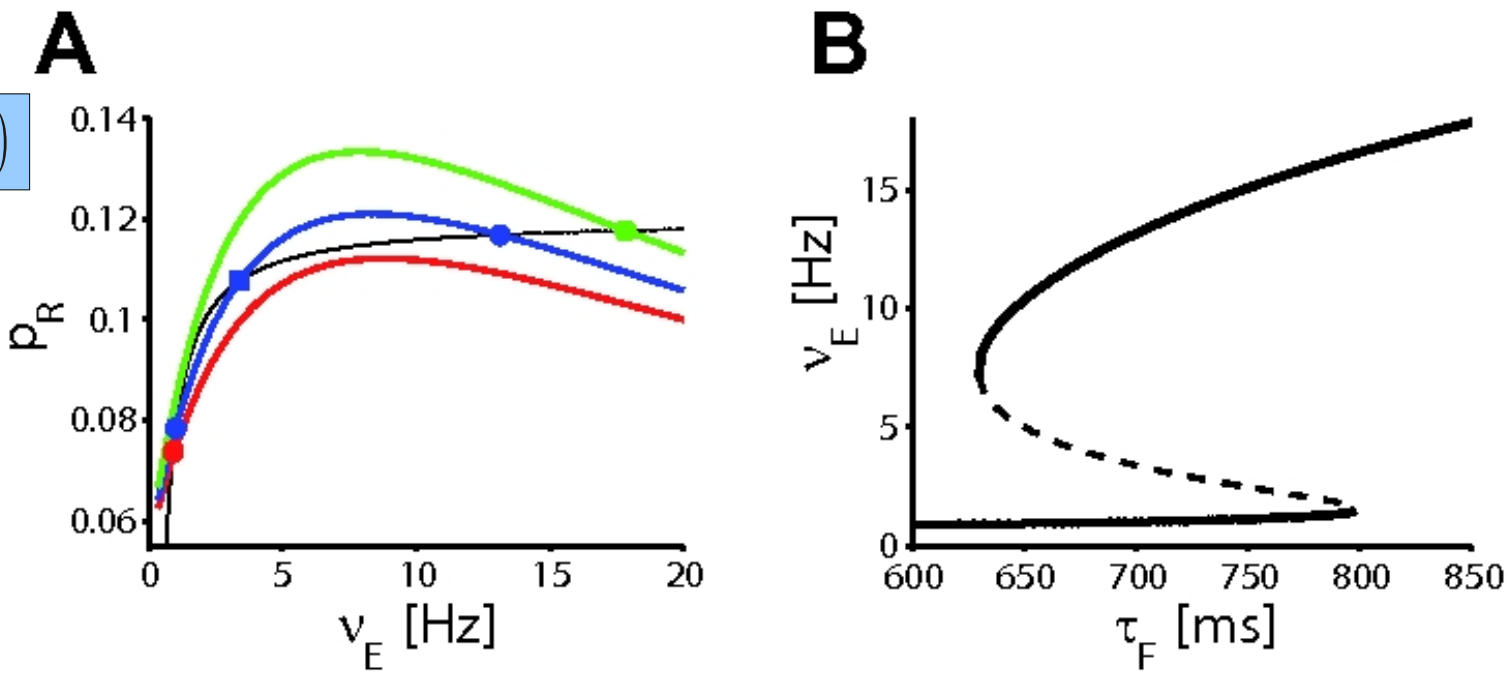
$$\nu_E = \Phi(\mu_E, \sigma_E(\nu_E)) \;\to\; \mu_E = \mu_E(\nu_E)$$

$$\hat{q}_{EE}(\nu_E) = q_{EE}(\mu_E(\nu_E), \sigma_E(\nu_E)) = \hat{p}_R(\nu_E)\cdot\nu_E$$

$$\hat{p}_R(\nu_E) = K\cdot(1 - \nu_0/\nu_E)$$

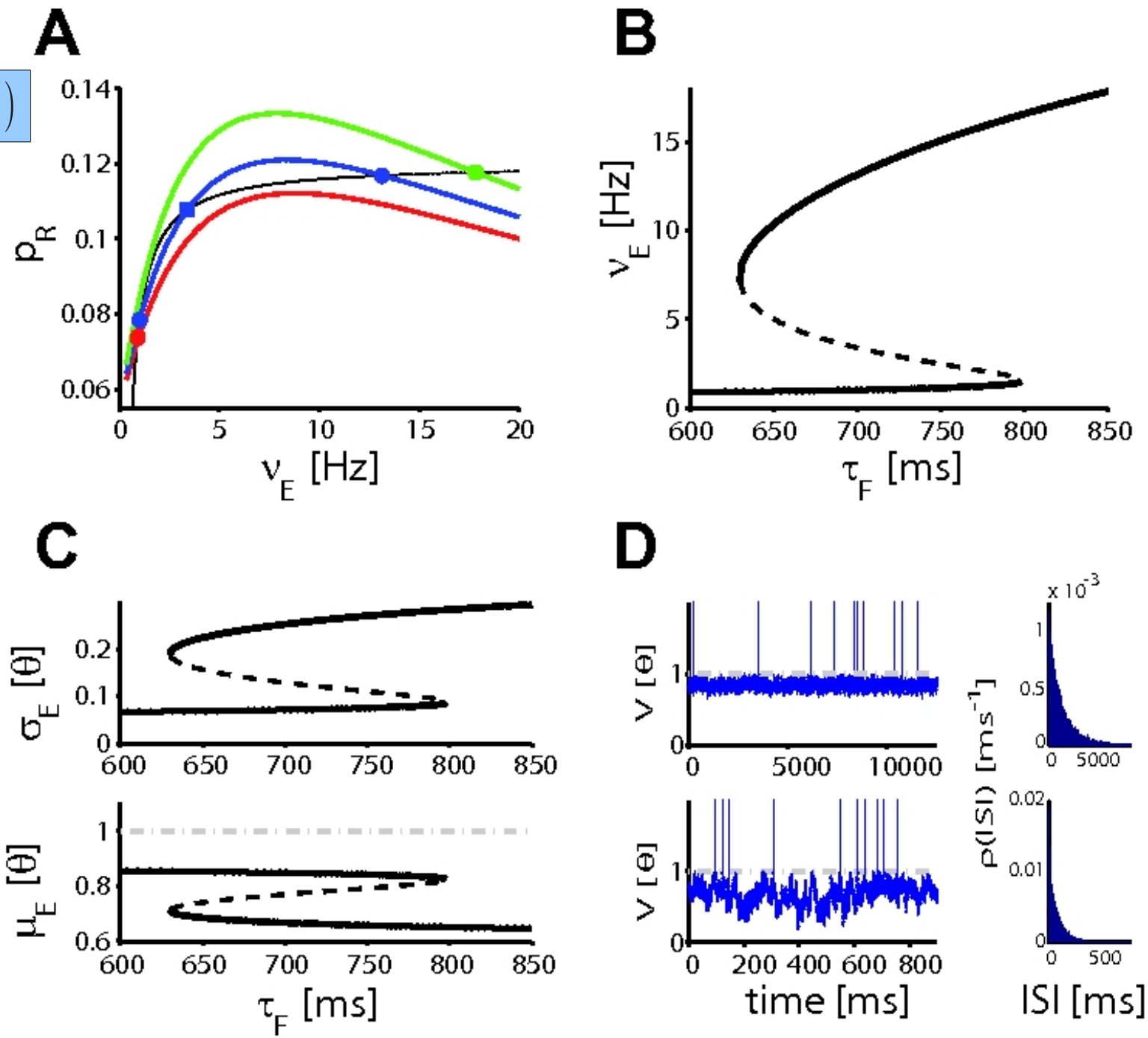
Bi-stability in the *C*–>∞ limit

$$\hat{p}_R(\nu_E) = K \cdot (1 - \nu_0/\nu_E)$$



Bi-stability in the C–>∞ limit

$$\hat{p}_R(\nu_E) = K \cdot (1 - \nu_0/\nu_E)$$



Robustness to Fast/Quenched Noise

$$\hat{p}_R(\nu_E) = K \cdot (1 - \nu_0/\nu_E)$$

$$\begin{cases} J_{\alpha\beta} \rightarrow g_s \cdot J_{\alpha\beta} \\ I_\alpha^{(ext)} \rightarrow g_s \cdot I_\alpha^{(ext)} \end{cases}$$

at parity of activity

$$\begin{cases} K \rightarrow K; \quad \nu_0 \rightarrow \nu_0 \\ \sigma_\alpha^2 \rightarrow g_s^2 \cdot \sigma_\alpha^2 \end{cases}$$

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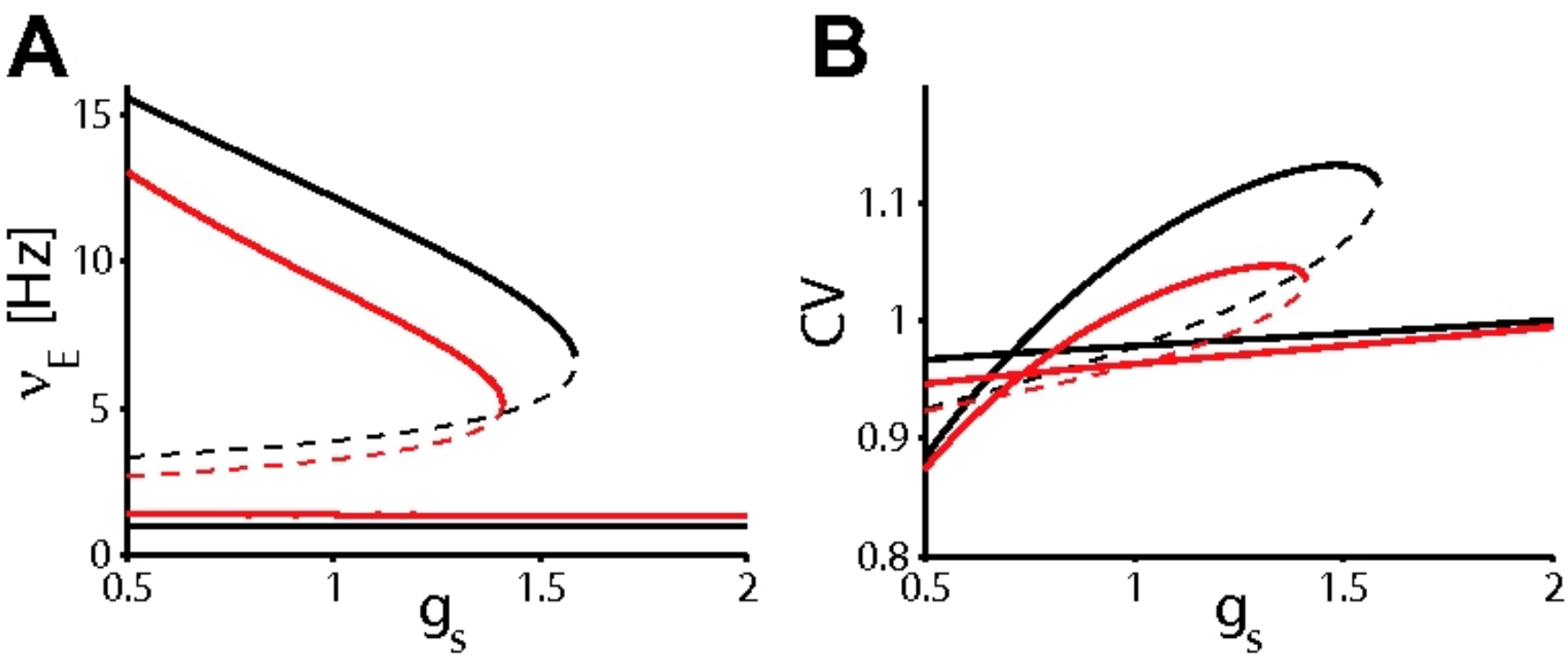
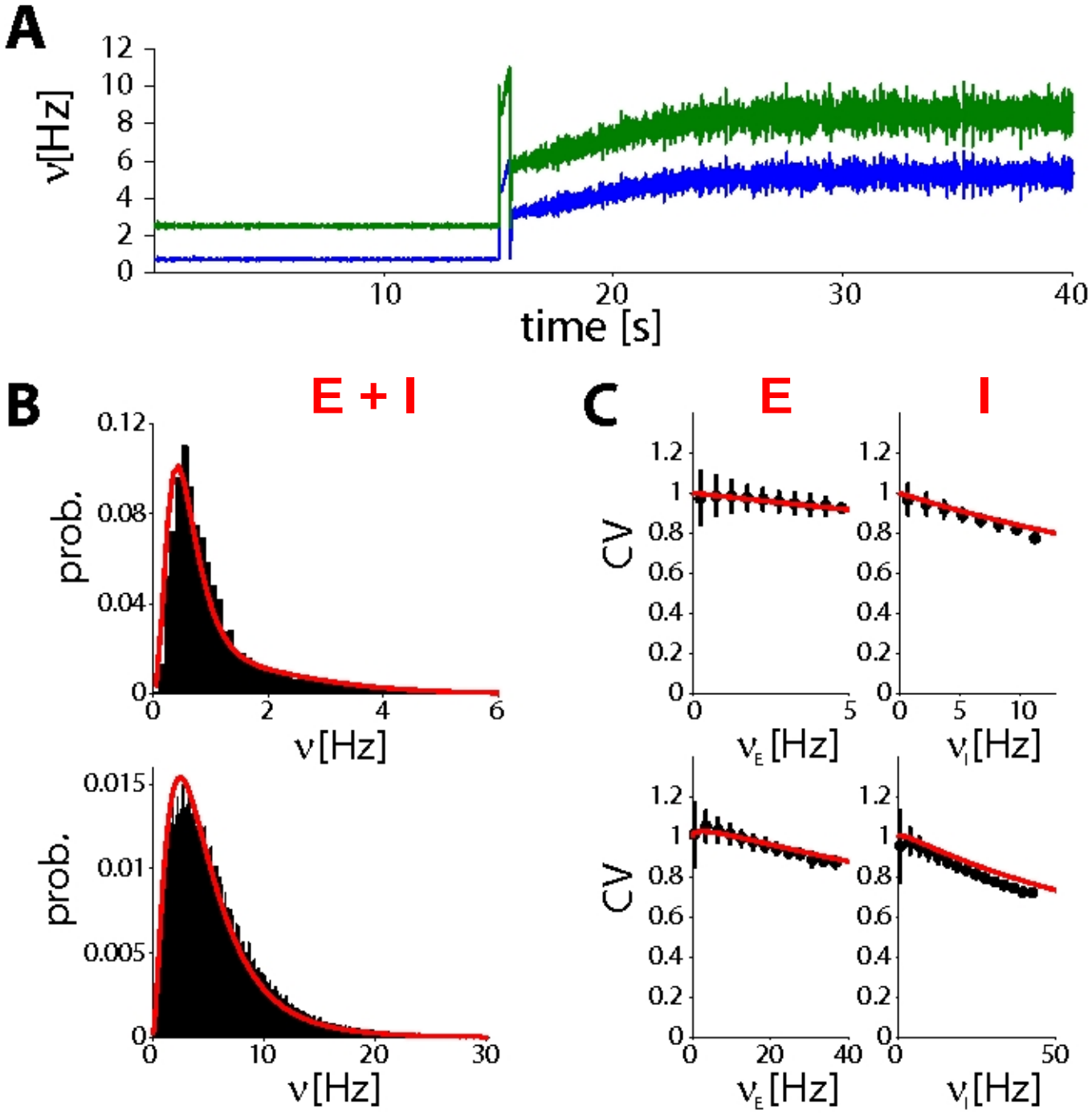


Numerical Simulations



LR state

HR state

Conclusions

- **Non-linearities originating from synaptic function support co-existing balanced states of activity.**

- **Self-generated fluctuations in combination with activity-dependent synaptic interactions stabilize the different steady-states.**

- **New functional role for short-term plasticity:**

  - **Supplying a robust source of non-linearity for networks operating In the presence of significant spatio-temporal irregularity.**