THESIS DEFENSE

Centralized and Decentralized Multiuser Schemes for Wireless Communications

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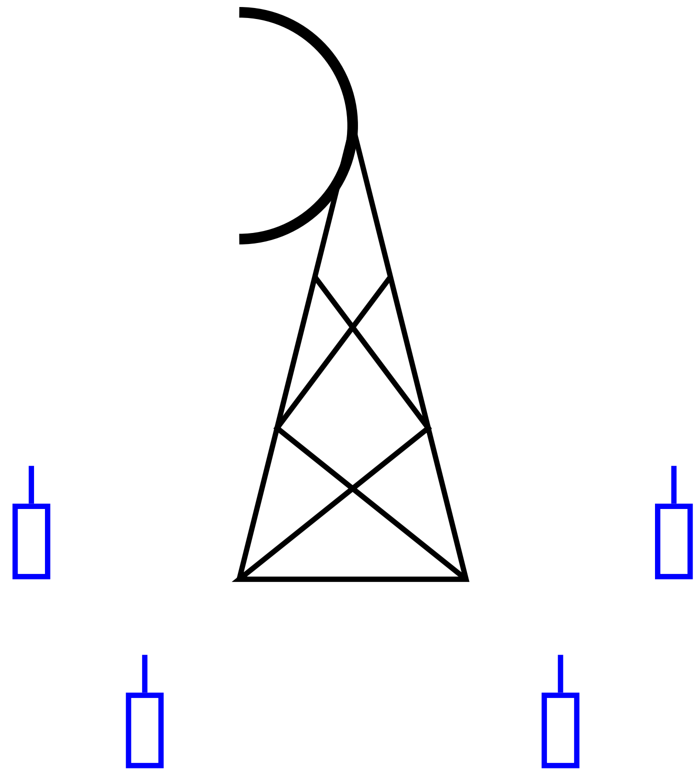
Motivation and Overview

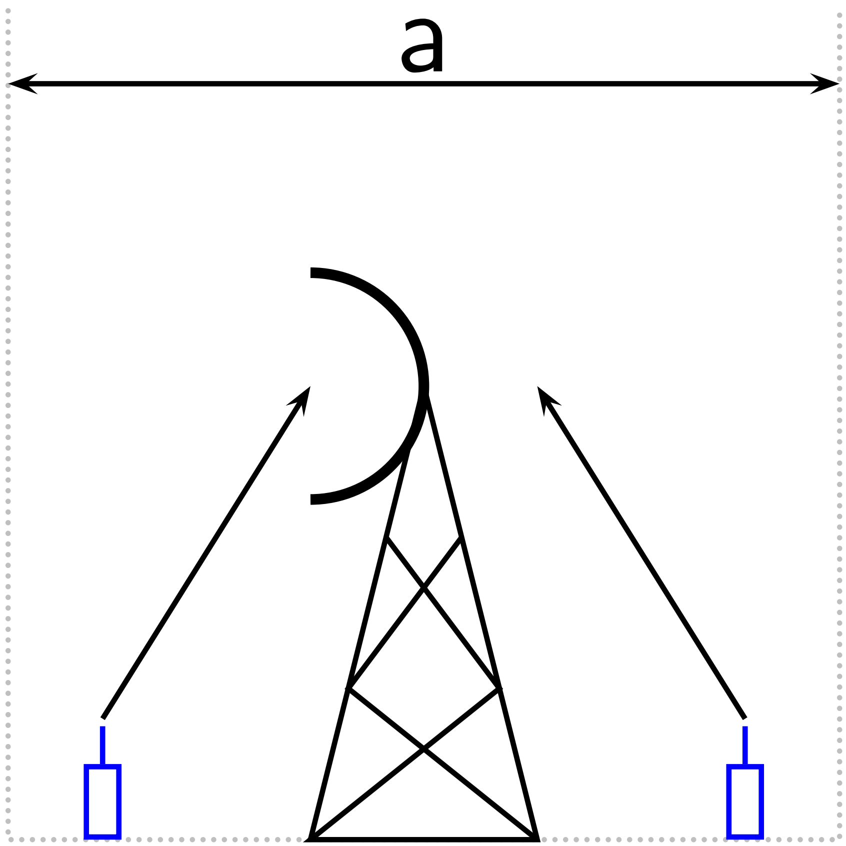
- Goal: Design and deployment of an infrastructure.
- Study centralized and decentralized systems.
- Several types of multiple access protocols: focus on CDMA in this presentation.
- Evaluation of the performance of large cellular systems.

Outline

- Motivation
- Single Cell CDMA
- Cellular network
- Distributed scheme
- Conclusions



Single Cell CDMA



- The spreading length is N.
- The number of users is K.
- Communication over the multipath channel: frequency selective fading.

Previous results on Single Cell CDMA

- Downlink CDMA framework:

  Asymptotic expressions of the spectral efficiency with orthogonal codes have been provided taking into account the code orthogonality structure and the channel statistics (Debbah et al. (03), Chauffray et al. (04)).

- Uplink CDMA framework:

  Asymptotic expressions of the spectral efficiency with i.i.d. random spreading codes have been provided in the case of

  - no fading (Verdú & Shamai (01)),
  - flat fading (Tse & Hanly (99), Shamai & Verdú (01)),
  - frequency selective fading (Tulino, Li & Verdú (05)).

- Why not orthogonal codes in uplink CDMA?

Intuition

- Intuitively:
  - In the case of flat fading, orthogonality is preserved.
  - In the case of frequency selective fading, orthogonality is destroyed.
- How can we translate this intuition into theoretical assessments?

Communication Model: MC-CDMA

- Spreading length N, number of users K.
- The received signal $\mathbf{y}$ at the base station is of the form:

$$\mathbf{y} = (\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W})\mathbf{s} + \mathbf{n}.$$

  - $\mathbf{H}$ is the frequency selective fading matrix, of size $N \times K$.
  - $\sqrt{\mathbf{P}}$ is the root square of the diagonal path loss matrix, of size $K \times K$.
  - $\mathbf{W}$ is an $N \times K$ spreading matrix.
  - $\mathbf{s}$ is the signal sent by the users, of size $K \times 1$.
  - $\mathbf{n}$ is the $N \times 1$ additive white Gaussian noise vector, with mean zero and variance σ^2.

- Remark: asymptotically equivalent to direct-sequence CDMA (Hachem (04)).
- How to analyze this system?

Performance Analysis

- Signal to Interference plus Noise ratio (SINR):

$$\mathsf{SINR}_k = \frac{\text{User } k\text{'s signal power}}{\text{Other users' signal power} + \text{Noise power}}.$$

- Spectral Efficiency: number of information bits per time, frequency and distance unit that the system is able to deliver.

$$C \propto \sum \log(1 + \mathsf{SINR}_k).$$

- In the finite size case: untractable computations.

- Asymptotic setting: when N and K tend to infinity with fixed ratio α.

- Application of Random Matrix Theory results.

Assumptions: Multipath Channel

- For user k, the channel impulse response is

$$c_k(\tau) = \sum_{p=0}^{L-1} c_{pk}\phi(\tau - \tau_{pk})$$

where $\phi(\cdot)$ is the transmit pulse filter.

- The Fourier transform of c_k after pulse matched filtering at the receiver is

$$h_k(f) = \sum_{p=0}^{L-1} c_{pk} e^{-j2\pi f \tau_{pk}} |\Psi(f)|^2 \text{ where } \Psi(f) = \begin{cases} 1 & \text{if } -\frac{W}{2} \leq f \leq \frac{W}{2} \\ 0 & \text{otherwise.} \end{cases}$$

- For $x \in [0, \alpha]$, define $h(f, x) = h_k(f)$ if $\frac{k}{N} \leq x < \frac{k+1}{N}$.

Code Structure Model

- Orthogonal Case:
  - Case when users are synchronized.
  - User codes $\mathbf{w}_k$ are extracted from a Haar distributed unitary matrix.
  - Definition: A random matrix $\mathbf{V}$ is Haar distributed if it takes its values uniformly in $\mathcal{U}(N)$.
  - Generation: $\mathbf{V} = \mathbf{X}(\mathbf{X}^H\mathbf{X})^{-1/2}$, where $\mathbf{X}$ is an i.i.d. Gaussian matrix, is Haar distributed.

- i.i.d. Random Case:
  - Case when users are not synchronized.
  - Entries of $\mathbf{W}$ are i.i.d. Gaussian, with zero mean and variance $1/N$.

Proposition

- Proposition: when $N \to \infty$ and $\frac{K}{N} \to \alpha$, the SINR with matched filter is

$$\text{SINR}_k^{\text{orth}} = \frac{P(x_k)\left(\int_0^1 |h(f,x)|^2 df\right)^2}{\sigma^2 \int_0^1 |h(f,x)|^2 df + \left(\int_0^\alpha \int_0^1 P(y)|h(f,x)|^2 |h(f,y)|^2 df\, dy - \mu(x)\right)}$$

with $\mu(x) = \int_0^\alpha P(y) |\int_0^1 h(f,x) h^*(f,y) df|^2 dy$.

$$\text{SINR}_k^{\text{rand}} = \frac{P(x_k)\left(\int_0^1 |h(f,x)|^2 df\right)^2}{\sigma^2 \int_0^1 |h(f,x)|^2 df + \left(\int_0^\alpha \int_0^1 P(y)|h(f,x)|^2 |h(f,y)|^2 df\, dy\right)}.$$

- The asymptotic SINR depends only on a few meaningful parameters: α, σ^2 and the distribution of the elements of $\mathbf{H}$.

- $\text{SINR}_k^{\text{rand}}$ is always inferior to $\text{SINR}_k^{\text{orth}}$.

Under Simplifying Assumptions. . .

- Assume the delays are uniformely distributed according to the bandwidth.
- Comparison with the SINR with i.i.d. random codes: orthogonality gain.

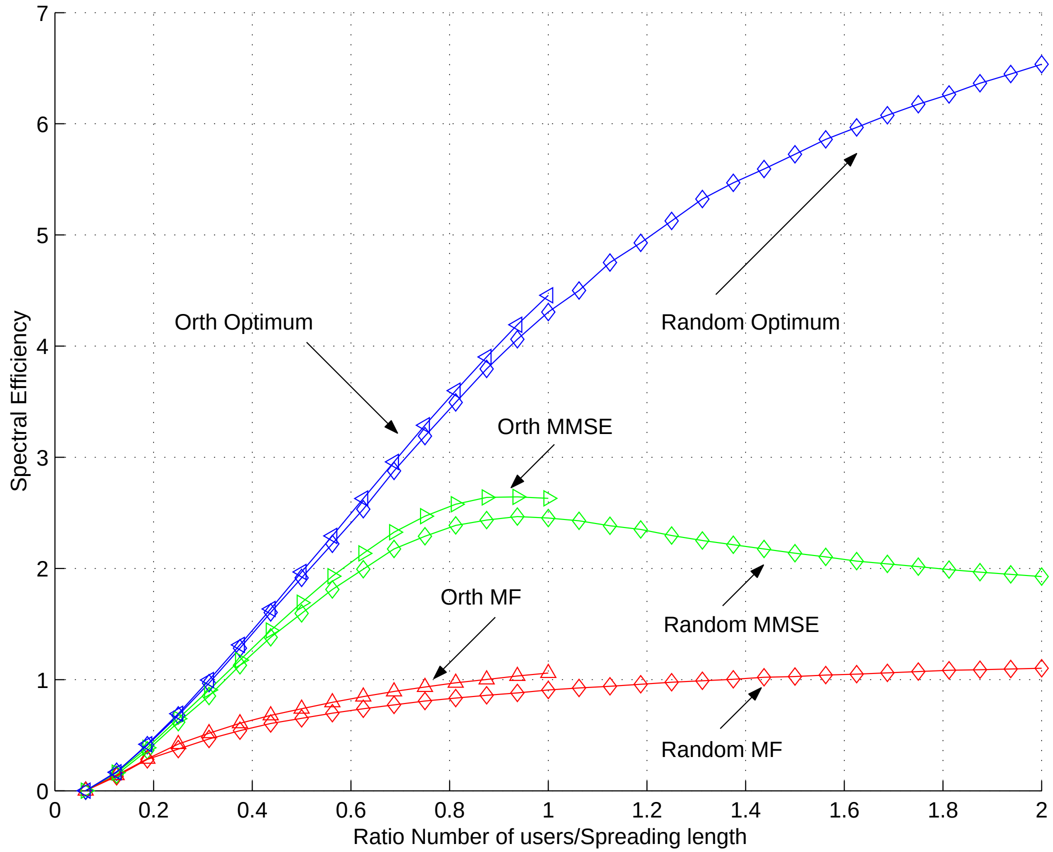
$$\frac{\mathsf{SINR}_k^{\mathsf{orth}}}{\mathsf{SINR}_k^{\mathsf{rand}}} = \frac{\sigma^2 + \alpha}{\sigma^2 + \alpha\left(1 - \frac{1}{L}\right)}.$$

- Remarkably, at high SNR ($\sigma^2 \to 0$), the SINR gain is given by:

$$\frac{\mathsf{SINR}_k^{\mathsf{orth}}}{\mathsf{SINR}_k^{\mathsf{rand}}} = \frac{L}{L-1}.$$

Simulations

Spectral efficiency C in function of the load α, $L = 5$



As L increases, the orthogonality gain decreases for any receiver.

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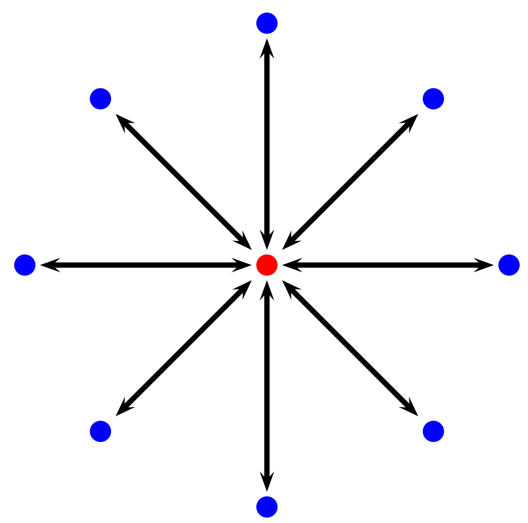
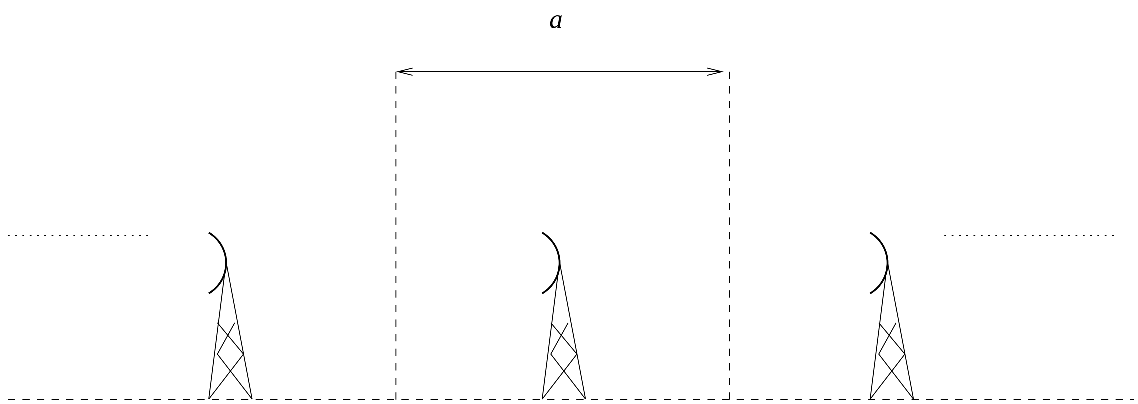


One Dimensional (1D) Cellular Network



- Infinite base station deployment ($2L+1$ with $L \to \infty$).
- Each base station is supposed to cover a region of length a (inter-cell distance).
- $\alpha = \frac{d}{N}$ is the load per meter.
- The number of users per cell is $K = da$. As the size of the cell increases, each cell accommodates more users.

Previous Results in the multi-cell case

- Downlink multi-cell CDMA framework:
  - For a finite dense network, results show that there is an optimal inter-cell distance maximizing the spectral efficiency (Debbah (04)).
- Uplink multi-cell CDMA framework:
  - Previous works have studied i.i.d. random CDMA cellular networks with simple interference models (Wyner's model) in the case of no fading (Zaidel et al. (01)) and flat fading (Dai et al. (03)).
- In the case when all interferers are taken into account, is there an optimal inter-cell distance that maximizes the spectral efficiency?

Intuition

- In the downlink, with orthogonal codes.
- As a increases, more users are orthogonalized, and the spectral efficiency increases.
- If a becomes too large, orthogonality can not be maintained.
- Two effects: path loss and fading.
- If a is large, path loss affects the communications.
- If frequency selective fading, orthogonality is destroyed.

Proposition

- Proposition: As $N \to \infty$ with $\frac{d}{N} = \alpha$, the spectral efficiency of downlink CDMA with random orthogonal spreading codes, general path loss, and matched filter is given by:

$$C(a) = \frac{\alpha}{a}\mathbb{E}_h\left[\int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2\left(1 + \frac{P(x)\left(\frac{1}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^2 df\right)^2}{I(x) + \frac{\sigma^2}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^2 df}\right) dx\right]$$

with

$$I(x) = \frac{\alpha a}{W}P(x)\left(\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^4 df - \frac{1}{W}\left(\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^2 df\right)^2\right)$$

$$+ \frac{\alpha a}{W}\sum_{q\neq 0} P_q(x)\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^2|h_q(f)|^2 df.$$

Path loss versus orthogonality

- When there is no fading,

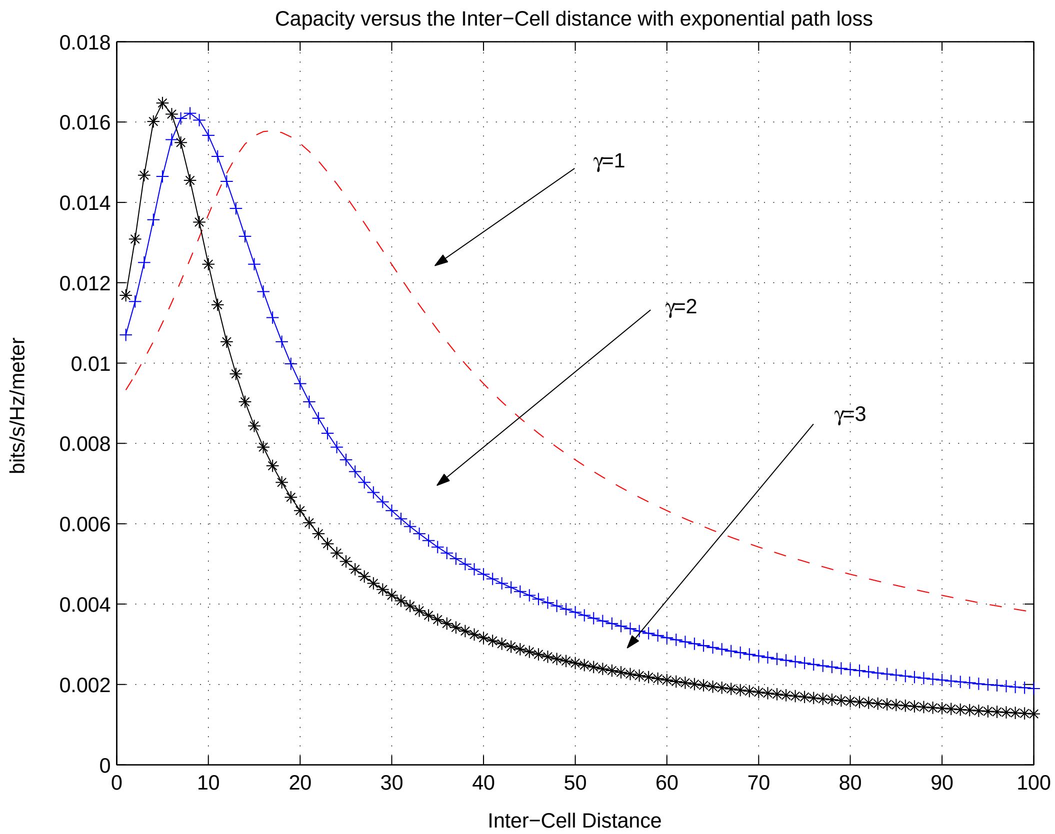
$$C(a) = \frac{\alpha}{a} \int_{-a/2}^{a/2} \log_2 \left(1 + \frac{P(x)}{\sigma^2 + \alpha a \sum_{q \neq 0} P_q(x)} \right) dx.$$

- Orthogonality is preserved: there is no intra-cell interference term.

- Simulations show that there is a nonzero optimal inter-cell distance a.

Path loss versus orthogonality

Spectral efficiency C in function of the inter-cell distance a



The optimal inter-cell distance depends on the path loss.

Fading versus orthogonality

- When path loss tends to 1,

$$\frac{\partial C}{\partial a} \propto \left(\frac{3}{2} - \frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2} \right).$$

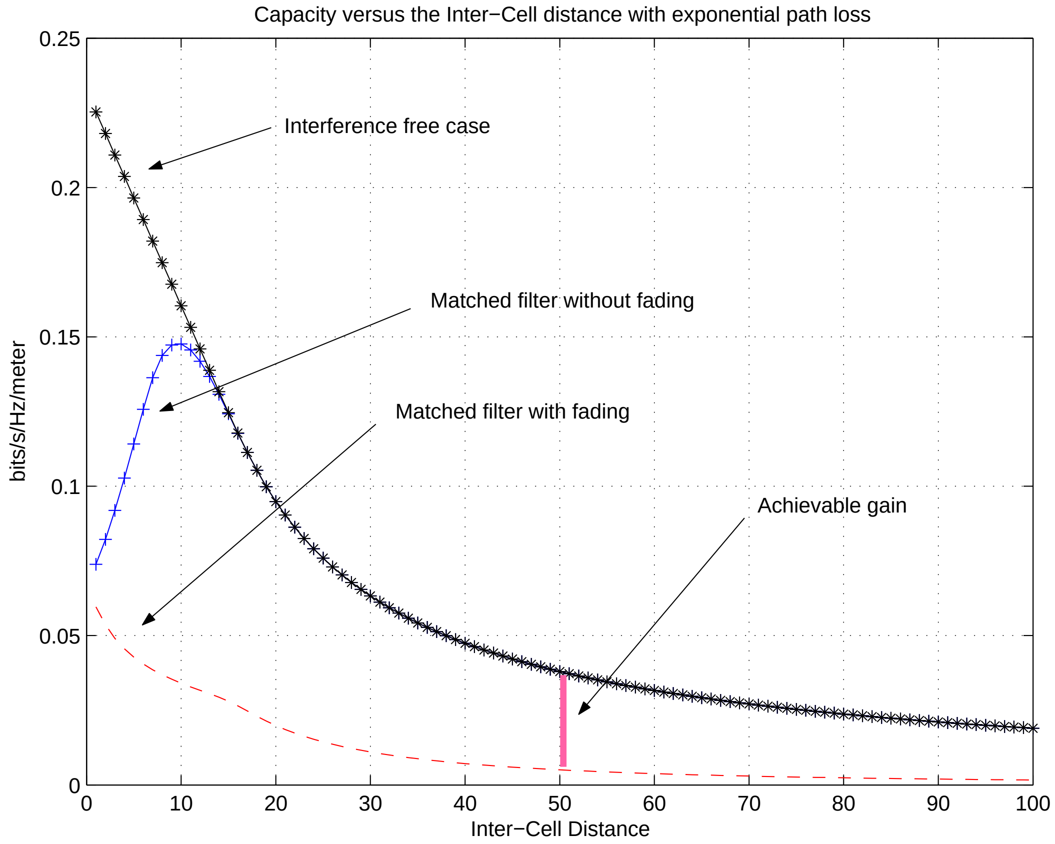
- The optimal inter-cell distance depends on how “peaky” the channel is through the kurtosis T.

- If $T > \frac{3}{2}$, the orthogonality is severely destroyed, the optimal inter-cell distance is $a = 0$.

- If $T < \frac{3}{2}$, the inter-cell distance can be increased to accomodate as many users as possible.

Fading versus orthogonality

Spectral efficiency C in function of the inter-cell distance a



It pays off to restore orthogonality.

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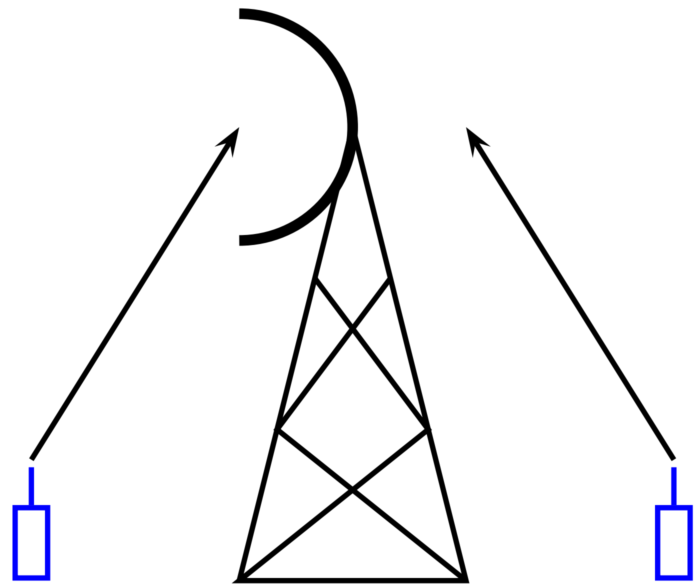


Power Allocation

- Importance of efficient Power Allocation mechanisms
- Until 1998, two paradigms:
- Centralized: some sort of central controller
  - Advantages: algorithms exist (Zander (92), Goodman (93), Wu (99))
  - Drawbacks: complexity, difficult to implement, high feedback load
  - Centralized power control is mostly analysed to obtain theoretical limits
- Distributed: local adaptation by the transmitters
  - Advantages: easy to implement, limited or no feedback
  - Drawbacks: limited knowledge of the system (iterative algorithms)
- In 1998, a new framework: game theory (Shah et al. (98), Alpcan et al. (02))
- What is game theory?

A Non-cooperative Game

- Set of players.
- Set of strategies.
- Preference relation on the outcomes. Generally a function called utility.
- Assumption of rationality.
- Assumption of strategic reasoning.
- Solution concept: Nash Equilibrium.
- Translation in the wireless communications domain?

Game Model

- Strategy: P_k, power allocation of user k.
- Utility: (Shah et al. (98), Meshkati et al. (05))

$$u_k = \frac{\gamma(\beta_k)}{P_k}.$$

- γ is a relevant performance measure such as the goodput $(1 - e^{-\beta})^M$ (unfortunately the capacity can not be used).
- β_k is the SINR of user k.
- SINR expression in frequency selective fading?

SINR expression: example of the MMSE receiver

- Proposition (Tulino (05)) As $N, K \to \infty$ with $K/N \to \alpha$, the SINR of user k at the output of the MMSE receiver is given by:

$$\mathsf{SINR}_k = \beta(\frac{k}{N})$$

where $\beta : [0, \alpha] \to \mathbb{R}$ is a function defined by the implicit equation

$$\beta(x) = P(x) \int_0^1 \frac{|h(f,x)|^2 df}{\sigma^2 + \int_0^\alpha \frac{P(y)|h(f,y)|^2 dy}{1+\beta(y)}}.$$

- Averaging effect: the codes do not have any importance.
- SINR_k is equal to P_k times something that does not depend on P_k.
- The result is similar for other receivers (Matched Filter, Optimum).

Nash equilibrium power allocation

- Nash Equilibrium: Occurs at $\beta^\star$ solution of

$$\beta_k \gamma'(\beta_k) - \gamma(\beta_k) = 0.$$

- Fixed target SINR $\beta^\star$ determines an equilibrium power allocation.

- For the MMSE receiver, the Nash equilibrium power allocation is

$$P_k = \frac{\beta^\star}{\frac{1}{N}\sum_{n=1}^{N} \frac{|h_{nk}|^2}{\sigma^2 + \frac{1}{1+\beta^\star}\frac{1}{N}\sum_{j=1, j\neq k}^{K} P_j |h_{nj}|^2}}.$$

- It depends on all the other users' fading and power allocations!

- How to alleviate this dependency?

Non-atomic games

- Interactions among numerous players (Wardrop (52), Haurie (85)).
- Primary application: road traffic, flows over a network.
- The game is considered to be played by infinitely many users, so that each user has a negligible impact on the global equilibrium.
- The power allocation of one user does not affect the power allocation of the others.

Results

- Assumption: The channel of each user k has L multipaths and $h_\ell(\frac{k}{N})$ is the fading on the ℓ-th path of user k.

- Define $E_k = \sum_{\ell=1}^{L} |h_\ell(\frac{k}{N})|^2$.

- Matched filter

$$P_k = \frac{1}{E_k} \frac{\sigma^2 \beta^\star}{1 - \alpha \beta^\star} \text{ for } \alpha < \frac{1}{\beta^\star}.$$

- MMSE filter

$$P_k = \frac{1}{E_k} \frac{\sigma^2 \beta^\star}{1 - \alpha \frac{\beta^\star}{1+\beta^\star}} \text{ for } \alpha < 1 + \frac{1}{\beta^\star}.$$

- In the case of flat-fading, same results as Meshkati et al. (05).

Results

- Optimum filter

$$P_k = \frac{1}{E_k} \frac{\sigma^2 \beta^+}{1 - \alpha \frac{\beta^+}{1+\beta^+}} \text{ for } \alpha < 1 + \frac{1}{\beta^+}$$

where β^+ is the solution to

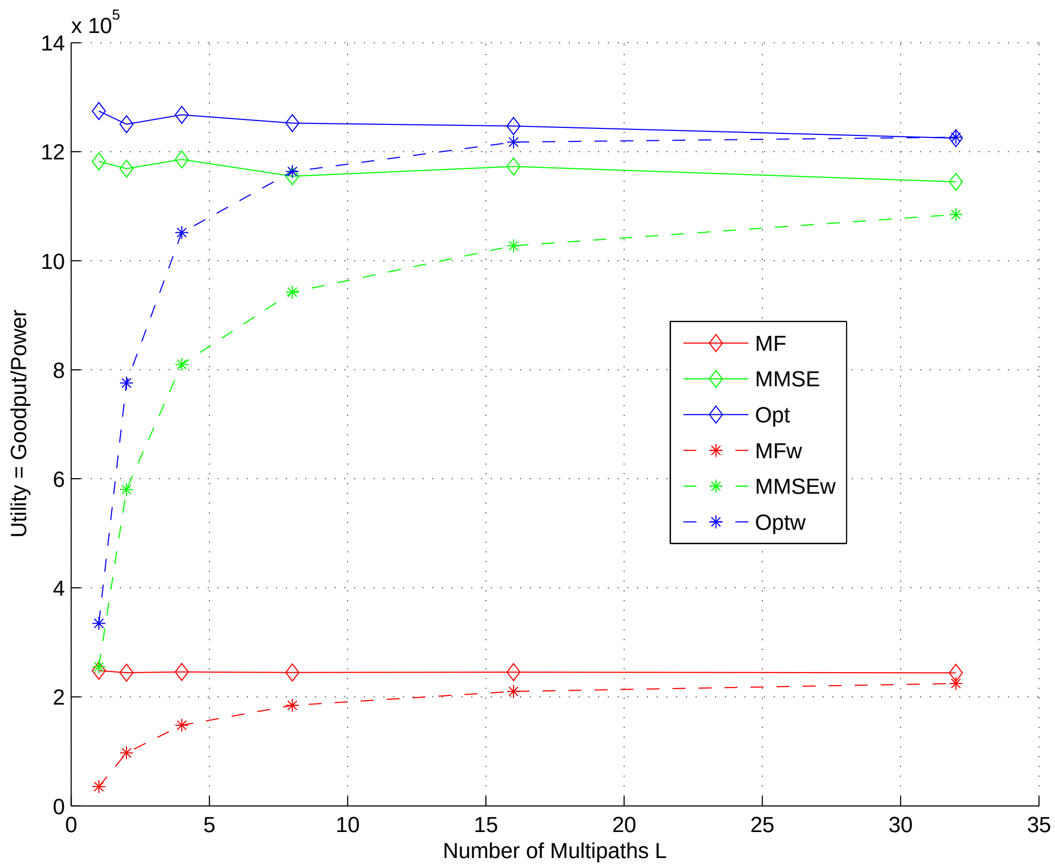
$$\alpha \log_2 \left(1 + \beta^+\right) - \alpha \log_2(e) \frac{\beta^+}{1 + \beta^+} + \log_2 \left(1 + \frac{1}{1+\beta^+} \frac{\alpha \beta^+}{1 - \alpha \frac{\beta^+}{1+\beta^+}}\right) = \alpha \log_2 \left(1 + \beta^\star\right).$$

- Power allocation is a constant times the inverse of the total channel energy E_k.

- Effect similar to "Channel Hardening": power allocation becomes uniform when the number of paths increases.

Simulations

Utility in function of the number of multipaths L



As L increases, the gap decreases, as the variance of E_k decreases, and the equilibrium power allocation becomes uniform.

Successive Interference Cancellation

- Coordination (Aumann (74)).
  - Two people have to choose a positive integer, without communicating.
  - If they choose the same number, both receive a reward, otherwise, neither receives anything.
  - People tend to choose 1 spontaneously!

- In the asymptotic regime: optimal ordering, based on a lemma by Verdú and Shamai (02).

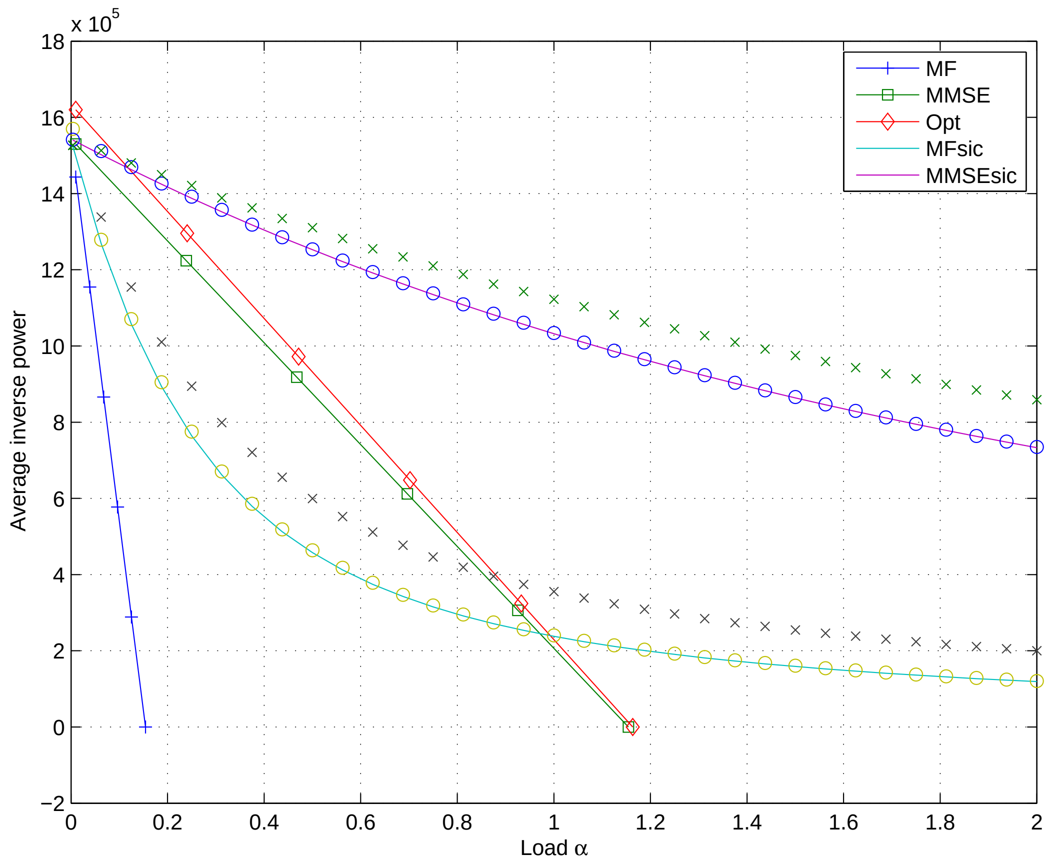
- Successive Interference Cancellation filters

$$P_k^{\mathrm{MF}} = \frac{\sigma^2 \beta^\star}{E_k} \left(1 + \frac{1}{N}\beta^\star\right)^{K-k},$$

$$P_k^{\mathrm{MMSE}} = \frac{\sigma^2 \beta^\star}{E_k} \left(1 + \frac{1}{N}\frac{\beta^\star}{1+\beta^\star}\right)^{K-k}.$$

Simulations

Average utility in function of the load K/N



SIC filters can accommodate an unlimited number of users.

Conclusions

- Single cell CDMA: impact of the number of multipaths.
- Cellular network: impact of the kurtosis.
- Distributed scheme: impact of coordination.

Other Works and Future Work

- Other Works featured in the thesis
  - Performance analysis of uplink CDMA cellular networks.
  - Application of Evolutionary Games to networking.
  - Application of Correlated Equilibrium to networking.
  - Distributed power allocation for OFDMA.

- Future work
  - Focus is on cellular networks but results can be extended to other architectures like MIMO or ad-hoc networks.
  - Compute the cost of centralization.
  - Network information theory.

List of Publications

- Published works

  1. "Spectral Efficiency of CDMA Uplink Cellular Networks", N. Bonneau, M. Debbah, E. Altman and G. Caire, *IEEE ICASSP 2005, Philadelphia, USA*, March 2005.

  2. "An Evolutionary Game Perspective to ALOHA with Power Control", N. Bonneau, E. Altman, M. Debbah and G. Caire, *19th ITC, Beijing, China*, Aug. 29–Sep. 2, 2005.

  3. "When to Synchronize in Uplink CDMA", N. Bonneau, M. Debbah, E. Altman and G. Caire, *IEEE ISIT 2005, Adelaide, Australia*, Sep. 4–9, 2005.

  4. "Performance of Channel Inversion Schemes for Multi-User OFDMA", N. Bonneau, M. Debbah, A. Hjørungnes and E. Altman, *2nd ISWCS, Siena, Italy*, Sep. 2005.

  5. "Spectral Efficiency of CDMA Downlink Cellular Networks with Matched Filter", N. Bonneau, M. Debbah, E. Altman, *EURASIP Journal on Wireless Communications and Networking*, 2006.

List of Publications (continued)

- Published works

6. "Correlated Equilibrium in Access Control for Wireless Communications", E. Altman, N. Bonneau and M. Debbah, *Networking 2006, Coimbra, Portugal*, May 15–19, 2006.

7. "Wardrop Equilibrium for CDMA Systems", N. Bonneau, M. Debbah, E. Altman and A. Hjørungnes, *RAWNET 2007, Limassol, Cyprus*, April 16, 2007.

8. "Constrained Cost-Coupled Stochastic Games with Independent State Processes", E. Altman, K. Avrachenkov, N. Bonneau, M. Debbah, R. El-Azouzi and D. Sadoc Menasche, accepted for publication in *Operations Research Letters*, 2007.

- Submitted works

9. "Non-Atomic Games for Multi-User Systems", N. Bonneau, M. Debbah, E. Altman and A. Hjørungnes, submitted to *IEEE JSAC Special Issue on "Game Theory in Communication Systems"*, 2007.

Last Slide

THANK YOU!

Theorem (Girko)

Let $\mathbf{Y} = \mathbf{V} \odot \mathbf{W}$ be a $N \times K$ matrix, where $\odot$ is the Hadamard (element-wise) product and $\mathbf{V}$ and $\mathbf{W}$ are independent $N \times K$ random matrices. Assume that $\mathbf{V}$ behaves ergodically with channel profile $\rho^{\mathbf{V}}(x, y)$ and that $\mathbf{W}$ has i.i.d. entries with zero mean and variance $\frac{1}{N}$. Then, as $N, K \to \infty$ with $K/N \to \alpha$, the empirical eigenvalue distribution of $\mathbf{YY}^H$ converges almost surely to a non-random limit distribution function whose Stieltjes transform is given by:

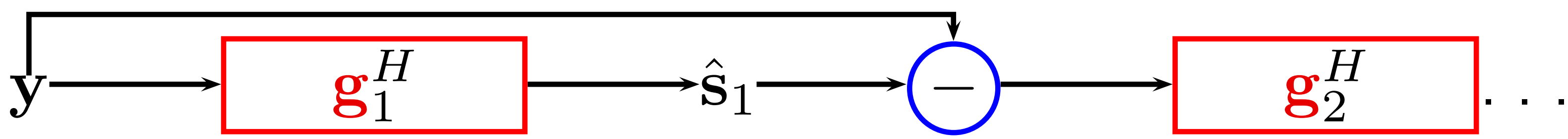
$$
\begin{aligned}
m^{\mathbf{YY}^H}(z) &= \lim_{N \to \infty} \frac{1}{N} \mathrm{Trace}\left(\left(\mathbf{YY}^H - z\mathbf{I} \right)^{-1} \right) \\
&= \int_0^1 u(x, z) dx
\end{aligned}
$$

and $u(x, z)$ satisfies the fixed point equation:

$$
u(x, z) = \frac{1}{\int_0^\alpha \frac{\rho^{\mathbf{V}}(x,y)dy}{1+\int_0^1 \rho^{\mathbf{V}}(x',y)u(x',z)dx'} - z}.
$$

The solution exists and is unique in the class of functions $u(x, z) \geq 0$, analytic for $\mathrm{Im}(z) > 0$, and continuous on $x \in [0, 1]$.

Successive Interference Cancellation



- Principle: users are ordered and are decoded successively.
- Assumption of perfect decoding.
- At each step, the signal is decoded and its contribution to the interference is then perfectly subtracted.
- This removes interference and increases the SINR of the following decoded users.