An Evolutionary Game Perspective to ALOHA with power control

N. Bonneau[a], E. Altman[a], M. Debbah[b] and G. Caire[b]

[a]MAESTRO, INRIA Sophia-Antipolis,
2004 Route des Lucioles, B.P.93, 06902 Sophia-Antipolis, France

[b]Mobile Communications Group, Institut Eurecom,
2229 Route des Cretes, B.P.193, 06904 Sophia-Antipolis, France

Abstract: We study a large population of communicating terminals using an ALOHA protocol with two possible levels of transmission power. We pose the problem of how to choose between these power levels. We study two non-cooperative optimization concepts: the Nash equilibrium and the Evolutionary Stable Strategy. The latter was introduced in mathematical biology in the context of Evolutionary Games, which allows to describe and to predict properties of large populations whose evolution depends on many local interactions, each involving a finite number of individuals. We compare the performances of these non-cooperative notions with the global cooperative solution. The payoffs that we consider are functions of the throughputs and of the cost for the power levels. We study in particular the impact of the pricing for the use of the power levels on the system performance.
Key words: ALOHA, Evolutionary Games, Evolutionary Stable Strategies

1. INTRODUCTION

Interest has been growing in recent years in studying competition aspects of networking in general, access to a common medium in particular, within the frame of non-cooperative game theory, see e.g. the survey paper [1]. In this paper we focus on the ALOHA access scheme [2].

Several previous papers have already studied ALOHA or slotted ALOHA in a non-cooperative context. The papers [3–7] have studied ALOHA for a non-cooperative choice of transmission probabilities. Several papers study slotted ALOHA with power diversities in the context of the cooperative formulation [8–10]. In [11] the authors have studied the performance of slotted ALOHA in a non-cooperative context, modeled as a game, in which both retransmission probabilities as well as power levels are controlled. A Markov chain formulation has been obtained, whose numerical solutions enable to study the system performance. In the current paper, in contrast, we propose an alternative simpler modeling approach which allows us to obtain explicit analytical expressions for the performance measures. This allows us then to compute analytically the solutions for various non-cooperative optimization criteria.

We study in this paper two equilibria concepts: the Nash equilibria and the Evolutionary Stable Strategies (ESS). ESS have been introduced in the context of mathematical biology in order to describe and to predict properties of large populations whose evolution depends on many local interactions, each involving a finite number of individuals. An ESS is characterized by a robustness property (that need not be satisfied by a Nash equilibrium): under an ESS, the populations becomes immune to the proliferations of mutations. In our context, when using ESS, not only is there no incentive to deviate for any single user (which is the definition of a Nash equilibrium), but also a whole (small) fraction of users cannot benefit by deviating.

ESS have first been defined in 1972 by Maynard Smith in [12]. In 1982, Smith's seminal text Evolution and the Theory of Games [13] appeared. Although ESS have been defined in the context of biological systems, it is highly relevant to engineering as well (see [14]). In particular, in the context of competition in the access to a common medium, we can expect that a technology that provides better performance will gain more market shares on the expense of less performant technologies. To the best of our knowledge, our paper is the first to apply evolutionary games to study non-cooperative behavior in wireless networks.

In section 2, we present the model. In section 3, we compute performance measures for this model, as well as consider some optimization issues in a cooperative setting. In section 4, we study the non-cooperative setting and exhibit Nash equilibria. In section 5, we introduce the concept of Evolutionary Stable Strategies. Finally, we present some numerical results in section 6 and draw our conclusions.

2. THE MODEL

We consider an infinite population of mobile terminals. We use a model similar to [15] for unslotted ALOHA where the global arrival of new packets from all mobiles follows a Poisson process with intensity λ. The time required to transmit a packet is one unit. If a packet is transmitted at time t, then any other transmission during the so-called vulnerable period $[t-1, t+1]$ will cause a collision.

We assume that for each packet, its source can choose the transmitted power among two levels. All packets of the lower power level involved in a collision are assumed to be lost and will have to be retransmitted later. In addition, if more than one packet of the higher power level is involved in a collision then all packets are lost. The power differentiation thus allows one packet of the higher power level to be successfully transmitted in collisions that do not involve other packets of the higher power level. This is the capture phenomenon (see e.g. [16,9]).

In this paper we study the choice of power levels. A strategy for a mobile corresponds to the choice of a power level. This can be a deterministic choice or a randomized one. We assume that the power level choice for a retransmitted packet is the same as the power level at which it was transmitted the first time. Thus, if the whole population uses the same strategy q for transmissions (meaning that the higher power level is chosen with probability q, and the lower with probability $\bar{q} = 1 - q$) then the rate of arrival of packets that will be transmitted with higher power level is given by λq.

We consider a non-cooperative approach in which each mobile determines its power level so as to maximize its payoff, which has two components:

(i) $P_{succ}(p, q)$, which is the success probability when it chooses the higher power level (level 1) with probability p and the lower one (level 2) with probability $\bar{p} = 1 - p$, given that all other mobiles choose the higher power level with probability q and the lower one with probability $\bar{q} = 1 - q$ (we shall keep using q below as the strategy of other mobiles).

(ii) $\pi(p)$, which represents the cost of a packet transmission when choosing the higher power level with probability p. $\pi(p)$ can be linear: if a is the cost for the higher power level and b ($b < a$) for the lower one then we have $\pi(p) = ap + b\bar{p}$. In this case $\pi(p)$ can represent in particular the *expected transmission power*. $\pi(p)$ can also be chosen as an arbitrary function $\pi_2(p)$ representing the *pricing* paid by the users. In this case, $\pi_2(p)$ is generally assumed to be strictly convex and increasing.

We shall consider the following payoff function, given as the ratio between the packet success probability and expected consumed power: $J_r(p, q) = \frac{P_{succ}(p,q)}{\pi(p)}$. This type of payoff can represent in particular the *power efficiency*, i.e., the expected number of packets that can be transmitted per a unit power transmitted, see e.g. [17].

Using the approach of [15], we assume that the point process describing packets that are either transmitted with power level i or retransmitted at a power level i ($i = 1, 2$) is a Poisson process with intensity $g_i = g_i(\lambda, q)$ (it depends on the arrival process of packets as well as on the fraction of packets sent with each power level).

3. COMPUTING THE PERFORMANCE MEASURES

3.1. Retransmission rates

Remark 3.1 *In the sequel, we shall need the following result. Recall that if $z \geq -\exp(-1)$, $x = \mathrm{LambertW}(z)$ is the root greater than or equal to -1 of the equation $z = x\exp(x)$. This implies that*

$$\exp(\mathrm{LambertW}(z)) = \frac{z}{\mathrm{LambertW}(z)}. \tag{1}$$

Theorem 3.1 *(i) Assuming that $\lambda q \leq \frac{1}{2}\exp(-1)$, we have*

$$g_1(\lambda, q) = -\frac{1}{2}\mathrm{LambertW}(-2\lambda q). \tag{2}$$

(ii) Assume $\lambda q \leq \frac{1}{2}\exp(-1)$ and $\left(\text{if } q \leq \frac{1}{1+\exp(-1)}, \text{ then } \lambda\bar{q}\exp\left(\exp(-1)\frac{q}{\bar{q}}\right) \leq \frac{1}{2}\exp(-1)\right)$. Then

$$g_2(\lambda, q) = -\frac{1}{2}\mathrm{LambertW}\left(\frac{-2\bar{q}g_1}{q}\right) = -\frac{1}{2}\mathrm{LambertW}\left(\frac{\bar{q}}{q}\mathrm{LambertW}(-2\lambda q)\right). \tag{3}$$

(iii) Under the same conditions, we have

$$P_{succ}(p, q) = p\exp(-2g_1) + \bar{p}\exp(-2(g_1 + g_2)) = \lambda\left(\frac{pq}{g_1} + \frac{\bar{p}\bar{q}}{g_2}\right). \tag{4}$$

(iv) If the conditions on (λ, q) are not met, then there is no possible steady state.

Proof: The success probability of a higher power level (re)transmission is given by $\exp(-2g_1)$. Thus the rate of departure of type 1 packets (i.e., the rate of successful packet transmissions and retransmissions of higher power level) is given by $g_1 \exp(-2g_1)$.

Since at steady state, λq, the rate of arrival of type 1 packets equals to the rate of departure of type 1 packets, we have

$$\lambda q = g_1 \exp(-2g_1). \tag{5}$$

This equation has a solution if $2\lambda q \leq \exp(-1)$, thus we obtain (2). It follows from (2) that $g_1(\lambda, 0) = 0$ and $g_1(\lambda, 1) = -\frac{1}{2}\,\mathrm{LambertW}(-2\lambda)$.

The success probability for type 2 packets is given by $\exp\left(-2[g_1 + g_2]\right)$. Thus the rate of departure of type 2 packets is given by $g_2 \exp\left(-2[g_1 + g_2]\right)$. Hence at steady state:

$$\lambda \bar{q} = g_2 \exp\left(-2[g_1 + g_2]\right) \tag{6}$$

Using (5), we can write $\exp(-2g_1) = \dfrac{\lambda q}{g_1}$, and substituting in equation (6), we obtain

$$\frac{-2\bar{q}g_1}{q} = -2g_2 \exp(-2g_2).$$

This equation has a solution if $\frac{2\bar{q}g_1}{q} \leq \exp(-1)$. Since $g_1 \leq \frac{1}{2}$, this is always verified if $q \geq \frac{1}{1+\exp(-1)}$. If $q \leq \frac{1}{1+\exp(-1)}$, this condition becomes $\mathrm{LambertW}(-2\lambda q) \geq \exp(-1)\frac{q}{\bar{q}}$, and since the function $x \mapsto x\exp(x)$ is increasing for $x \geq -1$, we can apply it to both sides of the inequality and get (3). It follows from (6) that $g_2(\lambda, 1) = 0$ and $g_2(\lambda, 0) = -\frac{1}{2}\,\mathrm{LambertW}(-2\lambda)$.

As (6) implies $\exp(-2[g_1 + g_2]) = \dfrac{\lambda \bar{q}}{g_2}$, we obtain the global success probability (4). ∎

We observe that $P_{succ}(p, q)$ is a linear function in p:

$$P_{succ}(p, q) = \lambda p \left(\frac{q}{g_1} - \frac{\bar{q}}{g_2}\right) + \frac{\lambda \bar{q}}{g_2}$$

The coefficient multiplicating p is zero for $q = 1$ and strictly positive otherwise. More specifically, when $q = 1$, we have $\exp(-2g_1(\lambda, 1)) = -\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)}$ and $g_2(\lambda, 1) = 0$, so that

$$P_{succ}(p, 1) = -\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)} \tag{7}$$

When $q = 0$, we have $g_1(\lambda, 0) = 0$ and $\exp(-2g_2(\lambda, 0)) = -\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)}$, so that

$$P_{succ}(p, 0) = p - \bar{p}\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)} \tag{8}$$

3.2. Optimization issues

We first seek to find the maximum throughput that can be achieved (through the choice of λ and q). (5) together with (6) imply that the global throughput of the system is

$$\Theta = g_1 \exp(-2g_1) + g_2 \exp(-2[g_1 + g_2]) \tag{9}$$

To obtain g_2 that gives the maximum throughput for a fixed g_1, we differentiate (9) with respect to g_2 and equate to zero. This gives $g_2^* = \frac{1}{2}$. g_2^* does not depend on g_1 and the optimization of Θ corresponds to the optimization of the single-variable function $g_1 \exp(-2g_1) + \frac{1}{2}\exp(-2g_1 - 1)$. Therefore, $g_1^* = \frac{1}{2}(1 - \exp(-1))$, and

$$\Theta^* = \frac{1}{2}\exp\left(\exp(-1) - 1\right) \tag{10}$$

These values are obtained for $\lambda = \Theta^* = \frac{1}{2}\exp\left(\exp(-1) - 1\right)$ and $q = 1 - \exp(-1)$, which satisfy the conditions of Theorem 3.1. We observe that this throughput is higher (by a factor $\exp(\exp(-1))$) than the maximum stable throughput with a single power level, which is equal to $\frac{1}{2}\exp(-1)$ for unslotted ALOHA. Such optimal performance can usually be obtained only in a cooperative setting, for example when a regulator enforces a common policy for all mobiles.

4. NASH EQUILIBRIUM

4.1. Power Efficiency case

The payoff function for a mobile using strategy p while the population uses strategy q is given by

$$J_r(p,q) = \frac{P_{succ}(p,q)}{\pi(p)} = \frac{\lambda\left[\left(\frac{q}{g_1} - \frac{\bar{q}}{g_2}\right)p + \frac{\bar{q}}{g_2}\right]}{(a-b)p+b} = \frac{\lambda\left(\frac{q}{g_1} - \frac{\bar{q}}{g_2}\right)}{a-b}\left[1 + \frac{\frac{\bar{q}g_1}{qg_2 - \bar{q}g_1} - \frac{b}{a-b}}{p + \frac{b}{a-b}}\right] \tag{11}$$

We begin by checking whether the boundary cases $q = 1$ and $q = 0$ are Nash equilibria. When $q = 1$, we obtain from (7)

$$J_r(p,1) = \frac{-\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)}}{(a-b)p+b} \tag{12}$$

$J_r(p,1)$ is strictly decreasing over $p \in [0,1]$. Thus $p = 0$ optimizes $J_r(p,1)$, so that $q = 1$ is not a Nash equilibrium.

When $q = 0$, we obtain from (8)

$$J_r(p,0) = \frac{p - \bar{p}\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)}}{(a-b)p+b} \tag{13}$$

As

$$\frac{\partial J_r}{\partial p}(p,0) = \frac{b + a\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)}}{((a-b)p+b)^2} \tag{14}$$

we conclude the following:

- If $-\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)} \geq \frac{b}{a}$ (i.e. $\lambda \leq -\frac{b}{2a}\ln\frac{b}{a}$) then $J_r(p,0)$ is non-increasing over $p \in [0,1]$. Therefore $q=0$ is a Nash equilibrium.
- If $-\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)} < \frac{b}{a}$ (i.e. $\lambda > -\frac{b}{2a}\ln\frac{b}{a}$) then $J_r(p,0)$ is strictly increasing over $p \in [0,1]$. Thus $q=0$ is not a Nash equilibrium.

Other equilibria: we now consider $q \in (0,1)$. (11) implies the following:

Theorem 4.1 *$J_r(p,q)$ does not depend on p if $\bar{q}g_1(a-b) = (qg_2 - \bar{q}g_1)b$. If $\frac{b}{a} \geq \exp(-1)$, there are three solutions to this equation, $q=0$, $q=1$, and*

$$q^* = 1 - \frac{\frac{b}{a}\ln\left(\frac{b}{a}\right)}{\mathrm{LambertW}\left(-2\lambda\left(\frac{b}{a}\right)^{b/a}\right)}$$

When $q^* \in (0,1)$ (which is not necessarily the case), q^* is a Nash equilibrium.

Note that if we allow the parameters $\frac{b}{a} \in [\exp(-1),1)$ and $\lambda \in (0, \frac{1}{2}\exp(-1))$ to fluctuate independently, then it is rather straightforward that the expression of q^* can take any value in $(-\infty,1)$. However, we observe that for $\lambda = -\frac{b}{2a}\ln\frac{b}{a}$, $q^*=0$, which implies that if $\lambda > -\frac{b}{2a}\ln\frac{b}{a}$ then $q^* \in (0,1)$, and if $\lambda \leq -\frac{b}{2a}\ln\frac{b}{a}$ then $q^* \in (-\infty,0]$.

To find other possible equilibria, we compute

$$\frac{\partial J_r}{\partial p}(p,q) = \lambda\frac{qg_2b - \bar{q}g_1a}{g_1g_2\left((a-b)p+b\right)^2} \tag{15}$$

The solutions to $\frac{\partial J_r}{\partial p}(p,q)=0$ reduce to the case studied in Theorem 4.1.

4.2. Pricing case

When the cost function $\pi_2(p)$ is strictly convex, and we consider the payoff function

$$J_r(p,q) = \frac{P_{succ}(p,q)}{\pi_2(p)} = \frac{\lambda\left(\frac{pq}{g_1}+\frac{\bar{p}\bar{q}}{g_2}\right)}{\pi_2(p)},$$

non-trivial potential Nash equilibria are determined by the following implicit equation:

$$q^* = 1 - \frac{r(q^*)\ln(r(q^*))}{\mathrm{LambertW}\left(-2\lambda(r(q^*))^{r(q^*)}\right)} \quad \text{where } r(p) = \frac{\frac{\pi_2(p)}{\pi_2{}'(p)} - p}{\frac{\pi_2(p)}{\pi_2{}'(p)} - p + 1}.$$

5. EVOLUTIONARY STABLE STRATEGY

5.1. Background and definitions

In the biological context, the amount of reward for an individual is related to its reproduction capability. A higher reward to some behavior (which can represent more food or more chances to mate) implies a higher growth rate of individuals that adopt it.

More precisely, assume that at a given time, a population uses one (possibly mixed) strategy q^* (this could be obtained either by a fraction q^* of the population playing one strategy and the remainder $\bar{q}^*$ playing the other, or by each individual randomizing between the strategies.) Suppose a small fraction (identified as mutations) adopts another distribution p over the two strategies. If for all $p \neq q^*$,

$$J(q^*, q^*) > J(p, q^*) \tag{16}$$

then the fraction of the mutations in the population will tend to decrease (as it has a lower reward, meaning a lower growth rate). q^* is then immune to mutations.

If there are n pure strategies ($n = 2$ in our case) denoted by $s_1, \ldots, s_n$, then a sufficient condition for (16) is that

$$J(q^*, q^*) > J(s_i, q^*), \quad s_i \neq q^*, \ i = 1, \ldots, n. \tag{17}$$

In the special case that the following holds,

$$J(q^*, q^*) = J(p, q^*) \text{ and } J(q^*, p) > J(p, p) \ \ \forall p \neq q^*, \tag{18}$$

a population using q^* is "weakly" immune against a mutation using p since if the mutant's population grows, then we shall frequently have individuals with strategy q^* competing with mutants; in such cases, the condition $J(q^*, p) > J(p, p)$ ensures that the growth rate of the original population exceeds that of the mutations. q^* that satisfies (16) or (18) is called an Evolutionary Stable Strategy (ESS).

5.2. Computing ESS: power efficiency case

Evolutionary Stable Strategies constitute a subset of the Nash equilibria, therefore we only have to check whether the Nash equilibria satisfy either condition (16) or (18).

Assume that $\lambda < -\frac{b}{2a} \ln \frac{b}{a}$ then the Nash equilibrium $q^* = 0$ (see (14)) is also an ESS, since (14) implies that for all $p \neq 0$, condition (16) holds.

Assume that $\frac{b}{a} \geq \exp(-1)$ and $\lambda > -\frac{b}{2a} \ln \frac{b}{a}$ then the Nash equilibrium in Theorem 4.1 exists, and is $q^* \in (0, 1)$. We note that the utility of a player does not depend on his choice of p: $J_r(q^*, q^*) = J_r(p, q^*)$, $\forall p$. Thus condition (16) does not hold. To check whether condition (18) holds, we recall (15):

$$\frac{\partial J_r}{\partial q}(q, p) = \lambda \frac{p g_2 b - \bar{p} g_1 a}{g_1 g_2 \left((a - b)\, q + b\right)^2}$$

If $p < q^*$, then $\frac{\partial J_r}{\partial q}(q, p) > 0$ and if $p > q^*$, then $\frac{\partial J_r}{\partial q}(q, p) < 0$, which means that for all $p \neq q^*$, condition (18) holds. Therefore q^* is an ESS.

Finally, for $\lambda = -\frac{b}{2a} \ln \frac{b}{a}$, (16) does not hold, but (18) does and $q^* = 0$ is an ESS.

Therefore, all Nash equilibria previously exhibited are ESS as well. As stated in the previous subsection, this fact has the following interpretation. All these equilibria are robust against small perturbations either in the "immune" or the "weakly immune" sense.

6. NUMERICAL RESULTS

Figures 1 and 2 show curves delimiting the atteignable regions of (λ, q) in steady state. As a means of comparison, the bound on the throughput of an ALOHA scheme with a single power level has been plotted. This bound is equal to $\frac{1}{2} \exp(-1)$. The atteignable

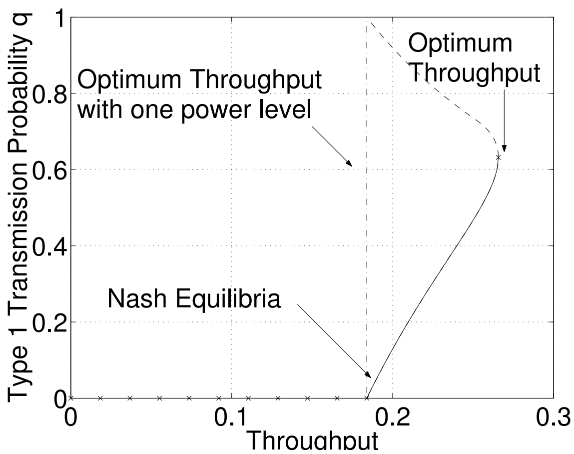

Figure 1. Atteignable throughput, Nash equilibria points with power efficiency payoff function, $b/a = \exp(-1)$.

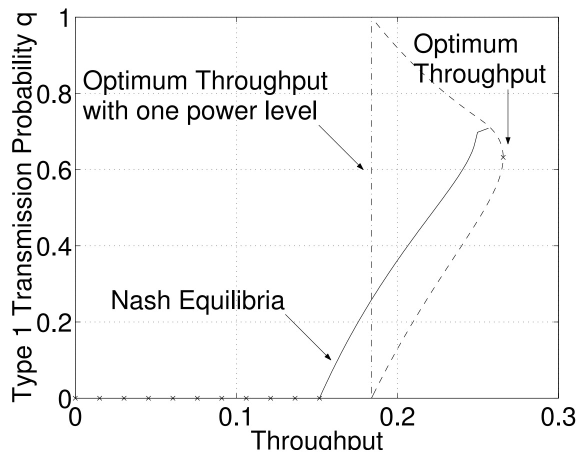

Figure 2. Atteignable throughput, Nash equilibria points with power efficiency payoff function, $b/a = \exp(-0.5)$.

region with one power level lies on the left of the dashdotted straight line. For two power levels, (λ, q) must satisfy the two conditions of Theorem 3.1 and the bound on the obtainable throughput is always larger than $\frac{1}{2}\exp(-1)$. In particular, the point defined by equation (10) appears as the rightmost point on the dashed curve in both figures. The atteignable region using two power levels lies on the left of the dashed curve. It is to be noted that these theoretical results can be applied to any ratio $b/a < 1$ of the two powers; however, if b/a is too close to 1, then the hypothesis of capture phenomenon becomes questionable.

In the case of a linear cost function, an interesting result occurs when $b/a = \exp(-1)$. For this particular value, the Nash equilibria curve will merge with the bound curve up to the point defined by equation (10), as seen in figure 1. It can be shown using the theoretical formulas, that this is the only case in which the optimum throughput represents a Nash equilibrium. For $b/a > \exp(-1)$, the Nash equilibria (and ESS) points are always on the left of the bound. However, there still exists Nash equilibria points up to a certain throughput greater than $\frac{1}{2}\exp(-1)$. In addition, for all b/a, the highest λ such that there exists a corresponding Nash equilibrium which lies on the bound curve, as can be seen on figure 2.

The case of an exponential pricing is shown in figure 3. The curve obtained for the Nash equilibria shows a similar behaviour as the case of a linear payoff.

In figure 4, we show the curves of the payoff function at the Nash equilibrium for some values of the parameter b/a, as well as for an exponential pricing, with regard to the rate of arrival λ (which is also the throughput at steady state). For the linear cost function, two distinct decreasing regions can be distinguished in each case, the first one corresponding to the trivial Nash equilibria and the second one to the equilibria of Theorem 4.1. For the exponential pricing, the curve is plotted only for the non-trivial Nash equilibria we have found. These curves show that even though the throughput with a lower value of b/a is

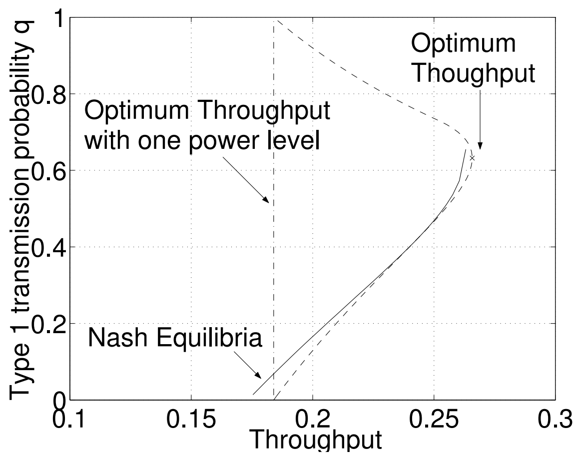

Figure 3. Atteignable throughput, Nash equilibria points with exponential pricing $\pi_2(p) = \exp(p)$.

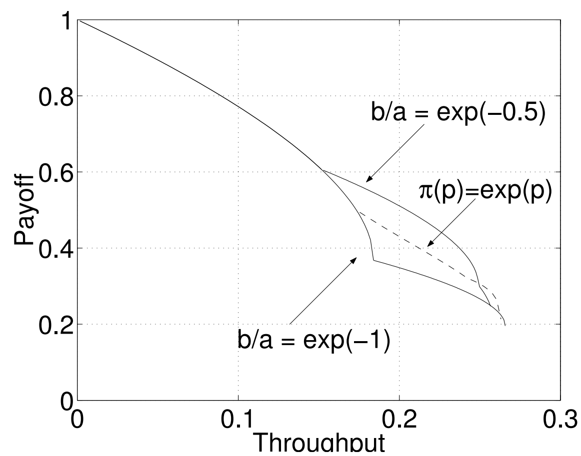

Figure 4. Values of the payoff function, power efficiency $b/a = \exp(-1)$ and $b/a = \exp(-0.5)$, and exponential pricing.

higher, the payoff for a mobile is lower.

7. CONCLUSION

We have introduced a variant of ALOHA involving two transmission power levels. Explicit expressions for the optimal throughput as well as for Nash equilibria points associated with the power level choice have been derived. The concept of Evolutionary Stable Strategies has been introduced, and the Nash equilibria points have been shown to be ESS points. The throughput obtained in a Nash equilibrium compared to the optimum throughput depends essentially on the ratio of the powers, in the case of a linear cost function. These results provide an insight on the potential gains when using several power levels in wireless communications.

REFERENCES

1. E. Altman, T. Boulogne, R. El Azouzi, T. Jimenez, and L. Wynter, "A survey on networking games," *Computers and Operations Research*, 2005.
2. A. Abramson, "The ALOHA system – another alternative for computer communications," in *AFIPS Conference Proceedings*, 1970, vol. 36, pp. 295–298.
3. E. Altman, R. El Azouzi, and T. Jimenez, "Slotted ALOHA as a Stochastic Game with Partial Information," in *Proceedings of WiOpt'03*, Sophia Antipolis, France, Mar. 2003.
4. E. Altman, D. Barman, R. El Azouzi, and T. Jimenez, "A game theoretic approach for delay minimization in slotted ALOHA," in *Proceedings of IEEE ICC*, Paris, France, June 2004.
5. A. B. MacKenzie and S. B. Wicker, "Selfish users in ALOHA: A game theoretic approach," in *IEEE Transactions on Vehicular Technology*, 2001.

6. A. B. MacKenzie and S. B. Wicker, "Stability of Slotted ALOHA with Multipacket Reception and Selfish Users," in *Proceedings of Infocom*, Apr. 2003.
7. Y. Jin and G. Kesidis, "Equilibria of a noncooperative game for heterogeneous users of an ALOHA network," *IEEE Comm. Letters*, vol. 6, no. 7, pp. 282–284, 2002.
8. J. J Metzner, "On improving utilization in ALOHA networks," *IEEE Trans. on Communications*, vol. 24, no. 4, 1976.
9. R. O. LaMaire, A. Krishna, and M. Zorzi, "On the randomization of transmitter power levels to increase throughput in multiple access radio systems," *Wireless Networks*, vol. 3, pp. 263–277, 1998.
10. J. H. Sarker, M. Hassan, and S. Halme, "Power level selection schemes to improve throughput and stability of slotted ALOHA under heavy load," *Computer Communication*, vol. 25, 2002.
11. E. Altman, D. Barman, A. Benslimane, and R. El Azouzi, "Slotted ALOHA with priorities and random power," *http://www.inria.fr/mistral/personnel/Eitan.Altman/mobile.html*.
12. M. Smith, *Game Theory and the Evolution of Fighting*, Edinburgh University Press, Edinburgh, 1972.
13. M. Smith, *Evolution and the Theory of Games*, Cambridge University Press, Cambridge, UK, 1987.
14. T. L. Vincent and T. L. S. Vincent, "Evolution and control system design," *IEEE Control Systems Magazine*, vol. 20, no. 5, pp. 20–35, Oct. 2000.
15. D. Bertsekas and R. Gallager, *Data Networks*, Prentice Hall, Englewood Cliffs, New Jersey, 1987.
16. Y. E. Sagduyu and A. Ephremides, "Power control and rate adaptation as stochastic games for random access," in *Proc. 42nd IEEE Conference on Decision and Control*, Hawaii, Dec. 2003.
17. C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Pricing and power control in a multicell wireless data network," *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 10, pp. 1883–1892, 2001.