Discussion on “A Gradient-based Repetitive Control Algorithm Combining ILC and Pole Placement”

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Iterative Learning Control (ILC) and Repetitive Control (RC) are relatively new research methodologies for improving the performance of systems that operate in a cyclic mode. The general form of Iterative Learning Control can be given as follows:

$$u_k(t) = \mathcal{L}\{e_i(\tau), u_i(\tau), \tau \in [0,T], i = 1, ..., k-1\},$$
$$t \in [0,T], \quad k = 1, 2, \dots \ , \tag{1}$$

where $\mathcal{L}$ is a learning operator that generates a command for the next cycle based on the information about the errors and commands from previous cycles. The error is defined by $e_k(t) = r(t) - y_k(t)$, where $r(t), t \in [0,t]$, is the desired output which the system must track repetitively. The time parameter t can be either continuous or discrete. The general form of Repetitive Control can be given as follows:

$$u(t) = \mathcal{L}\{e(\tau), u(\tau), \tau \in [0,t)\}, \quad t \in [0, \infty). \tag{2}$$

In the case of Repetitive Control the system has to track a periodic signal r with period T, i.e., $r(t) = r(t-T)$. Even though ILC and RC appear to be very similar, there is an important difference between them: In ILC the system initial conditions are reset at the beginning of each trial, whereas in RC the initial conditions are set only once at the beginning of the system operation. In orther words, the transients have an important influence on every cycle of ILC and in the case of RC the effect of transient fades away with time. It turns out that the following linear forms of ILC and RC are sufficiently versatile

$$u_k(t) = u_{k-1}(t) + Le_{k-1}, \quad t \in [0,T], \quad k = 1, 2, \dots \ , \tag{3}$$
$$u(t) = u(t-T) + Le, \quad t \in [0, \infty). \tag{4}$$

Many well known ILC and RC schemes can be represented in the above forms. In particular, the gradient-based RC algorithm studied in the paper by Hätönen *et al* can be presented as follows:

$$u(t) = u(t-T) + \beta G^* e, \quad t \in [0, \infty), \tag{5}$$

where G^* is the adjoint system operator in the case of FIR system or the truncated adjoint system operator in the case of IIR system, and where β is some tunable parameter. The gradient-based algorithms have been studied before in the context of ILC (see [5], [9], [4]). The gradient-based ILC algorithms are more complicated than the basic P-type ILC algorithm with $L = \gamma I$ and moreover they require a good system identification. However, the gradient-based ILC algorithms provide a monotonous convergence of the ILC from trial to trial, whereas the basic P-type ILC often suffers from very bad transients [6], [7]. It is natural to expect (and as observed in [7], it is often the case) that if some algorithm performs well in the context of ILC its analog will perform also well in the context of RC. Thus, one might expect that the gradient-based RC algorithm will provide good transient behavior when the system has to track a periodic signal. The paper by Hätönen *et al* initiates the following series of very interesting questions about the transient behavior of Repetitive Control:

1) What are differences between ILC and RC in terms of convergence and transient behavior?
2) Is it true that if some algorithm performs well for ILC its analog will perform also well for RC?
3) What are the conditions for monotonic convergence of RC? By monotonic convergence we mean the monotonic convergence in some function norm from cycle to cycle.
4) Is it possible that RC has very bad transient but converges eventually, as it is often the case for ILC?
5) What is the measure of good transient behavior or good convergence behavior in the case of RC?

Below let us elaborate further on the above questions. First, we discuss the differences between convergence conditions for ILC and RC. If the system operates in discrete time, as considered in the paper by Hätönen *et al*, the necessary and sufficient convergence conditions for the basic P-type ILC are as follows [8], [7]:

$$0 < \gamma C \Gamma < 2, \tag{6}$$

with matrices C and Γ defined in equation (1) of the paper by Hätönen *et al*. However, even if the above conditions are satisfied, the basic P-type ILC often has a very bad transient behavior. In contrast, the gradient-based ILC algorithm exhibits monotonic convergence [5]. There are simple frequency domain sufficient conditions that guarantee the monotonic convergence of ILC defined by (3) (see [7] and references therein)

$$\sup_{\omega \in [0,2\pi]} |1 - L(e^{i\omega}) G(e^{i\omega})| < 1. \tag{7}$$

In the case of the gradient-based ILC algorithm the above condition can always be met if $\beta < 2/\sup_{\omega \in [0,2\pi]} |G(e^{i\omega})|^2$.

In the paper by Hätönen *et al*, the authors require additional conditions to (7) in order to guarantee the monotonic convergence of the gradient-based algorithm in the context of RC. Specifically, the authors prove that if the impulse response of the system is finite and positive, with an appropriate choice of β the gradient-based RC algorithm produces monotonic convergence. Let us discuss how restrictive these additional conditions are. In fact, the authors mitigate the FIR condition themselves either using the truncation of the adjoint system operator or with the help of auxiliary feedback and pole placement. Then, the positivity condition remains. This seems to be a very restrictive condition and furthermore it is not clear if it is needed at all. The authors write that the positivity condition is "quite common in practical applications". However, their very practical experimental test facility does not satisfy the positivity assumption (see Figure 12 in Hätönen *et al*). Nevertheless, all runs of the gradient-based RC algorithms demonstrate monotonic convergence (see Figures 8-10 in Hätönen *et al*). Of course, one has to consider only the runs when the algorithms indeed converge with the appropriate choice of β and auxiliary feedback. Since in the context of ILC the single condition (7) is sufficient to guarantee monotonic convergence, it is very tempting to ask the following question: Is the condition (7) also sufficient to guarantee monotonic convergence for RC? As was pointed out in [7], the condition (7) is a sufficient condition of convergence (not necessarily monotonic) for RC and it is very close to be a necessary condition. Therefore, if the condition (7) guarantees monotonic convergence for RC, almost any RC algorithm would either converge monotonically or diverge. It is very likely that this supposition is too simplistic and, as usually the case, the reality is more subtle. Nevertheless, the above contemplation might suggest that if an RC algorithm satisfies condition (7), it converges with a "good" transient. And furthermore, in contrast to ILC, there could be virtually no RC algorithms that converge eventually but have very bad transients.

Now we come to the question what is the good performance measure for RC. In [2], [11] it is shown that for continuous time Repetitive Control systems the condition

$$||1 - L(s)G(s)||_{\mathcal{H}_\infty} = \sup_{\omega \in [0,\infty)} |1 - L(i\omega)G(i\omega)| < 1,$$

which is by the way the continuous time analog of (7), assures the $\mathcal{L}_2$ input/output stability. This suggests that the $\mathcal{L}_2$-norm could be a good measure of RC performance. It is interesting to note that in [10] to measure the performance of ILC the authors recommend to use the sum of mean-square-errors from several first consecutive learning trials. The latter measure becomes the $\mathcal{L}_2$-norm when it is applied to RC. It would be very interesting to compare several RC schemes, including the gradient-based scheme, using the $\mathcal{L}_2$-norm. It is a pity that Hätönen *et al* did not provide the comparison of their gradient-based RC algorithm with the basic P-type RC algorithm.

Finally, a very interesting point for the discussion is the application of discrete time algorithm to the continuous time system. Note that Hätönen *et al* developed the gradient-based RC algorithm and its theory for the discrete time plant but while making experiments they applied it to the continuous time plant. As shown in [3], one has to be alert to "ripple effect" when applying the discrete time RC algorithm to a continuous plant. The combination of appropriate choice of the sampling frequency and careful pole placement apparently limited the "ripple effect". It would be very nice to see some analytical bounds describing the effect of discretization. On the other hand, the use of discretization can serve as a low-pass filter to limit the influence of singular perturbations or unmodelled high frequency dynamics [1], [7]. The analysis of discretization effects in RC and ILC algorithms is an appealing future research direction.

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