Slotted Aloha as a stochastic game with partial information

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Abstract

This paper studies distributed choice of retransmission probabilities in slotted ALOHA. Both the cooperative team problem as well as the noncooperative game problem are considered. Unlike some previous work, we assume that mobiles do not know the number of backlogged packets at other nodes. A Markov chain analysis is used to obtain optimal and equilibrium retransmission probabilities and throughput. We then investigate the impact of adding retransmission costs (which may represent the disutility for power consumption) on the equilibrium and show how this pricing can be used to make the equilibrium throughput coincide with the optimal team throughput.

1 Introduction

Aloha [4] and slotted Aloha [12] have long been used as random distributed medium access protocols for radio channels. They are in use in both satellite as well as cellular telephone networks for the sporadic transfer of data packets. In these protocols, packets are transmitted sporadically by various users. If packets are sent simultaneously by more than one user then they collide. After the end of the transmission of a packet, the transmitter receives the information on whether there has been a collision (and retransmission is needed) or whether it was well received. All packets involved in a collision are assumed to be corrupted and are retransmitted after some random time. We focus in this paper on the slotted Aloha (which is known to have a better achievable throughput than the unslotted version, [5]) in which time is divided into units. At each time unit a packet may be transmitted, and at the end of the time interval, the sources get the feedback on whether there was zero, one or more transmissions (collision) during the time slot. A packet that arrives at a source is immediately transmitted. Packets that are involved in a collision are backlogged and are scheduled for retransmission after a random time.

The determination of the above random time can be considered as a stochastic control problem. The information structure, however, is not a classical one: sources do not have full state information as they do not know how many packets are backlogged. Nor do they know how many packets have been involved in a collision.

We study this control problem in two different frameworks:
1. as a team problem, i.e. where there is a common goal to all nodes in the network (such as maximizing the system throughput).
2. as a problem in a noncooperative framework: each node wishes to maximize its own throughput. This gives rise to a game theoretical formulation.

Our main finding is that as the workload increases (i.e. as the packet arrival rate increases), sources become more aggressive at equilibrium in the game setting (in comparison with the team problem) and this results in a dramatic decrease in the total system's throughput. To avoid this collapse of system's throughput, we study the effect of adding a cost for transmissions and retransmissions (which can, in particular, represent the battery power cost). We show that this additional cost improves the system's performance and that an appropriate pricing can be chosen that yields an equilibrium performance that coincides with the team one. We finally propose and study a distributed stochastic algorithm for dynamically adjusting the

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retransmission probabilities according to the congestion at the network.

Previous game formulations of the slotted ALOHA have been proposed in [11, 9]. In the first reference, a full information game is considered, in which each user knows how many backlogged packets there are. Moreover, it is assumed in [11] that a packet that is to be transmitted for the first time waits for a random time in the same way as a backlogged packet. Our goal is to study the slotted Aloha in the way it is actually deployed thus avoiding these two assumptions. In [9] it is assumed that nodes have always packets to send. Thus there is only one trivial state in the system (all nodes are backlogged) which is known to all users.

For more background on the use of stochastic control and of game theory in communication networks, see [1, 2, 3]. We note that the game formulation of our problem is similar to game formulation of retrial queues, in which customers retry to make a call after some random time if they find the line busy [6, 8]. The difference is, however, that in retrial queues there are no collisions.

The structure of the paper is as follows. We begin by introducing in Section 2 the general model and formulate the team and the game problems. We provide a Markov analysis for both the team and the game problem. This analysis is used in Section 3 to numerically study and compare the properties of the team and the game solutions. The model with pricing is then introduced in Section 4 and is investigated numerically in Section 5. The stochastic algorithm is presented in Section 6.

2 Model and problem formulation

We use a Markovian model based on [5, Sec. 4.2.2]. We assume that there are a finite number of sources without buffers. The arrival flow of packets to source i follows a Bernoulli process with parameter q_a (i.e. at each time slot, there is a probability q_a of a new arrival at a source, and all arrivals are independent). As long as there is a packet at a source (i.e. as long as it is not successfully transmitted) new packets to that source are blocked and lost.[1] The arrival processes to different sources are independent. A backlogged packet at source i is retransmitted with probability q_r^i. We shall restrict in our control and game problems to simple policies in which q_r^i does not change in time. Since sources are symmetric, we shall further restrict to finding a symmetric optimal solution, that is retransmission probabilities q_r^i that do not depend on i.

Remark 1. Other models for ALOHA have been also studied in the literature. A commonly used model is one with infinite many sources [5] with no buffers, in which the process of total number of (non-blocked) arrivals at a time slot is Poisson with parameter λ and the process of combined transmissions and retransmissions attempts forms a Poisson process with parameter G. Analysis of this model shows that it has two quasi-stable operation modes (as long as $\lambda < \exp(-1)$), one corresponding to a congested system (in which there are many backlogged packets and many retransmissions) and one corresponding to an uncongested system (with small amount of backlogged packets). The weakness of this model is that both operation points turn out to have the same throughput. Our model also has two quasi-stable operation modes but the throughput during congestion periods is lower than in the noncongested periods [5], which seems more realistic. We also note that in the case of infinitely many nodes, retransmissions with a fixed positive probability renders the system unstable [7].

Remark 2. Quite frequently one uses the ALOHA protocol for sporadic transmissions of signaling packets such as packets for making reservation for a dedicated channel for other transmissions (that do not use ALOHA), see e.g. the description of the SPADE on demand transmission protocol for satellite communications in [13]. In the context of signaling, it is natural to assume that a source does not start generating a new signaling packet (e.g. a new reservation) as long as the current signaling packet is not transmitted. In that case, the process of attempts to retransmit a new packet from a source after the previous packet has been suc-

[1] In considering the number of packets in the system, this assumption is equivalent to saying that a source does not generate new packets as long as a previous packet is not successfully transmitted.

cessfully transmitted coincides with our no buffer model.

For any choice of values $q_r^i \in (0,1]$, we obtain a Markov chain that contains a single ergodic chain (and possibly transient states as well). Let $\pi(q)$ be the corresponding steady state probabilities. We shall use as the state of the system the number of backlogged packets at the beginning of a slot, and denote it frequently with n.

We introduce further notation. Assume that there are n backlogged packets, and all use the same value q_r as retransmission probability. Let $Q_r(i,n)$ be the probability that i out of the n backlogged packets retransmit at the slot. Then

$$Q_r(i,n) = \binom{n}{i}(1-q_r)^{n-i}[q_r]^i. \tag{1}$$

Assume that m is the number of nodes and let $Q_a(i,n)$ be the probability that i unbacklogged nodes transmit packets in a given slot (i.e. that i arrivals occurred at nodes without backlogged packets). Then

$$Q_a(i,n) = \binom{m-n}{i}(1-q_a)^{m-n-i}[q_a]^i. \tag{2}$$

And let $Q_r(1,0)=0$ and $Q_a(1,m)=0$.
In case all nodes use the same value of q_r, the transition probabilities of the Markov chain are given by [5, eq. 4.3]:

$$P_{n,n+i}(q) = \begin{cases} Q_a(i,n), & 2 \le i \le m-n, \\ Q_a(1,n)[1-Q_r(0,n)], & i=1, \\ Q_a(1,n)Q_r(0,n) + Q_a(0,n)[1-Q_r(1,n)], & i=0, \\ Q_a(0,n)Q_r(1,n), & i=-1, \end{cases}$$

The system throughput (defined as the sample average of the number of packets that are successfully transmitted) is given almost surely by the constant

$$\begin{aligned} thp(q) &= \sum_{n=1}^{m} \pi_n(q)[P_{n,n-1}(q) + Q_a(1,n)Q_r(0,n)] \\ &\quad + \pi_0(q)Q_a(1,0) = q_a \sum_{n=0}^{m} \pi_n(q)(m-n). \end{aligned}$$

Note: the first equality follows from the fact that if the state at the beginning of the slot is $n>0$ then there is a departure of a backlogged packet during that slot with probability $P_{n,n-1}(q)$, and of a new arriving packet with probability $Q_a(1,n)Q_r(0,n)$; Moreover, if the state is 0 then there is a departure with probability $Q_a(1,0)$. The second equality simply expresses the expected number of arrivals at a time slot (which actually enter the system), which should equal to the expected number of departures (and thus the throughput) at stationary regime.

The team problem is therefore given as the solution of the optimisation problem:

$$\max_q \; thp(q) \; s.t. \begin{cases} \pi(q) = \pi(q)P(q), \\ \pi_n(q) \ge 0, n = 0, ..., m \\ \sum_{n=0}^{m} \pi_n(q) = 1. \end{cases}$$

A solution to the team problem can be obtained by computing recursively the steady state probabilities, as in Problem 4.1 in [5], and thus obtain an explicit expression for $thp(q)$ as a function of q.

Singularity at $q=0$ The only point where P does not have a single stationary distribution is at $q=0$, where it has two absorbing states: $n=m$ and $n=m-1$. All other states are transient (for any $q_a>0$), and the probability to end at one of the absorbing states depend on the initial distribution of the Markov chain. We note that if the state $m-1$ is reached then the throughput is q_a w.p.1, where as if the state m is reached then the throughput equals 0. It is thus a deadlock state. For $q_a>0$ and $q_r=0$, the deadlock state is reached with positive probability from any initial state other than $m-1$. We shall therefore exclude $q_r=0$ and optimize only on the range $\epsilon \le q_r \le 1$. We choose throughout the paper $\epsilon = 10^{-4}$.

Existence of a solution The steady state probabilities $\pi(q)$ are continuous over $0<q\le 1$. Since this is not a close interval, a solution need not exist. However, as we restrict to the closed interval $q \in [\epsilon, 1]$ where $\epsilon > 0$, an optimal solution indeed exists. Note also that the limit $\lim_{q\to 0}\pi(q)$ exists since $\pi(q)$ is a rational function of q at the neighborhood of zero. Therefore for any $\delta>0$, there exists some $q>0$ which is δ-optimal. ($q^*>0$ is said to be δ-optimal if it satisfies $thp(q^*) \geq thp(q) - \delta$ for all $q \in (0,1]$.)

Next, we formulate the game problem. For a given policy vector $\mathbf{q_r}$ of retransmission probabilities for all users (whose jth entry is q_r^j), define $([\mathbf{q_r}]^{-i}, \hat{q}_r^i)$ to be a retransmission policy where user j retransmits at a slot with probability q_r^j for all $j \neq i$ and where user i retransmits with probability $\hat{q}_r^i$. Each user i seeks to maximize his own throughput thp_i. The problem we are interested in is then to find a symmetric equilibrium policy $\mathbf{q_r^*} = (q_r, q_r, .., q_r)$ such that for any user i and any retransmission probability q_r^i for that user,

$$thp_i(\mathbf{q}^*) \geq thp_i([\mathbf{q_r^*}]^{-i}, q_r^i). \tag{3}$$

Since we restrict to symmetric $\mathbf{q_r^*}$, we shall also identify it (with some abused of notation) with the actual transmission probability (which is the same for all users). Next we show how to obtain an equilibrium policy. We first note that due to symmetry, to see whether $\mathbf{q_r^*}$ is an equilibrium it suffices to check (3) for a single player. We shall thus assume that there are $m+1$ users all together, and that the first m users retransmit with a given probability $\mathbf{q}^{-(m+1)} = (q^o, .., q^o)$ and user $m+1$ retransmits with probability $q_r^{(m+1)}$. Define the set

$$\mathcal{Q}^{m+1}(\mathbf{q_r^o}) = argmax\Big(thp_{m+1}([\mathbf{q_r^o}]^{-(m+1)}, q_r^{(m+1)})\Big),$$

where $\mathbf{q_r^o}$ denotes (with some abuse of notation) the policy where all users retransmit with probability q_r^o, and where the maximization is taken with respect to $q_r^{(m+1)}$. Then q_r^* is a symmetric equilibrium if

$$q_r^* \in \mathcal{Q}_r^{m+1}(q_r^*).$$

To compute $thp_{m+1}([\mathbf{q_r^o}]^{-i}, q_r^i)$, we introduce again a Markov chain with a two dimensional state. The first state component corresponds to the number of backlogged packets among the users 1,...,m, and the second component is the number of backlogged packets (either 1 or 0) of user $m+1$. The transition probabilities are given by

$$P_{(n,i),(n+k,j)}(q_r^o, q_r^{(m+1)}) =$$

$$\begin{cases}
\left.\begin{array}{ll} Q_a(k,n), & i=j=1 \\ Q_a(k,n)(1-q_a), & i=j=0 \\ Q_a(k,n)q_a, & i=0, j=1 \end{array}\right\} 2 \leq k \leq m-n \\[2ex]
\left.\begin{array}{l} Q_a(1,n)[1-Q_r(0,n)(1-q_r^{(m+1)})] \\ \hfill i=j=1 \\ Q_a(1,n)[1-Q_r(0,n)](1-q_a) \\ \hfill i=j=0 \\ Q_a(1,n)q_a, \hfill i=0, j=1 \end{array}\right\} k=1 \\[2ex]
\left.\begin{array}{l} (1-q_r^{(m+1)})Z + q_r(1-Q_r(0,n))Q_a(0,n) \\ \hfill i=j=1 \\ (1-q_a)Z + q_a Q_a(0,n)Q_r(0,n), \\ \hfill i=j=0 \\ q_a Q_a(0,n)[1-Q_r(0,n)], \hfill i=0, j=1 \\ q_r^{(m+1)} Q_a(0,n)Q_r(0.n), \hfill i=1, j=0 \end{array}\right\} k=0 \\[2ex]
\left.\begin{array}{ll} Q_a(0,n)Q_r(1,n)(1-q_r^{(m+1)}), & i=j=1 \\ Q_a(0,n)Q_r(1,n)(1-q_a), & i=j=0 \end{array}\right\} k=-1 \\[2ex]
0 \qquad \qquad \qquad \qquad \qquad otherwise
\end{cases}$$

where $Z = (Q_a(1,n)Q_r(0,n) + Q_a(0,n)[1 - Q_r(1,n)]$ and where Q_a and Q_r are given in (1) and (2), respectively (with q_r^o replacing q_r).

The throughput of user $m+1$ is given by

$$\begin{aligned} & thp_{m+1}([\mathbf{q_r^o}]^{-(m+1)}, q_r^{(m+1)}) \\ & \quad = \quad q_a \sum_{n=0}^{m} \pi_{n,0}([q_r^o]^{-(m+1)}, q_r^{(m+1)}). \end{aligned} \tag{4}$$

3 Numerical investigation

In this section we shall obtain the retransmission probabilities which solve the team and the game problem. We investigate their dependence and the dependence of the throughput that they imply on the arrival probabilities q_a and on the number of nodes.

Figures 1 and 2 provide the total throughput and optimal retransmission probabilities q_r for $m = 2$, $m = 3$ and $m = 4$ for the team problem, as a function of the arrival probability q_a.

Figure 3 shows both the retransmission probability and total throughput (obtained by multiplying the expression in eq. (4) by the number of mobiles) as function of the arrival probability for the game scenario.

Comparing the two figures, we see in particular that the game solution is very inefficient for large arrival probabilities: the total throughput converges to zero as the arrival probability increases, where as in the team case, the total throughput increases with q_a and attains more than 0.55 (for $m = 2, 3, 4$). The inefficiency is seen also through the optimal retransmission policy: as the system becomes more congested (larger arrival probabilities) the retransmission probability decreases in the team case so as to counter expected collisions. The game scenario gives rise, in contrast, to an equilibrium that becomes more and more aggressive as the arrival probabilities increase: the equilibrium retransmission probability is seen to increase with q_a (for $q_a > 0.2$) which explains the dramatic decrease in the system's throughput. In particular, as q_a approaches 1, so does q_r at equilibrium!

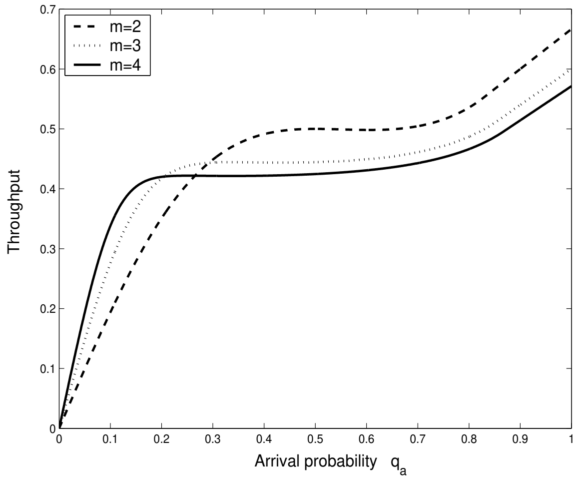

Figure 1: Optimal throughput for the team case as a function of the arrival probabilities q_a for $m = 2, 3, 4$

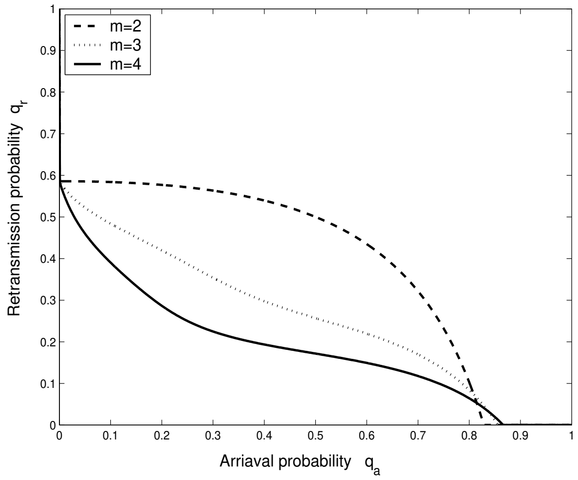

Figure 2: The optimal retransmission probabilities in the team case as a function of the arrival probabilities q_a for $m = 2, 3, 4$.

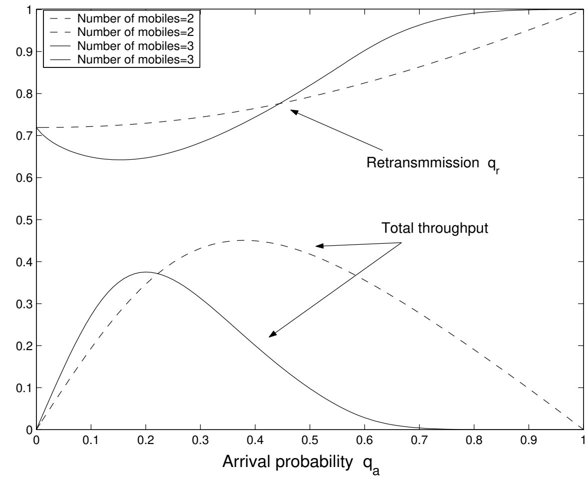

Figure 3: The equilibrium retransmission probabilities in the game case as a function of the arrival probabilities q_a.

4 Adding costs for retransmissions

In this section we consider the problem where there is an extra cost θ per each transmission and retransmission. This can represent the disutility for the consumption of battery energy, which is a scarce resource. For a given symmetric q for all users, the steady-state retransmission cost is $\theta q \sum_{n=0}^{m} \pi_n(q) n$, where as the transmission cost of arriving packets (i.e. packets that enter the system and are not rejected) is $\theta thp(q)$. (This is because the expected number of arrival packets equals to the expected number of departing packets at steady-state, and each time a packet arrives at the system it is immediately transmitted.)

Thus the new team problem is

$$\max_q \left\{ thp(q)(1-\theta) - \theta q \sum_{n=0}^{m} \pi_n(q)n \right\}.$$

For the non-cooperative problem, the retransmission cost for a symmetric retransmission policy q_r^o of users 1,...,m and a retransmission probability $q_r^{(m+1)}$ of user $m+1$ is:

$$\theta q_r^{(m+1)} \sum_{n=0}^{m} \pi_{n,1}([q_r^o]^{-(m+1)}, q_r^{(m+1)}).$$

User $m+1$ is thus faced with the problem:

$$\max_{q_r^{m+1}} J_{m+1}(q_r^o, q_r^{(m+1)})$$

where

$$\begin{aligned} & J_{m+1}(q_r^o, q_r^{(m+1)}) \\ = \quad & thp_{m+1}([q_r^o]^{-(m+1)}, q_r^{(m+1)})(1-\theta) \\ & -\theta q_r^{(m+1)} \sum_{n=0}^{m} \pi_{n,1}([q_r^o]^{-(m+1)}, q_r^{(m+1)}). \end{aligned}$$

Define as we did before

$$\overline{\mathcal{Q}}_r^{m+1}(q_r^o) = argmax\Big(J_{m+1}([q_r^o]^{-(m+1)}, q_r^{(m+1)})\Big).$$

Then we seek for the value q_r^* of retransmission probability that satisfies

$$q_r^* \in \overline{\mathcal{Q}}_r^{m+1}(q_r^*),$$

which is the Nash equilibrium for the game problem.

5 Numerical investigation

In this section we shall obtain the retransmission probabilities which solve the team are the game problems with the extra transmission costs. We shall investigate the dependence of the solution on the value θ.

In Figures 4 and 5 we depict the throughput obtained at the optimal solution and the optimal retransmission probabilities, respectively, as a function of the arrival probability, for the team problem with $m = 2$, for various values of θ. We see that both the throughput as well as the retransmission probabilities are monotone decreasing in the cost. This can be expected since retransmissions become more costly with increasing θ. An interesting feature is that for any fixed $\theta \neq 0$, the retransmission probabilities first increase in the arrival probability and then decrease. For $\theta = 0$, in contrast, the optimal retransmission probability decreases in the arrival probability (which is natural since congestion in the system increases as q_a increases).

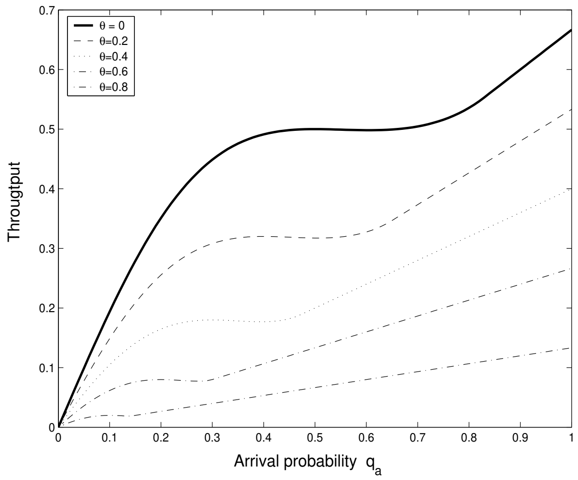

Figure 4: Throughput at optimal q_r for the team case as a function of the arrival probabilities q_a for $m = 2$ and $\theta = 0, 0.2, 0.4, 0.6, 0.8$.

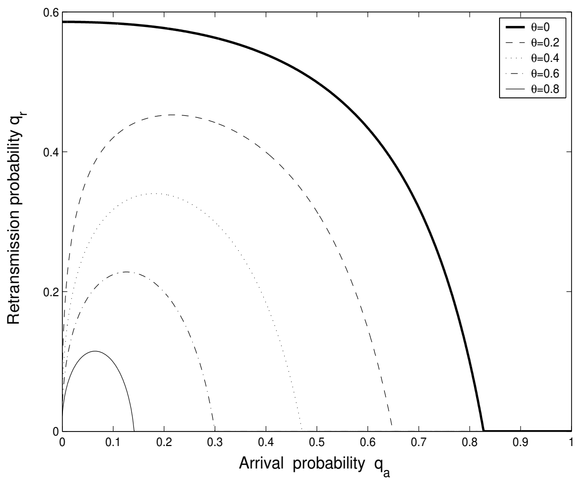

Figure 5: The optimal retransmission probabilities in the team case as a function of the arrival probabilities q_a for $m = 2$ and $\theta = 0, 0.2, 0.4, 0.6, 0.8$.

Next we consider the game problem with 2 mobiles. Figure 6 shows the impact of θ on the equilibrium retransmission probability q_r, as a function of the arrival q_a. We see that increasing the cost θ results in decreasing the retransmission probabilities.

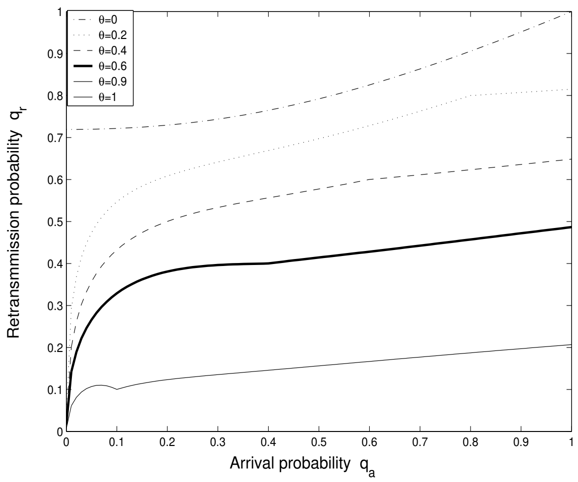

Figure 6: The equilibrium retransmission probabilities in the game case as function of the arrival probabilities q_a for $m = 2$ (number of mobiles) and $\theta = 0, 0.2, 0.4, 0.6, 0.9, 1$.

Figure 7 provides the total throughput at equilibrium as a function of the arrival probabilities for the case of two mobiles. We see that indeed the throughput is improved considerably by adding a cost on retransmission, especially for large arrival probabilities.

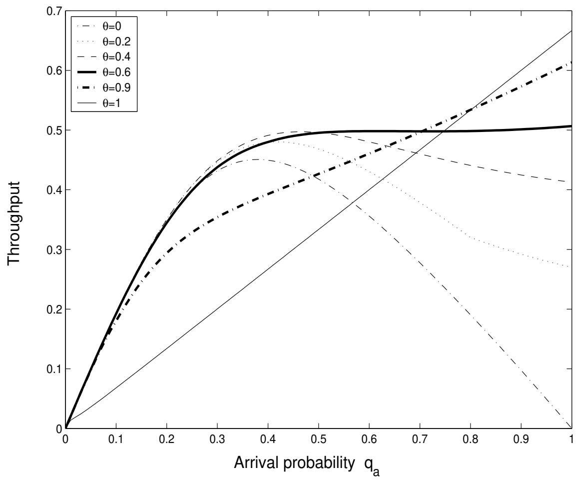

Figure 7: Total throughput for the game case as a function of the arrival probabilities q_a for $m = 2$ (number of mobiles) and $\theta = 0, 0.2, 0.4, 0.6, 0.9, 1$.

We then compute the pricing θ that is necessary for the equilibrium retransmission probabilities to coincide with those obtained for the team problem. This is the value of θ that will yield the optimal system throughput. The results are presented in Fig. 8.

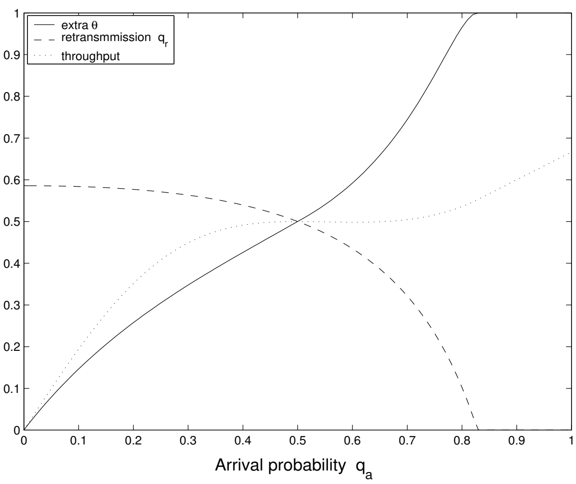

Figure 8: The retransmission cost θ such that the optimal retransmission in the game coincides with that of the original team problem, as function of the arrival probabilities q_a for $m = 2$.

6 A stochastic adaptive algorithm

The solutions of both the team as well as the game problems have been computed by using some fixed parameters (the arrival probabilities and number of mobiles m) which might not be available. It is thus of interest to study algorithms that do not need these parameters. We thus relax the assumption of time independent retransmission probabilities, and allow each $q_r^i(t)$, the retransmission probability of mobile i at time slot t, to vary according to the information it receives. Specifically, we propose the following distributed algorithm:

$$q_r^i(t+1) = q_r^i(t) + \epsilon(t)\xi(t),$$

where $\xi(t) = -1$ if there has been a collision at time slot t and is 1 otherwize. Note that even if a mobile does not transmit, a feedback is received if there has been a collision (involving other packets), and this information is used to infer that there is congestion. $\epsilon(t)$ are some constants. If we assume that the systems parameters are unknown but fixed, then $\epsilon(t)$ are chosen so as to satisfy $\lim_{t\to\infty} \epsilon(t) = 0$ and $\sum_{t\to\infty} \epsilon(t) = \infty$. This is inspired by standard stochastic approximation theory [10].

Remark 3. (i) Note that our algorithm is not a result of an optimization procedure.
(ii) Although the algorithm is decentralized, since all mobiles use the same algorithm and all mobiles receive the same feedback, $q_r^i(t)$ does not depend on i. We shall thus omit the superscript i.

We investigate below the case of three mobiles with $\epsilon(t) = 1/(20t)$. We ran the distributed algorithm for 10^4 slots. The evolution of $q_r(t)$ is depicted in Figure 9. The initial value of q_r was taken to be 0.1111.

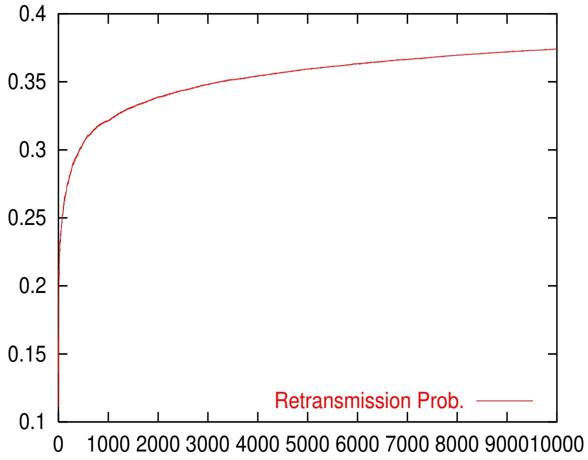

Figure 9: Evolution of $q_r(t)$ for the case of three mobiles using the distributed adaptive algorithm

We ran three times the same simulation and obtained convergence of $q_r(t)$ in all three cases; the final value was 0.365, 0.392 and 0.374 in the three simulations, and the averaged value over the simulations is 0.377. The total number of packets successfully transmitted in the simulations are respectively 4433, 4348 and 4462. This gives an average throughput of 0.4414. We note that this, as well as the limit obtained for the retransmission probabilities are very close to the corresponding optimal team values.

7 Concluding remarks

We have studied three approaches for choosing retransmission probabilities in a slotted Aloha system. First, we studied the team problem, then the non-cooperative game problem. The objective was initially to maximize the throughput. We saw that as the arrival probabilities increased, the behavior of mobiles became more and more aggressive (as compared to the team problem) which resulted in a global deterioration of the system throughput. This is in contrast to the team problem in which throughput increased with the arrival probabilities. We also considered additional costs on transmissions and showed numerically that pricing could be used to enforce an equilibrium whose throughput corresponds to the team optimal solution. We finally considered a distributed adaptive algorithm for updating the retransmission probabilities that did not require the knowledge of the system's parameters (number of mobiles and arrival probabilities). In the example we tested numerically, we obtained rapid convergence to a value close to the team optimal solution.

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