A Stochastic Evolutionary Game Approach to Energy Management in a Distributed Aloha Network

Eitan Altman*, Yezekael Hayel†

*INRIA, 2004 Route des Lucioles, 06902 Sophia Antipolis, France. E-mail: altman@sophia.inria.fr
† LIA, Université d'Avignon 339, chemin des Meinajaries, Agroparc BP 1228, 84911 Avignon Cedex 9, France.
E-mail: yezekael.hayel@univ-avignon.fr

***Abstract*—A major contribution of biology to competitive decision making is the area of evolutionary games. It describes the evolution of sizes of large populations as a result of many local interactions, each involving a small number of randomly selected individuals. An individual plays only once; it plays in a one shot game against another randomly selected player with the goal of maximizing its utility (fitness) in that game. We introduce here a new more general type of games: a Stochastic Evolutionary Game where each player may be in different states; the player may be involved in several local interactions during his life time and his actions determine not only the utilities but also the transition probabilities and his life duration. This is used to study a large population of mobiles forming a sparse ad-hoc network, where mobiles compete with their neighbors on the access to a radio channel. We study the impact of the level of energy in the battery on the aggressiveness of the access policy of mobiles at equilibrium. We obtain properties of the ESS (Evolutionary Stable Strategy) equilibrium which, Unlike the Nash equilibrium concept, is robust against deviations of a whole positive fraction of the population. We further study dynamical properties of the system when it is not in equilibrium.**

I. Introduction

The evolutionary games formalism is a central mathematical tools developed by biologists for predicting population dynamics in the context of *interactions between populations*. This formalism identifies and studies two concepts: The ESS (for *Evolutionary Stable Strategy*), and the *Replicator Dynamics* [10].

The ESS is characterized by a property of robustness against invaders (mutations). More specifically,

- (i) if an ESS is reached, then the proportions of each population do not change in time.
- (ii) at ESS, the populations are immune from being invaded by other small populations. This notion is stronger than Nash equilibrium in which it is only requested that a single user would not benefit by a change (mutation) of its behavior.

ESS has first been defined in 1972 by M. Smith [3], who further developed it in his seminal text *Evolution and the Theory of Games* [4], followed shortly by Axelrod's famous work [1].

Although ESS has been defined in the context of biological systems, it is highly relevant to engineering as well (see [5]). In the biological context, the replicator dynamics is a model for the change of the size of the population(s) as biologist observe, where as in engineering, we can go beyond characterizing and modelling existing evolution. The evolution of protocols can be engineered by providing guidelines or regulations for the way to upgrade existing ones and in determining parameters related to deployment of new protocols and services. In doing so we may wish to achieve adaptability to changing environments (growth of traffic in networks, increase of speeds or of congestion) and yet to avoid instabilities that could otherwise prevent the system to reach an ESS.

Evolutionary Stable Strategies Consider a large population of players. Each individual needs occasionally to take some action (such as power control decisions, or forwarding decision). We focus on some (arbitrary) tagged individual. Occasionally, the action of some N (possibly random number of) other individuals interact with the action of that individual (e.g. other neighboring nodes transmit at the same time). In order to make use of the wealth of tools and theory developed in the biology literature, we shall often restrict, as they do, to interactions that are limited to pairwise, i.e. to $N = 1$. This will correspond to networks operating at light loads, such as sensor networks that need to track some rare events such as the arrival at the vicinity of a sensor of some tagged animal.

We define by $J(p, q)$ the expected payoff for our tagged individual if it uses a strategy (also called policy) p when meeting another individual who adopts the strategy q. This payoff is called "fitness" and strategies with larger fitness are expected to propagate faster in a population.

p and q belong to a set K of available strategies. In the standard framework for evolutionary games there are a finite number of so called "pure strategies", and a general strategy of an individual is a probability distribution over the pure strategies. An equivalent interpretation of strategies is obtained by assuming that individuals choose pure strategies and then the probability distribution represents the fraction of individuals in the population that choose each strategy. Note that J is linear in p and q.

Suppose that the whole population uses a strategy q and that a small fraction ϵ (called "mutations") adopts another strategy p. Evolutionary forces are expected to select against p if

$$J(q, \epsilon p + (1-\epsilon)q) > J(p, \epsilon p + (1-\epsilon)q) \tag{1}$$

Definition 1: A strategy q is said to be ESS if for every $p \neq q$ there exists some $\overline{\epsilon}_y > 0$ such that (1) holds for all $\epsilon \in (0, \overline{\epsilon}_y)$.

In fact, we expect that if

$$\text{for all } p \neq q, \quad J(q,q) > J(p,q) \tag{2}$$

then the mutations fraction in the population will tend to decrease (as it has a lower reward, meaning a lower growth rate). q is then immune to mutations. If it does not but if still the following holds,

$$\text{for all } p \neq q, \quad J(q,q) = J(p,q) \text{ and } J(q,p) > J(p,p) \tag{3}$$

then a population using q are "weakly" immune against a mutation using p since if the mutant's population grows, then we shall frequently have individuals with strategy q competing with mutants; in such cases, the condition $J(q,p) > J(p,p)$ ensures that the growth rate of the original population exceeds that of the mutants. We shall need the following characterization:

Theorem 1: [9, Proposition 2.1] or [2, Theorem 6.4.1, page 63] A strategy q is ESS if and only if it satisfies (2) or (3).

Corollary 1: (2) is a sufficient condition for q to be an ESS. A necessary condition for it to be an ESS is

$$\text{for all } p \neq q, \quad J(q,q) \geq J(p,q) \tag{4}$$

The conditions on ESS can be related to an interpreted in terms of Nash equilibrium in a matrix game The situation in which an individual, say player 1, is faced with a member of a population in which a fraction p chooses strategy A is then translated to playing the matrix game against a second player who uses mixed strategies (randomizes) with probabilities p and $1-p$, resp. The central model that we shall use to investigate protocol and service evolution is introduced in the next Subsection along with its matrix game representation.

We can learn and adopt notions from biology not only through the concept of evolutionary game, but also in applications related to energy issues that have a central role both in biology as well as in mobile networking. The long term animal survival is directly related to its energy strategies (competition over food etc), and a population of animals that have good strategies for avoiding starvation is more fit and is expected to survive [11], [12], [13]. By analogy, we may expect mobile terminals which adopt efficient energy strategies to live longer. Networks with more efficient nodes then have more chances to survive longer [14].

Another element of evolutionary games is the replicator dynamics. It has been used for describing the evolution of road traffic congestion in which the fitness is determined by the strategies chosen by all drivers [7]. It has also been studied in the context of the association problem in wireless networks in [8]. We shall propose a replicator dynamics that extends the standnard one and study its convergence.

The structure of the paper is as follows. In the next section we present the model and provide some notation and definitions. We then compute in Section III the ESS as a function of the system's parameters, and then study its properties in Section IV. In Section V we compare the performance at ESS which characterizes the non-coopoerative behavior to that obtained by a cooperative approach. We then propose and study in Section VI the replicator dynamics and end with a section on numerical examples.

II. Model

We consider a large population of mobile terminals. We assume that the density of the network is low, so that if a terminal attempts transmission one can neglect the probability of interference from more than one other mobile (called "neighbor"). Interference occurs as in the Aloha protocol: if more than one neighbors transmit a packet at the same time then there is a collision. To avoid loosing both packets at a collision, we assume that a mobile whose battery energy level is high can transmit at a high power. This allows to recover the transmission whenever a collision occurs with a packet transmitted at a low energy level.

More precisely, we consider non-cooperative game among a large population of users of mobile terminals. A terminal i attempts transmissions during some sequence $\{t_n^i\}$ of times, called time opportunities or time slots. At each attempt, it has to take a decision on the transmission power based on his battery energy state. To simplify, we assume that the state can take three values: $\{F, A, E\}$ for Full, Almost empty or Empty.

The transmission signal power of a terminal can be High (h) or Low (l). We call these the "actions" that a terminal can take. Transmission at high power is possible only when the mobile is in state F. At state A only action l is available, and at E transmission is not possible any more. The lifetime of the mobile is defined as the number of slots during which its battery is nonempty.

We consider an Aloha-type game where a mobile transmits a packet with success during a slot if:

- the mobile is the only one to transmit during this slot
- the mobile transmits with high power and all others transmitting nodes use low power

A. Some notation

We introduce the following notations:

- p is the probability for a mobile to be the only transmitter during a slot
- $Q_i(a)$ is the probability of remaining at energy level i when using action a. Since state A only action l is available, we write Q_A instead of $Q_A(l)$
- α is the fraction of the population who use the action h at any given time. Note that for a given α, the proportion of the population that choose action h at state F is higher than α, since at state A a mobile cannot choose h.

B. Policies

In standard evolutionary games, a player takes only one action. The result of the action is represented using a matrix R whose entry (i,j) represent the fitness of the individual if it plays action i when it meets an individual that plays action j. Then, if a fraction α_j of the population plays action j, then the expected fitness of and individual playing action i is $\sum_j \alpha_j R_{ij}$.

The game we face here is different since an individual can take several actions during its lifetime. Hence the fitness cannot be directly represented as an entry of a matrix whose rows corresponds to the players' actions only. Instead, an entry would correspond to the total expected fitness that the player gets in his lifetime as a function of its policy.

A general policy u is a sequence $u = (u_1, u_2, ...)$ where u_i is the probability of choosing h if at time i the state is F. A stationary policy is a policy for which at any time that the individual is at state F, the probability u_i of choosing h is the same. We shall use a single real number β to denote a stationary policy of an individual. β stands for the probability that a mobile uses h when at state F.

A stationary policy is called pure if it does not use randomization (in our case there exist two pure stationary policies: the one that always choose h at state F and the one that always choose l). Other stationary policies are called *randomized* stationary policies. (They play the role of mixed policies in standard evolutionary games.)

C. Fitness

Choose some individual terminal and let R_t denote the number of packets (zero or one) successfully transmitted at time slot t by this terminal. We define the *fitness* of the terminal to be given by $\sum_{t=1}^{\infty} R_t$, i.e. the total number of packets successfully transmitted during its lifetime.

Assume that α is fixed and does not change in time (we shall consider later the case in which it may vary in time). Then the expected optimal fitness of an individual starting at a given initial state can be computed using the standard theory of total-cost dynamic programming, that states in particular that there exist optimal stationary policy (i.e. a policy for which at any time that the individual is at state F, the probability u_i of choosing h is the same). We shall therefore *restrict to stationary policies* unless stated otherwise.

Some more definitions:

- $V_\beta(i,\alpha)$ is the total expected fitness (i.e. reward or valuation) of a user given that it uses policy β, that it is in state i and given the parameter α.
- We define with some abuse of notation $V_h(i,\alpha)$ (respectively $V_l(i,\alpha)$) to be the total expected fitness of a user given that it is in state i, that it uses action h when at state F (action l, respectively) and given the parameter α.

III. Computing fitness and expected sojourn times

We proceed by computing the individual's expected total utility and remaining lifetime that correspond to a given initial state and a stationary policy β. We note however that only at state F there will be a dependence on β.

State E
When the level of energy is in state E, the valuation is equal to $V(E) = 0$.

State A
When the state is A, the valuation is

$$V(A) = p + Q_A V(A),$$

which gives that

$$V(A) = \frac{p}{1 - Q_A}.$$

The expected time during which a mobile spends in state A is given by

$$T(A) = 1 + Q_A T(A)$$

which implies that

$$T(A) = \frac{1}{1 - Q_A}.$$

State F
Define the dynamic programming operator $Y(v,a,\alpha)$ to be the *total expected fitness of an individual starting at state F*, if

- It takes action a at time 1,
- If at time 2 the state is F then the total sum of expected fitness from time 2 onwards is v.
- At each time the mobile attempts transmission, the probability that another interfering mobile uses action h, given that there is a simultaneous transmission with another mobile, interferes with another transmission, is α.

Below, α is omitted for the case $a = l$ since the dependence on α appears only with the action $a = h$.

We have

$$\begin{aligned} Y(v,l) &= p + Q_F(l)v + (1 - Q_F(l))V(A), \\ &= p + Q_F(l)v + p\frac{1 - Q_F(l)}{1 - Q_A}. \end{aligned} \quad (5)$$

and

$$\begin{aligned} Y(v,h,\alpha) &= \alpha(p + Q_F(h)v + (1 - Q_F(h))V(A)) \\ &\quad +(1-\alpha)(1 + Q_F(h)v + (1 - Q_F(h))V(A)), \\ &= \alpha p + (1-\alpha) + Q_F(h)v + p\frac{1 - Q_F(h)}{1 - Q_A}. \end{aligned} \quad (6)$$

Assume that a mobile uses h w.p. β at state F. Then the expected time it spends at state F is

$$T_\beta(F) = 1 + \beta Q_F(h) T_\beta(F) + (1-\beta) Q_F(l) T_\beta(F)$$

which gives

$$T_\beta(F) = \frac{1}{1 - \beta Q_F(h) - (1-\beta)Q_F(l)}$$

The fraction of time that the mobile uses action h is then

$$\begin{aligned}\widehat{\alpha}(\beta) &= \beta\frac{T(F)}{T(F)+T(A)} \\ &= \beta\frac{1-Q_A}{2-Q_A-\beta Q_F(h)-(1-\beta)Q_F(l)}\end{aligned} \quad (7)$$

Denote by $V_\beta(F,\alpha)$ the total expected utility the mobile gains starting from state F. Then $V_\beta(F,\alpha)$ is the unique solution of

$$v=(1-\beta)Y(v,l)+\beta Y(v,h,\alpha).$$

This gives

$$\begin{aligned}V_\beta(F,\alpha) &= \frac{(\frac{p}{1-Q_A})[(1-\beta)(1-Q_F(l)+\beta(1-Q_F(h)))]}{1-(1-\beta)Q_F(l)-\beta Q_F(h)} \\ &\quad+\frac{(1-\beta)p+\beta(\alpha(p-1)+1)}{1-(1-\beta)Q_F(l)-\beta Q_F(h)} \\ &= \frac{p}{1-Q_A}+\frac{(1-\beta)p+\beta(\alpha(p-1)+1)}{1-(1-\beta)Q_F(l)-\beta Q_F(h)} \\ &= V(A)+\frac{p+\beta(1-p)(1-\alpha)}{1-Q_F(l)+\beta(Q_F(l)-Q_F(h))}\end{aligned}$$

In the special case of the aggressive policy $\beta=1$, we obtain

$$\forall\alpha,\quad V_1(F,\alpha)=V(A)+\frac{1-\alpha(1-p)}{1-Q_F(h)}, \quad (8)$$

and for $\beta=0$, we obtain

$$\forall\alpha,\quad V_0(F,\alpha)=V(A)+\frac{p}{1-Q_F(l)}. \quad (9)$$

Remark 1: Differentiating (8) with respect to β, we obtain

$$\frac{\mathrm{d}V_\beta(F,\alpha)}{\mathrm{d}\beta}= \quad (10)$$

$$\frac{(1-p)(1-\alpha)(1-Q_F(l))-p(Q_F(l)-Q_F(h))}{(1-Q_F(l)+\beta(Q_F(l)-Q_F(h)))^2}$$

This is either strictly positive for all $\beta\in[0,1]$ or strictly negative for all $\beta\in[0,1]$ or equals zero over all the interval. We conclude that $V_\beta(F,\alpha)$ is either constant or strictly monotone in β over the whole interval $[0,1]$.

IV. Properties and Characterization of the ESS

A. Preliminaries

As already mentioned, our game is different and has a more complex structure than a standard evolutionary game. In particular, the fitness that is maximized is not the outcome of a single interaction but of the sum of fitnesses obtained during all the opportunities in the mobile's lifetime. In spite of this difference, we shall still use the definition 1 or the equivalent conditions (2) or (3) for the ESS but with the following changes:

- β replaces the strategy q in the initial definition,
- β' replaces the strategy p in the initial definition,
- We use for $J(q,p)$ the total expected fitness $V_\beta(F,\alpha)$, where $\alpha=\widehat{\alpha}(\beta')$ is given in (7).

We obtain the following characterizations of ESS.

Corollary 2: A necessary condition for β^* to be an ESS is

$$\text{for all } \beta'\neq\beta^*,\quad V_{\beta^*}(F,\widehat{\alpha}(\beta^*))\geq V_{\beta'}(F,\widehat{\alpha}(\beta^*)) \quad (11)$$

A sufficient condition for β^* to be an ESS is that (11) holds with strict inequality for all $\beta'\neq\beta^*$.

We present next a structural property of ESS. It is an adaptation to our setting of the fact that in standard evolutionary games, only pure policies can be ESS satisfying (2); where as a non-pure stationary policy can be an ESS only if it satisfies (3), that is

Theorem 2: A necessary condition for a non-pure stationary policy β^* to be an ESS is that

$$\text{for all } \beta'\neq\beta^*,\quad V_{\beta^*}(F,\widehat{\alpha}(\beta^*))=V_{\beta'}(F,\widehat{\alpha}(\beta^*)). \quad (12)$$

Equivalently, assume that for some stationary policies β^* and $\beta'\neq\beta^*$ we have

$$V_{\beta^*}(F,\widehat{\alpha}(\beta^*))\neq V_{\beta'}(F,\widehat{\alpha}(\beta^*))$$

Then a necessary condition for β^* to be an ESS is that it is a pure policy.

Proof: If $V_{\beta^*}(F,\widehat{\alpha}(\beta^*))<V_{\beta'}(F,\widehat{\alpha}(\beta^*))$ then β^* is not ESS due to Corollary 2. Assume that β^* is not pure (it is neither 0 nor 1) and that $V_{\beta^*}(F,\widehat{\alpha}(\beta^*))>V_{\beta'}(F,\widehat{\alpha}(\beta^*))$ for some $\beta'\neq\beta^*$. Applying Remark 1 with $\alpha=\widehat{\alpha}(\beta^*)$ and $\beta=\beta'$ we conclude that

$$V_1(F,\alpha(\beta^*))>V_{\beta^*}(F,\alpha(\beta^*)) \text{ or } V_0(F,\alpha(\beta^*))>V_{\beta^*}(F,\alpha(\beta^*))$$

Hence the necessary condition for β^* to be an ESS (see Corollary 2) does not hold, which establishes the proof. ■

We shall call an ESS policy β^* that satisfies (12) *weakly immuned ESS*, and one satisfying (4) with strict inequality for all $\beta'\neq\beta^*$ a *strongly immuned ESS*. Remark 1 implies that an ESS is either strongly immuned or weakly immuned. In other words, there does not exist an ESS β for which (4) holds with equality for some $\beta'\neq\beta$ and with strict inequality for some other β'.

We proceed by identifying range of parameters for which various ESS structures are obtained.

B. Pure equilibrium: high power at F

We define an aggressive multi-policy to be the one in which all mobiles use high power each slot at state F.

A mobile that always uses high power at state F spends an expected amount of time of

$$T(F)=\frac{1}{1-Q_F(h)}$$

at state F. The fraction of time it spends at state F is thus

$$\alpha(1)=\frac{T(F)}{T(F)+T(A)}=\frac{1-Q_A}{2-Q_A-Q_F(h)} \quad (13)$$

Theorem 3: Define

$$\Delta_h:=\frac{1-Q_F(h)}{2-Q_A-Q_F(h)}(1-p)-\frac{Q_F(l)-Q_F(h)}{1-Q_F(l)}p$$

Let u be the pure aggressive strategy that uses always h at state F.
(i) $\Delta_h > 0$ is a sufficient condition for u to be an ESS.
(ii) $\Delta_h \geq 0$ is a necessary condition for u to be an ESS.

Proof: We substitute the value of α from eq. (13) in (8) to obtain

$$\begin{aligned} V_1(F, \alpha(1)) &= V(A) \\ &+ \frac{1}{1 - Q_F(h)} \left(1 - (1-p)\frac{1 - Q_A}{2 - Q_A - Q_F(h)}\right) \end{aligned} \tag{14}$$

To prove (i), we have to check that if $\Delta_h > 0$ then for all $\beta \neq 1$ we have

$$V_1(F, \alpha(1)) > V_\beta(F, \alpha(1)). \tag{15}$$

For the case $\beta = 0$, we obtain when taking the difference between (9) and (8) that

$$V_1(F, \alpha(1)) - V_0(F, \alpha(1)) = \frac{\Delta_h}{1 - Q_F(h)}$$

so (15) indeed holds for $\beta = 0$ if $\Delta_h > 0$. We show next that this holds for all other $\beta \in (0, 1)$ as well. It follows from Remark 1 that $V_1(F, \alpha(1)) > V_\beta(F, \alpha(1))$ either holds for every $\beta \in [0, 1)$ or for no β in this interval. Since we showed that if $\Delta_h > 0$ then it holds for $\beta = 0$ then we conclude that $\Delta_h > 0$ is a sufficient condition for (15) to hold for all β.
The necessary condition is proved by showing that $\Delta_h \geq 0$ implies that $V_1(F, \alpha(1)) - V_\beta(F, \alpha(1)) \geq 0$ for all $\beta \neq 1$. This is done in the same way as the sufficient condition is established, and then we apply Corollary 2. ∎

Conclusions. We can draw many qualitative conclusions from the Theorem; here are some of them.

- Note that we have $Q_F(l) > Q_F(h)$ because using less power, the mobile has more probability to stay in state full than using high power. If, for any reason, those probabilities are the same, i.e. $Q_F(l) = Q_F(h)$, then the strategy high power is obviously an ESS.
- If $p = 0$ (i.e. the probability of having another simultaneous transmission is null) then there is no benefit from transmission with high power. Indeed we see that in this case $\Delta < 0$. In fact, we can conclude that there is a threshold p_h^* given by

$$\frac{(1 - Q_F(h))(1 - Q_F(l))}{(1 - Q_F(h))(1 - Q_F(l)) + (2 - Q_A - Q_F(h))(Q_F(l) - Q_F(h))}$$

  so that u is an ESS for $p > p_h^*$ and is not ESS for $p < p_h^*$.
- $\Delta_h > 0$ is a sufficient and necessary condition for u to be a strongly immune ESS.

C. Pure equilibrium: low power at F.

We investigate conditions for which in the full state, each user decides to transmit always with low power.

Theorem 4: Define

$$\Delta_l := p(1 - Q_F(h)) - (1 - Q_F(l))$$

Let v be the pure strategy that uses always l at state F.
(i) $\Delta_l > 0$ is a sufficient condition for v to be an ESS.
(ii) $\Delta_l \geq 0$ is a necessary condition for v to be an ESS.

Proof: Note that $\alpha(0) = 0$. We have

$$V_0(F, 0) - V_1(F, 0) = \frac{\Delta_l}{(1 - Q_F(H))(1 - Q_F(l))}.$$

Hence $\Delta_l > 0$ implies that $V_0(F, 0) > V_\beta(F, 0)$ for $\beta = 1$. As in the proof of Theorem 3, we can use Remark 1 to show that this implies that $V_0(F, 0) > V_\beta(F, 0)$ for all $\beta \neq 0$. Applying Corollary 2, we conclude that $\Delta_l > 0$ is a sufficient condition for the policy v (i.e. for $\beta = 0$) to be ESS. ∎

Conclusions. We can draw many qualitative conclusions from the Theorem; here are some of them.

- Assume that $Q_F(l) < 1$ or that $p < 1$. If $Q_F(h) = Q_F(l)$ then $\Delta_l < 0$ so v is not ESS.
- Define
$$p_l^* = \frac{1 - Q_F(l)}{1 - Q_F(h)}.$$
  Then it is seen that for all $p < p_l^*$, the policy v is an ESS, and for all $p > p_l^*$ it is not an ESS.
- Note that the conditions for the policy u (in Theorem 3) to be ESS depended on Q_A where as the conditions for the policy v (Theorem 4) to be ESS does not. The reason is that the condition for ESS of a policy β involves $\alpha(\beta)$ which depends on Q_A for all β's except $\beta = 0$.
- $\Delta_l > 0$ is a sufficient and necessary condition for v to be a strongly immune ESS.

D. Weakly Immune ESS and Mixed Equilibrium

In the previous Subsections we characterized all strongly immune ESS. In view of Theorem 2, a necessary condition for β^* to be a weakly immune ESS is that $V_\beta(F, \widehat{\alpha}(\beta^*))$ be independent of β. Equivalently, we need β^* to be such that $\mathrm{d}V_\beta(F, \alpha)/\mathrm{d}\beta = 0$ where $\alpha = \widehat{\alpha}(\beta^*)$. Using (10) this condition provides:

$$1 - \alpha = \frac{p(Q_F(l) - Q_F(h))}{(1 - p)(1 - Q_F(l))}$$

which yields

$$\alpha = \frac{(1 - Q_F(l)) - p(1 - Q_F(h))}{(1 - p)(1 - Q_F(l))}. \tag{16}$$

For a real number ζ we denote below $\overline{\zeta} := 1 - \zeta$.

Theorem 5: (a) Each one of the following conditions is necessary for there to exist a Weakly Immune ESS:

- *Condition (i):* $\Delta_l \leq 0$,
- *Condition (ii):* $\Delta_h \leq 0$,

(b) Assume that Condition (i) and (ii) hold. Then there exists a unique weakly immune ESS given by

$$\beta^* = \frac{(\overline{Q_A} + \overline{Q_F(l)})[\overline{Q_F(l)} - p\overline{Q_F(h)}]}{\overline{Q_A}\,\overline{p}\,\overline{Q_F(l)} - (Q_F(l) - Q_F(h))(\overline{Q_F(l)} - p\overline{Q_F(h)})} \tag{17}$$

Proof: A necessary condition for α in (16) to correspond to an ESS is that $0 \le \alpha \le \widehat{\alpha}(1)$. First, we observe that the enumerator of (16) is $-\Delta_l$. This provides that $\Delta_l \le 0$ implies $\alpha \ge 0$. Second, the condition $\alpha \le \widehat{\alpha}(1)$ is

$$\frac{(1-Q_F(l)) - p(1-Q_F(h))}{(1-p)(1-Q_F(l))} \le \frac{1-Q_A}{2-Q_A-Q_F(h)}.$$

This condition is equivalent to

$$\Delta_m := \frac{1-Q_A}{2-Q_A-Q_F(h)} - \frac{(1-Q_F(l)) - p(1-Q_F(h))}{(1-p)(1-Q_F(l))} \ge 0.$$

But we have:

$$\begin{aligned}(1-p)\Delta_m &= \frac{(1-p)(1-Q_A)}{2-Q_A-Q_F(h)} - \frac{(1-Q_F(l)}{(1-Q_F(l)} + p\frac{1-Q_F(h)}{1-Q_F(l)},\\ &= p\frac{1-Q_F(h)}{1-Q_F(l)} - \frac{1-Q_F(h)+p(1-Q_A)}{2-Q_A-Q_F(h)},\\ &= -\Delta_h.\end{aligned}$$

We can deduce that if $\Delta_h \le 0$ then $\alpha \le \widehat{\alpha}(1)$. This proves (a).

Next we establish (b). Assume conditions (i) and (ii) hold. Inverting (7) and substituting there (16), we conclude that if β is a Weakly Immune ESS then it is given by expression (17). This provides the uniqueness of a Weakly Immune ESS. It remains to show that β^*, as defined in (17), is indeed a Weakly Immune ESS. We first observe that

$$\text{for all } \beta' \neq \beta^*, \quad V_{\beta^*}(F, \widehat{\alpha}(\beta^*)) = V_{\beta'}(F, \widehat{\alpha}(\beta^*))$$

so to conclude it suffices to show that

$$\forall \beta \neq \beta^*,\ V_{\beta^*}(F, \widehat{\alpha}(\beta)) > V_\beta(F, \widehat{\alpha}(\beta)). \tag{18}$$

Define $H(\beta) = V_{\beta^*}(F, \widehat{\alpha}(\beta)) - V_\beta(F, \widehat{\alpha}(\beta))$ and after some algebras we have the following expression:

$$H(\beta) = \frac{(\beta^* - \beta)(\overline{Q_F(l)}(1-p)(1-\widehat{\alpha}(\beta)) - xp)}{(\overline{Q_F(l)} + \beta^* x)(\overline{Q_F(l)} + \beta x)},$$

with $x = Q_F(l) - Q_F(h)$. Thus (18) is satisfied if the following two conditions are satisfied:

1) $\forall \beta > \beta^*$, $\widehat{\alpha}(\beta) > \frac{x(1-p)-yp}{x(1-p)}$,
2) $\forall \beta < \beta^*$, $\widehat{\alpha}(\beta) < \frac{x(1-p)-yp}{x(1-p)}$.

After some calculous, we observe that

$$\widehat{\alpha}(\beta) < \frac{x(1-p)-yp}{x(1-p)} \quad \Leftrightarrow \quad \beta < \beta^*.$$

Then we deduce that $\forall \beta \neq \beta^*$ we have $H(\beta) > 0$ so that (18) holds. We conclude that β^* is indeed a weakly immune ESS, which establishes (b). ∎

We finally mention the following useful property whose proof is immediate:

$$V_\beta(F, \widehat{\alpha}(\gamma)) \text{ is strictly monotone decreasing in } \gamma. \tag{19}$$

We next present a last result about existence and unicity of the ESS.

Theorem 6: For all Q_A, $Q_F(l)$, $Q_F(h)$ and p, the ESS β^* of the stochastic evolutionary game exists and is unique.

Proof: The expression of the ESS of the stochastic evolutionary game depends on the sign of Δ_l and Δ_h.

- If both are negative, we have the weakly immune ESS β^* defined in theorem 5 (a) and is unique.
- If Δ_l or Δ_h is positive, we have a strongly immune ESS $\beta^* = 0$ or $\beta^* = 1$.
- We prove now that we cannot have both Δ_l and Δ_h positive. We assume that Δ_l strictly positive, that is $p > \frac{1-Q_F(l)}{1-Q_F(h)}$. Now , we have Δ_h strictly positive if and only if

$$\begin{aligned}\frac{1-Q_F(h)}{2-Q_A-Q_F(h)}(1-p) &> \frac{Q_F(l)-Q_F(h)}{1-Q_F(l)}p,\\ \min_{Q_A\in[0,1]} \frac{1-Q_F(h)}{2-Q_A-Q_F(h)}(1-p) &> \frac{Q_F(l)-Q_F(h)}{1-Q_F(l)}p,\\ 1-p &> \frac{Q_F(l)-Q_F(h)}{1-Q_F(l)}p,\\ \Delta_l &< 0.\end{aligned}$$

We then have proved that there is always one weakly immune ESS β^* or one of the two strongly immune ESS (exclusively). ∎

V. The globally optimal solution and the price of Anarchy

We next compute the cooperative optimal strategy for users and compare it with the ESS. $\widetilde{\beta}$ is globally optimal if it maximizes $V_\beta(F, \widehat{\alpha}(\beta))$. Substituting (7) into (8) this amounts to maximize

$$Z(\beta) = V(A) + \frac{p + \beta\overline{p}(1-\widehat{\alpha}(\beta))}{\overline{Q_F(l)} + \beta(Q_F(l) - Q_F(h))},$$

with $\widehat{\alpha}(\beta) = \frac{\beta\overline{Q_A}}{\overline{Q_A}+\overline{Q_F(l)}+\beta(Q_F(l)-Q_F(h))}$. We have

$$\begin{aligned}Z'(\beta) &= \frac{[\overline{Q_F(l)} + \beta(Q_F(l)-Q_F(h))][\overline{p}(1-\widehat{\alpha}(\beta)) - \beta\overline{p}\widehat{\alpha}'(\beta)]}{(\overline{Q_F(l)} + \beta(Q_F(l)-Q_F(h)))^2}\\ &\quad - \frac{[p+\beta\overline{p}(1-\widehat{\alpha}(\beta))][(Q_F(l)-Q_F(h))]}{(\overline{Q_F(l)} + \beta(Q_F(l)-Q_F(h)))^2}\end{aligned}$$

In order to find the optimum point, we have to solve $Z'(\beta) = 0$, that is

$$\begin{aligned}&\overline{Q_F(l)}\overline{p}(1-\widehat{\alpha}(\beta))\\ &-\beta\overline{p}\widehat{\alpha}'(\beta)[\overline{Q_F(l)} + \beta(Q_F(l)-Q_F(h))]\\ &-p(Q_F(l)-Q_F(h)) = 0.\end{aligned}$$

We introduce the following notations:

$$x = Q_F(l) - Q_F(h).$$

Moreover, after some calculus we obtain

$$\widehat{\alpha}'(\beta) = \frac{\overline{Q_A}(\overline{Q_A} + \overline{Q_F(l)})}{(\overline{Q_A} + \overline{Q_F(l)} + \beta(Q_F(l)-Q_F(h)))^2} > 0.$$

The equation $Z'(\beta) = 0$ is equivalent to

$$(1-\widehat{\alpha}(\beta))\overline{Q_F(l)} = \beta\widehat{\alpha}'(\beta)\overline{Q_F(l)} + \beta^2 x\widehat{\alpha}'(\beta) + \frac{p}{\overline{p}}x,$$

and given the derivative of $\widehat{\alpha}$ we obtain after some algebras the following second order equation:

$$\beta^2\left(\overline{Q_F(l)}x(x-\overline{Q_A})-x\overline{Q_A}(\overline{Q_A}+\overline{Q_F(l)})-\frac{p}{\overline{p}}x^3\right)$$
$$+2\beta\left(\overline{Q_F(l)}(\overline{Q_A}+\overline{Q_F(l)})(x-\overline{Q_A})-\frac{p}{\overline{p}}x^2(\overline{Q_A}+\overline{Q_F(l)})\right)$$
$$+(\overline{Q_F(l)}-\frac{p}{\overline{p}}x)(\overline{Q_A}+\overline{Q_F(l)})^2=0.$$

The discriminant of this equation is

$$\Delta=4(\overline{Q_A}+\overline{Q_F(l)})^2\overline{Q_A}^2(\overline{Q_F(l)}^2+x\overline{Q_F(l)}-\frac{p}{\overline{p}}x^2).$$

We observe that depending on $Q_F(l)$, $Q_F(h)$ and p, the sign of the discriminant is changing. We shall use the following equivalence:

$$\Delta>0\Leftrightarrow p<\frac{u}{1+u},\text{ where } u=\frac{\overline{Q_F(l)Q_F(h)}}{(Q_F(l)-Q_F(h))^2}. \quad (20)$$

Theorem 7: If $p>\frac{u}{1+u}$, where u is given in (20), then the global optimal strategy is

$$\widetilde{\beta}=0.$$

Proof: If $p>\frac{u}{1+u}$ then after some calculus we obtain:

$$\frac{p}{\overline{p}}>\frac{\overline{Q_F(l)}}{x}+\left(\frac{\overline{Q_F(l)}}{x}\right)^2.$$

Also the discriminant is strictly negative, then the sign of the derivative is the same for all $\beta\in[0,1]$ and we have

$$Z'(0)=\frac{\overline{p}\overline{Q_F(l)}-p(Q_F(l)-Q_F(h))}{\overline{Q_F(l)}^2}.$$

Observing that $\frac{p}{\overline{p}}>\frac{\overline{Q_F(l)}}{x}+(\frac{\overline{Q_F(l)}}{x})^2$ implies $\frac{p}{\overline{p}}>\frac{\overline{Q_F(l)}}{x}$ leads to $Z'(0)<0$. Then we conclude that the global optimal strategy is obtain with $\widetilde{\beta}=0$ because the function Z is strictly decreasing when $p>\frac{u}{1+u}$. ■

When $p<\frac{u}{1+u}$, the solutions β^+ and β^- of the second order equation are given by

$$\beta^-=\frac{-\overline{Q_F(l)}(x-\overline{Q_A})+\frac{p}{\overline{p}}x^2-\overline{Q_A}\sqrt{\Delta'}}{\frac{-\overline{Q_F(l)}(x-\overline{Q_A})+\frac{p}{\overline{p}}x^2}{\overline{Q_F(l)}+\overline{Q_A}}-\overline{Q_A}}\frac{1}{x},$$

and

$$\beta^+=\frac{-\overline{Q_F(l)}(x-z)+\frac{p}{\overline{p}}x^2+\overline{Q_A}\sqrt{\Delta'}}{\frac{-\overline{Q_F(l)}(x-\overline{Q_A})+\frac{p}{\overline{p}}x^2}{\overline{Q_F(l)}+\overline{Q_A}}-\overline{Q_A}}\frac{1}{x},$$

with $\Delta'=\overline{Q_F(l)}^2+x\overline{Q_F(l)}-\frac{p}{\overline{p}}x^2$. The global optimal strategy in the cooperative case is then given by the following theorem.

Theorem 8: Let β^- and β^+ be defined as above. Then the global optimal strategy $\widetilde{\beta}$ is

$$\widetilde{\beta}=\arg\max_{\beta\in\{0,\beta^-,\beta^+,1\}}Z(\beta).$$

Proof: The derivative of the global objective function Z is a second order polynomial function depending in β. After some algebra, we obtain β^- and β^+ as the two roots of this function. Then, depending on those values are in the interval $[0,1]$ and the sign of Z', the optimal global strategy is either 0, 1 or one of the roots of the function. ■

Next, we obtain some relations between the ESS and the globally optimal solution.

Theorem 9: ESS strategy is more aggressive than the social optimum strategy, i.e.

$$\beta^*\geq\widetilde{\beta}.$$

Proof: If $\beta^*=0$ a strong immune ESS, then we have for all $\beta>0$:

$$V_0(F,\widehat{\alpha}(0))>V_\beta(F,\widehat{\alpha}(0)\geq V_\beta(F,\widehat{\alpha}(\beta),$$

because $\widehat{\alpha}(\beta)$ is strictly increasing in β. Then we have proved that $\beta=0$ is the global optimum,i.e. $\widetilde{\beta}=0$ and $\beta^*=\widetilde{\beta}$.
If β^* is weakly immune then $\beta^*\in]0,1[$ and

$$V_0(F,\widehat{\alpha}(0))=V_0(F,\widehat{\alpha}(\beta^*))=V^*_\beta(F,\widehat{\alpha}(\beta^*))$$
$$=V_1(F,\widehat{\alpha}(\beta^*))>V_1(F,\widehat{\alpha}(1))$$

(the last inequality follows from (19)). From theorem 7, we have:

$$Z'(0)=\frac{\overline{p}\overline{Q_F(l)}-p(Q_F(l)-Q_F(h))}{\overline{Q_F(l)}^2}=\frac{-\Delta_l}{\overline{Q_F(l)}^2}.$$

As β^* is weakly immune, $\Delta_l\leq 0$ and thus $Z'(0)\geq 0$. Moreover, as $V_0(F,\widehat{\alpha}(0))=V^*_\beta(F,\widehat{\alpha}(\beta^*))$, there exists $\beta_1\in]0,\beta^*[$ which maximizes V over $]0,\beta^*[$. Assuming that $\widetilde{\beta}\in]\beta^*,1[$, then there are necessary three local optima, which is not possible because V is a quadratic function in β and has two local optima. Thus, $\widetilde{\beta}=\beta_1<\beta^*$. ■

VI. Dynamics

We extend the definition of the well known replicator dynamics [9] to the context of stochastic evolutionary game, and study its convergence to the various ESS obtained in the last section.

In the biological context, a replicator dynamics is a differential equation that describes the way strategies change in time as a function of the fitness. Roughly speaking they are based on the idea that the average growth rate per individual that uses a given action is proportional to the excess of fitness of that action with respect to the average fitness. In engineering, the replicator dynamics could be viewed as a rule for updating policies by individuals. It is a decentralized rule since it only requires knowing the average utility of the population rather than the action of each individual.

Although each individual terminal i attempts transmission at some distinct times t^i_n, these times are assumed to be sufficiently variable from one terminal to another so that the update rule of the whole population can be written as the following continuous time differential equation. Define $\vec{\beta} = (\beta, 1-\beta)$ and set for $a = h$ and $a = l$:

$$\begin{aligned}\frac{\mathrm{d}\vec{\beta}(a)}{\mathrm{d}t} &= K\vec{\beta}(a)\left[V_a(F,\widehat{\alpha}(\beta)) - \sum_{b=h,l}\vec{\beta}(b)V_b(F,\widehat{\alpha}(\beta))\right], \quad (21)\\ &:= W(\vec{\beta}).\end{aligned}$$

Note that when summing the above over a we get simply

$$\frac{\mathrm{d}\vec{\beta}(h)}{\mathrm{d}t} + \frac{\mathrm{d}\vec{\beta}(l)}{\mathrm{d}t} = 0$$

and hence $\vec{\beta}(h)+\vec{\beta}(l) = 1$ at any time, as expected. In order to prove the convergence and relation between equilibria ad rest points of the dynamics, we consider the property of *positive correlation*. Following [16], we define the property of positive correlation or net monotonicity [8] of our dynamic equation $\frac{d\vec{\beta}}{dt} = W(\vec{\beta})$ as

$$\sum_{b=h,l} V_b(F,\widehat{\alpha}(\beta))W_b(\vec{\beta}) > 0 \quad \text{whenever} \quad W(\vec{\beta}) \neq 0.$$

By a straightforward adaptation of the argument in [16] we conclude that the positive correlation insures that all equilibria of our game are the stationary points of the replicator dynamics. Thus a non-stationary point of the dynamics cannot be an equilibrium. We therefore prove the following:

Proposition 1: The replicator dynamics given by $\frac{d\vec{\beta}}{dt} = W(\vec{\beta})$ described in (21) satisfies the property of *positive correlation*.

Proof: We have

$$\sum_{b=h,l} V_b(F,\widehat{\alpha}(\beta))W_b(\vec{\beta}) = \sum_{b=h,l} V_b(F,\widehat{\alpha}(\beta))\frac{d\vec{\beta}}{dt} =$$

$$\sum_{b=h,l} V_b(F,\widehat{\alpha}(\beta))\left(K\vec{\beta}(b)\left[V_b(F,\widehat{\alpha}(\beta)) - \sum_{b=h,l}\vec{\beta}(b)V_b(F,\widehat{\alpha}(\beta))\right]\right),$$

$$= K\sum_{b=h,l}\vec{\beta}(b)V_b(F,\widehat{\alpha}(\beta))^2 - K\left(\sum_{b=h,l}\vec{\beta}(b)V_b(F,\widehat{\alpha}(\beta))\right)^2.$$

Using Jensen's inequality, we have that this expression is strictly positive and thus the replicator dynamics, in this stochastic context, has the property of *positive correlation*. ∎

We show next that the replicator dynamics converges almost globally.

Theorem 10: For all interior starting points $\beta_0 \in]0,1[$, the replicator dynamics defined be Equation (21) converges to the ESS β^*.

Proof: We rewrite the replicator dynamics for the high power level as:

$$\frac{\mathrm{d}\vec{\beta}(h)}{\mathrm{d}t} = K\vec{\beta}(h)(1-\vec{\beta}(h))\left[V_h(F,\widehat{\alpha}(\beta)) - V_l(F,\widehat{\alpha}(\beta))\right].$$

Hence the sign of the derivative of the evolution of the h strategy is defined by the sign of

$$V_h(F,\widehat{\alpha}(\beta)) - V_l(F,\widehat{\alpha}(\beta)).$$

Notice that $V_l(F,\widehat{\alpha}(\beta)) = V_0(F,\widehat{\alpha}(\beta)) = V(A) + \frac{p}{1-Q_F(l)}$, which does not depend on β and so is constant. Then only $V_h(F,\widehat{\alpha}(\beta)) = V_1(F,\widehat{\alpha}(\beta))$ depends on β and we have already proved that $\widehat{\alpha}(\beta)$ is strictly increasing in β, thus $V_1(F,\widehat{\alpha}(\beta))$ is strictly decreasing in β. Then we have just to compare $V_1(F,\widehat{\alpha}(0)) - (V(A) + \frac{p}{1-Q_F(l)})$ and $V_1(F,\widehat{\alpha}(1)) - (V(A) + \frac{p}{1-Q_F(l)})$ in order to know the sign of the difference for all $\beta \in]0,1[$.

First, we have after some calculous:

$$\begin{aligned}V_1(F,\widehat{\alpha}(0)) &\leq V(A) + \frac{p}{1-Q_F(l)},\\ V(A) + \frac{1}{1-Q_F(h)} &\leq V(A) + \frac{p}{1-Q_F(l)},\end{aligned}$$

which is equivalent to $\Delta_l \geq 0$. Then, we prove that if $\beta^* = 0$, using theorem 4 we have $\Delta_l \geq 0$, which implies that for all $\beta \in]0,1[$,

$$V_1(F,\widehat{\alpha}(\beta)) < V_1(F,\widehat{\alpha}(0)) \leq V(A) + \frac{p}{1-Q_F(l)},$$

that leads $\frac{\mathrm{d}\vec{\beta}(h)}{\mathrm{d}t} < 0$ and the replicator dynamics converge to $\beta^* = 0$.

Second, after some analog calculus we obtain:

$$\begin{aligned}V_1(F,\widehat{\alpha}(1)) &\geq V(A) + \frac{p}{1-Q_F(l)},\\ V(A) + \frac{1-(1-p)\frac{1-Q_A}{2-Q_A-Q_F(h)}}{1-Q_F(h)} &\geq V(A) + \frac{p}{1-Q_F(l)},\end{aligned}$$

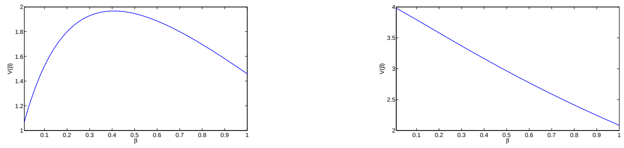
which is equivalent to $\Delta_h \geq 0$, which is a necessary condition to have $\beta^* = 1$. Then, from the same arguments that in the previous case, the replicator dynamics converge here to the ESS, $\beta^* = 1$, because $\frac{\mathrm{d}\vec{\beta}(h)}{\mathrm{d}t} > 0$ for all $\beta \in]0,1[$.

Finally, if we have a mixed ESS, from theorem 5 we have $\Delta_l \leq 0$ and $\Delta_h \leq 0$, then $V_1(F,\widehat{\alpha}(0)) \geq V(A) + \frac{p}{1-Q_F(l)}$, $V_1(F,\widehat{\alpha}(1)) \leq V(A) + \frac{p}{1-Q_F(l)}$ and $V_1(F,\widehat{\alpha}(\beta^*)) = V(A) + \frac{p}{1-Q_F(l)}$. Thus, for all $\beta \in]0,\beta^*[$, we have $V_1(F,\widehat{\alpha}(\beta)) > V_0(F,\widehat{\alpha}(\beta))$, i.e. $\frac{\mathrm{d}\vec{\beta}(h)}{\mathrm{d}t} > 0$. And for all $\beta \in]\beta^*[,1$, we have $V_1(F,\widehat{\alpha}(\beta)) < V_0(F,\widehat{\alpha}(\beta))$, i.e. $\frac{\mathrm{d}\vec{\beta}(h)}{\mathrm{d}t} < 0$. Thus, we have also convergence of the replicator dynamics to the unique ESS β^*. ∎

VII. Numerical Illustrations

In this section we present several numerical results illustrating theorems of previous sections. For all numerical applications we use the following variables: $Q_A = 0.5$, $Q_F(l) = 0.9$ and $Q_F(h) = 0.7$. First, we plot on figures 1(a) and 1(b) the valuation function $V(\beta)$ depending on the strategy β for different value of the probability p. We observe that the global optimal strategy is equal to $\widetilde{\beta} = 0.4081$ and becomes $\widetilde{\beta} = 0$ as the probability p grows from 0.1 to 0.4.

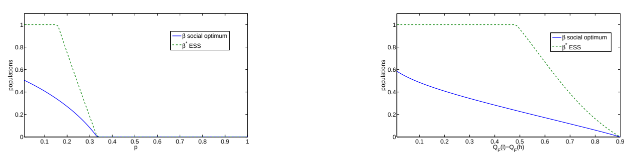
On figures 2(a) and 2(b) we show both equilibria: non-cooperative one β^* and the global optimum cooperative one $\widetilde{\beta}$. We observe on both figures that the result of the theorem 9, saying that the ESS is always more aggressive than the social optimum, is verified. Also, we describe numerically both



(a) $p = 0.1$ (b) $p = 0.4$

Fig. 1. Cooperative valuation function.

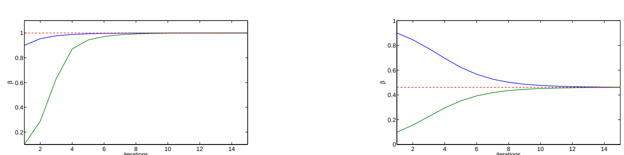
equilibria depending on the probability p on figure 2(a) and we observe that when the probability for a mobile to be the only transmitter is small, the ESS is $\beta^* = 1$. That is the strategy is every one uses high power because the probability to have competition with other mobiles is high. We observe also the different equilibria depending on the battery quality, that is the difference $Q_F(l) - Q_F(h)$, on figure 2(b). Hence, this difference gives difference in probability to stay in the state F using high or low power. We observe that both equilibria are less aggressive as the difference increases because the probability to stay in state F using low power becomes very important compare to the one using high power.



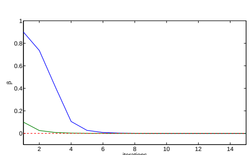
(a) Depending on the probability p. (b) Depending on the difference $Q_F(l) - Q_F(h)$.

Fig. 2. ESS and global optimum.

Finally, on figures 3(a), 3(b) and 3(c), we show the convergence of our replicator dynamics with two initial points $\beta_0 = 0.9$ and $\beta_0 = 0.1$. We consider the three possible ESS, the two pure and the mixed one; we take $K = 1$. We observe the convergence property defined in theorem 10 and we can also have an idea of the speed of convergence (after 15 iterations, the system attains the equilibrium).



(a) $p = 0.1$ (b) $p = 0.25$



(c) $p = 0.4$

Fig. 3. Convergence of the replicator dynamics.

VIII. Conclusions and Perspectives

Biological models and tools have inspired a growing number of studies and of designs of decentralized wireless ad-hoc networks. In various ways, this paper falls in this category of biology-inspired research. First, the problem we have studied, concerning competitive energy management in wireless terminals, is inherent to biological models as well where animals compete over food. A second biology-inspired feature of this work is the evolutionary game paradigm that we have adopted and extended. Our convergence results concerning the replicator dynamics renders the ESS equilibrium concept more relevant and meaningful than the standard Nash equilibrium, as the ESS is shown to be actually achieved as a limit of a replicator dynamics, and if achieved, this equilibrium point turns out to be more stable than the Nash equilibrium. We have presented a problem that extends the standard evolutionary games by (i) modeling the possibility of individuals to take several actions during their lifetime, (ii) allowing these actions to have an impact not only on the instantaneous fitness but also on the future individual's state. The state dependence allows in turn to distinguish between individuals. In our case, the state of an individual gave an indication on its expected time to live and on its available set of actions. We plan to extract the generic features present in our problem in order to develop a generic theory of stochastic evolutionary game.

References

[1] Axelrod, R. The Evolution of Cooperation. New York: Basic Books, 1984.

[2] Josef Hofbauer and Karl Sigmund, " Evolutionary Games and Population Dynamics", Cambridge University Press 1998.

[3] M. Smith, "Game Theory and the Evolution of Fighting." In John Maynard Smith, On Evolution (Edinburgh: Edinburgh University Press), pp.8-28, 1972.

[4] M. Smith, *Evolution and the Theory of Games*, Cambridge University Press, UK, 1982.

[5] T. L. Vincent and T. L. S. Vincent, "Evolution and control system design", *IEEE Control Systems Magazine*, Vol 20 No. 5, Oct. 2000.

[6] L. Samuelson, "Evolution and Game Theory", *The journal of Economics Perspectives*, Vol 16, No 2, 2002.

[7] W.H. Sandholm, "Potential games with continuous player sets", *Journal of Economic Theory*, vol. 97, 2001.

[8] S. Shakkottai, E. Altman and A. Kumar, "The Case for Non-cooperative Multihoming of Users to Access Points in IEEE 802.11 WLANs", *proceedings of IEEE Infocom*, 2006.

[9] J.W. Weibull, *Evolutionary Game Theory*, Cambridge, MA: MIT Press, 1995.

[10] J. Hofbauer and K. Sigmund, "Evolutionary game dynamics", *American Mathematical Society*, Vol 40, No. 4, 2003.

[11] A. I. Houston and J. M. McNamara, "Evolutionarily stable strategies in the repeated hawk-dove game", *Behavioral Ecology*, 1991.

[12] J. M. McNamara, "The policy which maximizes long-term survival of an animal faced with the risks of starvation and predation", *Advances of Applied Probability*, vol. 22, 1990.

[13] J. M. McNamara, S. Merad and E. J. Collins, "The hawk-dove game as an average cost problem", *Advances of Applied Probability*, vol. 23, 1991.

[14] V. Mhatre and C. Rosenberg, "Energy and cost optimizations in wireless sensor networks: A survey", Kluwer Academic Publishers, Jan 2004.

[15] M. Nowak, "Five Rules for the Evolution of Cooperation" *Science*, vol. 314, 2006.

[16] W. Sandholm, "Evolutionary Implementation and Congestion Pricing", *Review of Economic Studies*, vol. 69, 2002.