Admission and Flow Control in Telecommunication Networks as a Hybrid Control Problem

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ABSTRACT

We consider the problem of optimally controlling piecewise deterministic (or equivalently jump parameter) linear systems, where the transition rate matrix of the underlying Markov jump process is also controlled. We first develop a general theory for the existence and characterization of optimal feedback controllers, and then apply the specific results obtained for the scalar case to a problem that arises in high speed telecommunication networks. This involves combined admission and rate-based flow control, where the former corresponds to control of the jump Markov process, and the latter to control of the continuous linear system.

1. A Motivating Example

To motivate the class of control problems studied in this paper, consider the following communication network, which can be viewed as a modified version of the models studied recently in [1] and [2][2]. It is assumed that the network has linearized dynamics (for the control of queue length), and all performance measures (such as throughput, delays, loss probabilities, etc.) are determined essentially by a bottleneck node. Both these assumptions have theoretical as well as experimental justifications; see, [1].

Let q_t denote the queue length at a bottleneck link, and r_t denote the effective service rate available for traffic of the given source at that link at time t. We let r_t be arbitrary, but assume that the controllers have perfect measurements of it. Let ξ_t denote the (controlled) source rate at time t, and $u_t^1 := \xi_t - r_t$ (called flow control) its shifted version. Consider the following linearized dynamics for the queue length:

$$\frac{dq}{dt} = u^1 , \tag{1}$$

which is called *linearized* because the end-point effects have been ignored. The objectives of the flow controller are *(i)* to ensure that the bottleneck queue size stays around some desired level $\bar{Q}$, and *(ii)* to achieve good tracking between input and output rates. In particular, the choice of $\bar{Q}$ and the variability around it have direct impact on loss probabilities and throughput. We therefore define a shifted version of q:

$$x_t := q_t - \bar{Q},$$

in view of which (1) now becomes

$$\frac{dx}{dt} = u^1. \tag{2}$$

An appropriate local cost function that is compatible with the objectives stated above would be the one that penalizes variations in x_t and u_t^1 around *zero* — a candidate for which is the weighted quadratic cost function.

Suppose that there are several types (say s) of possible traffic, with different kinds of requirements on the performance measures. Associated with type i traffic are the positive constants $Q(i)$ and $R(i)$ appearing in the immediate cost: $L(x,i,u) = |x|^2_{Q(i)} + |u^1|^2_{R(i)}$. Typically, traffic requiring higher quality of service (QoS) might have a larger $Q(i)$, which reflects the fact that it might require lower loss probabilities and higher throughput. It could be receiving a higher priority from the network in the sense that larger variations in u^1 will be tolerated so as to achieve the required QoS; thus the corresponding $R(i)$ might be smaller. The occurrence of these different types of traffic will be governed by a continuous-time Markov jump process, with transitions controlled by a second controller with a finite action set.

A typical admission control problem is the following: by default the system always accepts traffic of some given type, say 1. Traffic of type 1 is transmitted until a session consisting of another type of traffic of higher priority, say type i ($i = 2, 3, \ldots, s$), is accepted. When it is accepted, the session cannot be interrupted until it ends. Thus at states $\theta = i > 1$, there are no (admission) control actions available.

The controller 2 is thus effective only at state 1, at which the rate of arrival of sessions of high priority traffic is to be determined. To each traffic type there corresponds two admission decisions (that are part of the action to be chosen by controller 2): 0 - corresponding to low admission rate $\underline{\lambda}(j)$, and 1- corresponding to a high admission rate $\bar{\lambda}(j)$. The control action at state 1 is thus of the form $u^2 = (u_1^2, \ldots, u_s^2)$, $u_j^2 \in \{0,1\}$. The controlled transition rates have the form

$$\lambda_{1,u^2,j} \quad = \quad \underline{\lambda}(j) 1_{\{u_j^2=0\}} + \bar{\lambda}(j) 1_{\{u_j^2=1\}} ,$$

where $1_{\{.\}}$ denotes the set indicator function. Note that, if at state 1 the control action u^2 is fixed, then the next type of session to be accepted will be $j (j>1)$ with probability

$$[\underline{\lambda}(j) 1_{\{u_j^2=0\}} + \bar{\lambda}(j) 1_{\{u_j^2=1\}}] / \cdot \sum_{k=2}^{s} \left[\underline{\lambda}(k) 1_{\{u_k^2=0\}} + \bar{\lambda}(k) 1_{\{u_k^2=1\}} \right].$$

The problem then is to minimize the expected discounted or average long term cost (with instantaneous cost being L

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[2] These earlier papers dealt with the flow control problem only, whereas the present paper addresses both admission and flow control; on the other hand, the models in the earlier papers were more general, accounting for the possibilities of imperfect, delayed measurements, and partially unknown statistics for noise.

above) with respect to the multi-strategy $\mu := (\mu^1, \mu^2)$, where μ^1 is the flow controller and μ^2 the admission controller, both having as arguments the current and past values of ξ, r, and θ.

2. General Model

A general model that captures the telecommunication network problem formulated above as a special case is the following: Consider a system that evolves according to

$$\frac{dx}{dt} = A(\theta)x + B(\theta)u^1, \quad x(0) = x_0 \tag{3}$$

where $x \in I\!R^n$, x_0 is a fixed (known) initial state, u^1 is a control, applied by controller 1, taking values in $\mathbf{U}_1 = I\!R^r$, and $\theta(t)$ is a controlled, continuous time Markov jump process, taking values in a finite state space S, with cardinality s. Transitions from state i to j occur at a rate controlled by controller 2, who chooses at time t an action $u^2(t)$ among a finite set $\mathbf{U}_2(i)$ of actions available at state i. Let $\mathbf{U}_2 := \cup_{i\in\mathcal{S}}\mathbf{U}_2(i)$. The controlled rate matrix (of transitions within $\mathcal{S}$) is

$$\Lambda = \{\lambda_{i,a,j}\}, \qquad i, j \in S, a \in \mathbf{U}_2(i),$$

where henceforth we drop the "commas" in the subscripts of λ. The λ_{iaj}'s are real numbers such that for any $i \neq j$, and $a \in \mathbf{U}_2(i)$, $\lambda_{iaj} \geq 0$, and for all $i \in S$ and $a \in \mathbf{U}_2$, $\lambda_{iai} = -\sum_{j\neq i}\lambda_{iaj}$. Fix some initial state i_0 of the controlled Markov chain S, and the final time t_f (which may be infinite). Consider the class of policies $\mu^k \in \mathcal{U}_k$ for controller k ($k = 1, 2$), whose elements (taking values in $\mathbf{U}_k$) are of the form

$$u^k(t) = \mu^k(t, x_{[0,t]}; \theta_{[0,t]}), \qquad t \in [0, t_f),$$

Here, μ^k is taken to be piecewise continuous in its first argument, and piecewise Lipschitz continuous in its second argument.

Define $\mathcal{X} = I\!R^n \times \mathcal{S}$ to be the global state space of the system and $\mathcal{U} := \mathcal{U}_1 \times \mathcal{U}_2$ to be the class of multi-strategies. Define the immediate cost $L : \mathcal{X} \times \mathbf{U}_1 \to I\!R$, where $Q(\cdot) \geq 0$ and $R(\cdot) > 0$:

$$L(x, i, u^1) = |x|^2_{Q(i)} + |u^1|^2_{R(i)}.$$

To any fixed initial state (x_0, i_0) and a multi-strategy $\mu \in \mathcal{U}$, there corresponds a unique probability measure $P^{\mu}_{x_0,i_0}$ on the canonical probability space Ω of the states and actions of the players, equipped with the standard Borel σ-algebra. Denote by $\mathbf{E}^{\mu}_{x_0,i_0}$ the expectation operator corresponding to $P^{\mu}_{x_0,i_0}$. Denote by $(X(t), S(t)), U(t), t \in [0, t_f)$, the stochastic processes corresponding to the states and actions, respectively.

Consider a discount factor $\beta \geq 0$, and introduce the following discounted cost function corresponding to an initial state (x_0, i_0), a multi-strategy $\mu \in \mathcal{U}$, and a horizon of duration t_f (where $Q_f(\cdot) \geq 0$, and we take $Q_f \equiv 0$ when $t_f = \infty$):

$$J_\beta(t_f, x_0, i_0, \mu) := \mathbf{E}^{\mu}_{x_0,i_0}\Big\{|X(t_f)|^2_{Q_f(S(t_f))} + \int_0^{t_f} e^{-\beta t} L(X(t), S(t), U^1(t))dt\Big\}.$$

The optimal control problem is then the minimization of J_β over $\mu \in \mathcal{U}$, which is what we address in this paper.

3. Main Results

Introduce the backward controlled Markov operator $\mathcal{A}^v$ associated with the system above as follows: for each $\psi(t, \cdot, i)$ such that $\psi(\cdot, \cdot, i) \in \mathcal{C}^1$ for all $i \in \mathcal{S}$, and for each $v = (u, a) \in \mathbf{U}$,

$$\mathcal{A}^v\psi(t, x, i) := \frac{\partial\psi(t, x, i)}{\partial t} + f(x, u, i) \cdot D_x\psi(t, x, i) + \sum_{j\in\mathcal{S}} \lambda_{iaj}\psi(t, x, j)$$

where D_x stands for the gradient operator, and $f(x, u, i) := A(i)x + B(i)u$. Further introduce, for each function $\psi(x, i)$ for which $\psi(\cdot, i) \in \mathcal{C}^1$ for all $i \in \mathcal{S}$, and for each $v \in \mathbf{U}$,

$$G^v\psi(x, i) := -f(x, u, i) \cdot D_x\psi(x, i) - \sum_{j\in\mathcal{S}} \lambda_{iaj}\psi(x, j).$$

Let

$$\hat{J}_\beta(t_f, x_0, i_0) := \inf_{\mu\in\mathcal{U}} J_\beta(t_f, x_0, i_0, \mu), \tag{4}$$

denote its counterpart when $t_f = \infty$ by $\hat{J}_\beta(x_0, i_0)$, and the minimizing multi-strategy in each case by μ^* (assuming that it exists). For finite t_f, consider, subject to the boundary condition $\psi(t_f, x, i) = |x|^2_{Q_f(i)}$, the HJB equation:

$$0 = \min_{v\in\mathbf{U}} \left[\mathcal{A}^v\psi(t, x, i) + e^{-\beta t}L(t, x, i, u)\right]. \tag{5}$$

For infinite t_f, consider its infinite-horizon version:

$$\beta\psi(x, i) = \min_{v\in U} \left[-G^v\psi(x, i) + L(x, i, v)\right]. \tag{6}$$

Associate with (5) and (6) the corresponding sets, $\mathcal{D}$, of functions ψ having continuous first-order partial derivatives. We first have:

Theorem 3.1 *(i) Consider the case of finite t_f, and assume that (5) has a solution ψ in $\mathcal{D}$. Then the value of (4) equals ψ. Moreover, any Markov policy that chooses at time t, for all $t \in [0, t_f]$, actions that achieve the argmin in (5), given that the state at that time is (x, i), is optimal.*
(ii) Assume that (6) has a solution ψ in $\mathcal{D}$. Then,

(a) $\psi(x, i) \leq J_\beta(x, i, \mu)$ for every $\mu \in \mathcal{U}$ that satisfies

$$\underline{\lim}_{t_1\to\infty}\, e^{-\beta t_1}\mathbf{E}^{\mu}_{x,i}\psi(X(t_1), S(t_1)) \leq 0. \tag{7}$$

(b) Any stationary policy g that chooses at state (x, i), actions that achieve the argmin in (6) satisfies $\psi(x, i) \geq J_\beta(x, i, g)$, provided that

$$\overline{\lim}_{t_1\to\infty}\, e^{-\beta t_1}\mathbf{E}^{g}_{x,i}\psi(X(t_1), S(t_1)) \geq 0. \tag{8}$$

Equation (5) does not generally admit a closed-form solution, but it does in some special cases. To investigate these cases, let us first stipulate a structure for $\psi(t, x, i)$ that is quadratic in x:

$$\psi(t, x, i) := x^T\tilde{P}(i, t)x, \quad t \in [0, t_f], i \in \mathcal{S}, \tag{9}$$

where $\tilde{P}(i, t)$ is an $n \times n$ matrix for each $i \in \mathcal{S}$, $t \in [0, t_f]$. Substituting this structural form into (5), we obtain:

$$\begin{aligned} 0 &= x^T\tilde{P}_t(i, t)x + x^TQ(i)xe^{-\beta t} \\ &+ \min_{u^1}\left[2(A(i)x + B(i)u^1)^T\tilde{P}(i, t)x \right. \\ &\left. + e^{-\beta t}|u^1|^2_{R(i)}\right] + \min_{u^2}\sum_{j\in\mathcal{S}}\lambda_{iu^2j}x^T\tilde{P}(j, t)x \end{aligned} \tag{10}$$

The minimizing control u^1 in (10) is

$$\mu^1_{opt}(x,i,t) = -e^{\beta t} R^{-1}(i) B^T(i) \tilde{P}(i,t) x, \tag{11}$$

whose substitution into (10) leads to:

$$\begin{aligned} 0 &= x^T \big(P_t(i,t) - \beta P(i,t) + Q(i) + 2A^T(i)P(i,t) \\ &\quad - P^T(i,t)B(i)R^{-1}(i)B^T(i)P(i,t) \big) x \\ &\quad + \min_{u^2} \sum_{j\in\mathcal{S}} \lambda_{iu^2j} x^T P(j,t) x \,, \end{aligned} \tag{12}$$

where $P(i,t) := \tilde{P}(i,t)e^{\beta t}$. Hence, the quadratic structure is the right one provided that the minimization over u^2 is independent of x – which is clearly the case for the scalar problem (that is, when $n=1$). Let $P(i,t)$ be a nonnegative solution to the following set of linearly coupled scalar Riccati equations, subject to the boundary condition $P(i,t_f) = Q_f(i)e^{\beta t_f}$:

$$\begin{aligned} \beta P(i,t) &= P_t(i,t) + Q(i) + 2A(i)P(i,t) \\ &\quad - P(i,t)^2 B(i)^2 R^{-1}(i) + \min_{u^2} \sum_{j\in\mathcal{S}} \lambda_{iu^2j} P(j,t) \end{aligned} \tag{13}$$

Then we have the following result:

Theorem 3.2 *Assume that $n=1$, i.e. x is one dimensional.*
(i) Let t_f be finite, and assume that there exists a nonnegative function $P(i,t), i\in\mathcal{S}$, that satisfies (13) for all $i\in\mathcal{S}, t\in[0,t_f]$. Then $\hat{J}_\beta(t_f,x,i) = \tilde{P}(i,t)x^2$ is a solution of (5), where $\tilde{P}(i,t) = P(i,t)\exp(-\beta t)$. A Markov policy μ^{2*} that uses at time t an action (depending on i and t, but not on x) that achieves the minimum in (13) is optimal. The nonnegative solution P of (13) determines an optimal Markov policy μ^{1*}:

$$\mu^{1*}(x,i,t) = -\big[B(i)P(i,t)/R(i)\big]\, x \tag{14}$$

(ii) Consider the infinite-horizon cost case, and assume that there exist nonnegative functions $P(i), i\in\mathcal{S}$ satisfying the linearly coupled Riccati equations

$$\begin{aligned} \beta P(i) &= 2A(i)P(i) - P(i)^2 B(i)^2 R^{-1}(i) \\ &\quad + Q(i) + \min_{u^2} \sum_{j\in\mathcal{S}} \lambda_{iu^2j} P(j), \ i\in\mathcal{S}. \end{aligned} \tag{15}$$

Then $\hat{J}_\beta(x,i) = P(i)x^2$ is a solution of (6). A stationary policy μ^{2*} obtained as the argument that achieves the minimum in (15) is optimal, and the solution P determines an optimal stationary policy μ^{1*} through (14).

Proof. (i) follows directly from Theorem 3.1 (i).
(ii) Let $\psi(x,i) := x^T P(i) x$ which is in $\mathcal{D}$ (see the definition below (6)). We now make use of Theorem 3.1 (ii); since ψ is nonnegative, (8) holds for $\mu^* = (\mu^{1*},\mu^{2*})$. Let $\mu\in\mathcal{U}$ be an arbitrary policy, and suppose that it does not satisfy (7). It then follows that there exists some $\xi>0$ such that

$$\varliminf_{t_1\to\infty} e^{-\beta t_1} \mathbf{E}^\mu_{x,i} |X(t_1)|^2_{Q(S(t_1))} \geq \xi. \tag{16}$$

Since $Q(i)$ are positive, this implies that $J_\beta(x,i,\mu)=\infty$, so that

$$J_\beta(x,i,\mu^*) \leq J_\beta(x,i,\mu). \tag{17}$$

Finally, if μ satisfies (7), then (17) follows from Theorem 3.1 (ii). ∎

Next we show that (15) admits a unique nonnegative solution, which can further be obtained as the value of a (finite) quadratic program. First we introduce a useful definition:

Definition 3.1 *The set of* **superharmonic functions** Γ *is the class of functions $\phi : \mathcal{X} \to \mathbb{R}$ that satisfy for all $i\in\mathcal{S}$ and $\mu\in\mathcal{U}$:*

$$\begin{aligned} \beta\phi(x,i) &\leq f(x,i,\mu)\cdot D_x\phi(x,i) \\ &\quad + \sum_{j\in\mathcal{S}} \lambda_{iaj}\phi(x,j) + L(x,i,\mu) \end{aligned} \tag{18}$$

and that grow at most polynomially fast in $|x|$.

Theorem 3.3 *The value function $\hat{J}_\beta$ is the largest superharmonic function (componentwise).*

Proof. Consider an arbitrary superharmonic function ϕ, and let μ^* be the optimal (stationary) policy. Then, by applying the Dynkin formula (see [4], p. 146, eq.(9.7)) we get

$$\begin{aligned} \phi(x,i) &\leq \mathbf{E}^\mu_{x,i} \int_0^{t_1} e^{-\beta t} L(X(t),S(t),U(t))\,dt \\ &\quad + e^{-\beta t_1}\mathbf{E}^\mu_{x,i}\phi(X(t_1),S(t_1)). \end{aligned}$$

Since system (3) is scalar and μ is optimal, it stabilizes the stochastic system (as there exists a common Lyapunov function), and therefore by taking the limit as t_1 goes to infinity we obtain

$$\phi(x,i) \leq \mathbf{E}^\mu_{x,i} \int_0^\infty e^{-\beta t} L(X(t),S(t),U(t))dt = \hat{J}_\beta(x,i).$$

Since, by Theorem 3.2, $\hat{J}_\beta(x,i)$ is a super-harmonic function, this completes the proof. ∎

Theorem 3.3 enables us to formalize a mathematical program to compute the solution $P(\cdot)$ of (15):

Theorem 3.4 *For any arbitrarily fixed $k\in\mathcal{S}$, the solution $P(k)$ of (15) is given by the following quadratic program:*
$\mathbf{QP1}(k)$**:** *Find $P(i)$, $i\in\mathcal{S}$, to maximize $P(k)$ subject to*

$$\begin{aligned} 0 &\leq -\beta P(i) + Q(i) + 2A(i)P(i) \\ &\quad - [P(i)]^2[B(i)]^2R^{-1}(i) \\ &\quad + \sum_{j\in\mathcal{S}} \lambda_{ivj}P(j), \ \forall i\in\mathcal{S}, v\in\mathbf{U}_2(i) \end{aligned} \tag{19}$$

Proof. Direct consequence of Theorem 3.3, obtained by specializing it to functions ϕ of the form $\phi(x,i) = x^2 Q(i)$. ∎

It is appropriate here to list some useful properties of the mathematical program $\mathbf{QP1}(k)$. First note that the feasible region satisfying the constraints (19) is nonempty; indeed, $P(i)=0$ is feasible. Moreover it is a closed region; if the $B(i)$'s are strictly positive then the feasible region is bounded, and an optimal solution for $\mathbf{QP1}(k)$ exists. Let $P^*(i)$ be the optimal solution of $\mathbf{QP1}(i)$, for $i\in\mathcal{S}$. Then $P^*(i)$, $i\in\mathcal{S}$, are feasible for $\mathbf{QP1}(k)$ for any $k\in\mathcal{S}$ (this follows from Theorem 3.3). Consequently, if optimal solutions $P^*(i)$ have already been computed for $i\in\mathcal{S}'\subset\mathcal{S}$, then one can substitute these for $P(i)$ in (19), when computing $P^*(j)$ for $j\notin\mathcal{S}'$.

4. An iterative solution

We now present an iterative solution procedure for (15), which is an alternative to the one discussed in the previous section. Consider the following value iteration algorithm:

(A1) Set $P_0(j) := 0$, $j \in \mathcal{S}$. Choose some constants $\overline{\lambda}_i, i \in \mathcal{S}$, that satisfy:

$$\overline{\lambda}_i \geq \max_{u^2} \sum_{j \neq i} \lambda_{iu^2j} = \max_{u^2} |\lambda_{iu^2i}|.$$

(A2) Compute $P_n(j), j \in \mathcal{S}, n \geq 1$, iteratively by solving the Riccati-type equation:

$$\begin{aligned} 0 &= -(\overline{\lambda}_i + \beta)P_{n+1}(i) + Q(i) + 2A(i)P_{n+1}(i) \\ &\quad - P_{n+1}(i)^2 B(i)^2 R^{-1}(i) \\ &\quad + \min_{u^2} \sum_{j \in \mathcal{S}} \lambda_{iu^2j} P_n(j) + \overline{\lambda}_i P_n(i). \end{aligned} \tag{20}$$

Theorem 4.1 *The solution $P(j), j \in \mathcal{S}$, of (15) is obtained as the limit of the nondecreasing sequences $\{P_n(j)\}_{n \geq 0}$.*

Proof. To see that the $P_n(j)$'s are nondecreasing, write (20) as

$$\begin{aligned} 0 &= -(\overline{\lambda}_i + \beta)P_{n+1}(i) + Q_n(i) + 2A(i)P_{n+1}(i) \\ &\quad - P_{n+1}(i)^2 B(i)^2 R^{-1}(i) \end{aligned} \tag{21}$$

where

$$Q_n(i) := Q(i) + \min_{u^2} \sum_{j \in \mathcal{S}} \lambda_{iu^2j} P_n(j) + \overline{\lambda}_i P_n(i). \tag{22}$$

Set $Q_0(i) = Q(i)$. It follows from (22) that for any i, $n \geq 0$,

$$\text{if } \forall j \in \mathcal{S} \;\; P_{n+1}(j) \geq P_n(j), \;\; \textit{then} \;\; Q_{n+1}(i) \geq Q_n(i). \tag{23}$$

This is because under **(A1)** coefficients of the $P_n(j)$'s in

$$m(u^2; P_n) := \sum_{j \in \mathcal{S}} \lambda_{iu^2j} P_n(j) + \overline{\lambda}_i P_n(i) \tag{24}$$

are each nonnegative, for every $u^2 \in \mathbf{U}_2(i)$. Hence, under the hypothesis of (23),

$$m(u^2; P_{n+1}) \geq m(u^2; P_n) \quad \text{for each } u^2 \in \mathbf{U}_2(i), \tag{25}$$

from which (23) follows. Now, (21) is a standard Riccati equation that corresponds to a system that always remains in state i, and where the weighting on the quadratic cost for the state is $Q_n(i)$. Therefore, its solution $P_{n+1}(i)$ will be increasing in $Q_n(i)$. It then follows that

$$\textit{if} \;\; Q_{n+1}(i) \geq Q_n(i), \;\; \textit{then} \;\; P_{n+2}(i) \geq P_{n+1}(i). \tag{26}$$

In view of the fact $P_1(i) \geq P_0(i) = 0$, (23)-(26) establish by induction the desired result that the sequences $\{P_n(i)\}$ and $\{Q_n(i)\}$ are nondecreasing for each $i \in \mathcal{S}$, and therefore have respective limits, with the former satisfying (15). ∎

5. The Admission-Flow Control Example

The results presented in the two previous sections now readily apply to the admission-flow control problem formulated in Section 1. We simply let $A(i) = 0$ and $B(i) = 1$, for all $i \in \mathcal{S}$. To obtain some explicit results in this context, we took $\beta = 0$ and $s = 3$ (i.e., $\mathcal{S} = \{1, 2, 3\}$). We further took $\mathbf{U}_2(1) = \{00, 01, 11, 10\}$, $\mathbf{U}_2(2) = \{0\}$, $\mathbf{U}_2(3) = \{0\}$; $\lambda_{1002} = \lambda_{1012} = 1$, $\lambda_{1003} = \lambda_{1103} = 2$, $\lambda_{1102} = \lambda_{1112} = 10$, $\lambda_{1013} = \lambda_{1113} = 20$; $Q(1) = 1$, $Q(2) = 10$, $Q(3) = 20$; $R(1) = 10$, $R(2) = 2$, $R(3) = 1$. For each possible choice of μ^2, we computed the corresponding optimal policy for controller 1 (i.e., μ^1), and the associated optimal cost function. By comparing the optimal value functions obtained for all possible (four) admission policies, we found the unique optimal controller 2 to be $\mu^{2*}(1) = 00$, and the solution to (13) to be: $P^*(1) = 4.141$, $P^*(2) = 4.439$, $P^*(3) = 4.352$. The unique optimal flow controller is then

$$\mu^{1*}(q, i) = \begin{cases} -0.4141\, q & i = 1 \\ -2.220\, q & i = 2 \\ -4.352\, q & i = 3 \end{cases}$$

Typical system responses under all four admission control policies are depicted in Figures 1–4. In each of the figures, we have plotted the time history of the queue size, traffic type, the flow control and the integral cost incurred. For illustration purposes, we have taken the initial queue size to be 30 units, which is 10 units larger than the desired queue length of 20 units. Since the high priority traffic types (2 and 3) have stringent QoS specifications, the corresponding flow control is more aggressive to maintain the queue size around its desired value. On the other hand, for the low priority type (1), the control is smoother in order to minimize the jitter (i.e., variability) in the network. These observations are consistent with what we would have expected from the design. We further observe that by admitting the high priority traffic at a higher rate, the queue size reaches its desired value more quickly, at the expense of a larger control jitter. Although they only represent one sample path of the stochastic system in each of the four cases, the simulations corroborate the theory very well. The smallest integral cost is achieved under $\mu^2(1) = 00$, which is actually the optimal admission rule for the average (expected) value of the cost function.

We next considered the case when $R(1)$ was increased to $R(1) = 100$, with all parameter values remaining the same as above. In this case, the optimal admission controller turned out to be $\mu^{2*}(1) = 11$, and the solution to (13) was: $P^*(1) = 4.508$, $P^*(2) = 4.476$, $P^*(3) = 4.485$. The unique optimal flow controller was then

$$\mu^{1*}(q, i) = \begin{cases} -0.0451\, q & i = 1 \\ -2.238\, q & i = 2 \\ -4.485\, q & i = 3 \end{cases}$$

Typical system responses for this case are depicted in Figures 5–8. Qualitative behaviors similar to those in the previous case are also observed here. Because of the increased weighting of $R(1)$, the flow control magnitude for the type 1 traffic is reduced significantly. The integral costs incurred under the four admission control laws are ranked differently from the previous case. Coincidentally, the cost incurred under the optimal admission control $\mu^{2*}(1) = 11$ is the smallest among the four particular sample paths simulated.

Finally, while holding $R(1)$ at 100, we increased $Q(3)$, from 20 to $Q(3) = 30$. With this change, the optimal controller 2 turned out to be $\mu^{2*}(1) = 10$, and the solution to (13) was: $P^*(1) = 4.680$, $P^*(2) = 4.493$, $P^*(3) = 5.223$. The unique optimal flow controller was then

$$\mu^{1*}(q, i) = \begin{cases} -0.0468\, q & i = 1 \\ -2.246\, q & i = 2 \\ -5.223\, q & i = 3 \end{cases}$$

Due to space limitations, we have included here only the figure corresponding to the optimal μ^2 (see Figure 9). In all four simulations we have observed qualitative behavior similar to the previous two cases. We have seen a significant increase in the flow control magnitude for type 3 traffic; this is due to the increased weighting of $Q(3)$, which corresponds to a more stringent QoS for type 3 traffic than in the previous cases. Again coincidentally, the cost incurred under the optimal admission control $\mu^{2*}(1) = 10$ turned out to be the smallest among the four particular sample paths simulated.

6. Conclusions

Several extensions of the results of this paper can be envisioned, both for the general theoretical model and for the special telecommunication network application. For the former, one question that this paper has left unanswered is the structure of the solution to the HJB equation (5), or its infinite-horizon version (6), when the minimization over u^2 in (10) depends on the state x. There is also the issue of developing computational tools for the solutions of (5) and (6) when general structural information is lacking. One extension of the general model of this paper would be the inclusion of an additional additive term in (3), which would represent an unknown disturbance – modeled either as a stochastic process with known statistics (such as a Brownian motion process) or as a completely unknown deterministic process (as in H^∞ control [3]). In the latter case, one chooses as performance index the ratio of J_β introduced here, to the energy of the unknown deterministic process, whose maximum over the unknown input will now have to be minimized over the multi-strategy μ. For discussions on the solution to this problem for the special case when the transition rate matrix is not controlled, but under various types of measurements for the controller (including the noise-perturbed measurement scheme not covered in this paper), see [5].

As far as the specific telecommunication network model of Section 1 is concerned, there is the potential to extend it to the more general case where the effective service rate r_t is not known, but is measured in some additive noise. Furthermore, r_t could be generated by a stochastic ARMA process, or by a deterministic linear model driven by an unknown deterministic process with finite energy. Such models have been considered before in [2], but for a single type traffic (i.e., with $s = 1$), and the extensions to the cases where there are multiple types of traffic (as in this paper) with fixed or controlled transition rates remain today as interesting but challenging research topics to pursue.

7. References

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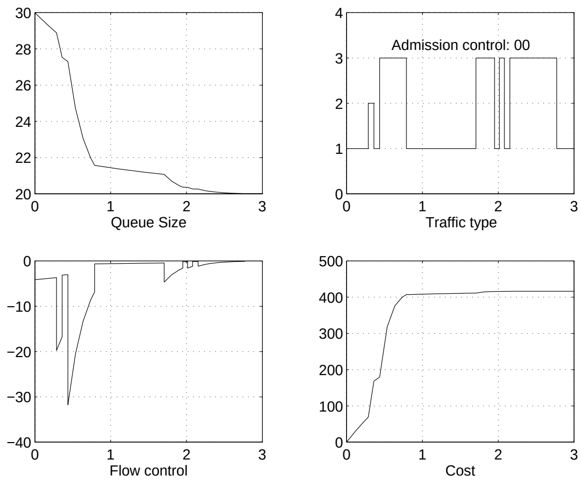

Figure 1: Case 1: Performance under μ^{1*} w.r.t. $\mu^{2*} = 00$.

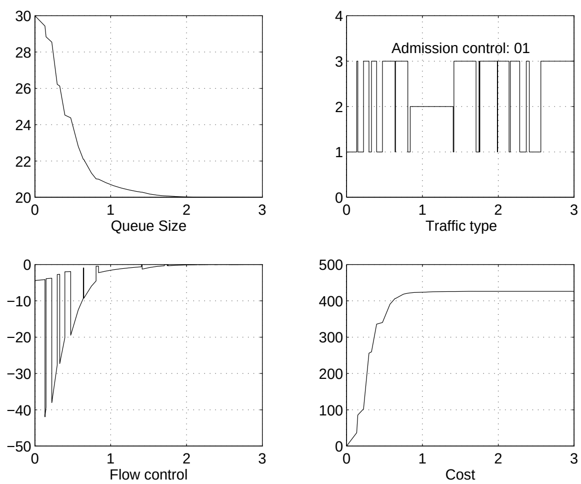

Figure 2: Case 1: Performance under μ^{1*} w.r.t. $\mu^{2*} = 01$.

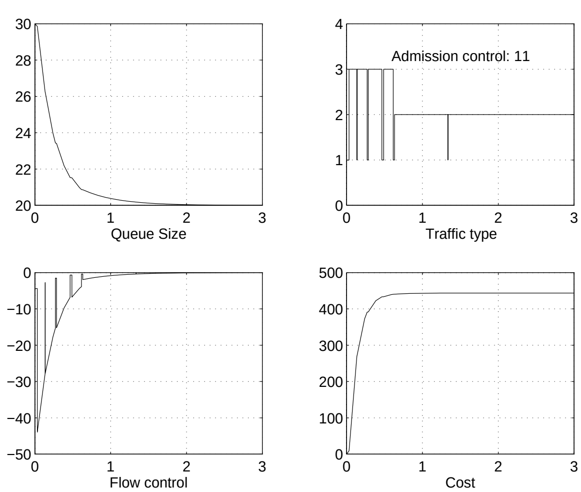

Figure 3: Case 1: Performance under μ^{1*} w.r.t. $\mu^{2*} = 11$.

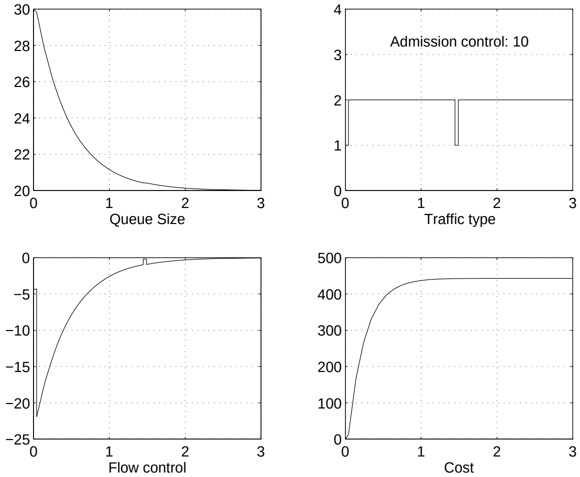

Figure 4: Case 1: Performance under μ^{1*} w.r.t. $\mu^{2*} = 10$.

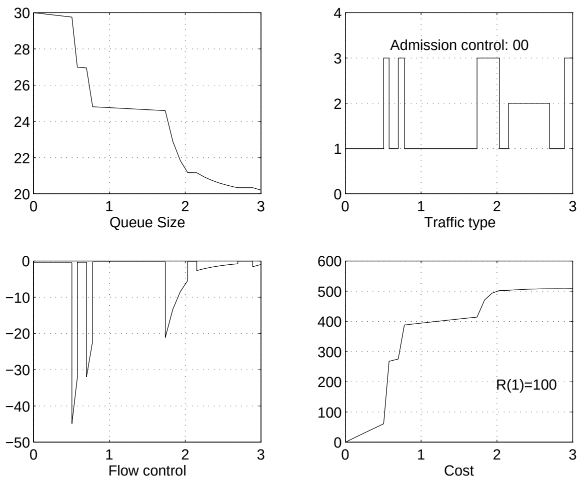

Figure 5: Case 2: Performance under μ^{1*} w.r.t. $\mu^{2*} = 00$.

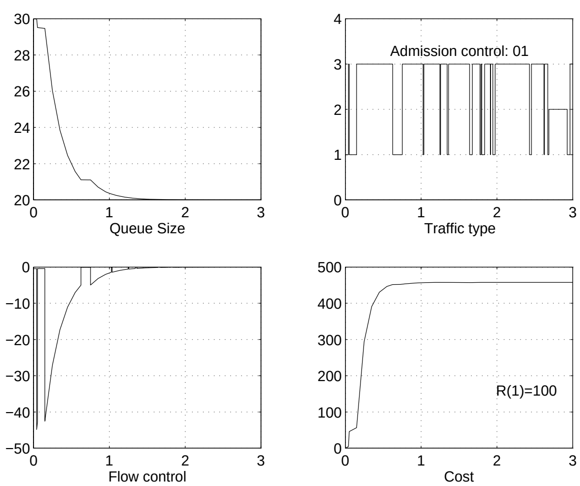

Figure 6: Case 2: Performance under μ^{1*} w.r.t. $\mu^{2*} = 01$.

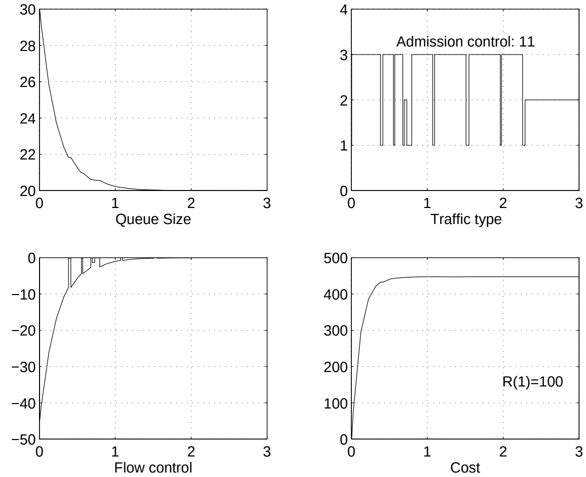

Figure 7: Case 2: Performance under μ^{1*} w.r.t. $\mu^{2*} = 11$.

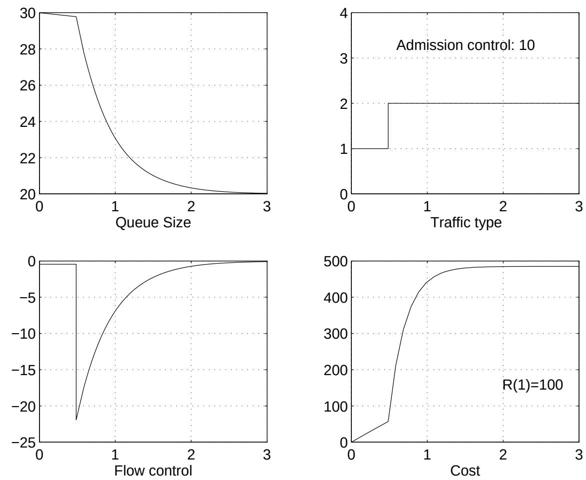

Figure 8: Case 2: Performance under μ^{1*} w.r.t. $\mu^{2*} = 10$.

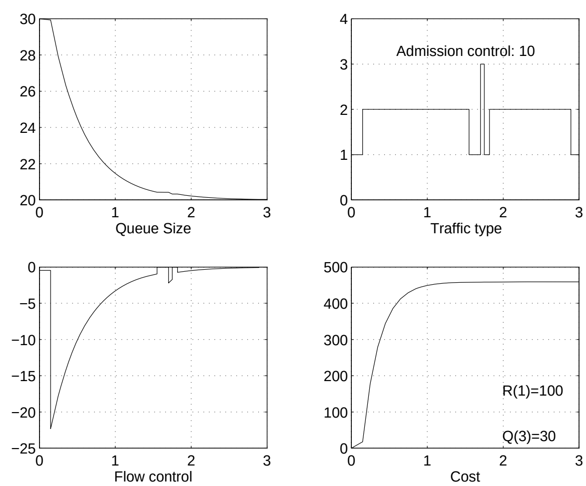

Figure 9: Case 3: Performance under μ^{1*} w.r.t. $\mu^{2*} = 10$.