Slotted Aloha with priorities and random power

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***Abstract*— This paper studies distributed choice of retransmission probabilities in slotted Aloha under power differentiation schemes. We consider random sets of possible transmission powers and further study the role of priorities (through power control) given either to new arriving packets or to backlogged packets. We study both the cooperative team problem in which a common objective is jointly optimized as well as the noncooperative game problem in which mobiles optimize individually their own objectives. We consider as objectives both maximizing throughputs as well as minimizing delays, and study the tradeoff between these objectives. We show that the new proposed schemes not only improve the average performances considerably but are also able in some cases to eliminate the bi-stable nature of the slotted Aloha.**

Keywords: Mathematical Programming, Optimization, Nash equilibrium, Economics

I. Introduction

Aloha [1] and slotted Aloha [10] have long been used as random distributed medium access protocols for radio channels. They are used in satellite networks, sensor networks and cellular telephone networks for the sporadic transfer of data packets. In these protocols, packets are transmitted sporadically by various users. If packets are sent simultaneously by more than one user then they collide. After the end of the transmission of a packet, the transmitter receives the information on whether there has been a collision (and retransmission is needed) or whether it was well received. All packets involved in a collision are assumed to be corrupted and are retransmitted after some random time. We focus in this paper on the slotted Aloha (which is known to have a better achievable throughput than the unslotted version, [4]) in which time is divided into units. At each time unit a packet may be transmitted, and at the end of the time interval, the sources get the feedback on whether there was zero, one or more transmissions (collision) during the time slot. A packet that arrives at a source is immediately transmitted. Packets that are involved in a collision are backlogged and are scheduled for retransmission after a random time.

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In this paper we introduce two new schemes in which multiple power levels are used for transmission. When several packets are sent simultaneously, one of them can often be successfully received due to the power capture effect. We assume that the packet with the largest received power captures the channel [9], [6], [11]; if two or more packets are transmitted simultaneously with the same power, we assume that neither one of them can be captured. In addition to the power diversity which had already been proposed in [9], [6], [11] we consider differentiation between new packets and backlogged packets and allow for prioritization of one or the other in terms of transmitted power. We study and compare in the paper the following schemes:

1) the one with power diversity and without prioritization,
2) a new packet is transmitted with the lowest power, and backlogged packets transmit at a random power selected among N larger distinct levels.
3) a new packet is transmitted with the highest power, and backlogged packets transmit at a random power selected among N lower distinct levels.
4) standard Aloha: all transmit with the same power.

We first consider the problem of optimal selection of transmission probabilities for the various schemes so as to achieve the maximum throughput or the minimum expected delay. We discover however that in heavy load, the optimality is obtained at the expense of huge expected delay of backlogged packets (EDBP). We therefore consider the alternative objective of minimizing the EDBP. We study both the throughput as well as the delay performance of the global optimal policy. We also solve the multicriteria problem in which the objective is a convex combination of the throughput and the EDBP. This allows, in particular, to compute the transmission probabilities that maximize the throughput under a constraint on EDBP, which could be quite useful for delay-sensitive applications.

We show that the new schemes we propose not only improve the average performances considerably but are also able in some cases to eliminate the bi-stable nature of the slotted Aloha.

In addition to the global optimization problem, we also study the game problem in which each mobile chooses its transmission probability selfishly so as to optimize its individual objective. This gives rise to a game theoretic model of which we study the equilibrium properties. We show that the power diversity and the prioritization profit to mobiles also in this competitive scenario.

Related work: Various game formulations of the standard slotted Aloha (with a single power) have been derived and studied in [3], [2], [7], [8], [5] for the non-cooperative choice of transmission probabilities. Several papers study slotted Aloha with power diversities but without differentiating between transmitted and backlogged packets, and without the game formulation. In [9] it is shown that the system capacity could be increased from 0.37 to 0.53 if one class of terminals always used high power and the other always used low power level. In [6], power diversity is studied with the capture model that we use as well as with another capture model based on signal to noise ratio. [11] studies power diversity under three types of power distribution between the power levels and provides also stability analysis.

The rest of the paper is organized as follows. In Section II, we describe the problem and the model. In Section III, we discuss the team formulation of the problems. In Section IV, we discuss the game formulation of the problems. We evaluate the performance of different schemes numerically in Section V. We conclude the paper in Section VI.

II. Model and problem formulation

In this section we describe the new mechanisms of slotted aloha and associated assumptions used in this paper. We consider one central receiver and m sources without buffer. We assume a perfect capture model where a successful capture of a packet at the receiver occurs when the power level selected of this packet is greater than those of all other packets transmitted in the same slot. A mobile terminal can transmit a packet using a power from N different levels.

We use a Markovian model extending [4, Sec. 4.2.2]. The arrival flow of packets to source i follows a Bernoulli process with parameter q_a (i.e. at each time slot, there is a probability q_a of a new arrival at a source, and all arrivals are independent). As long as there is a packet at a source (i.e. as long as it is not successfully transmitted) new packets to that source are blocked and lost.[1] The arrival processes to different sources are independent. A backlogged packet at source i is retransmitted with probability q_r^i. We shall restrict in our control and game problems to simple policies in which q_r^i does not change in time. Since sources are symmetric, we shall further restrict to finding a symmetric optimal solution, that is retransmission probabilities q_r^i that do not depend on i.

We shall use as the state of the system the number of backlogged nodes (or equivalently, of backlogged packets) at the beginning of a slot, and denote it frequently with n. For any choice of values $q_r^i \in (0,1]$, the state process is a Markov chain that contains a single ergodic chain (and possibly transient states as well). Define $\mathbf{q}_r$ to be the vector of retransmission probabilities for all users (whose jth entry is q_r^j). Let $\pi(\mathbf{q}_r)$ be the corresponding vector of steady state probabilities where its nth entry, $\pi_n(\mathbf{q}_r)$, denotes the probability of n backlogged nodes. When all entries of $\mathbf{q}_r$ are the same, say q, we shall write (with some abuse of notation) $\pi(q)$ instead of $\pi(\mathbf{q}_r)$.

We introduce further notation. Assume that there are n backlogged packets, and all use the same value q_r as retransmission probability. Let $Q_r(i,n)$ be the probability that i out of the n backlogged packets retransmit at the slot. Then

$$Q_r(i,n) = \binom{n}{i} (1-q_r)^{n-i} [q_r]^i. \tag{1}$$

Let $Q_a(i,n)$ be the probability that i unbacklogged nodes transmit packets in a given slot (i.e. that i arrivals occurred at nodes without backlogged packets). Then

$$Q_a(i,n) = \binom{m-n}{i} (1-q_a)^{m-n-i} [q_a]^i. \tag{2}$$

Let $Q_r(1,0) = 0$ and $Q_a(1,m) = 0$.

III. Team problem

In this section we propose and analyze three different schemes. We observe that standard slotted Aloha is a special case of these proposed schemes.

A. Scheme 1 : Random power levels without priority scheme

In this subsection, we describe the use of a scheme in which multiple power levels are used at transmitter. A mobile terminal can transmit a packet (new arrival packets or backlogged packets) using one of N distinct available power levels. In case all nodes use the same value q and q_r, the transmission probabilities of the Markov chain is given by $P_{n,n+i}(q) =$

$$\begin{cases} Q_a(i,n)[\sum_{j=0}^{n} Q_r(j,n)(1-A_{j+i})], & i = m-n, i \geq 2 \\ Q_a(i,n)[\sum_{j=0}^{n} Q_r(j,n)(1-A_{j+i})] + \\ \quad Q_a(i+1,n)[\sum_{j=0}^{n} Q_r(j,n)A_{j+i+1}], & 2 \leq i < m-n \\ Q_a(1,n)\sum_{j=1}^{n} Q_r(j,n)(1-A_{j+1}) + \\ \quad Q_a(2,n)\sum_{j=0}^{n} Q_r(j,n)A_{j+2}], & i = 1 \\ Q_a(0,n)[Q_r(0,n) + \sum_{j=2}^{n} Q_r(j,n)(1-A_j)] + \\ \quad Q_a(1,n)[Q_r(0,n) + \sum_{j=1}^{n} Q_r(j,n)A_{j+1}] & i = 0 \\ Q_a(0,n)[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)A_j], & i = -1 \end{cases}$$

where the probability of a successful transmission among $k \geq 2$ packets, is denoted and given by $A_k = k\sum_{l=0}^{N-1} X_{N-k}(1-\sum_{i=N-l}^{N} X_i)^{k-1}$, $A_0 = 0, A_1 = 1$ and X_i is the probability that a packet (new arrival or backlogged) will choose power level i for retransmission.

[1] In considering the number of packets in the system, this assumption is equivalent to saying that a source does not generate new packets as long as a previous packet is not successfully transmitted.

B. Scheme 2: Retransmission with more power

In this scheme, a backlogged packet retransmits with a power from N different distinct levels. A new arrival packet uses a lower power than any one these N levels. The random power levels are chosen uniformly. Successful capture occurs if one of the backlogged packet transmits with a power level which is larger than that chosen by all others transmitters or a single new arrival occurs and there is no retransmission attempt of any backlogged packet. The transition matrix is given by $P_{n,n+i}(q) =$

$$\begin{cases} Q_a(i,n)[Q_r(0,n)+\sum_{j=2}^{n}Q_r(j,n)(1-A_j)], & i=m-n, i\geq 2\\ Q_a(i,n)[Q_r(0,n)+\sum_{j=2}^{n}Q_r(j,n)(1-A_j)]+ & \\ \quad Q_a(i+1,n)[Q_r(1,n)+\sum_{j=2}^{n}Q_r(j,n)A_j], & 2\leq i<m-n\\ Q_a(0,n)[Q_r(0,n)+\sum_{j=2}^{n}Q_r(j,n)(1-A_j)]+ & \\ \quad Q_a(1,n)[Q_r(0,n)+Q_r(1,n)+\sum_{j=2}^{n}Q_r(j,n)A_j], & i=0\\ Q_a(i,n)\sum_{j=2}^{n}Q_r(j,n)(1-A_j)+ & \\ \quad Q_a(i+1,n)[Q_r(1,n)+\sum_{j=2}^{n}Q_r(j,n)A_j], & i=1\\ Q_a(0,n)[Q_r(1,n)+\sum_{j=2}^{n}Q_r(j,n)A_j], & i=-1 \end{cases}$$

C. Scheme 3 : Retransmission with less power

In this scheme, a new transmitted packet has the highest power. Backlogged packets attempt retransmissions with a random power choice among N distinct lower power levels. The random power levels are chosen uniformly. The transition matrix is given by: $P_{n,n+i}(q) =$

$$\begin{cases} Q_a(i,n) & i\geq 2\\ 0 & i=1\\ Q_a(1,n)+Q_a(0,n)[\sum_{j=2}^{n}Q_r(j,n)(1-B_j)+Q_r(0,n)] & i=0\\ Q_a(0,n)[Q_r(1,n)+\sum_{j=2}^{n}Q_r(j,n)B_j] & i=-1, \end{cases}$$

where the probability of a successful retransmission among $j\geq 2$ is given by $B_j = j\sum_{k=1}^{N-1}P_k(1-\sum_{i=1}^{k}P_i)$, $j\leq 2$.

D. Performance Metrics

We present the performance measures of interest for optimization as a function of the steady state probabilities. Denote by $\pi_n(q)$ the equilibrium probability that the network is in state n (number of backlogged packets at the beginning of a slot). Then we have the equilibrium state equations:

$$\begin{cases} \pi(q)=\pi(q)P(q),\\ \pi_n(q)\geq 0, n=0,...,m\\ \sum_{n=0}^{m}\pi_n(q)=1. \end{cases} \tag{3}$$

The average number of backlogged packets which is given by

$$S(q)=\sum_{n=0}^{m}\pi_n(q)n \tag{4}$$

The system throughput (defined as the sample average of the number of packets that are successfully transmitted) is given almost surely by the constant, $thp(q) =$

$$\begin{cases} \sum_{n=1}^{m}\pi_n(q)[Q_a(0,n)(Q_r(1,n)+\sum_{j=2}^{n}Q_r(j,n)A_j)+ & \\ \quad Q_a(1,n)\sum_{j=0}^{n}Q_r(j,n)A_{j+1}+ & \\ \quad \sum_{i=2}^{m-n}Q_a(i,n)\sum_{j=0}^{n}Q_r(j,n)A_{i+j}\Big]+ & \\ \quad \pi_0(q)Q_a(1,0) & \text{scheme 1}\\ \sum_{n=1}^{m}\pi_n(q)\Big[Q_a(1,n)Q_r(0,n)+Q_r(1,n)+ & \\ \quad \sum_{j=2}^{n}Q_r(j,n)A_j\Big]+\pi_0(q)Q_a(1,0) & \text{scheme 2}\\ \sum_{n=0}^{m}\pi_n(q)\Big[Q_a(1,n)+Q_r(0,n)(Q_r(1,n)+ & \\ \quad \sum_{j=2}^{n}Q_r(j,n)A_j)\Big] & \text{scheme 3} \end{cases}$$

The throughput satisfies (and thus can be computed through)

$$thp(q)=q_a\sum_{n=0}^{m}\pi_n(q)(m-n)=q_a(m-S(q)). \tag{5}$$

Indeed, the throughput is the expected number of arrivals at a time slot (which actually enter the system), and this is expressed in the equation for $thp(q)$ by conditioning on n. The throughput should equal to the expected number of departures (and thus the throughput) at stationary regime, which is expressed in (5).

The expected delay of transmitted packets D, is defined as the average time, in slots, that a packet takes from its source to the receiver. Applying Little's result, this is given by

$$D(q)=1+\frac{S(q)}{thp(q)}=1+\frac{S(q)}{q_a(m-S(q))} \tag{6}$$

Note that the first term accounts for the first transmission from the source.

Combining the last equality in (5) with (6) it follows that maximizing the global throughput is equivalent to minimizing the average delay of transmitted packets. We shall therefore restrict in our numerical investigation to maximization of the throughput. However, we shall consider the delay of *backlogged packets* as yet another objective to minimize.

Performance measures for backlogged packets. The throughput of the backlogged packets for each scheme is given by: $thp^c = thp(q) - \Delta$ where Δ is given by

$$\begin{cases} \sum_{n=0}^{m} \sum_{i=1}^{m-n} \sum_{j=0}^{n} (\frac{i}{i+j} Q_a(i,n) Q_r(j,n) A_{i+j}) \pi_n(q) & \text{scheme 1} \\ \sum_{n=0}^{m} Q_a(1,n) Q_r(0,n) \pi_n(q), & \text{scheme 2} \\ \sum_{n=0}^{m} Q_a(1,n) \pi_n(q) & \text{scheme 3} \end{cases}$$

Another relevant quantity in this context is the expected delay of backlogged packets D^c which is defined as the average time, in slots, that a backlogged packet takes to go from the source to receiver. Applying Little's result, the expected delay of packets that arrive and become backlogged is given by

$$D^c(q) = 1 + S(q)/thp^c(q) \tag{7}$$

The team problem is therefore given as the solution of the optimization problem:

$$\max_q \ objective(q) \ s.t. \ \begin{cases} \pi(q) = \pi(q)P(q), \\ \pi_n(q) \geq 0, n = 0, ..., m \\ \sum_{n=0}^{m} \pi_n(q) = 1. \end{cases}$$

A solution to the team problem can be obtained by computing recursively the steady state probabilities, as in Problem 4.1 in [4], and thus obtain an explicit expression for $thp(q)$ as a function of q.

Stability. Another qualitative way to compare schemes is in the stability characteristics of the protocol. Slotted Aloha is known to have a bi-stable behavior, and we shall check whether this is also the case in our new schemes.

Let us define the *drift* in state $n(D_n)$ as the expected change in backlog over one slot time, starting in state n. Thus, D_n is the expected number of new arrivals accepted into the system [i.e., $(m-n)q_a$] less the expected number of successful transmissions in the slot; the expected number of successful transmissions is just the probability of a successful transmission, defined as P_{succ}. Thus,

$$D_n = (m-n)q_a - P_{succ} \tag{8}$$

where P_{succ} is given by

$$\begin{cases} Q_a(1,n)Q_r(0,n) + Q_a(0,n)Q_r(1,n), & \text{same-power} \\ \sum_{i=0}^{m-n} Q_a(i,n) \sum_{j=0}^{n} Q_r(j,n) A_{i+j} & \text{Scheme 1} \\ Q_a(1,n)Q_r(0,n) + Q_a(0,n)[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)A_j], & \\ & \text{Scheme 2} \\ Q_a(1,n) + Q_a(0,n)[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)B_j], & \text{Scheme 3} \end{cases}$$

For standard slotted Aloha it has been observed (see [4]) that there are three equilibria, where an equilibrium is defined as a state n in which the arrival rate $(m-n)q_a$ equals the departure rate P_{succ}. Moreover, among those three, the two extreme ones (the one corresponding to the smallest state and the one corresponding to the largest one) are stable.[2] A bi-stable situation as in the standard Aloha is undesirable since it means in practice that the system spends long time in each of the stable equilibria including in the one with large n corresponding to a congestion situation (low throughput and large delays). We shall study numerically the stability behavior of the various schemes.

Next, we discuss some properties related to the optimization problem.

Singularity at $q = 0$. The only point where P does not have a single stationary distribution is at $q = 0$, where it has two absorbing states: $n = m$ and $n = m-1$. All other states are transient (for any $q_a > 0$), and the probability to end at one of the absorbing states depends on the initial distribution of the Markov chain. We note that if the state $m-1$ is reached then the throughput is q_a w.p.1, where as if the state m is reached then the throughput equals 0. It is thus a deadlock state. For $q_a > 0$ and $q_r = 0$, the deadlock state is reached with positive probability from any initial state other than $m-1$. We shall therefore exclude $q_r = 0$ and optimize only on the range $\epsilon \leq q_r \leq 1$. We choose throughout the paper $\epsilon = 10^{-4}$.

Existence of a solution. The steady state probabilities $\pi(q)$ are continuous over $0 < q \leq 1$. Since this is not a close interval, a solution need not exist. However, as we restrict to the closed interval $q \in [\epsilon, 1]$ where $\epsilon > 0$, an optimal solution indeed exists. Note also that the limit $\lim_{q \to 0} \pi(q)$ exists since $\pi(q)$ is a rational function of q at the neighborhood of zero. Therefore for any $\delta > 0$, there exists some $q > 0$ which is δ-optimal. ($q^* > 0$ is said to be δ-optimal for the throughput maximization if it satisfies $thp(q^*) \geq thp(q) - \delta$ for all $q \in (0, 1]$. A similar definition holds for the EDBP minimization.)

IV. Game Problem

Next, we formulate the game problem. This formulation is of interest as it is more appropriate in decentralized scenarios in which mobiles may not be controllable by a centralized entity (and so the team approach does not hold any more). The equilibrium concept then replaces the optimality concept from the team problem. It possesses a robustness property: at equilibrium, no mobile has incentive to deviate.

For a given policy vector $\mathbf{q_r}$ of retransmission probabilities for all users (whose jth entry is q_r^j), define $([\mathbf{q_r}]^{-i}, \hat{q}_r^i)$ to be a retransmission policy where user j retransmits at a slot with probability q_r^j for all $j \neq i$ and where user i retransmits with probability $\hat{q}_r^i$. Each

[2] Recall that an equilibrium is stable if the drift corresponding to a small deviation (increasing or decreasing n) from the equilibrium is in the direction opposite to the deviation (e.g. if we take n slightly larger than the equilibrium then the drift should be negative, tending to decrease again n).

user i seeks to maximize his own $objective_i(\mathbf{q})$. The problem we are interested in is then to find a symmetric equilibrium policy $\mathbf{q_r^*} = (q_r, q_r, .., q_r)$ such that for any user i and any retransmission probability q_r^i for that user,

$$objective_i(\mathbf{q_r^*}) \geq objective_i([\mathbf{q_r^*}]^{-i}, q_r^i) \qquad (9)$$

where the objective function is the throughput or minus the delay[3]. Since we restrict to symmetric $\mathbf{q_r^*}$, we shall also identify it (with some abuse of notation) with the actual transmission probability (which is the same for all users). Next we show how to obtain an equilibrium policy. We first note that due to symmetry, to see whether $\mathbf{q_r^*}$ is an equilibrium it suffices to check (9) for a single player. We shall thus assume that there are $m+1$ users all together, and that the first m users retransmit with a given probability $\mathbf{q_r}^{-(m+1)} = (q^o, .., q^o)$ and user $m+1$ retransmits with probability $q_r^{(m+1)}$. Define the set

$$\mathcal{Q}^{m+1}(\mathbf{q_r^o}) = \arg\max_{q_r^{(m+1)} \in [\epsilon, 1]} \Big(objective_{m+1}([\mathbf{q_r^o}]^{-(m+1)}, q_r^{(m+1)}) \Big),$$

where $\mathbf{q_r^o}$ denotes (with some abuse of notation) the policy where all users retransmit with probability q_r^o, and where the maximization is taken with respect to $q_r^{(m+1)}$. Then q_r^* is a symmetric equilibrium if

$$q_r^* \in \mathcal{Q}_r^{m+1}(q_r^*).$$

To compute the performance measures of interest $objective_{m+1}([\mathbf{q_r^o}]^{-i}, q_r^i)$, we introduce again a Markov chain with a two dimensional state. The first state component corresponds to the number of backlogged packets among the users 1,...,m, and the second component is the number of backlogged packets (either 1 or 0) of user $m+1$.

Scheme 1: Retransmission with more power but with no priority. We consider the game problem in which packets are transmitted/retransmitted with random power uniformly selected from N levels. There is no priority for any packet. The transition probabilities when the m other mobiles use q_r^o and a given other mobile uses $q_r^{(m+1)}$ are given in Appendix VIII.

Scheme 2: Retransmission with more power. We consider the game problem in which backlogged packets are retransmitted with random power uniformly selected from N levels. A new arriving packet is always transmitted with less power than any retransmitted packet. The transition probabilities when the m other mobiles use q_r^o and a given other mobile uses $q_r^{(m+1)}$ are given in Appendix VII.

Scheme 3 : Retransmission with less power. We consider the game problem in which backlogged packets are retransmitted with a random power from chosen among N distinct levels. Here the new arrival packets are transmitted with higher power. Define $C_j = \sum_{k=1}^{N-1} P_k (1 - \sum_{i=1}^{k} P_i)^{j-1}$. Then the transition probabilities when the m other mobiles use q_r^o and a given other mobile uses $q_r^{(m+1)}$ are given by (10).

$$P_{(n,i),(n+k,j)}(q_r^o, q_r^{(m+1)}) = \qquad (10)$$

$$\begin{cases} \left.\begin{array}{ll} Q_a(k,n), & i=j=1 \\ Q_a(k,n)(1-q_a), & i=j=0 \\ Q_a(k,n)q_a, & i=0, j=1 \end{array}\right\} & 2 \leq k \leq m-n, k \geq 2 \\ \left.\begin{array}{l} (1-q_r^{(m+1)})H + \\ \quad q_r^m [Q_a(0,n) \sum_{j=1}^{n} Q_r(j,n)(1-C_{j+1}) + Q_a(1,n)], \\ \hfill i=j=1 \\ q_a Q_a(0,n) + (1-q_a)H \hfill i=j=0 \\ q_r^{(m+1)} Q_a(0,n)[Q_r(0,n) + \sum_{j=1}^{n} Q_r(j,n) C_{j+1}], \\ \hfill i=1, j=0 \end{array}\right\} & k=0 \\ \left.\begin{array}{l} Q_a(0,n)(1-q_r^{m+1})[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n) C_j], \\ \hfill i=j=1 \\ Q_a(0,n)(1-q_a)[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n) C_j], \\ \hfill i=j=0 \end{array}\right\} & k=-1 \\ Q_a(1,n)q_a, & i=0, j=1, k=1 \\ 0 & \text{otherwise} \end{cases}$$

where $H = Q_a(1,n) + Q_a(0,n)[Q_r(0,n) + \sum_{j=2}^{n} Q(j,n)(1-C_j)]$

Performance Metrics. In the game problem, the average number of backlogged packets of source $m+1$ is given by:

$$S_{m+1}([\mathbf{q_r^o}]^{-1(m+1)} q_r^{m+1}) = \sum_{n=0}^{m} \pi_{n,1}([\mathbf{q_r^o}^{-(m+1)}], q_r^{(m+1)}) \qquad (11)$$

and the average throughput of user $m+1$ is given by

$$thp_{m+1}([\mathbf{q_r^o}]^{-(m+1)}, q_r^{(m+1)}) = q_a \sum_{n=0}^{m} \pi_{n,0}([q_r^o]^{-(m+1)}, q_r^{(m+1)}) \qquad (12)$$

Hence the expected delay of transmitted packets of user $m+1$ for both scheme is given by

$$D_{m+1}(\mathbf{q_r^o}]^{-1(m+1)}, q_r^{m+1}) = 1 + \frac{S_{m+1}([\mathbf{q_r^o}]^{-1(m+1)} q_r^{m+1})}{thp_{m+1}(\mathbf{q_r^o}]^{-1(m+1)}, q_r^{m+1})} \qquad (13)$$

Let us denote by thp_{m+1}^c the throughput of backlogged packets (i.e. of the packets that arrive and become backlogged) at source $m+1$:

$$thp_{m+1}^c(\hat{\mathbf{q}}_{m+1}) = \sum_{n=0}^{m} \sum_{n'=0}^{m} P_{(n,0),(n',1)}(\hat{\mathbf{q}}_{m+1}) \pi_{n,0}(\hat{\mathbf{q}}_{m+1})$$

[3]Several definitions of delays will be given. In case delay is optimized, this formulation is equivalent to minimizing delays

Thus, the expected delay of backlogged packets at source $m+1$, is given by

$$D^c_{m+1}(\hat{\mathbf{q}}_{m+1}) = 1 + S_{m+1}(\hat{\mathbf{q}}_{m+1})/thp^c_{m+1}(\hat{\mathbf{q}}_{m+1}) \tag{14}$$

V. Numerical Investigation

We describe next numerical investigation of the team and the game problems for the three new schemes as well as the standard Aloha.

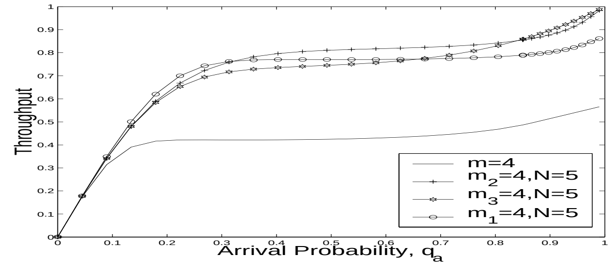
A. Maximizing the global throughput

As already mentioned, we shall not study the minimizing of the expected delay as it is equivalent to maximizing throughput. In Figure 1 (a) and (b), we plot the throughput and expected delay of backlogged packets (EDBP) for all schemes under the objective of maximizing the global throughput for $m = 4$ (and with $N = 5$ for schemes 1-3). Throughout we use the notation $m_i = x$ in the figure to indicate that scheme i is used with number of nodes x. When we write $m = x$ (without subscript) we mean that standard Aloha is used with x nodes.

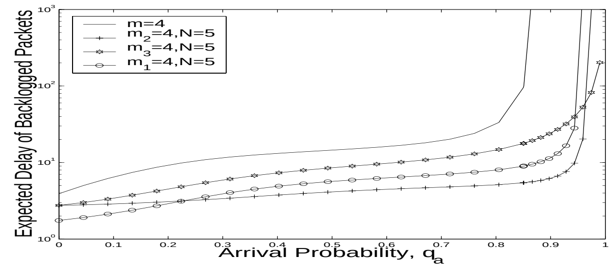
We observe that when load is high, scheme 2 performs better than other schemes in terms of throughput. This is due to the fact that scheme 2 prioritizes the retransmission of backlogged packets. But when load is very high: $q_a > 0.8$, the throughput of scheme 3 is highest. Under low load for $m = 4$, scheme 1 performs a little better than other schemes because prioritizing backlogged packets may not result in gain when there are few backlogged packets. All schemes outperform standard Aloha.

We next observe the result of maximizing throughput on the EDBP (Figure 1 (b)). At moderate and high load scheme 2 performs best, and at very high load scheme 3 performs best. All the schemes perform very bad at heavy load. Standard Aloha gives the worse performance at all loads.

We plot the corresponding figures for $m = 10$ in Figure 2 and observe similar trends.

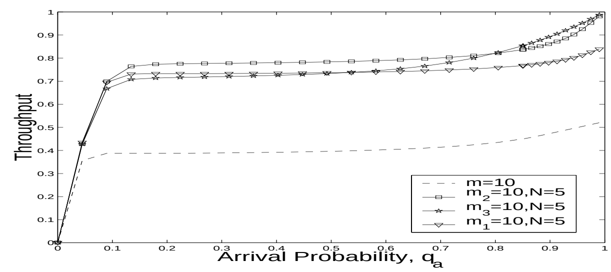


(a)

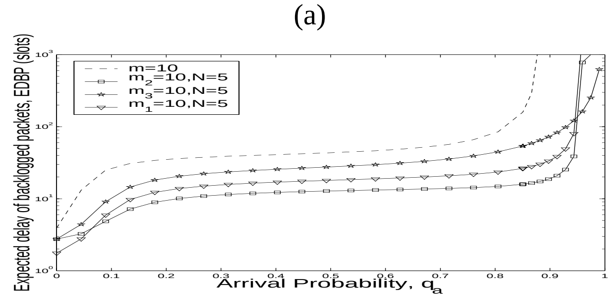


(b)

Fig. 1. (a) and (b) show the throughput and delay of backlogged packets. The objective under all the schemes is to maximize the throughput.The number of mobiles is 4 and number of levels is 5.



(a)



(b)

Fig. 2. (a) and (b) show the throughput and delay of backlogged packets. The objective under all the schemes is to maximize the throughput.The number of mobiles is 10 and the number of levels is 5.

B. Minimizing EDBP

When maximizing the global throughput we observed a huge EDBP under all schemes in heavy load. Such large delay may be very harmful in many applications. We thus investigate directly the problem of minimizing EDBP. We shall also investigate the impact of this optimization on the throughput performance. We shall show in particular that throughput performance in the new schemes improves considerably with respect to standard Aloha even when standard Aloha uses the previous objective of maximizing throughput (this will correspond to obj_1 in the figures, whereas obj_2 will correspond to minimizing EDBP).

In Figure 3, we plot the performance of the various schemes for $m = 4$. Part (a) considers the impact on the throughput while minimizing the EDBP. It is seen that all three schemes outperform standard Aloha even when the latter uses throughput maximization as optimization objective. Scheme 1 is the best in terms of throughput only at light load, scheme 2 is the best in medium load whereas scheme 3 remarkably outperforms the other schemes at high loads ($q_a > 0.8$). In part (b) of the figure we see that Scheme 2 outperforms all others in terms of EDBP for both medium and high load. Part (c) provides the optimal retransmission probabilities. We observe the phenomenon of tiny optimal retransmission probabilities for standard Aloha when throughput is maximized, which explains its corresponding huge EDBP. In contrast, when EDBP is minimized standard Aloha has optimal retransmission probabilities of around 0.3 in heavy load whereas all other versions have much higher retransmission probabilities.

The case of 10 mobiles is presented in Figure 4 which provides similar trends.

Figure 5 illustrates the effect of all the new schemes as a function of N. We observe that performance improves generally with increasing value of N and the returns diminishes with increase in N.

Table I summarizes the performance of the team problem in terms of throughput and EDBP.

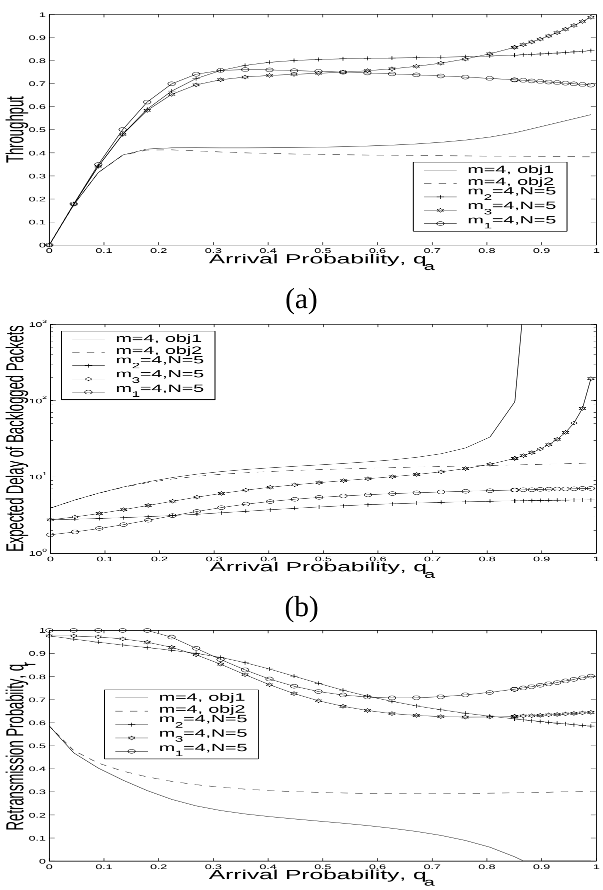


Fig. 3. (a), (b) and (c) show the throughput, EDBP and retransmission probabilities. The objective under schemes 1-3 is to minimize EDBP (obj_2). m, m_1, m_2 and m_3 refer to standard Aloha, schemes 1, 2 and 3 respectively. The number of mobiles is 4. obj_1 refers to the objective of maximizing the throughput.

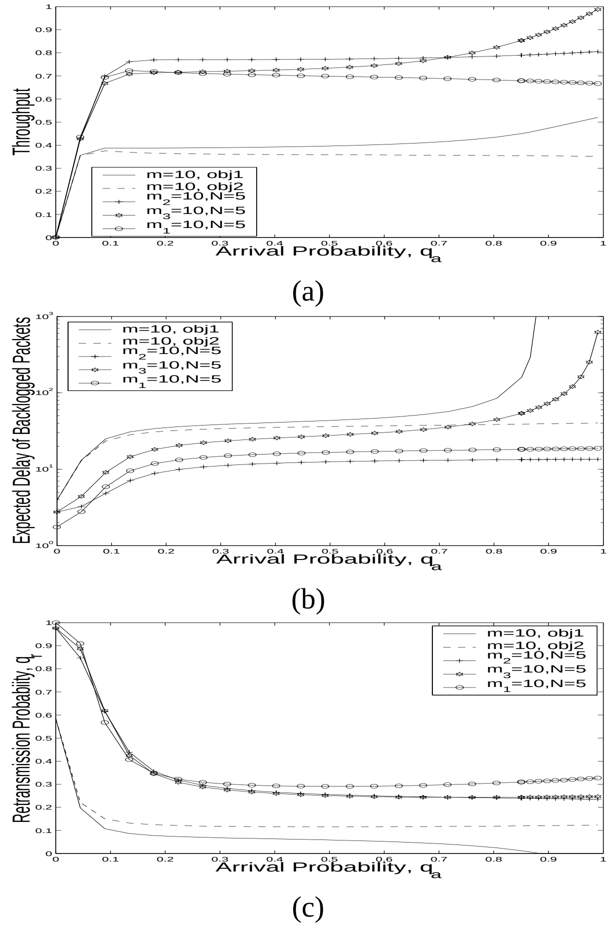


Fig. 4. (a), (b) and (c) show the throughput, EDBP and retransmission probabilities. The objective under schemes 1-3 is to minimize EDBP (obj_2). The number of mobiles is 10.

C. Stability

In Figure 6, we illustrate the stability behavior for $q_r = 0.15, q_a = 0.01, m = 40, N = 5$. The drift is the difference between the curves (representing the departure rate or P_{succ}) and the straight line representing the arrival rate $(m-n)q_a$. Since the drift is the expected change in state from one slot to the next, the system, although fluctuating, tends to move in the direction of the drift and consequently tends to cluster around the two stable points with rate excursions between the two (for same-power scheme). We see that slotted Aloha is the only scheme that suffers from the bi-stability problem.

We see that for standard slotted Aloha, the departure is at most $1/e$ whereas for different power schemes it is quite higher.

By choosing a large value of retransmission probabilities, we can obtain situations where schemes 1 and 2 acquire a bi-stable regime, and the scheme 3 remains stable for all values of retransmission probability q_r. For example, with 40 mobiles and $q_a = 0.15$, scheme 1 and 2 suffer from the bi-stable problem when $q_r = 0.8$ (see Figure 7). It should be noted that the bi-stability can occur in all schemes, which is the case when the number of mobiles becomes large. For example, with 60 mobiles

Scheme	m = 4 (delay, throughput)		m = 10 (delay, throughput)	
	$q_a = 0.4$	$q_a = 0.9$	$q_a = 0.4$	$q_a = 0.9$
same-power, $\max \lambda$	(13.15, 0.4219)	$(1.82 * 10^3, 0.5141)$	(40.92, 0.392)	($1.83 * 10^3$, 0.473)
same-power, $\min$ EDBP	(11.78, 0.397)	(14.67, 0.384)	(35.27, 0.3592)	(39.40, 0.3536)
scheme 1, $N = 1$	(8.617, 0.3972)	(10.306, 0.384)	(26.35, 0.359)	(28.19, 0.353)
scheme 1, $N = 5$	(4.582, 0.759)	(6.40, 0.709)	(13.16, 0.703)	(15.12, 0.675)
scheme 1, $N = 7$	(4.24, 0.811)	(6.14, 0.755)	(12.349, 0.755)	(14.24, 0.726)
scheme 2, $N = 1$	(5.6374, 0.5641)	(6.0039, 0.6541)	(17.7847, 0.5185)	(16.7333, 5937)
scheme 2, $N = 5$	(3.6385, 0.7916)	(4.7153, 0.8288)	(11.4713, 0.7709)	(12.4741, 0.7946)
scheme 2, $N = 7$	(3.4276, 0.8265)	(4.5284, 0.8622)	(10.7758, 0.8146)	(11.9088, 0.832)
scheme 3, $N = 1$	(12.34, 0.5895)	(40.904, 0.8574)	(42.2731, 0.574)	(134.118, 0.8587)
scheme 3, $N = 5$	(7.3, 0.735)	(23.7439, 0.8954)	(25.5891, 0.7251)	(73.6986, 0.8931)
scheme 3, $N = 7$	(6.9134, 0.7523)	(22.4717, 0.90)	(24.2433, 0.7444)	(69.0206, 0.8983)

TABLE I

COMPARISON OF THE THROUGHPUTS AND THE EDBP FOR DIFFERENT SCHEMES AT DIFFERENT LOADS FOR TEAM PROBLEMS, $m = 4$ AND $m = 10$.

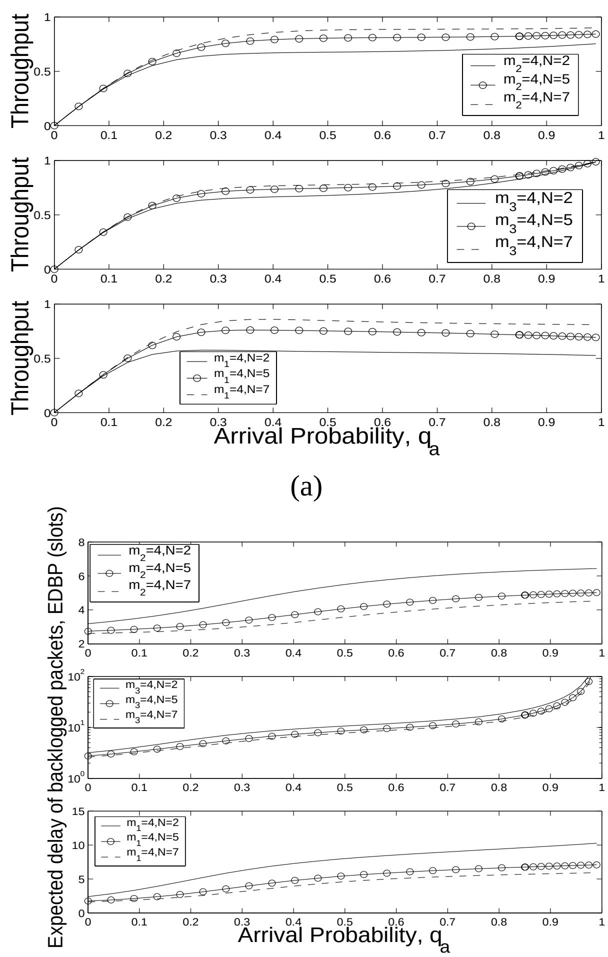


Fig. 5. (a) and (b) show the throughput and the EDBP when the objective is to minimize the delay of backlogged packets. The figures show the diminishing effect of increasing power levels for $m = 4$.

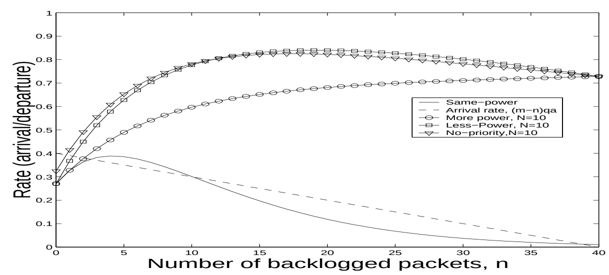


Fig. 6. Stability of slotted Aloha schemes 1, 2, and 3: The arrival probability $q_a = 0.01$, number of mobiles $m = 40$, number of levels $N = 5$ and retransmission probability $q_r = 0.15$

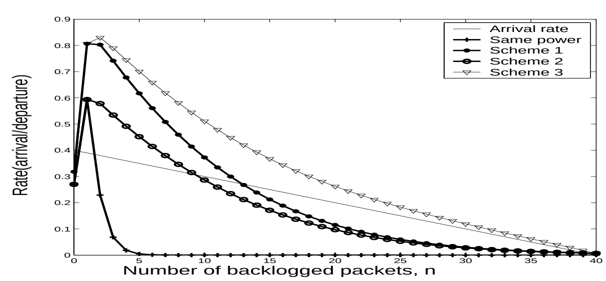


Fig. 7. Stability and instability of slotted Aloha schemes : The arrival probability $q_a = 0.01$, number of mobiles $m = 40$, number of levels $N = 5$ and retransmission probability $q_r = 0.8$.

and $q_a = 0.005$, the standard slotted Aloha is bi-stable already for $q_r = 0.1$, Scheme 1 and 2 are bi-stable with $q_r = 0.5$ and Scheme 3 becomes bi-stable with $q_r = 0.95$ (see Fig 8). Here, as well as in all other examples (not reported here) scheme 3 always turned out to have the largest region of parameters for which a unique stable point is obtained.

The average number of backlogged packets for different schemes which correspond to their equilibrium points are given in Table II with $m = 60$, $q_a = 0.005$ and $N = 5$. This is compared to the expected number of backlogged packets. In the case of a single equilibrium, a good match is seen for schemes 1, 2 and 3, which means that the simple computation of the stable equilibrium can be used to approximate the expected number of backlogged packets. In standard Aloha we see that the congested stable equilibrium provides a very good

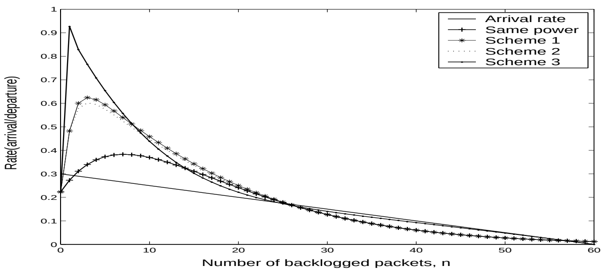
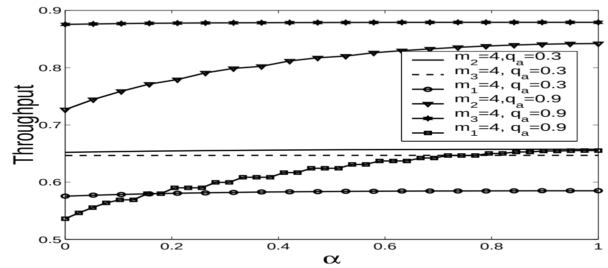
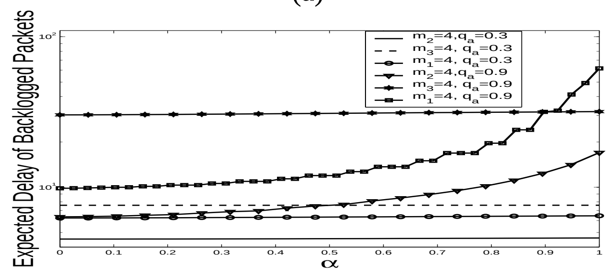


Fig. 8. Stability and instability of slotted Aloha schemes: The arrival probability $q_a = 0.005$, number of mobiles $m = 60$, number of levels $N = 5$ and retransmission probability $q_r = 0.8$ for all schemes



(a)



(b)

Fig. 9. (a) and (b) show throughput and delay of backlogged packets when the multi-criteria objective is optimized for $N = 2, m = 4$.

approximation for the expected number of backlogged packets, which suggests that the system spends most of the time at that equilibrium.

We observe also that we have same behavior in scheme 1 and 2 when the retransmission probability increases (around 0.5). Scheme 1 and 2 acquire a bi-stable with $q_r = 0.5$. But contrary to standard aloha, we see from Table II that the expected number of backlogged packets for scheme 1 and 2 *can be approximated by the desired stable equilibrium*. That means that in the bi-stability case for scheme 1 and 2, the system spends most of the time at that equilibrium. Now if the mobiles becomes aggressive (q_r around 0.9), we see that the congested stable equilibrium provides a very good approximation for the expected number of backlogged packets in all schemes, which suggests that the system spends most of the time at that equilibrium.

schemes	same-power	no-prior	more-power	less-power
ABP $q_r = 0.1$	56.8	0.71	1.04	1.09
(un)stable eq. $q_r = 0.1$	1.51, 25.47 56.88	0.67	1.449	1
ABP $q_r = 0.5$	60	0.28	0.28	0.287
(un)stable eq. $q_r = 0.5$	1.51, 25.47 56.88	0.136, 28.85 56.83	0.10, 24.33 56.89	0.208
ABP $q_r = 0.9$	60	59.98	59.98	57.58
(un)stable eq. $q_r = 0.9$	0.16, 1.72 60	0.07, 12.43 59.98	0.16, 10.46 59.98	0.11, 26.70 56.99

TABLE II

AVERAGE NUMBER OF BACKLOGGED PACKETS (ABP) AND EQUILIBRIUM POINT (S)

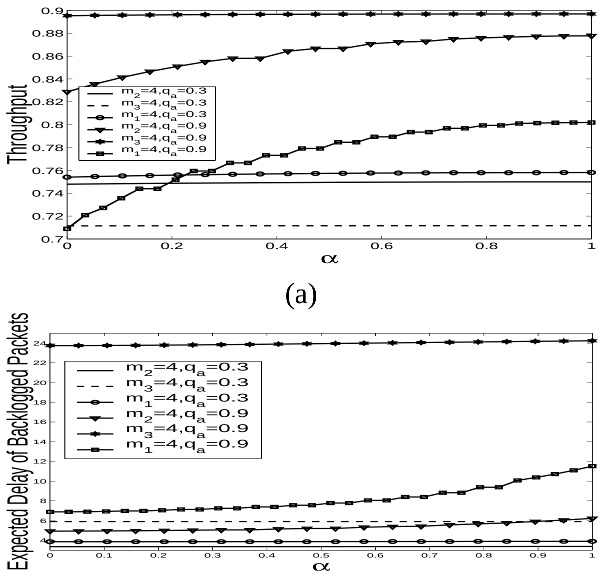
D. Multi-criteria Objective

So far we have considered the extreme cases of maximizing the throughput or minimizing the EDBP. In practice it may be of interest to have a multi-criteria optimization in which a combination of both is optimized. The objective is given by $\alpha thp(q) + (1-\alpha)\frac{1}{D^c}, 0 \leq \alpha \leq 1$. This allows in particular to handle QoS constraints: By varying α one can find appropriate tradeoff between the throughput and delays, so that the throughput be maximized while keeping the EDBP bounded by some constant.

First we plot the performance when $N = 2$ and the number of mobiles is 4, for all the new power schemes in Figure 9. We observe that scheme 3 (less-power for retransmission) is insensitive to the value of α under different load. The optimal retransmission probabilities for scheme 3 are very close under both the objectives: when throughput is maximized and when EDBP is minimized. When α increases (i.e., gives more weight to throughput), the throughput of more-power and no-priority schemes increase with α at higher load. But the delay also increases for more-power (scheme 2) and no-priority schemes (scheme 1). Next, we plot the performance when $N = 5, m = 4$ in Figure 10. The performance for all the schemes improve with increasing N.

E. Game problem: maximizing individual throughputs

Next, we evaluate the performance of distributed game version of the team problems mentioned in previous sections. The notation $m_i = x$ in the figures will mean that the *total* number of mobiles is x and they use scheme i. We first consider the criterion of throughput maximization. For $m = 3$ mobiles, Fig 11 (a), (b) and (c) show the global equilibrium throughput (i.e. the expression in Eq. (12) times the number of mobiles), the equilibrium EDBP and the equilibrium retransmission probabilities, respectively. We observe that the performance of more-power scheme (scheme 2) is the best in terms of EDBP. But at high load, the throughput of the less-power scheme (scheme 3) is best. We observe that the equilibrium retransmission probability is high (close to 1) and thus most aggressive for schemes 1-3 for any arrival probability. Standard Aloha at low arrival rates is less aggressive at equilibrium. A possible explanation



(a)

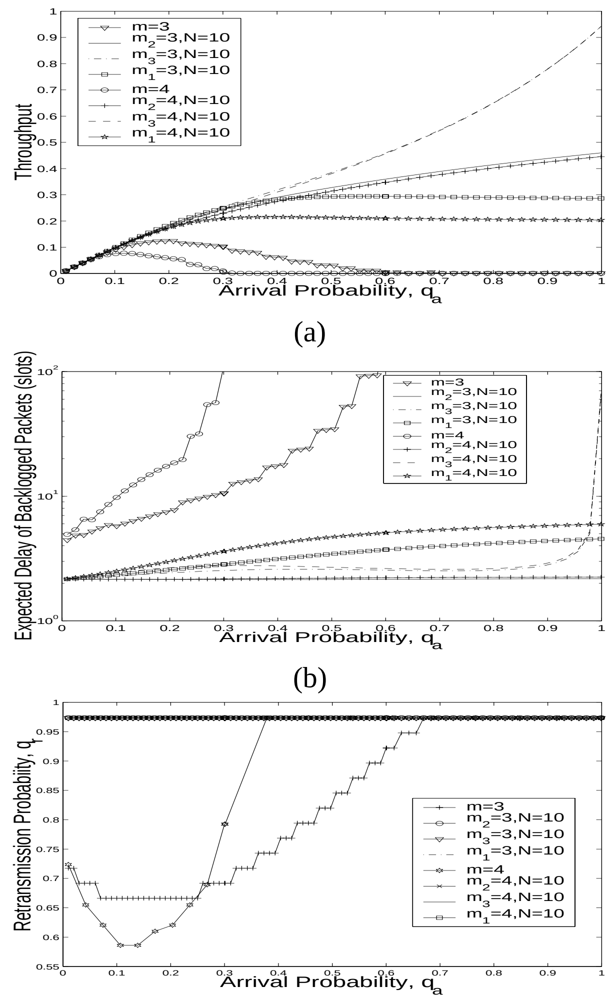
(b)

Fig. 10. (a) and (b) show throughput and delay of backlogged packets when the multi-criteria objective is optimized for $N = 5$.

for this behavior is the following. If an individual tagged mobile were very aggressive in standard Aloha (retransmission probabilities close to 1) then eventually all other mobiles would become backlogged which could increase the collision rate and thus decrease the throughput of the tagged mobile. Hence for some values of arrival probabilities the equilibrium behavior of standard Aloha is not very aggressive. In contrast, the new schemes suffer less from other mobiles becoming backlogged since they can reduce collisions due to the randomization and priorities. Hence increasing backlog of other mobiles does not penalize the tagged station anymore, so it has incentive to become more aggressive. The equilibrium transmission probabilities for schemes 1-3 are constants as function of q_a given by 0.9734 (for $m = 3, 4$).

Similar trends are obtained when increasing the number of mobiles to 4 in Figure 11. The improved performance of the new schemes with respect to standard Aloha appears even with a small number N of levels. We reduce N to 2 in Figure 12, and observe that the new schemes still outperform both standard Aloha as well as scheme 1 for all q_a. The equilibrium throughput of scheme 3 is seen to outperform considerably all other schemes.

A remarkable feature of the new schemes is that the equilibrium throughput is *increasing* in the arrival probabilities, which is a similar behavior as we had in the team problem. In contrast, for high loads the throughput decreases for Scheme 1 and it also contains a decreasing behavior in standard Aloha. Thus the competition in the game formulation does not allow to benefit from increased input rates for standard Aloha and Scheme 1 (except for low values of q_a) whereas the new schemes do benefit from that. Finally, we note that schemes 1-3 all avoid the throughput collapse of standard Aloha (for which we see in Fig. 11 (a) that the equilibrium throughput vanishes for both $m = 4$ at $q_a > 0.3$ and for $m = 3$ at $q_a > 0.65$).



(a)

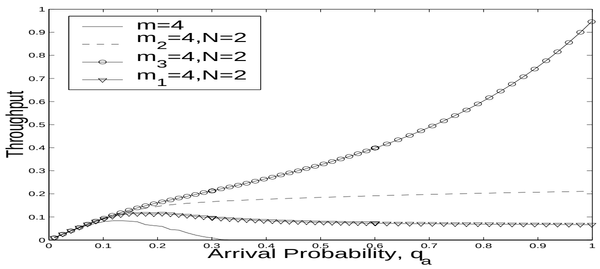
(b)

(c)

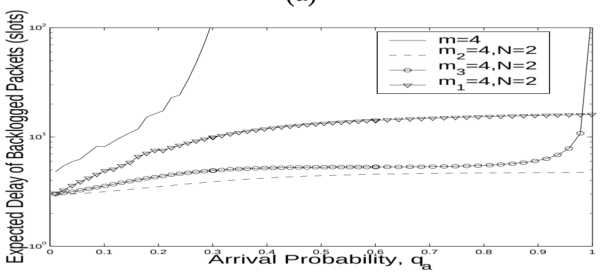
Fig. 11. (a), (b), and (c) show the throughput, EDBP and retransmission probability when the objective is to maximize the throughput for all the schemes. for 3 and 4 mobiles.

F. Game problem: Minimizing individual EDBP

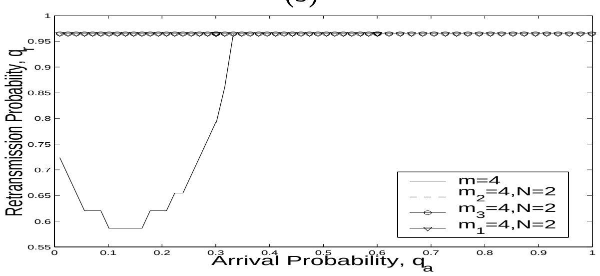
Next, we evaluate the performance of the distributed game problems of minimizing EDBP. We notice again from Figures 13(a) and 14 (for $N = 10$ and $N = 2$ respectively) that the equilibrium throughput decreases in the arrival rates for Scheme 1 (for arrival probabilities larger than 0.5) and for standard Aloha (for arrival probabilities larger than 0.2). In both Schemes 2 and 3 it increases (for both $N = 10$ and $N = 2$) yet the increase is much larger in Scheme 3. This scheme outperforms all others for any q_a. Schemes 1-3 all avoid the throughput



(a)

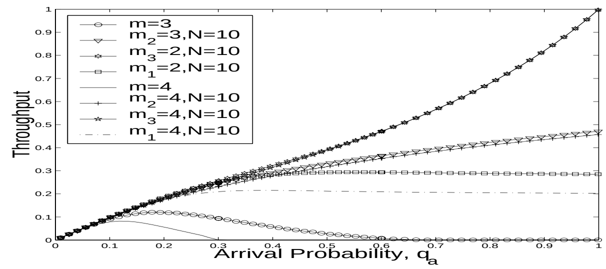


(b)

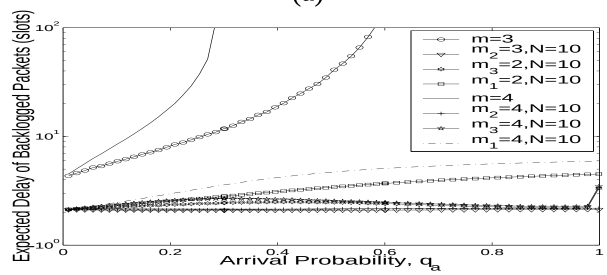


(c)

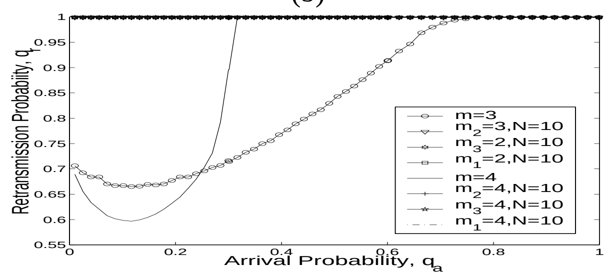
Fig. 12. (a) (b) and (c) show the throughput, EDBP and retransmission probability when the objective is to maximize the throughput for all the schemes for 4 mobiles and 2 levels.



(a)



(b)



(c)

Fig. 13. (a), (b), (c) show the throughput, EDBP and retransmission probability when the objective is to minimize the delay of backlogged packets for all the schemes for 3 and 4 mobiles.

collapse of standard Aloha. We observe the decrease in throughput for schemes 1-3 when N decreases from 10 to 2. This is due to the fact that now the number of mobiles is more than that of power levels and there are more collision events.

We observe a non-monotonic behavior of the equilibrium EDBP for scheme 3 in Figs 13(b) for $N = 10$. According to Eq. (14), this means that as the arrival rate increases, the throughput grows faster than the expected number of backlogged packets. This nonmonotonicity does not occur for $N = 2$ (Fig. 15(a)). Scheme 2 and 3 have very close EDBP which is better than scheme 1 and standard Aloha for all q_a.

As in the throughput maximization, we see that schemes 1-3 are very aggressive in terms of retransmission probabilities for $N = 10$ and $N = 2$ (Figs 13(c) and 15(b), respectively).

An interesting feature to note is that the throughput obtained when maximizing the individual throughput is less than that obtained when minimizing the EDBP. This is due to the fact that we are in a non-cooperative game setting, for which the equilibria are known not to be efficient (as is the case in the famous prisoner's dilemma

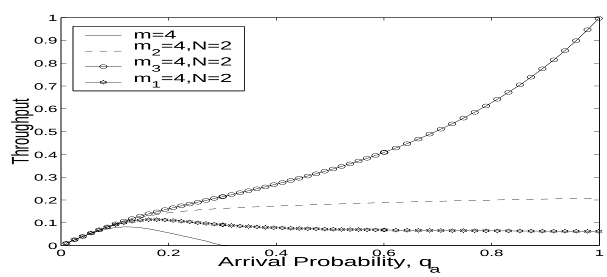


Fig. 14. Equilibrium throughput when the objective is to minimize the individual EDBP for all the schemes for 4 mobiles and $N = 2$ levels.

paradox).

Table III summarizes the performance of the game problems in terms of throughput and EDBP.

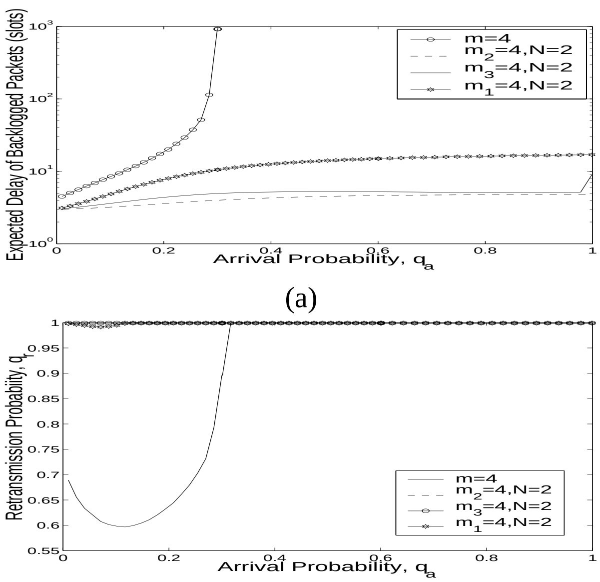
VI. Conclusions

We have studied in this paper two new schemes that involve both prioritization as well as power diversity for increasing the throughput and decreasing the EDBP. We studied optimal choices of transmission probabilities both in a cooperative team setting as well as in a non-cooperative setting modeled using game theory. We saw

Scheme	m = 4 (delay, throughput)		m = 5 (delay, throughput)	
	$q_a = 0.4$	$q_a = 0.9$	$q_a = 0.4$	$q_a = 0.9$
same-power, $\max\lambda$	$(6.8 \cdot 10^3, 1.5 \cdot 10^{-4})$	$(6.9 \cdot 10^3, 1.4 \cdot 10^{-4})$	$(1.3 \cdot 10^5, 7.8 \cdot 10^{-6})$	$(1.3 \cdot 10^5, 7.8 \cdot 10^{-6})$
same-power, $\min$ EDBP	$(6.7 \cdot 10^8, 1.5 \cdot 10^{-9})$	$(6.7 \cdot 10^8, 1.5 \cdot 10^{-9})$	$(5.8 \cdot 10^{11}, 1.7 \cdot 10^{-12})$	$(5.8 \cdot 10^{11}, 1.7 \cdot 10^{-12})$
scheme 1, $N = 1$	$(6.6 \cdot 10^8, 1.5 \cdot 10^{-9})$	$(6.6 \cdot 10^8, 1.5 \cdot 10^{-9})$	$(5.8 \cdot 10^{11}, 1.7 \cdot 10^{-12})$	$(5.8 \cdot 10^{11}, 1.7 \cdot 10^{-12})$
scheme 1, $N = 5$	(5.1, 0.18)	(7.06, 0.16)	(7.34, 0.13)	(9.6, 0.11)
scheme 1, $N = 7$	(4.5, 0.20)	(6.27, 0.18)	(6.23, 0.14)	(8.2, 0.14)
scheme 2, $N = 1$	(17.75, 0.06)	$(6.7 \cdot 10^8, 1.5 \cdot 10^{-9})$	$(5.81 \cdot 10^{11}, 1.7 \cdot 10^{-12})$	$(5.8 \cdot 10^{11}, 1.7 \cdot 10^{-12})$
scheme 2, $N = 5$	(2.33, 0.26)	(2.47, 0.39)	(2.68, 0.359)	(2.53, 0.25)
scheme 2, $N = 7$	(2.21,0.27)	(2.29, 0.42)	(2.32, 0.26)	(2.41, 0.396)
scheme 3, $N = 1$	(42.86,0.07)	$(5.6 \cdot 10^3, 0.09)$	$(4.9 \cdot 10^3, 0.015)$	$(1.8 \cdot 10^5, 0.065)$
scheme 3, $N = 5$	(2.96,0.307)	(2.6,0.814)	(3.18,0.303)	(2.8, 0.813)
scheme 3, $N = 7$	(2.7, 0.31)	(2.4,0.82)	(2.9, 0.31)	(2.5, 0.82)

TABLE III

COMPARISON OF THROUGHPUTS AND THE EDBP DIFFERENT SCHEMES AT DIFFERENT LOADS FOR GAME PROBLEMS, $m = 4$ AND $m = 5$



(b)

Fig. 15. (a) and (b) show the EDBP and retransmission probability when the objective is to minimize the EDBP for all the schemes for 4 mobiles and 2 levels.

that our new Scheme 3 has the best stability properties and the best throughput performance in the game setting. The throughput performance of both new schemes 2 and 3 benefit from increasing the arrival rate in the game scenario, in contrast with the standard Aloha which suffers a throughput collapse, and with the power diversity scheme 1 (without priorities) whose equilibrium throughput decreases in high load. In the team case, our new Scheme 3 is the best in very high load and Scheme 2 is the best in medium load when maximizing throughput; Scheme 2 is best for both medium and high load when minimizing EDBP.

A remarkable feature of our scheme 3 is that it performs very well in the game setting as compared to the team problem. In particular, when maximizing the throughput, we see that in heavy traffic it attains the maximum achievable throughput as is the case for the team formulation.

As a part of future work, we would like to study the impact of pricing on the performance of game problems.

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VII. APPENDIX: TRANSITION PROBABILITIES FOR THE GAME PROBLEM UNDER SCHEME 2

The transition probabilities $P_{(n,a),(n+k,b)}(q_r^o, q_r^{(m+1)}) =$ are given by the following expression:

$$
\begin{cases}
\left.\begin{array}{ll}
Q_a(k,n)q_r^{m+1}[Q_r(0,n) + \sum_{j=1}^{n} Q_r(j,n)A_{j+1}], & a=1, b=0 \\
Q_a(k,n)q_a[Q_r(0,n) + \sum_{j=2}^{n} Q_r(j,n)(1-A_j)], & a=0, b=1 \\
Q_a(k,n)(1-q_a)[Q_r(0,n) + \sum_{j=2}^{n} Q_r(j,n)(1-A_j)], & a=0, b=0 \\
Q_a(k,n)[(1-q_r^{m+1})(Q_r(0,n) + \sum_{j=2}^{n} Q_r(j,n)(1-A_j))+ & \\
q_r^{m+1} \sum_{j=1}^{n} Q_r(j,n)(1-A_{j+1})], & a=1, b=1
\end{array}\right\} k=(m-n)\geq 2 \\
\left.\begin{array}{ll}
Q_a(k,n)q_r^{m+1}[Q_r(0,n) + \sum_{j=1}^{n} Q_r(j,n)A_{j+1}], & a=1, b=0 \\
Q_a(k,n)q_a[Q_r(0,n) + \sum_{j=2}^{n} Q_r(j,n)(1-A_j)]+ & \\
Q_a(k+1,n)q_a[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)A_j], & a=0, b=1 \\
Q_a(k,n)(1-q_a)[Q_r(0,n) + \sum_{j=2}^{n} Q_r(j,n)(1-A_j)]+ & \\
Q_a(k+1,n)(1-q_a)[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)A_j], & a=0, b=0 \\
Q_a(k+1,n)(1-q_r^{m+1})[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)A_j] + Q_a(k,n)[(1-q_r^{m+1})\{Q_r(0,n)+ & \\
\sum_{j=2}^{n} Q_r(j,n)(1-A_j)\} + q_r^{m+1} \sum_{j=1}^{n} Q_r(j,n)(1-A_{j+1})], & a=1, b=1
\end{array}\right\}, 2\leq k < m-n \\
\left.\begin{array}{ll}
q_aQ_a(0,n)Q_r(0,n) + (1-q_a)[Q_a(1,n)\{Q_r(0,n)+ & \\
Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)A_j\} + Q_a(0,n)\{Q_r(0,n) + \sum_{j=2}^{n} Q_r(j,n)(1-A_j)\}] & a=b=0 \\
(1-q_r^{(m+1)})[Q_a(1,n)\{Q_r(0,n) + Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)A_j\} + Q_a(0,n)\{Q_r(0,n)+ & \\
\sum_{j=2}^{n} Q_r(j,n)(1-A_j)\}] + q_r^{m+1}Q_a(0,n)[\sum_{j=1}^{n} Q_r(j,n)(1-A_{j+1})], & a=1, b=1 \\
q_aQ_a(0,n)[\sum_{j=2}^{n} Q_r(j,n)(1-A_j)] + q_aQ_a(1,n)[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)A_j] & a=0, b=1 \\
q_r^{(m+1)}Q_a(0,n)[Q_r(0,n) + \sum_{j=1}^{n} Q_r(j,n)A_{j+1}], & a=1, b=0
\end{array}\right\} k=0 \\
\left.\begin{array}{ll}
(1-q_r^{(m+1)})Q_a(2,n)\Big[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)A_j\Big]+ & \\
Q_a(1,n)\Big[q_r^{m+1}(\sum_{j=1}^{n} Q_r(j,n)(1-A_{j+1})) + (1-q_r^{m+1}) \sum_{j=2}^{n} Q_r(j,n)(1-A_j)\Big], & a=b=1 \\
Q_a(2,n)(1-q_a)[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)A_j] + Q_a(1,n)(1-q_a) \sum_{j=1}^{n} Q_r(j,n)(1-A_j), & a=0, b=0 \\
Q_a(2,n)q_a[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)A_j] + Q_a(1,n)q_a[Q_r(0,n) + \sum_{j=2}^{n} Q_r(j,n)(1-A_j)], & a=0, b=1 \\
Q_a(1,n)q_r^{m+1}[Q_r(0,n) + \sum_{j=1}^{n} Q_r(j,n)A_{j+1}], & a=1, b=0,
\end{array}\right\} k=1 \\
\left.\begin{array}{ll}
q_aQ_a(0,n)[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)A_j], & a=0, b=1 \\
Q_a(0,n)(1-q_r^{m+1})[Q_r(1,n) + \sum_{j=2}^{n} Q_r(j,n)A_j], & a=b=1 \\
Q_a(0,n)(1-q_a)[Q_r(1,n) + \sum_{j=1}^{n} Q_r(j,n)A_j], & a=b=0
\end{array}\right\} k=-1 \\
0 \qquad \text{otherwise}
\end{cases}
$$

VIII. Appendix: Transition probabilities for the game problem under scheme 1

The transition probabilities, $P_{(n,a),(n+k,b)}$ are given by the following expression:

$$
\left\{
\begin{array}{l}
\left.\begin{array}{ll}
Q_a(k,n)q_r^{m+1}\sum\limits_{j=0}^{n}\frac{1}{j+k+1}Q_r(j,n)A_{j+k+1}, & a=1,b=0\\
Q_a(k,n)q_a\sum\limits_{j=0}^{n}Q_r(j,n)(1-A_{j+k+1}), & a=0,b=1\\
Q_a(k,n)[(1-q_a)\sum\limits_{j=0}^{n}Q_r(j,n)(1-A_{j+k})+q_a\sum\limits_{j=0}^{n}\frac{1}{j+k+1}Q_r(j,n)A_{j+k+1}], & a=0,b=0\\
Q_a(k,n)[q_r^{m+1}\sum\limits_{j=0}^{n}Q_r(j,n)(1-A_{j+k+1})+(1-q_r^{m+1})\sum\limits_{j=0}^{n}Q_r(j,n)(1-A_{j+k})], & a=1,b=1
\end{array}\right\} k=(m-n)\geq 2\\[2ex]
\left.\begin{array}{ll}
Q_a(k,n)q_r^{m+1}\sum\limits_{j=0}^{n}Q_r(j,n)\frac{A_{j+k+1}}{j+k+1}, & a=1,b=0\\
Q_a(k,n)q_a\sum\limits_{j=0}^{n}Q_r(j,n)(1-A_{j+k+1})+Q_a(k+1,n)q_a\sum\limits_{j=0}^{n}Q_r(j,n)\frac{(j+k+1)A_{j+k+2}}{j+k+2}, & a=0,b=1\\
Q_a(k,n)[(1-q_a)\sum\limits_{j=0}^{n}Q_r(j,n)(1-A_{j+k})+q_a\sum\limits_{j=0}^{n}Q_r(j,n)\frac{A_{j+k+1}}{j+k+1})]+ & \\
Q_a(k+1,n)(1-q_a)\sum\limits_{j=0}^{n}Q_r(j,n)A_{j+k+1}, & a=0,b=0\\
Q_a(k+1,n)[(1-q_r^{m+1})\sum\limits_{j=0}^{n}Q_r(j,n)A_{j+k+1}+q_r^{m+1}\sum\limits_{j=0}^{n}Q_r(j,n)\frac{j+k+1}{j+k+2}A_{j+k+2}]+ & \\
Q_a(k,n)[(1-q_r^{m+1})\sum\limits_{j=0}^{n}Q_r(j,n)(1-A_{j+k})+q_r^{m+1}\sum\limits_{j=0}^{n}Q_r(j,n)(1-A_{j+k+1})], & a=1,b=1
\end{array}\right\} 2\leq k<m-n\\[2ex]
\left.\begin{array}{ll}
q_r^{m+1}Q_a(0,n)(Q_r(0,n)+\sum\limits_{j=1}^{n}Q_r(j,n)\frac{A_{j+1}}{j+1}), & a=1,b=0\\
q_a[Q_a(0,n)\sum\limits_{j=1}^{n}Q_r(j,n)(1-A_{j+1})+Q_a(1,n)\sum\limits_{j=0}^{n}Q_r(j,n)\frac{(j+1)A_{j+2}}{j+2}], & a=0,b=1\\
q_aQ_a(0,n)(Q_r(0,n)+\sum\limits_{j=1}^{n}\frac{1}{j+1}Q_r(j,n)A_{j+1})+(1-q_a)[Q_a(1,n)(\sum\limits_{j=1}^{n}Q_r(j,n)A_{j+1}+Q_r(0,n))+ & \\
Q_a(0,n)(Q_r(0,n)+\sum\limits_{j=2}^{n}Q_r(j,n)(1-A_j))], & a=0,b=0\\
(1-q_r^{m+1})[Q_a(1,n)\sum\limits_{j=0}^{n}Q_r(j,n)A_{j+1}+Q_a(0,n)(\sum\limits_{j=2}^{n}Q_r(j,n)(1-A_j)+Q_r(0,n))]+ & \\
q_r^{m+1}Q_a(0,n)\sum\limits_{j=1}^{n}Q_r(j,n)(1-A_{j+1}), & a=1,b=1
\end{array}\right\} k=0\\[2ex]
\left.\begin{array}{ll}
(1-q_r^{m+1})Q_a(2,n)\sum\limits_{j=0}^{n}Q_r(j,n)A_{j+2}+Q_a(1,n)[q_r^{m+1}\sum\limits_{j=0}^{n}Q_r(j,n)(1-A_{j+2})+ & \\
(1-q_r^{m+1})\sum\limits_{j=1}^{n}Q_r(j,n)(1-A_{j+1})]i+q_r^{m+1}Q_a(2,n)\sum\limits_{j=0}^{n}Q_r(j,n)\frac{j+2}{j+3}A_{j+3}, & a=1,b=1\\
(1-q_a)[Q_a(2,n)\sum\limits_{j=0}^{n}Q_r(j,n)A_{j+2}+Q_a(1,n)\sum\limits_{j=1}^{n}Q_r(j,n)(1-A_{j+1})]+ & \\
q_aQ_a(1,n)\sum\limits_{j=0}^{n}Q_r(j,n)\frac{A_{j+2}}{j+2}, & a=0,b=0\\
Q_a(2,n)q_a\sum\limits_{j=0}^{n}Q_r(j,n)\frac{(j+2)A_{j+3}}{j+3}+Q_a(1,n)q_a\sum\limits_{j=0}^{n}Q_r(j,n)(1-A_{j+2}), & a=0,b=1\\
Q_a(1,n)q_r^{m+1}\sum\limits_{j=0}^{n}Q_r(j,n)\frac{A_{j+2}}{j+2}, & a=1,b=0
\end{array}\right\} k=1\\[2ex]
\left.\begin{array}{ll}
q_aQ_a(0,n)\sum\limits_{j=1}^{n}Q_r(j,n)\frac{jA_{j+1}}{j+1}, & a=0,b=1\\
(1-q_r^{m+1})Q_a(0,n)(Q_r(1,n)+\sum\limits_{j=2}^{n}Q_r(j,n)A_j)+q_r^{m+1}Q_a(0,n)\sum\limits_{j=1}^{n}Q_r(j,n)\frac{jA_{j+1}}{j+1}, & a=1,b=1\\
Q_a(0,n)(1-q_a)[Q_r(1,n)+\sum\limits_{j=2}^{n}Q_r(j,n)A_j], & a=0,b=0\\
0, & a=1,b=0
\end{array}\right\} k=-1
\end{array}
\right.
$$