Throughput Maximizing Optimal Hop Distance in Dense Mobile Ad Hoc Networks

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Abstract

I. Introduction

It is needless to mention the plenitude of research literature available today on Mobile Ad Hoc Networks (MANETs). Diverse issues about MANETs like medium access scheduling, routing protocols, transmission power control and performance analysis have been the focus of research in the past few years. In this paper, we study the optimal next hop distance that maximizes the *end-to-end flow throughput* in a mobile multi-hop wireless network environment subject to a network average power constraint. In our investigation we assume a spatially dense spreadout of nodes and we incorporate channel gain due to path-loss caused by the mobility of nodes. We consider a *periphery* limited mobility scenario in which nodes are restricted to move in their own local, approximately circular periphery and follow the random waypoint mobility model within this circle. For the calculation of the average throughput with path-loss, this kind of a mobility model leads us to compute the probability density function (PDF) of random distance between two nodes moving inside their local circular periphery. Computation of this PDF constitutes a problem in Geometric Probability Theory and to the best of our knowledge the derivation of PDF of random distance between two circles has never been investigated before. This is thus the first main contribution of this paper. The second main contribution of this paper is that, with reference to the vast literature available on MANETs, ours is the first attempt to derive a throughput maximizing optimal hop distance in a dense ad hoc network environment with mobility.

II. Network and Mobility Model

Consider a multi-hop wireless network with a dense collection of mobile nodes. A contention based distributed channel access mechanism is employed by nodes to be able to schedule packet transmissions.

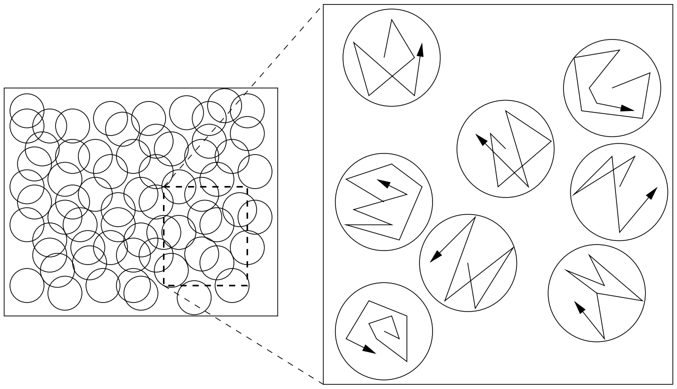

Fig. 1. *Periphery* limited mobility model

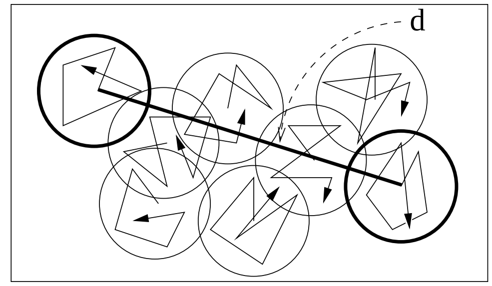

Fig. 2. Consecutive relay nodes donot overlap

The CSMA/CA based distributed coordination function (DCF) is such a mechanism that is commonly used in IEEE 802.11 technology. Assume that for control signalling (such as RTS/CTS in IEEE 802.11), nodes do not employ power control and hence use constant power. We assume a ”single cell” situation in which control packets are heard by all nodes constituting the network and only one transmitter-receiver pair can successfully transmit in any given time slot. It is further assumed that, during control signalling, channel gain estimation at the transmitter side is possible and each transmitter can select the power level for transmission of its data packets.

We assume a *periphery* limited mobility scenario in which the movement of each node is restricted to an approximate circular periphery around itself. Inside their confined area, the movement of nodes is in accordance with the famous random waypoint model. Figure 1 shows such a scenario. We could approximate the periphery by a square or any other shape but a circle is a more natural choice. For the sake of clarity, the magnified box shows only non-overlapping periphery nodes, but actually there are also neighbouring nodes with overlapping peripheries, present, as shown in the left box. We further impose that peripheries of consecutive relay nodes that form a route donot overlap (Figure 2). This kind of a mobility model can be readily applied to various situations. For example, an urban intra-city MANET formed by mobile nodes (people) restricted to move inside the buildings. A similar MANET formed across different rooms of a building by mobile nodes restricted to move in their rooms. Soldiers in a battlefield moving inside their own troops, a group of sensor robots moving in a mine field or nuclear establishments restricted to their confined areas, etc are other examples. With such a mobility model, our goal is to obtain an optimal hop distance d between the periphery centers of two relay nodes (Figure 2) that maximizes an end-to-end flow throughput measure.

Since a transmitter can estimate the channel gain to its intended receiver during control signalling (e.g., RTS/CTS), channel state information can be used by the transmitter to apply power control during each transmission oppurtunity. In our model, we assume that power control is applied in order to compensate for the channel gain due to varying path-loss caused by mobility of nodes. If the channel gain due to path-loss observed by a transmitter is denoted by h and $P(h)$ denotes the corresponding power control applied, the achieved transmission rate by Shannon's formula is then given by

$$C(h) = Wlog_2\left(1 + \frac{P(h)h\alpha}{\sigma^2}\right)$$

where, W is the RF bandwidth, σ^2 is AWGN power and α is a constant depending on the far field reference distance d_0. (**MENTION ABOUT d_0 and feasible hop distance and Node saturation assumption**.)

III. Background and Problem Objectives

With multi-hop communication between a source and destination taking place in such a scenario, there is an inherent tradeoff between employing shortest path (in terms of number of hops) routing with high transmission power and many small hops with low transmission power. Shortest path routing with high transmission power can account for minuscule end-to-end packet delays but at the same time can easily result in frequent link failures and fast depletion of battery energy. Instead, use of small hops with low transmission power can overcome the disadvantages of the former case, but on the other side, floods the network with relatively much higher number of packets. These arguments clearly illustrate the need for an optimal power control policy combined with an optimal hop distance choice. Such an optimization problem for fixed multi-hop networks has been studied by the authors in [1] in which they obtain optimal power control and hop distance that maximizes an end-to-end flow throughput measure in bit-metres/sec. In this paper we use a slightly modified version of the results obtained for fixed networks in [1]. Our network model is similar to the model used in [1] augmented by our mobility model described before in Section II. For the sake of clarity we briefly re-visit the derivation of results in [1] to deduce a slightly modified version for our use. In the rest of the paper, when we say that the distance between two nodes is d, we always actually mean that the distance between the centers of the periphery circles of the two nodes is d.

A. *Objective function with end-to-end flow throughput*

Consider a source and a destination that are distance D apart (actually whose periphery circle centers are distance D apart, as mentioned in the previous paragraph) and engage intermediate relay nodes for multi-hop communicaton. By the dense node spreadout assumption, we can always find a multihop path between a source and destination such that the periphery centers of the relay nodes lie on the straight line connecting them. We assume that for optimal performance, the distances between consecutive relay nodes are all equal to d metres. Let $\Theta(d)$ denote the aggregate throughput of all source-destination pairs in the network. We consider the *fixed transmission time* case [1] along with channel gain due to variable path loss caused by mobility of nodes. By fixed transmission time it is meant that, upon winning channel access, a node is allowed to transmit only for a fixed amount of time T irrespective of the channel condition. If power control is applied during a transmission oppurtunity then the node will be able to transmit only $L(h) := C(h)T$ amounts of data. Moreover, the data transmission rate in the network would be given by (see [2], [3])

$$\Theta(d) = \frac{p_s \int_h L(h)\, g_H(h)\ dh}{p_i T_i + p_c T_c + p_s \left(T_o + p_{tr}\frac{L(h)}{C(h)}\right)}$$

where, p_i is the probability that the contention period goes idle, p_s is the probability that there is a successful transmission, p_c is the probability that there is a collision, T_i is the average idle time, T_o and T_c are fixed overheads associated with a successful transmission and a collision, respectively, and $(1-p_{tr})$ corresponds to the fraction of T seconds long transmission oppurtunities that were left idle due to bad channel condition.

If we suppose that all source-destination pairs are distance D apart then there are approximately $\frac{D}{d}$ hops for a pair and the end-to-end aggregate flow throughput for the whole network is given by $\frac{\Theta(d)}{\frac{D}{d}} = \frac{\Theta(d)}{D}d$. However for each flow that is beamed across distance D, the end-to-end flow throughput is given by $\frac{\Theta(d)}{\frac{D}{d}} \times D = \Theta(d)\, d$ in bit-metres/sec. The objective function that is to be maximized can therefore be defined as $\phi(P(h), d) \triangleq \Theta(d)\, d$. Power control $P(h)$ is applied to indemnify the effects of randomly varying path-loss due to the random distance seperation between consecutive relay nodes. Hence $P(h)$ depends only on this random distance that we denote by l and not on the distance between the periphery centers d. Maximizing $\phi(P(h), d)$ can therefore be isolated into first maximizing over $P(h)$ and then maximizing over d. Consequently we seek to solve the following problem

$$\max_d \max_{P(h):E[P(h)]\leq \bar{P}} \quad \phi(P(h), d)$$

or

$$\max_d \max_{P(h):E[P(h)]\leq \bar{P}} \quad \frac{p_s \left(\int_h L(h)\, g_H(h)\ dh\right)\ d}{p_i T_i + p_c T_c + p_s\left(T_o + p_{tr} T\right)} \tag{1}$$

Consider the 'Channel Left Idle when Bad' case [1] when the channel is left idle for T seconds if the power $P(h)$ allocated by the transmitter is 0 for any channel state h or equivalently the case of $p_{tr} = 1$

for $P(h) = 0$. Then we see that the denominator of the objective function $\phi(P(h), d)$ in Equation 1 does not depend on $P(h)$ and d. Thus maximizing $\phi(P(h), d)$ over $P(h)$ reduces to maximizing

$$\int d \cdot L(h) \cdot g_H(h)\, dh = \frac{W\,T\,d}{ln2} \int ln \left(1 + \frac{P(h)h\alpha}{\sigma^2}\right) g_H(h)\, dh$$

over $P(h)$, subject to an average network power constraint given by $\int P(h) g_H(h)\, dh \;\le\; \bar{P}$. This is however a well known optimization problem that has a water-pouring form solution given by

$$P(h) \;=\; \left(\frac{W\,T\,d}{\lambda\, ln2} - \frac{\sigma^2}{h\alpha}\right)^+$$

where λ is obtained from $\int P(h) g_H(h)\, dh \;=\; \bar{P}$. For computing λ, in the following section we determine the path-loss distribution $g_H(h)$ that incorporates the effect of path-loss h due to nodes moving randomly according to our mobility model described previously.

IV. Obtaining the path-loss distribution

In the network model described before in Section II, we assume a spatially dense spreadout of nodes with the periphery of nodes overlapping with each other. However, according to the feasibility of hop distance assumption $d \ge d_o$, the circular peripheries of consecutive relay nodes, constituting a route in a multihop connection, donot overlap. For simplicity, let the radius of circular periphery of all nodes be same and equal to a meters. Consider two circles $\mathcal{C}_1$ and $\mathcal{C}_2$ of radius a centered at $(0, 0)$ and $(d, 0)$ with two consecutive relay nodes moving according to the random waypoint model, one in each one of them. Packets are beamed between this transmitter-receiver pair after the transmitter wins a transmission oppurtunity at the end of a contention attempt. At the next transmission oppurtunity, either only the transmitter or only the receiver or both or none would have moved to another randomly chosen point(s) in the circle(s). We assume that nodes have relatively large pause times as compared to the average time interval between two consecutive transmission oppurtunities of a node pair. We also assume that the period of movement (time interval between initial position and final position) is small as compared to the pause times. Then, the path-loss between a transmitter and receiver will be random owing to the random distance seperation phenomenon between the two nodes. In the following part of this section we first compute $f_L(l)$ the PDF of random distance between two circles and then later compute the path-loss distribution $g_H(h)$ from $f_L(l)$.

A. *PDF of random distance between two circles*

Let the distance between two randomly chosen points p_1 and p_2 in each of the two circles $\mathcal{C}_1$ and $\mathcal{C}_2$ shown in Figure **??** be denoted by l. Then the probability density for this random distance l can be

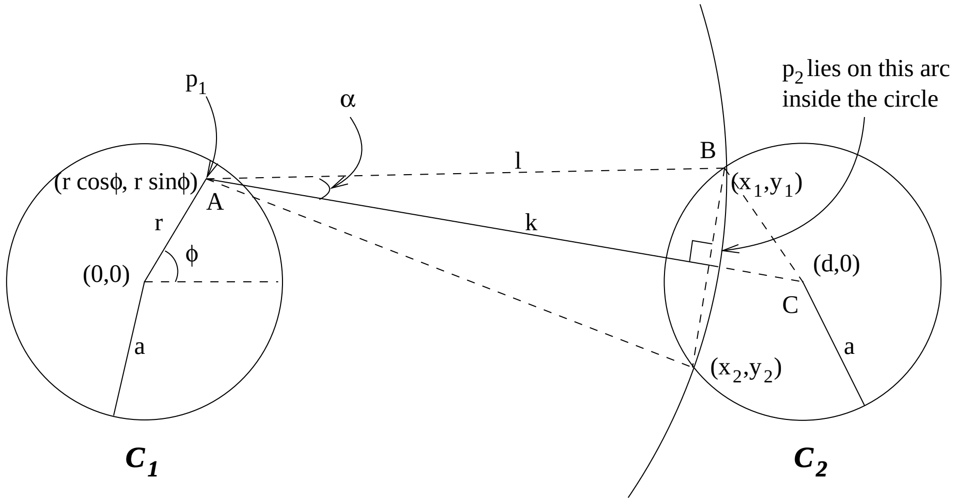

Fig. 3. Random distance between circular periphery of two consecutive relay nodes

computed as

$$f_L(l) = \frac{\int_{\mathcal{C}_1} d\vec{p}_1 \int_{\mathcal{C}_2} \delta(|\vec{p}_1 - \vec{p}_2| - l)\, d\vec{p}_2}{\int_{\mathcal{C}_1} d\vec{p}_1 \int_{\mathcal{C}_2} d\vec{p}_2}$$

where $\delta(\cdot)$ is Dirac's delta function. For a fixed l, the term $\int_{\mathcal{C}_2} \delta(|\vec{p}_1 - \vec{p}_2| - l)\, d\vec{p}_2$ in the numerator of the equation above, represents the length of an arc of the circumference of a circle of radius l centered at $\vec{p}_1$, that lies inside $\mathcal{C}_2$. Referring to the geometry shown in Figure 3, if k is the distance from the center of the circle of radius l, centered at $p_1 = (rcos\phi, rsin\phi)$, to the line joining points of its intersection with $\mathcal{C}_2$, then the length of the arc inside $\mathcal{C}_2$ is given by $2\,l\,\alpha$, where $\alpha = \arccos\left(\frac{k}{l}\right)$. Using polar coordinates and $d\vec{p}_1 = r\,dr\,d\phi$, the numerator can thus be written as $2\,l \int_{\mathcal{C}_1} \arccos\left(\frac{k}{l}\right) r\,dr\,d\phi$. The denominator is simply the product of the areas of $\mathcal{C}_1$ and $\mathcal{C}_2$ given by $\pi^2 a^4$. For computing k we proceed as follows. Consider the two circles centered at $(rcos\phi, rsin\phi)$ and $(d, 0)$ with radii l and a, respectively. Denote the difference between their x coordinates as $e = d - rcos\phi$, difference between their y coordinates as $f = 0 - rsin\phi$ and distance between their centers by $p = \sqrt{e^2 + f^2}$. Now in $\triangle$ABC, the cosine formula for triangles gives $a^2 = l^2 + p^2 - 2\,l\,p\,\cos\alpha$. But we also have $\cos\alpha = \frac{k}{l}$ and this gives the distance between center of first circle and line joining points of their intersection as

$$k = \frac{p^2 + l^2 - a^2}{2p}$$

Note that l can vary as $d - 2a \leq l \leq d + 2a$. Now, from previous discussion, the distribution $f_L(l)$ can be written as

$$f_L(l) = \frac{2\,l}{\pi^2 a^4} \int\int_{\mathcal{C}_1{}^o} r \;\arccos\left[\frac{d^2+r^2-2\,d\,r\;cos\phi+l^2-a^2}{2l\sqrt{d^2+r^2-2\,d\,r\;cos\phi}}\right]\;dr\;d\phi \tag{2}$$

where $\mathcal{C}_1{}^o$ is a sub-region of the circle $\mathcal{C}_1$ given by $cos\phi \geq \frac{d^2+r^2-(l+a)^2}{2dr}$ for $d-2a \leq l < d$ and $cos\phi \leq \frac{d^2+r^2-(l-a)^2}{2dr}$ for $d < l \leq d+2a$. The sub-regions are derived using the bounds $l-a \leq p$ and $l+a \geq p$ for the two circles to intersect. We have not been able to integrate $f_L(l)$ to obtain a closed form expression and hence we will pursue a numerical analysis in Section V.

B. Path-loss distribution as transformation of $f_L(l)$

The path-loss h for a transmission distance l is given by $h = \frac{1}{l^\eta}$. Since l is randomly changing due to mobility of nodes, the transmissions encounter random path-loss. For mathematical convenience let h be defined as $h = \left(\frac{d}{l}\right)^\eta$, then the path-loss distribution $g_H(h)$ can be computed as

$$g_H(h) = f_L\left(\frac{d}{h^{\frac{1}{\eta}}}\right)\left|\frac{-d}{\eta \cdot h^{1+\frac{1}{\eta}}}\right|$$

From Equation 2, $g_H(h)$ is thus given by

$$g_H(h) = \frac{2}{\pi^2 a^4}\,\frac{d}{h^{\frac{1}{\eta}}}\int\int_{\mathcal{C}_1{}^o} r\;arccos\left[\frac{d^2+r^2-2\,d\,r\;cos\phi+\frac{d^2}{h^{\frac{2}{\eta}}}-a^2}{2\,\frac{d}{h^{\frac{1}{\eta}}}\,\sqrt{d^2+r^2-2\,d\,r\;cos\phi}}\right]\;dr\;d\phi$$

where $\mathcal{C}_1{}^o$ is the region $cos\phi \geq \frac{d^2+r^2-\left(\frac{d}{h^{1/\eta}}+a\right)^2}{2dr}$ for $1 < h \leq \left(\frac{d}{d-2a}\right)^\eta$ and $cos\phi \leq \frac{d^2+r^2-\left(\frac{d}{h^{1/\eta}}-a\right)^2}{2dr}$ for $\left(\frac{d}{d+2a}\right)^\eta \leq h < 1$.

V. Optimal Hop Distance by Numerical Analysis

Having obtained the path-loss distribution in the previous section, we now obtain the optimal hop distance d with the help of numerical analysis since we have not been able to symbolically integrate $g_H(h)$ and obtain a closed form expression.

A. Graphs obtained from numerical analysis

VI. Conclusion

References

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