A game theoretic approach for delay minimization in slotted aloha

Eitan Altman[1], Dhiman Barman[2], Rachid El Azouzi[3], Tania Jiménez[4]

Abstract— **This paper studies distributed choice of retransmission probabilities in slotted ALOHA. Both the cooperative team problem as well as the noncooperative game problem are considered. In previous work that has focused on the maximization of throughput, it was shown that in heavy load, this maximization is obtained at the cost of a huge delay of backlogged packets. This motivates us to investigate the delay minimization problem as well as the multicriterion problem of minimizing the average expected delay (or maximizing the throughput) subject to constraints on the expected delay of backlogged packets. A Markov chain analysis is used to obtain optimal and equilibrium retransmission probabilities and expected delays. analysis.**

I. Introduction

Aloha [2] and slotted Aloha [7] have long been used as random distributed medium access protocols for radio channels. They are used in both satellite as well as cellular telephone networks for the sporadic transfer of data packets. In these protocols, packets are transmitted sporadically by various users. If packets are sent simultaneously by more than one user then they collide. After the end of the transmission of a packet, the transmitter receives the information if the transmission was successful. All packets involved in a collision are assumed to be corrupted and are retransmitted after some random time. We focus in this paper on the slotted Aloha (which is known to have a better achievable throughput than the unslotted version, [3]) in which time is divided into units. At each time unit a packet may be transmitted, and at the end of the time interval, the sources get the feedback on whether there was zero, one or more transmissions (collision) during the time slot. A packet that arrives at a source is immediately transmitted. Packets that are involved in a collision are backlogged and are scheduled for retransmission after a random time.

The determination of the above random time can be considered as a stochastic control problem. The information structure, however, is not a classical one: sources do not have full state information as they do not know how many packets are backlogged. Nor do they know how many packets have been involved in a collision. We study this control problem in two different frameworks:

a) as a team problem, i.e. where there is a common goal to all nodes in the network.

b) as a problem in a noncooperative framework: each node wishes to maximize its own performance measure. This gives rise to a game theoretical formulation.

In a previous paper [1], we have already considered both these frameworks where the optimization performance objective was the maximization of throughput. However we observed that the optimal retransmission probabilities in the team context in heavy traffic were close to zero, which meant that during most of the time, either all mobiles or all but one mobiles were backlogged and remained so during a very long periods. The expected delay of backlogged packets were very large. This motivates us to pose the problem of delay minimization in slotted ALOHA as well as the multicriterion problem of minimizing the average expected delay (or maximizing the throughput) subject to constraints on the expected delay of backlogged packets only.

Previous game formulations of the slotted ALOHA have been proposed in [4], [5], [6]. In the last two references, a full information game is considered, in which each user knows how many backlogged packets there are. Moreover, it is assumed in [5], [6] that a packet that is to be transmitted for the first time waits for a random time in the same way as a backlogged packet. Our goal is to study the slotted Aloha avoiding these two assumptions; relaxing the assumptions allows us to model more accurately the original versions of Aloha, and in particular, relaxing the first assumptions allows for more distributed implementations of Aloha.

The structure of the paper is as follows. In Section II we formulate and solve the team problem. This analysis is used in Section III to numerically study the team solutions. The multicriterion problem is studied in Section IV, and the game problem is then introduced and studied in Sections V and VI.

II. Model and problem formulation

A. Team Problem

We use a Markovian model based on [3, Sec. 4.2.2]. We assume that there are a finite number of sources without buffers. The arrival flow of packets to source i follows a Bernoulli process with parameter q_a (i.e. at each time slot, there is a probability q_a of a new arrival at a source, and all arrivals are independent). As long as there is a packet at a source (i.e. as long as it is not successfully transmitted) new packets to that source are blocked and lost. The arrival processes to different sources are independent. A backlogged packet at source i is retransmitted with probability q_r^i. We shall restrict in our control and game problems to simple policies in which q_r^i does not change in time. Since sources are symmetric, we shall further restrict to finding a symmetric optimal solution, that is retransmission probabilities q_r^i that do not depend on i.

For any choice of values $q_r^i \in (0, 1]$, we obtain a Markov

[1] INRIA,2004 Route des Lucioles, 06902 Sophia Antipolis Cedex, France. The work of this author was partially supported by the Euro NGI network of excellence.

[2] 111 Cummington Street, Department of Computer Science, Boston University, Boston, MA 02215

[3] LIA/CERI, Univesité d'Avignon, BP 1228, 84911, Avignon, France

[4] C.E.S.I.M.O., Facultad de Ingeniería, Universidad de Los Andes, Mérida, Venezuela

chain that contains a single ergodic chain (and possibly transient states as well). We shall use as the state of the system the number of backlogged packets at the beginning of a slot, and denote it frequently with n. Assume that there are n symmetric backlogged packets using the same value q_r as retransmission probability. Let $Q_r(i,n)$ be the probability that i out of the n backlogged packets retransmit at the slot:

$$Q_r(i,n) = \binom{n}{i}(1-q_r)^{n-i}q_r^i \qquad (1)$$

Assume that m is the number of nodes and let $Q_a(i,n)$ be the probability that i unbacklogged nodes transmit packets in a given slot (i.e. that i arrivals occurred at nodes without backlogged packets). Then

$$Q_a(i,n) = \binom{m-n}{i}(1-q_a)^{m-n-i}q_a^i. \qquad (2)$$

And let $Q_r(1,0) = 0$ and $Q_a(1,m) = 0$. In case all nodes use the same value of q_r, the transition probabilities of the Markov chain are given by [3, eq. 4.3]:

$$P_{n,n+i}(q) = \begin{cases} Q_a(i,n), \quad 2 \le i \le m-n, \\ Q_a(1,n)[1-Q_r(0,n)], \quad i=1, \\ Q_a(1,n)Q_r(0,n) + Q_a(0,n)[1-Q_r(1,n)], \quad i=0, \\ Q_a(0,n)Q_r(1,n), \quad i=-1, \end{cases}$$

Let us denote by $\pi_n(q)$ the equilibrium probability that the network is in state n (number of backlogged packets at the beginning of a slot). Then we have the equilibrium state equations:

$$\begin{cases} \pi(q) = \pi(q)P(q), \\ \pi_n(q) \ge 0, n = 0, ..., m \\ \sum_{n=0}^{m} \pi_n(q) = 1. \end{cases} \qquad (3)$$

A solution of (3) can be obtained by computing recursively the steady state probabilities, as in Problem 4.1 in [3]. The team problem is therefore given as the solution of the optimization problem:

$$\min_q \ Objective(q) \ s.t. \begin{cases} \pi(q) = \pi(q)P(q), \\ \pi_n(q) \ge 0, n = 0, ..., m \\ \sum_{n=0}^{m} \pi_n(q) = 1. \end{cases} \qquad (4)$$

When we wish to maximize the throughput, then objective to minimize is minus the throughput; we considered that in [1]. In this paper we shall consider two different objectives related to the expected delay, see Sections II-B-II-C. We shall therefore exclude $q_r = 0$ and optimize only on the range $\epsilon \le q_r \le 1$ (see [1]). We choose throughout the paper $\epsilon = 10^{-5}$. Next we give the performance measures of interest to optimize, as a function of the steady state probabilities:

B. Expected delay of transmitted packets (E.D.T.P)

We shall express the expected delay of transmitted packets of the team problem, as a function of the steady state probabilities. In order to calculate it, first we compute the average number of backlogged packets which is given by

$$S(q) = \sum_{n=0}^{m} \pi_n(q)n \qquad (5)$$

Second, the system throughput (defined as the sample average of the number of packets that are successfully transmitted) is given almost surely by the constant [1]:

$$thp(q) = q_a \sum_{n=0}^{m} \pi_n(q)(m-n) = q_a(m - S(q))$$

Last, the expected delay of transmitted packets D, is defined as the average time, in slots, that a packet takes from its source to the receiver. Applying Little's result, this is given by

$$D(q) = 1 + S(q)/thp(q) \qquad (6)$$

Note that the first term accounts for the first transmission from the source.

C. Expected delay of backlogged packets (E.D.B.P)

Another relevant quantity in this context is the expected delay of backlogged packets D^c which is defined as the average time, in slots, that a backlogged packet takes to go from the source to receiver. To compute it, we need to calculate the total throughput of backlogged packets which is defined as the sample average of the number of new arriving packets that become backlogged. This is given by

$$thp^c(q) = \sum_{n=0}^{m-1}\sum_{i=1}^{m-n} P_{n,n+i}(q) i \pi_n(q) \qquad (7)$$

Applying Little's result, the expected delay of packets that arrive and become backlogged is given by

$$D^c(q) = 1 + S(q)/thp^c(q) \qquad (8)$$

III. Numerical Investigation of the team problem

In this section we shall obtain the retransmission probabilities which solve the team problem. We investigate the dependence of the expected delay of packets that are successfully transmitted (i.e., all packets that are not rejected when arriving at the system) and of packets that upon arrival become backlogged, on the arrival probabilities q_a and on the number of nodes.

A. Expected Delay of Transmitted packets

We consider the team problem in which the performance to optimize is the expected delay of packets that are successfully transmitted (not rejected upon arrival) given by (6) for team problem. In Figure 1 and 2, the expected delay of transmitted packets and the optimal retransmission probabilities are plotted versus the arrival probabilities q_a for the team problem. In Figure 1, we observe that expected delay of transmitted packets decreases when q_a increases after certain value of q_a which depends on number of mobiles, for example, for $m = 2$ $q_a \approx 0.8$. In fact, in heavy traffic or for large number of mobiles, the optimal retransmission policy is seen to be ϵ (see figure 2) : as the system becomes more congested (larger arrival probability or large number of mobiles) the retransmission probability decreases so as to counter increased expected collisions. Thus, the steady probabilities π are then close to $\pi_m = 1/2$, $\pi_{m-1} = 1/2$ and $\pi_n = 0$, $\forall n < m-1$. Hence the expected delay of transmitted packets is given by:

$$D = 1 + \sum_{n=0}^{m} \pi_n n \Bigg/ \left(q_a(m - \sum_{n=0}^{m} \pi_n n) \right) \approx 1 + \frac{2m-1}{q_a}$$

which depends only on q_a and is seen to decrease in q_a.

B. Expected Delay of Backlogged packets

Now we consider the team problem in which the performance to optimize is the expected delay of backlogged packets which given by (8). In Figure 3 and 4, the expected delay of backlogged packets and the optimal retransmission probabilities are plotted versus the arrival probabilities q_a for the team problem.

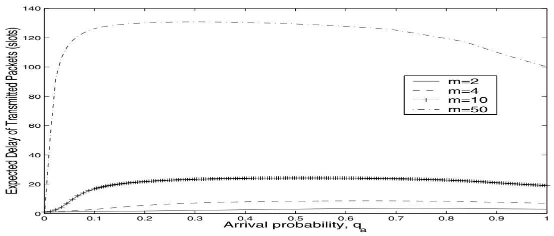

Fig. 1. Expected delay of packets that are successfully transmitted for the team case as a function of the arrival probabilities q_a for $m = 2, 4, 10, 50$.

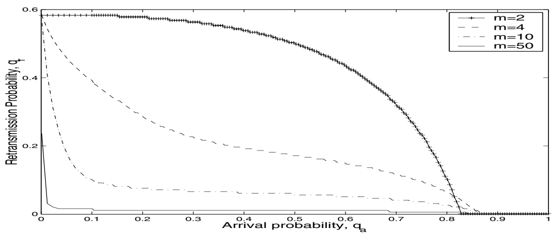

Fig. 2. The retransmission probability in the team case as a function of the arrival probabilities q_a for $m = 2, 4, 10, 50$.

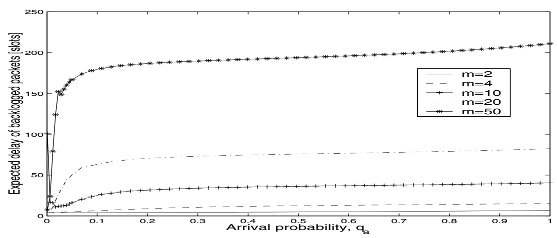

Fig. 3. Expected delay of backlogged packets for the team case as a function of the arrival probabilities q_a for $m = 2, 4, 10, 20, 50$.

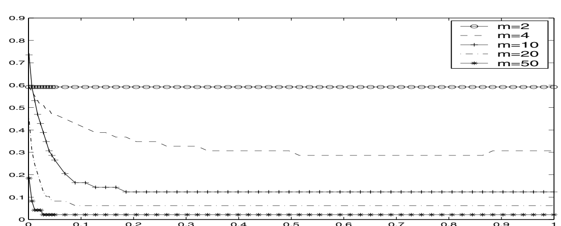

Fig. 4. The retransmission probability in the team case as a function of the arrival probabilities q_a for $m = 2, 4, 10, 20, 50$.

C. Comparison

Now we calculate and compare the performance in team problem for different scenarios: team problem with E.D.T.P and team problem with E.D.B.P. In Figure 5, we plot the throughput at optimal q_r for the team problem with E.D.T.P and E.D.B.P versus the arrival probability q_a for $m = 10$ respectively. As shown in Figure 5, the throughput of the team problem with E.D.B.P is inefficient for large arrival probabilities. The retransmission probability remain almost constant when the arrival probability increases, which explains the decrease in the system's throughput. This can be expected since in the team problem with E.D.B.P objective, we concentrate our objective to the delay of backlogged packets only and not on the average delay. Another interesting observation that can be made comparing the expected delay of backlogged packets in E.D.T.P problem and E.D.B.P problem. Figure 6 shows the expected delay of backlogged packets at optimal

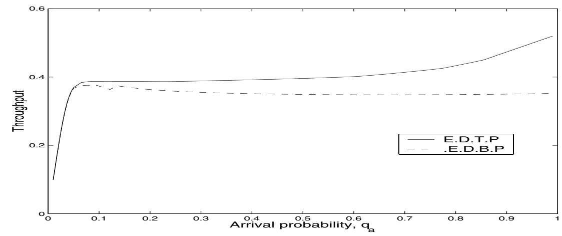

Fig. 5. Throughput at optimal retransmission q_r for the team case for the problem of minimizing E.D.T.P and E.D.B.P, respectively, as a function of the arrival probabilities q_a for $m = 10$

retransmission q_r obtained respectively by E.D.T.P problem and E.D.B.P problem, as function of the arrival probabilities q_a for $m = 10$. We observe that minimizing the expected delay of packets that are successfully transmitted results in a dramatic increase in the expected delay of backlogged packets when the arrival probability approaches 1; (recall that we constrained the retransmission probabilities to be larger than 10^{-5}, otherwise we may expect the average delay of backlogged packet to converge to infinity.) This can be expected since in heavy traffic, the delay of transmitted packets is minimized precisely at the expense of expected delay of the backlogged packets. When the number of mobiles increases, we observe that the expected delay of backlogged packets increases and difference in performance of E.D.T.P and E.D.B.P is more prominent. With more mobiles, the increased intensity of new arrivals of packets which are transmitted at the cost of backlogged packets leads to even higher delay of backlogged packets.

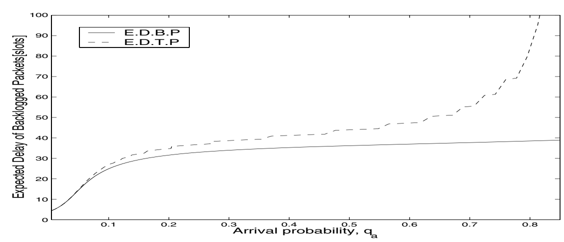

Fig. 6. The expected delay of backlogged packets at optimal retransmission q_r for the team case with the problems of minimizing E.D.T.P and E.D.B.P respectively, as a function of the arrival probabilities q_a for $m = 10$

IV. SEVERAL QOS CRITERIA

As shown in last section, the delay of backlogged packets become very large in heavy traffic when our objective is to minimize the average delay of all transmitted packets (or equivalently, when maximizing the average throughput). Thus we can distinguish two separate QoS criteria: the total average delay as well as the delay of backlogged packets only. We then consider the team problem of minimizing the expected average delay of transmitted packets subject to a constraint on the expected delay of backlogged packets. The new problem that has a new QoS constraint is therefore formulated as

$$\min_q \ D(q) \ s.t. \ D^c(q) \leq d \tag{9}$$

d is some constant. We shall denote by $d_{\max}$ the smallest value of d that insures that there is a solution to (9) for any value of q_a. (As we shall see, there are indeed values of d for which, for some q_a, there is no solution to (9)).

Note that due to (6), maximizing the throughput is equivalent to minimizing $S(q)$. Hence, we can deduce the fact that maximizing the throughput is equivalent to minimizing the expected delay. Therefore the retransmission probabilities that solve (9) also solve the problem $\max_q \; thp(q) \; s.t. \; D^c(q) \leq d$. In Figures 7-8, we plot the expected delay of transmitted

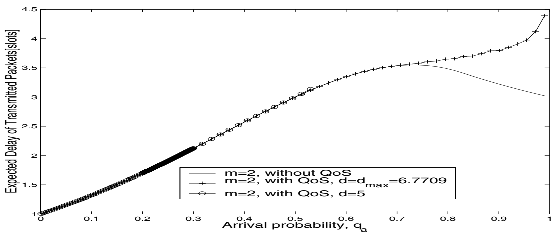

Fig. 7. Expected delay of packets that are successfully transmitted for the team case as function of arrival probabilities with/without delay constraint, m=2, and d=5, d$_{\max}$=6.7709

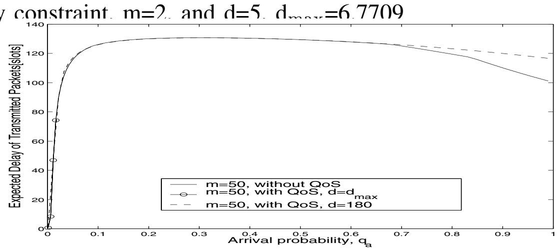

Fig. 8. The expected delay of transmitted packets of transmitted packets in the team E.D.T.P as function of arrival probabilities with/without delay constrain for $m = 50$ and d=180, d$_{\max}$=220

packets for different values of m (i.e., $m = 2, 50$) with and without QoS constraints (9). In all the above figures, we observe that beyond some value of q_a, the expected delay of transmitted packets without QoS constraint becomes less than that with QoS constraints. This is indeed expected since we know that in absence of the constraint, minimizing the average delay is possible by inducing very large delays of backlogged packets; the new constraint does not allow us to decrease the average delay at the expense of the backlogged packets. We note however that in spite of the QoS constraint, we don't observe much performance degradation in the expected delay of transmitted packets.

Also, we observe that for some values of d, we don't have any feasible strategy satisfying the QoS for some values of q_a. For example, for $m = 2$ and $d = 5$, the feasible region of strategies is not empty only for $q_a \leq 0.3$, but for $d = d_{\max}$, there exists a feasible region for all values of q_a because $d_{\max}$ is chosen to be the maximum expected delay of backlogged packets at optimal retransmission strategy for $q_a \in [\epsilon, 1]$ which explains that there exists at least one feasible strategy for every q_a.

V. The Game Problem

Next, we formulate the game problem. For a given policy vector $\mathbf{q_r}$ of retransmission probabilities for all users (whose jth entry is q_r^j), define $([\mathbf{q_r}]^{-i}, \hat{q}_r^i)$ to be a retransmission policy where user j retransmits at a slot with probability q_r^j for all $j \neq i$ and where user i retransmits with probability $\hat{q}_r^i$. Each user i seeks to optimize her own objective $Objective_i$. The problem we are interested in is then to find a symmetric equilibrium policy $\mathbf{q_r^*} = (q_r, q_r, .., q_r)$ such that for any user i and any retransmission probability q_r^i for that user,

$$Objective_i(\mathbf{q_r^*}) \leq Objective_i([\mathbf{q_r^*}]^{-i}, q_r^i) \tag{10}$$

Since we restrict to symmetric $\mathbf{q_r^*}$, we shall also identify it (with some abuse of notation) with the actual transmission probability (which is the same for all users). Next we show how to obtain an equilibrium policy. We first note that due to symmetry, to see whether $\mathbf{q_r^*}$ is an equilibrium it suffices to check (10) for a single player. We shall thus assume that there are $m+1$ users all together, and that the first m users retransmit with a given probability $\mathbf{q_r}^{-(m+1)} = (q_r^o, .., q_r^o)$ and user $m+1$ retransmits with probability $q_r^{(m+1)}$. Define the set

$$\mathcal{Q}^{m+1}(\mathbf{q_r^o}) = \arg\min \left(Objective_{m+1}([\mathbf{q_r^o}]^{-(m+1)}, q_r^{(m+1)}) \right) \tag{11}$$

where $\mathbf{q_r^o}$ denotes (with some abuse of notation) the policy where all users retransmit with probability q_r^o, and where the maximization is taken with respect to $q_r^{(m+1)}$. Then q_r^* is a symmetric equilibrium if

$$q_r^* \in \mathcal{Q}_r^{m+1}(q_r^*) \tag{12}$$

To compute $Objective_{m+1}([\mathbf{q_r^o}]^{-i}, q_r^i)$, we introduce again a Markov chain with a two dimensional state. The first state component corresponds to the number of backlogged packets among the users 1,...,m, and the second component is the number of backlogged packets (either 1 or 0) of user $m+1$. We use the transition probabilities $P_{(n,i),(n+k,j)}(q_r^o, q_r^{(m+1)})$ as given in ([1], Section 2). In the game problem, the average number of backlogged packets of source m+1 is given by:

$$S_{m+1}(\hat{\mathbf{q}}_{m+1}) = \sum_{n=0}^{m} \pi_{n,1}(\hat{\mathbf{q}}_{m+1}) \tag{13}$$

where $\hat{\mathbf{q}}_{m+1} = ([\mathbf{q_r^o}]^{-(m+1)}, q_r^{(m+1)})$ and the average throughput of user $m+1$ is given by

$$thp_{m+1}(\hat{\mathbf{q}}_{m+1}) = q_a \sum_{n=0}^{m} \pi_{n,0}(\hat{\mathbf{q}}_{m+1}) \tag{14}$$

Hence the E.D.T.P of user $m+1$, D_{m+1}, is given by:

$$D_{m+1}(\hat{\mathbf{q}}_{m+1}) = 1 + S_{m+1}(\hat{\mathbf{q}}_{m+1}) / thp_{m+1}(\hat{\mathbf{q}}_{m+1}) \tag{15}$$

For the game problem, let us denote by thp_{m+1}^c the throughput of backlogged packets (i.e. of the packets that arrive and become backlogged) at source $m+1$:

$$thp_{m+1}^c(\hat{\mathbf{q}}_{m+1}) = \sum_{n=0}^{m} \sum_{n'=0}^{m} P_{(n,0),(n',1)}(\hat{\mathbf{q}}_{m+1}) \pi_{n,0}(\hat{\mathbf{q}}_{m+1})$$

Thus, the expected delay of backlogged packets at source $m+1$, is given by

$$D_{m+1}^c(\hat{\mathbf{q}}_{m+1}) = 1 + S_{m+1}(\hat{\mathbf{q}}_{m+1}) / thp_{m+1}^c(\hat{\mathbf{q}}_{m+1}) \tag{16}$$

VI. Numerical investigation of the game problem

In this section we shall obtain the retransmission probabilities which solve the game problem. We investigate the dependence of the expected delay of packets that are successfully transmitted (i.e. all packets that are not rejected when arriving at the system) and of packets that upon arrival become backlogged, on the arrival probabilities q_a and on the number of nodes. We consider the game problem in which the performance to optimize are the expected delay of packets that are successfully transmitted (not rejected upon arrival) given by (15) and the the expected delay of backlogged packets . In Figure 9 ans 10 (resp. 11 and 12), the expected delay of transmitted packets (resp. expected delay of backlogged packets) and the optimal retransmission probabilities are plotted versus the arrival probability for the game problem.

Comparing the figures, we see in particular that the game solution is very inefficient for large arrival probabilities: the expected delay increases and attains large values, whereas in the team case, the expected delay is less for each q_a. The inefficiency is seen also through the optimal retransmission policy: as the system becomes more congested (larger arrival probabilities) the retransmission probability decreases in the team case so as to counter expected collisions. The game scenario gives rise, in contrast, to an equilibrium that becomes more and more aggressive as the arrival probabilities increase: the equilibrium retransmission probability is seen to increase with q_a (for $q_a > 0.2$) which explains the dramatic increase in the system's delay. In particular, as q_a approaches 1, so does q_r at equilibrium! In conclusion, the game solution is very inefficient for heavy traffic, and even for light traffic it becomes inefficient when the number of mobiles is larger than five.

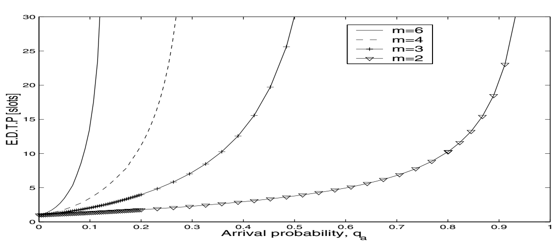

Fig. 9. Expected delay of transmitted packets in the game case (E.D.T.P) as a function of the arrival probabilities q_a for $m = 2, 3, 4, 6$.

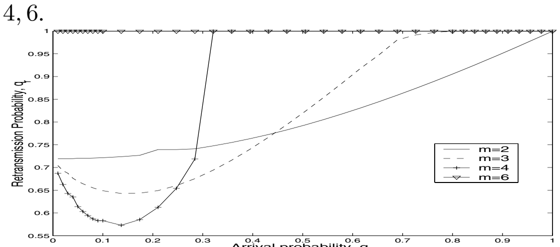

Fig. 10. The optimal retransmission probabilities in the game case (E.D.T.P) as a function of the arrival probabilities q_a for $m = 2, 3, 4, 6$.

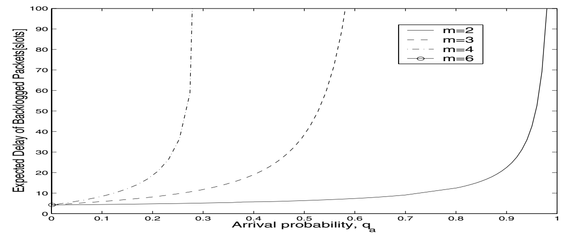

Fig. 11. Expected delay of backlogged packets for the game case as (E.D.B.P) a function of the arrival probabilities q_a for $m = 2, 3, 4, 6$.

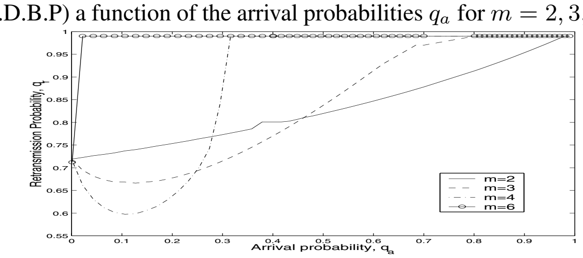

Fig. 12. The retransmission probability in the game case (E.D.B.P) as a function of the arrival probabilities q_a for $m = 2, 3, 4, 6$.

VII. Conclusion

To summarize, we present team and non-cooperative game formulations of slotted ALOHA. We study the distributed choice of retransmission probabilities so as to minimize expected delays of transmitted and backlogged packets. We found that game problem makes the system inefficient by increasing the delays unduly even under light traffic. There are still several avenues for further research in this area: for instance, considering slotted ALOHA with heterogeneous retransmission probabilties to capture different mobile devices with different capabilities. Pricing is often opted as a solution to make an inefficient game problem more efficient. It is interesting to investigate the impact of adding retransmission costs (which may represent the disutility for power consumption) on the equilibrium and show how this pricing affects the equilibrium.

References

[1] E. Altman, R. El-Azouzi and T. Jiménez, "Slotted Aloha as a stochastic game with partial information", *to appear in Computer Networks*, 2004.

[2] N. Abramson, "The Aloha system – another alternative for computer communications", *AFIPS Conference Proceedings*, Vol. 36, pp. 295-298, 1970.

[3] D. Bertsekas and R. Gallager, *Data Networks*, Prentice Hall, Englewood Cliffs, New Jersey, 1987.

[4] Y. Jin and G. Kesidis, "Equilibria of a noncooperative game for heterogeneous users of an ALOHA network", *IEEE Comm. Letters* Vol. 6, No. 7, pp. 282-284, 2002.

[5] A. B. MacKenzie and S. B. Wicker, "Selfish users in Aloha: A game theoretic approach", *Proceedings of the Fall 2001 IEEE Vehicular Technology Conference*, 2001.

[6] A. B. MacKenzie and S. B. Wicker, "Stability of Slotted Aloha with Multipacket Reception and Selfish Users", *Proceedings of IEEE Infocom*, April 2003.

[7] L. G. Roberts, "Aloha packet system with and without slots and capture", *Tech. Rep. Ass Note 8*, Stanford Research Institute, Advance Research Projects Agency, Network Information Center, 1972.

[8] M. Schwartz. *Information, transmission, modulation and noise*. McGraw-Hill, 3rd edition, 1980.