A Jamming Game in Wireless Networks with Transmission Cost*

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Abstract. We consider jamming in wireless networks with transmission cost for both transmitter and jammer. We use the framework of non-zero-sum games. In particular, we prove the existence and uniqueness of Nash equilibrium. It turns out that it is possible to provide analytical expressions for the equilibrium strategies. These expressions is a generalization of the standard water-filling. In fact, since we take into account the cost of transmission, we obtain even a generalization of the water-filling in the case of one player game. The present framework allows us to study both water-filling in time and water-filling in frequency. By means of numerical examples we study an important particular case of jamming of the OFDM system when the jammer is situated close to the base station.

Keywords: Wireless networks, Jamming, Non-zero-sum games, Nash Equilibrium, Water-filling.

1 Introduction and Problem Formulation

Power control in wireless networks became an important research area. Since the technology in the current state cannot provide batteries which have small weight and large energy capacity, the design of algorithms for efficient power control is crucial. For a comprehensive survey of recent results on power control in wireless networks an interested reader can consult [15]. It turns out that game theory provides a convenient framework for approaching the power control problem see for instance [9] and references therein. Most of the work on application of game theory to power control considers mobile terminals as players of the same type. Here we consider the jamming problem with two types of players. The first type of players are regular users of the wireless mobile network who want to use the available wireless channels in the most efficient way. The second type of players are jammers who want to prevent or to jam the communication of the regular users. The study of jamming in wireless networks is important in the context of military actions or fighting against terrorist activity. On a battlefield, it is very

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likely that one side will try to prevent the wireless communication of the other side. Thus, one side is interested in the best usage of power to overcome the artificial noise emitted by the other side. And conversely, the other side tries to use power to harm the communication in the most efficient way.

In [2] the authors have studied the application of dynamic stochastic zero sum game to the jamming problem in wireless networks. In the model of [2] the transmission power can be chosen from a discrete set. Here we suppose that the power level can be chosen from a continuous set. This allows us not only to prove the existence of the Nash equilibrium (NE) but also to show its uniqueness. Here, in addition to the power constraint we introduce the cost of power usage. This makes the problem a non-zero game. Furthermore, the current continuous model allows us to study not only temporal power distribution for one channel but also the distribution of power among different sub-channels.

In the works [7] and [14] the authors have analyzed the worst case wireless channel capacity when the noise variances are fixed (possibly unknown at the transmitter) and the carrier gains are allowed to vary while verifying a certain constraint. In that case, transmission at the worst rate guarantees error free communication under any possible conditions of the channel, although it might give a pessimistic result. This formulation leads to a minimax problem. In the works [7] and [14] as well as in [2] the cost of transmission is not taken into account. Other problem formulations involving jamming in which one wireless terminal wishes to maximize the mutual information and the other tries to minimize it, can be found at [3]. For other related work, see [5].

Let us specify the present model formulation. We consider two mobile terminals and one base station. Since we use the framework of game theory, we shall use the terms mobiles and players interchangably. Player 1 seeks to transmit information to the base station. We shall refer to it as “Transmitter”. Player 2 has an antagonistic objective: to prevent or to jam the transmissions of Player 1 to the base station. Thus, we shall call Player 2 “Jammer”. Both players have in addition a transmission cost (see below) which prevents us from using zero-sum games to model our problem.

We assume that there are n independent resources, each of which can be used simultaneously by both players. We further assume that resource i has a ”weight” of π_i.

Possible interpretations

(i) The resources may correspond to capacity available at different time slots; we assume that there is a varying environment whose state changes among a finite set of states $i \in [1, n]$, according to some ergodic stochastic process with stationary distribution $\{\pi_i\}_{i=1}^n$. We assume that both players have perfect knowledge of the environment state at the beginning of each time slot.

(ii) The resources may correspond to frequency bands (e.g. as in OFDM) where one should assign different power levels for different sub-carriers [15]. In that case we may take $\pi_i = 1/n$ for all i.

The pure strategy of Transmitter is $T = (T_1, \dots, T_n)$ where $T_i \geq 0$ for $i \in [1, n]$ and $\sum_{i=1}^n \pi_i T_i \leq \bar{T}$ where $\bar{T} > 0$, $\pi_i > 0$ for $i \in [1, n]$. The component T_i can be interpreted as the power level dedicated to resource of type i. If the resource i is the available capacity when the environment state is i, then T_i is the power level that is chosen whenever we visit state i, and $\bar{T}$ is a bound on the power averaged over time.

If the resources correspond to frequency bands, then T_i is the average power to be transmitted at the ith band. $\bar{T}$ is then the maximal average power level that can be used by Transmitter.

The pure strategy of Jammer is $N = (N_1, \dots, N_n)$ where $N_i \geq 0$ for $i \in [1, n]$ and $\sum_{i=1}^{n} \pi_i N_i \leq \bar{N}$ where $\bar{N} > 0$. The payoffs to Transmitter and Jammer are given as follows

$$
\begin{aligned}
v_T(T, N) &= \sum_{i=1}^{n} \pi_i \ln\left(1 + \frac{g_i T_i}{h_i N_i + N_i^0}\right) - c_T \sum_{i=1}^{n} \pi_i T_i, \\
v_N(T, N) &= -\sum_{i=1}^{n} \pi_i \ln\left(1 + \frac{g_i T_i}{h_i N_i + N_i^0}\right) - c_N \sum_{i=1}^{n} \pi_i N_i
\end{aligned}
\tag{1}
$$

where N_i^0 is the power level of the uncontrolled noise of the environement at state i, $c_T > 0$ and $c_N > 0$ are the costs of power usage for Transmiter and Jammer, and $g_i > 0$ and $h_i > 0$ are fading channel gains for Transmiter and Jammer when the environement is in state i. The first sum in payoff is the expected value of the Shanon capacity [6,10,15] and the second sum is the average cost of transmission.

We shall look for a NE, that is, we want to find $(T^*, N^*) \in A \times B$ such that

$$
\begin{aligned}
v_T(T, N^*) &\leq v_T(T^*, N^*) \text{ for any } T \in A, \\
v_N(T^*, N) &\leq v_N(T^*, N^*) \text{ for any } N \in B,
\end{aligned}
$$

where A and B are the sets of all the strategies of Transmitter and Jammer, respectively. In particular, we shall prove that the NE exists and is unique and we shall provide closed form analytic expressions for its calculation.

In the special case when c_T and c_N are zero in (1), the game is zero-sum. As v_T is convex in T_i and concave in N_i, we can apply Sion's minimax Theorem to conclude that it has a saddle point.

The structure of the paper is as follows: To complete the picture and to introduce notations, in Section 2 we consider single player water-filling game with the environment when the transmission cost is taken into account. Section 3 is the main part of the paper where we study the structure of the NE in the jamming game. Then, in Section 4, based on theoretical results of Section 3, we provide an algorithm for determination of the NE. We study some numerical examples in Section 5 and make conclusions in Section 6.

2 Water-Filling with Transmission Cost

In this section we consider the following single person game with the environment. There is one player named Transmitter. He/she wants to send information through a channel which state depends on the state of the environment or through n sub-channels. The goal of Transmitter is to maximize the sending rate of the transmitted information and to minimize the transmission cost. The pure strategy of Transmitter is $T = (T_1, \dots, T_n)$ where $T_i \geq 0$ for $i \in [1, n]$ and $\sum_{i=1}^{n} \pi_i T_i \leq \bar{T}$ where $\bar{T} > 0$ and $\pi_i > 0$ for $i \in [1, n]$. The payoff to Transmitter is given as follows

$$v(T) = \sum_{i=1}^{n} \pi_i \ln \left(1 + \frac{T_i}{N_i^0}\right) - c_T \sum_{i=1}^{n} \pi_i T_i,$$

where $N_i^0 > 0$ is the noise level when the environment is in state i, $i \in [1, n]$ and c_T is a cost for power usage. We would like to emphasize that this is a generalization of the standard water-filling scheme, see e.g., [8,13,15]. Following the standard water-filling approach we can get the following result.

Theorem 1. *Let* $1/N_1^0 = \max_{i\in[1,n]} 1/N_i^0$ *and* $T_i(\omega) = \left[1/(c_T + \omega) - N_i^0\right]_+$ *for* $i \in [1, n]$ *and* $H_T(\omega) = \sum_{i=1}^{n} \pi_i T_i(\omega)$.

If $c_T \geq 1/N_1^0$ *then* $T^* = (0, \dots, 0)$ *is the unique optimal strategy and its payoff is 0. If* $c_T < 1/N_1^0$ *then* $T(\omega^*) = (T_1(\omega^*), \dots, T_n(\omega^*))$ *is the unique optimal strategy and its payoff is* $v(T(\omega^*))$ *where for* $H_T(0) \leq \bar{T}$ $\omega^* = 0$ *and for* $H_T(0) > \bar{T}$ ω^* *is the unique root of the equation* $H_T(\omega) = \bar{T}$.

3 Jamming Game

In this section we consider a non-zero-sum game between Transmitter and Jammer with payoff functions defined by (1). We shall study the NE of this game, that is, we want to find $(T^*, N^*) \in A \times B$ such that

$$v_T(T, N^*) \leq v_T(T^*, N^*) \text{ for any } T \in A,$$
$$v_N(T^*, N) \leq v_N(T^*, N^*) \text{ for any } N \in B,$$

where A and B are the sets of all the strategies of Transmitter and Jammer, respectively.

Note that

$$\frac{\partial^2 v_T(T, N)}{\partial T_i^2} = -\frac{\pi_i g_i^2}{(g_i T_i + h_i N_i + N_i^0)^2 (h_i N_i + N_i^0)^2} < 0$$

and

$$\frac{\partial^2 v_N(T, N)}{\partial N_i^2} = -\frac{\pi_i T_i g_i h_i^2 (g_i T_i + 2h_i N_i + 2N_i^0)}{(g_i T_i + h_i N_i + N_i^0)^2 (h_i N_i + N_i^0)^2} < 0.$$

Thus, v_T and v_N are concave in T and N respectively. So, we can apply the Kuhn-Tucker Theorem to find the form that the NE has, namely, we will show in Theorem 3 that each NE is of the form $(T(\omega, \nu), N(\omega, \nu))$ for some nonnegative ω and ν where $T(\omega, \nu)$ and $N(\omega, \nu)$ are given in closed form in (5) and (6). These functions have a nice monotonous properties established in Lemma 1, namely, $N(\omega, \nu)$ is decreasing in ω and ν and $T(\omega, \nu)$ is decreasing in ω and is increasing in ν. This properties allow us to prove in Theorem 4 that there is at most one NE. Then, based on the monotonous properties of $T(\omega,\nu)$ and $N(\omega,\nu)$, we produce a NE in Theorems 5 and 6 in a way where the original two parametric problems in ω and ν reduces to one parametric problem either in ω or in ν where the optimal values of ω and ν can be found from solution of an equation with monotonous function. This in turn allows us in Section 4 to produce an effective algorithm based on the bisection method for numerical determination of NE.

Now we can pass on to our analysis. As it was noticed v_T and v_N are concave in T and N, thus, the Kuhn - Tucker Theorem implies the following theorem.

Theorem 2. *(T^*,N^*) is a NE if and only if there are non - negative ω and ν such that*

$$\frac{\partial}{\partial T_i} v_T(T^*,N^*) = \frac{g_i}{g_i T_i^* + h_i N_i^* + N_i^0} - c_T \begin{cases} = \omega & \text{for } T_i^* > 0, \\ \leq \omega & \text{for } T_i^* = 0, \end{cases} \tag{2}$$

$$\begin{aligned} \frac{\partial}{\partial N_i} v_N(T^*,N^*) = {} & \frac{g_i h_i T_i^*}{(g_i T_i^* + h_i N_i^* + N_i^0)(h_i N_i^* + N_i^0)} \\ & - c_N \begin{cases} = \nu & \text{for } N_i^* > 0, \\ \leq \nu & \text{for } N_i^* = 0, \end{cases} \end{aligned} \tag{3}$$

where

$$\omega \begin{cases} \geq 0 & \text{for } \sum_{i=1}^n \pi_i T_i^* = \bar{T}, \\ = 0 & \text{for } \sum_{i=1}^n \pi_i T_i^* < \bar{T} \end{cases} \quad \text{and} \quad \nu \begin{cases} \geq 0 & \text{for } \sum_{i=1}^n \pi_i N_i^* = \bar{N}, \\ = 0 & \text{for } \sum_{i=1}^n \pi_i N_i^* < \bar{N}. \end{cases} \tag{4}$$

For non-negative ω and ν let

$$\begin{aligned} I_{00}(\omega,\nu) &= I_{00}(\omega) = \{i \in [1,n] : h_i g_i / N_i^0 \leq h_i(\omega + c_T)\}, \\ I_{10}(\omega,\nu) &= \{i \in [1,n] : h_i(\omega + c_T) < h_i g_i / N_i^0 \leq h_i(\omega + c_T) + g_i(\nu + c_N)\}, \\ I_{11}(\omega,\nu) &= \{i \in [1,n] : h_i(\omega + c_T) + g_i(\nu + c_N) < h_i g_i / N_i^0\}, \end{aligned}$$

$$T_i(\omega,\nu) = \begin{cases} \dfrac{g_i}{(\omega + c_T)h_i + (\nu + c_N)g_i} \times \dfrac{\nu + c_N}{\omega + c_T} & \text{for } i \in I_{11}(\omega,\nu), \\ \dfrac{1}{c_T + \omega} - \dfrac{N_i^0}{g_i} & \text{for } i \in I_{10}(\omega,\nu), \\ 0 & \text{for } i \in I_{00}(\omega,\nu), \end{cases} \tag{5}$$

$$N_i(\omega,\nu) = \begin{cases} \dfrac{g_i}{(\omega + c_T)h_i + (\nu + c_N)g_i} - \dfrac{N_i^0}{h_i} & \text{for } i \in I_{11}(\omega,\nu), \\ 0 & \text{for } i \in I_{00}(\omega,\nu). \end{cases} \tag{6}$$

Theorem 3. *Each NE is of the form $(T(\omega,\nu),N(\omega,\nu))$ for some nonnegative ω and ν.*

Now we go on to finding optimal ω and ν. Let

$$H_T(\omega,\nu) = \sum_{i=1}^n \pi_i T_i(\omega,\nu), \quad H_N(\omega,\nu) = \sum_{i=1}^n \pi_i N_i(\omega,\nu).$$

Then Theorem 3 implies that

$$H_T(\omega,\nu) = \sum_{i\in I_{10}} \pi_i\left(\frac{1}{c_T+\omega} - \frac{N_i^0}{g_i}\right) + \frac{\nu+c_N}{\omega+c_T}\sum_{i\in I_{11}} \frac{\pi_i g_i}{(\omega+c_T)h_i+(\nu+c_N)g_i},$$

$$H_N(\omega,\nu) = \sum_{i\in I_{11}} \pi_i\Big(\frac{g_i}{(\omega+c_T)h_i+(\nu+c_N)g_i} - \frac{N_i^0}{h_i}\Big).$$

In the next lemma some monotonous properties of $T_i(\omega,\nu)$ and $N_i(\omega,\nu)$, $H_T(\omega,\nu)$ and $H_N(\omega,\nu)$ are obtained.

Lemma 1. **(i)** *For fixed $\omega > 0$ and $0 \le \nu_1 < \nu_2$ we have: (1) $T_i(\omega,\nu_1) \le T_i(\omega,\nu_2)$ where strict inequality holds if and only if $i \in I_{10}(\omega,\nu_1)$, (2) $N_i(\omega,\nu_1) \ge N_i(\omega,\nu_2)$ where strict inequality holds if and only if $i \in I_{10}(\omega,\nu_1)$, (3) $H_T(\omega,\nu_1) \le H_T(\omega,\nu_2)$ where equality holds if and only if $I_{10}(\omega,\nu_1) = \emptyset$, (4) $H_N(\omega,\nu_1) \ge H_N(\omega,\nu_2)$ where equality holds if and only if $I_{10}(\omega,\nu_1) = \emptyset$.*

(ii) *For fixed $\nu > 0$ and $0 \le \omega_1 < \omega_2$ we have: (1) $T_i(\omega_1,\nu) \le T_i(\omega_2,\nu)$ where equality holds if and only if $i \in I_{00}(\omega_1,\nu)$, (2) $N_i(\omega_1,\nu) \ge N_i(\omega_2,\nu)$ where equality holds if and only if $i \notin I_{10}(\omega_1,\nu)$, (3) $H_T(\omega_1,\nu_1) \ge H_T(\omega_2,\nu)$ where equality holds if and only if $I_{00}(\omega,\nu_1) = [1,n]$, (4) $H_N(\omega_1,\nu) \ge H_N(\omega_2,\nu)$ where equality holds if and only if $I_{10}(\omega_1,\nu) = \emptyset$.*

(iii) *$H_T(\omega,\nu)$ and $H_N(\omega,\nu)$ are non-negative and continuous in $[0,\infty)\times[0,\infty)$.*

(iv) *If $H_N(0,0) \le \bar{N}$ then $H_N(\omega,\nu) < \bar{N}$ for $\omega > 0$ and $\nu > 0$.*

Based on monotonous properties described in Lemma 1 we can establish the following result about the number of NE the game can have.

Theorem 4. *There is at most one NE.*

Note that

$$\begin{aligned} H_T(\omega,0) = &\sum_{i\in[1,n]:\, h_i(\omega+c_T)<h_i g_i/N_i^0 \le h_i(\omega+c_T)+g_i c_N} \pi_i\left(\frac{1}{c_T+\omega} - \frac{N_i^0}{g_i}\right) \\ &+ \frac{c_N}{\omega+c_T}\times \sum_{i\in[1,n]:\, h_i(\omega+c_T)+g_i c_N<h_i g_i/N_i^0} \pi_i \frac{g_i}{(\omega+c_T)h_i+c_N g_i}. \end{aligned} \tag{7}$$

The following lemma supplying some properties of $H_T(\omega,0)$ follows straighforward from (7) and Lemma 1.

Lemma 2. *(i) $H_T(\cdot,0)$ is non-negative and continuous in $(0,\infty)$,*
(ii) $H_T(\omega,0) = 0$ for enough big ω, namely, for $\omega \ge \max_i\{g_i/N_i^0 - g_i c_N/h_i\} - c_T$,
(iii) $H_T(\omega,0)$ is strictly decreasing on ω while $H_T(\omega,0) > 0$,

Lemma 2 implies that if $H_T(0,0) > \bar{T}$ that there exists the unique positive ω_{10}^* such that $H_T(\omega_{10}^*,0) = \bar{N}$ (indexes 10 mean that in this moment we look for the optimal solution where $\omega > 0$ and $\nu = 0$). If $H_T(0,0) \le \bar{T}$ then $H_T(\tau,0) < \bar{T}$ for $\tau > 0$. Then, from Theorems 2 and 3 and Lemmas 1(iv) and 2 we have the following theorem.

Theorem 5. *Let* $H_N(0,0) \le \bar{N}$ *then*
(a) if $H_T(0,0) \le \bar{T}$ *then* $(T(0,0), N(0,0))$ *is NE,*
(b) if $H_T(0,0) > \bar{T}$ *then* $(T(\omega_{10}^*,0), N(\omega_{10}^*,0)$ *is NE.*

By Lemma 1 the following Lemma holds

Lemma 3. *If* $H_N(0,0) > \bar{N}$ *then there is* ν_{01}^* *such that* $H_N(0,\nu_{01}^*) = \bar{N}$ *(subscript* 01 *signifies that we look for the optimal solution where* $\omega = 0$ *and* $\nu > 0$*) and there is* $\hat{\omega}$ *such that* $H_N(\hat{\omega},0) = \bar{N}$. *Thus,* $H_N(\omega,\nu) < \bar{N}$ *for each* $\omega > \hat{\omega}$ *and each non-negative* ν. *For each* $\omega \in (0,\hat{\omega}]$ *there is unique nonnegative* $\nu(\omega)$ *such that* $H_N(\omega,\nu(\omega)) = \bar{N}$. $\nu(\omega)$ *is continuous and strictly decreasing on* ω, $\nu(0) = \nu_{01}^*$ *and* $\nu(\hat{\omega}) = 0$.

Thus, by Lemma 3 we can introduce the following notation:

$$\bar{H}_T(\omega) = H_T(\omega,\nu(\omega)) = \sum_{i \in I_{10}(\omega,\nu(\omega))} \pi_i \left(\frac{1}{c_T + \omega} - N_i^0 \right) + \frac{\nu(\omega) + c_N}{\omega + c_T} \times \sum_{i \in I_{11}(\omega,\nu(\omega))} \pi_i \frac{g_i}{(\omega + c_T)h_i + (\nu(\omega) + c_N)g_i}.$$

Then by Lemma 1 $\bar{H}_T$ is continuous and strictly decreasing in $(0,\hat{\omega})$. Thus, if $\bar{H}_T(0) \le \bar{T}$ then $\bar{H}_T(\omega) < \bar{T}$ for $\omega \in (0,\hat{\omega})$. If $\bar{H}_T(\hat{\omega}) > \bar{T}$ then $\bar{H}_T(\omega) > \bar{T}$ for $\omega \in (0,\hat{\omega})$. If $\bar{H}_T(\hat{\omega}) < \bar{T}$ and $\bar{H}_T(0) > \bar{T}$ then there is unique $\omega_{11}^* \in (0,\hat{\omega})$ such that $\bar{H}_T(\omega_{11}^*) = \bar{T}$ (subscript 11 signifies that we look for the optimal solution where $\omega, \nu > 0$). Then, from Theorems 2 and 3 we have the following theorem.

Theorem 6. *Let* $H_N(0,0) > \bar{N}$ *then*
(a) if $\bar{H}_T(0) = H_T(0,\nu_{01}^*) \le \bar{T}$ *then* $(T(0,\nu_{01}^*), N(0,\nu_{01}^*))$ *is NE,*
(b) if $\bar{H}_T(0) = H_T(0,\nu_{01}^*) > \bar{T}$ *and* $\bar{H}_T(\hat{\omega}) = H_T(\hat{\omega},0) > \bar{T}$ *then* $(T(\omega_{10}^*,0), N(\omega_{10}^*,0))$ *is NE,*
(c) if $\bar{H}_T(0) = H_T(0,\nu_{01}^*) > \bar{T}$ *and* $\bar{H}_T(\hat{\omega}) = H_T(\hat{\omega},0) \le \bar{T}$ *then* $(T(\omega_{11}^*,\nu(\omega_{11}^*)), N(\omega_{11}^*,\nu(\omega_{11}^*))$ *is NE.*

Theorems 4 – 6 imply the following main result.

Theorem 7. *There is unique NE given by Theorems 5 and 6.*

The case where there are no the costs of power usage for Transmiter and Jammer, namely, $c_T = c_N = 0$, is an important particular case of our model. For this case our model from non-zero sum game turns into zero-sum game. Then, it is clear, that $H_N(0+,0+) = \infty$ and $\bar{H}_T(0+) = \infty$ and we come under conditions of Theorem 6 (b) and (c). Thus, if $H_T(\hat{\omega},0) \le \bar{T}$ (where $\hat{\omega}$ is defined by equation $H_N(\hat{\omega},0) = \bar{N}$, see Lemma 3), then $(T(\omega_{11}^*,\nu(\omega_{11}^*)), N(\omega_{11}^*,\nu(\omega_{11}^*))$ is the equilibrium. If $H_T(\hat{\omega},0) > \bar{T}$ then $(T(\omega_{10}^*,0), N(\omega_{10}^*,0))$ is the equilibrium.

4 Algorithm

In this section we present an algorithm based on the bisection method and Theorems 5 and 6 and Lemmas 2 and 3 to find the optimal values of ω and ν and the corresponding optimal solution.

Algorithm

Step 1. If $H_N(0,0) \le \bar{N}$ and $H_T(0,0) \le \bar{T}$ then $\omega = \nu = 0$ and $(T(0,0), N(0,0))$ is NE and the algorithm is terminated.
Step 2. If $H_N(0,0) \le \bar{N}$ and $H_T(0,0) > \bar{T}$. Then call $\omega_{10}^* = BS_T^1(0)$, $(T(\omega_{10}^*, 0), N(\omega_{10}^*, 0))$ is NE and the algorithm is terminated.
Step 3. If $H_N(0,0) > \bar{N}$ then $BS_N^1(\hat{\omega}, 0)$ and $\nu_{01}^* = BS_N^2(0)$.
Step 4. If $H_T(0, \nu_{01}^*) \le \bar{T}$ then $(T(0, \nu_{01}^*), N(0, \nu_{01}^*))$ is NE and the algorithm is terminated.
Step 5. If $H_T(0, \nu_{01}^*) > \bar{T}$ and $H_T(\hat{\omega}, 0) > \bar{T}$ then $(T(\omega_{10}^*, 0), N(\omega_{10}^*, 0))$ is NE and the algorithm is terminated.
Step 6. If $H_T(0, \nu_{01}^*) > \bar{T}$ and $H_T(\hat{\omega}, 0) \le \bar{T}$ then $\omega^0 = 0$, $\omega^1 = \hat{\omega}$.
Step 6a. $\nu^0 = BS_N^2(\omega^0)$, $\nu^1 = BS_N^2(\omega^1)$.
Step 6b. Set $\bar{\omega} = (\omega^1 + \omega^0)/2$.
Step 6c. $\bar{\nu} = BS_N^2(\bar{\omega})$.
Step 6d. If $\omega^1 - \omega^0 \le \epsilon$, then $\omega_{11}^* = (\omega^1 + \omega^0)/2$, $\nu_{11}^* = BS_N^2(\omega_{11}^*)$ and $(T(\omega_{11}^*, \nu_{11}^*), N(\omega_{11}^*, \nu_{11}^*))$ is NE and the algorithm is terminated.
Step 6e. If $\omega^1 - \omega^0 > \epsilon$, then, if $H_T(\bar{\omega}, \bar{\nu}) < \bar{N}$ then $\omega^0 = \bar{\omega}$, if $H_N(\bar{\omega}, \bar{\nu}) > \bar{N}$ then $\omega^1 = \bar{\omega}$ and go to Step 6b.
Step 6f. Let $\omega^1 - \omega^0 > \epsilon$ and $H_N(\bar{\omega}, \bar{\nu}) = \bar{N}$ then $\omega_{11}^* = \bar{\omega}$, $\nu_{11}^* = \bar{\nu}$ and $(T(\omega_{11}^*, \nu_{11}^*), N(\omega_{11}^*, \nu_{11}^*))$ is NE and the algorithm is terminated.

Function $\omega = BS_T^1(\nu)$

Step 1. Let $\omega^0 = 0$, $\omega^1 = max_i\{g_i/N_i^0 - g_i c_N/h_i\} - c_T$
Step 2. Set $\bar{\omega} = (\omega^1 + \omega^0)/2$.
Step 3. If $\omega^1 - \omega^0 \le \epsilon$, then return $\omega = (\omega^1 + \omega^0)/2$.
Step 4. If $\omega^1 - \omega^0 > \epsilon$ then, if $H_T(\bar{\omega}, \nu) < \bar{T}$ set $\omega^0 = \bar{\omega}$, if $H_T(\bar{\omega}, \nu) > \bar{T}$ set $\omega^1 = \bar{\omega}$ and go to Step 2.
Step 5. Let $\omega^1 - \omega^0 > \epsilon$ and $H_T(\bar{\omega}, \nu) = \bar{N}$ then return $\bar{\omega}$.

Function $\omega = BS_N^1(\nu)$

Step 1. Let $\omega^0 = 0$, $\omega^1 = max_i\{g_i/N_i^0 - g_i c_N/h_i\} - c_T$
Step 2. Set $\bar{\omega} = (\omega^1 + \omega^0)/2$.
Step 3. If $\omega^1 - \omega^0 \le \epsilon$ then return $\omega = (\omega^1 + \omega^0)/2$.
Step 4. If $\omega^1 - \omega^0 > \epsilon$ then, if $H_N(\bar{\omega}, \nu) < \bar{N}$ set $\omega^0 = \bar{\omega}$, if $H_N(\bar{\omega}, \nu) > \bar{N}$ set $\omega^1 = \bar{\omega}$ and go to Step 2.
Step 5. Let $\omega^1 - \omega^0 > \epsilon$ and $H_N(\bar{\omega}, \nu) = \bar{N}$ then return $\bar{\omega}$.

Function $\nu = BS_N^2(\omega)$

Step 1. Let $\nu^0 = 0$, $\nu^1 = max_i\{h_i/N_i^0 - h_i c_T/g_i\} - c_N$

Step 2. Set $\bar{\nu} = (\nu^1 + \nu^0)/2$.
Step 3. If $\nu^1 - \nu^0 \leq \epsilon$ then return $\nu = (\nu^1 + \nu^0)/2$.
Step 4. If $\nu^1 - \nu^0 > \epsilon$ then, if $H_N(\omega, \bar{\nu}) < \bar{N}$ set $\nu^0 = \bar{\nu}$, if $H_N(\omega, \bar{\nu}) > \bar{N}$ set $\nu^1 = \bar{\nu}$ and go to Step 2.
Step 5. Let $\nu^1 - \nu^0 > \epsilon$ and $H_N(\omega, \bar{\nu}) = \bar{N}$ then return $\bar{\nu}$.

5 Numerical Examples

In this section we consider a few numerical examples. The numerical examples correspond to the OFDM scheme with five sub-channels ($n = 5$). Consequently, we take $\pi_i = 1/5$. Let us consider an important particular case of jamming in the OFDM system when the jammer is near the base station. In this scenario $h_i = 1$ for all $i \in [1, 5]$. First, we take $g_i = \kappa^{i-1}$ for $i \in [1, 5]$ where $\kappa \in (0, 1)$. This corresponds to Rayleigh fading. Also we set $N_i^0 = 0.1$, $i \in [1, 5]$, $\bar{N} = \bar{T} = 1$ and $c_T = c_N = 0.1$. The payoffs of the players as functions of κ is shown in Figure 1. As an example, we depict the optimal strategies of the players in Figure 2 for the case $\kappa = 1/2$. It is interesting to observe that Jammer spends more energy in the sub-channels with good quality and Transmitter tries to use the resources of the bad quality sub-channels. In other words, Jammer pays less attention to the sub-channel with bad quality and Transmitter takes an opportunity to send some part of information over bad quality sub-channels.

In the second example, we consider that the background noise is different in each sub-channel. Specifically, we take $N_i^0 = i/10$ for $i \in [1, 5]$ and $\kappa = 0.2$. Then, we obtain the optimal strategies $T^* = (3.66, 1.18, 0.16, 0, 0)$ and $N^* = (3.90, 1.10, 0, 0, 0)$ with the payoffs 0.07 and -0.27. This example illustrates the possibility of the situation when Transmitter uses more sub-channels than Jammer, see formulae (5) and (6).

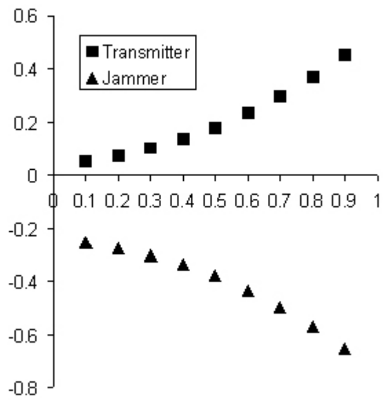

Fig. 1. The payoffs as functions of κ

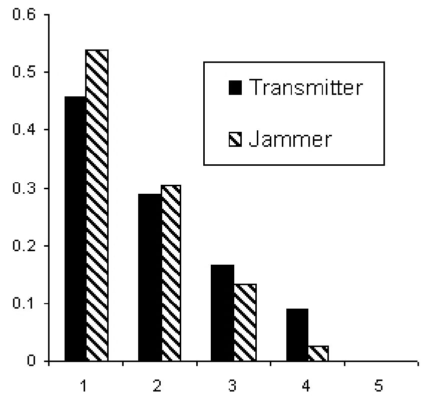

Fig. 2. The optimal strategies for $\kappa = 1/2$

6 Conclusions

In this paper we considered jamming in wireless networks with transmission cost for both transmitter and jammer from a game theoretical point of view. We proved the existence and uniqueness of NE. It turned out that it is possible to provide analytical expressions for the equilibrium strategies which depend on two parameters. We propose an efficient algorithm for finding these parameters, and hence, the optimal strategies. The presented jamming game is a generalization of the standard water-filling problem. In fact, since we take into account the cost of transmission, for the case of the single player, we obtain even the generalization of the water-filling optimization problem. The present framework allows us to study both water-filling in time and water-filling in frequency. By means of numerical examples we study an important particular case of jamming of the OFDM system when the jammer is situated close to the base station. These examples showed that Jammer pays less attention to the sub-channel with bad quality and Transmitter takes an opportunity to send some part of information over bad quality sub-channels.

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References

1. S. Alpern, and S. Gal, *The Theory of Search Games and Rendezvous*. International Series in Operations Research and Management Science, v. 55. Kluwer Academic Publishers, 2003.
2. E. Altman, K. Avrachenkov, R. Marquez, and G. Miller, "Zero-sum constrained stochastic games with independent state processes", *Math. Meth. Oper. Res.*, v.62, pp. 375-386, 2005.
3. Kashyap A, Basar T, Srikant R, "Correlated jamming on MIMO Gaussian fading channels", Information Theory, IEEE Transactions on, Vol. 50, No. 9. (2004), pp. 2119-2123.
4. V.J. Baston and A.Y. Garnaev, "A Search Game with a Protector", *Naval Research Logistics*, v. 47, pp. 85-96, 2000.
5. M. H. Brady amd J. M. .Cioffi, "The worst-case interference in DSL systems employing dynamic spectrum management", Eurasip Journal on Applied Signal Processing, Vol. 2006, pages 1-11.
6. T. Cover and J. Thomas, *Elements of Information Theory*, Wiley, 1991.
7. E.A. Jorswieck and H. Boche, "Performance analysis of capacity of MIMO systems under multiuser interference based on worst case noise behavior", EURASIP Journal on Wireless Communications and Networking, v.2, pp.273-285, 2004.
8. S. Kasturia, J.T. Aslanis and J.M. Cioffi, "Vector coding for partial response channels", *IEEE Trans. Information Theory*, v.36(4), pp.741-762, 1990.

9. L. Lai and H. El Gamal, "The water-filling game in fading multiple access channels", submitted to *IEEE Trans. Information Theory*, November 2005, available at `http://www.ece.osu.edu/ helgamal/`.
10. R.G. Gallager, *Information Theory and Reliable Communication*, Wiley, 1968.
11. A. Garnaev, *Search Games and Other Applications of Game Theory*, Springer, 2000.
12. A. Garnaev, "Find a "Hidden" Treasure", *Naval Research Logistics*, 2006 (to apprear).
13. A.J. Goldsmith and P.P. Varaiya, "Capacity of fading channels with channel side information", *IEEE Trans. Information Theory*, v.43(6), pp.1986-1992, 1997.
14. A. Suarez-Real, *Robust Waterfilling strategies for the fading channel*, INRIA. Master SICOM Thesis. 2006.
15. D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2005.

Appendix

Proof of Theorem 3. Let (T^*, N^*) be a NE. Then for each $i \in [1, n]$ the following four cases are possible: (a) $T_i^* = N_i^* = 0$, (b) $T_i^* = 0$, $N_i^* > 0$, (c) $T_i^* > 0$, $N_i^* > 0$ and (d) $T_i^* > 0$, $N_i^* = 0$.

(a) Let $T_i^* = 0$ and $N_i^* = 0$ then by (2) we have that $g_i/N_i^0 - c_T \le \omega^*$. Thus, $i \in I_{00}(\omega^*, \nu^*)$ and $T^* = T(\omega^*, \nu^*)$, $N^* = N(\omega^*, \nu^*)$.

(b) Let $T_i^* > 0$ and $N_i^* = 0$ then by (2) we have that

$$\frac{g_i}{g_i T_i^* + N_i^0} - c_T = \omega^*.$$

Thus, $\frac{g_i}{N_i^0} > \omega^* + c_T$ and $T_i^* = \frac{1}{\omega^* + c_T} - \frac{N_i^0}{g_i}$. Then, by (3) we have that

$$\begin{aligned}\nu^* &\ge \frac{g_i h_i T_i^*}{(h_i T_i^* + N_i^0) N_i^0} - c_N = \left(\frac{1}{\omega^* + c_T} - \frac{N_i^0}{g_i}\right) \frac{h_i}{N_i^0}(\omega^* + c_T) - c_N \\ &= \frac{h_i}{N_i^0} - \frac{h_i}{g_i}(\omega^* + c_T) - c_N.\end{aligned}$$

Thus, $i \in I_{10}(\omega^*, \nu^*)$ and $T^* = T(\omega^*, \nu^*)$, $N^* = N(\omega^*, \nu^*)$.

(c) Let $T_i^* > 0$ and $N_i^* > 0$ then by (2) and (3) we have that

$$\begin{aligned}\omega^* &= \frac{g_i}{g_i T_i^* + h_i N_i^* + N_i^0} - c_T, \\ \nu^* &= \frac{g_i h_i T_i}{(g_i T_i^* + h_i N_i^* + N_i^0)(h_i N_i^* + N_i^0)} - c_N.\end{aligned}$$

(d) Let $T_i^* = 0$ and $N_i^* > 0$ then by (3) $\nu^* = -c_N < 0$. This contradiction proves that the assumption that $T_i^* = 0$ and $N_i^* > 0$ cannot take place and the result follows.

Proof of Lemma 1. (i1) For fixed $\omega > 0$ and $0 \le \nu_1 < \nu_2$ we have $I_{10}(\omega, \nu_1) \subseteq I_{10}(\omega, \nu_2)$ and $I_{11}(\omega, \nu_1) \supseteq I_{11}(\omega, \nu_2)$. Since for any ν $I_{10}(\omega, \nu) \cup I_{11}(\omega, \nu) =$

$[1,n]\backslash I_{00}(\omega)$ does not depend on ν we have to consider separately the cases $i \in I_{00}(\omega,\nu_1)$, $i \in I_{10}(\omega,\nu_1)$, $i \in I_{11}(\omega,\nu_2)$ and $i \in I_{11}(\omega,\nu_1) \cap I_{10}(\omega,\nu_2)$, and then (i1) now follows easily from the definitions.

Proof of Theorem 4. Suppose there are at least two NE, say $(T(\omega_1,\nu_1), N(\omega_1,\nu_1))$ and $(T(\omega_2,\nu_2), N(\omega_2,\nu_2))$.

Suppose that $\nu_1 = \nu_2 = \nu$. We can assume that $0 \le \omega_1 < \omega_2$. Thus, by Theorem 2, $H_T(\omega_2,\nu) = \bar{T}$. Thus, by Lemma 1 (ii3) $H_T(\omega_2,\nu) \le H_T(\omega_1,\nu)$ So, $H_T(\omega_1,\nu) = \bar{T} = H_T(\omega_2,\nu)$ and by Lemma 1 (ii3) $I_{00} = [1,n]$. Thus, $H_T(\omega_2,\nu) = 0$. This contradictions shows ω_1 has to be equal to ω_2.

Suppose that $0 \le \nu_1 < \nu_2 = \nu$. Thus, by Theorem 2, $H_T(\omega_2,\nu_2) = \bar{N}$. So, $I_{11}(\omega_2,\nu_2) \ne \emptyset$.

Assume that $\omega_1 \le \omega_2$. Then $I_{11}(\omega_2,\nu_2) \subseteq I_{11}(\omega_1,\nu_1)$ and $N_i(\omega_1,\nu_1) > N_i(\omega_2,\nu_2)$ for $i \in I_{11}(\omega_2,\nu_2)$. Thus, $H_N(\omega_1,\nu_1) > H_N(\omega_2,\nu_2) = \bar{N}$. This contradiction shows that the inequality $\omega_1 > \omega_2$ has to be held.

So, let $\omega_1 > \omega_2$. Thus, $I_{00}(\omega_2) \subseteq I_{00}(\omega_1)$. We can assume that $I_{00}(\omega_2) \ne [1,n]$ since otherwise the equilibrium coincides with each other, namely $T_i(\omega_k,\nu_k) = N_i(\omega_k,\nu_k) = 0$ for $k = 1,2$. So, $I_{00}(\omega_2) \ne [1,n]$. Thus, by Lemma $H_T(\omega_2,\nu_2) \ge H_T(\omega_2,\nu_1) > H_T(\omega_1,\nu_1) = \bar{T}$. This contradiction completes the proof of theorem.