Capacity Optimizing Hop Distance in a Mobile Ad Hoc Network with Power Control

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Abstract— In a dense multi-hop network of mobile nodes capable of applying adaptive power control, we consider the problem of finding the optimal hop distance that maximizes a certain throughput measure in *bit-metres/sec*, subject to average network power constraints. The mobility of nodes is restricted to a circular periphery area centered at the nominal location of nodes. We incorporate only randomly varying path-loss characteristics of channel gain due to the random motion of nodes, excluding any multi-path fading or shadowing effects. Computation of the throughput metric in such a scenario leads us to compute the probability density function of random distance between points in two circles. Using numerical analysis we discover that choosing the nearest node as next hop is *not always* optimal. Optimal throughput performance is also attained at non-trivial hop distances depending on the available average network power.

I. INTRODUCTION

¹ In this paper, we study the optimal next hop distance that maximizes the system end-to-end flow throughput in a mobile multi-hop wireless network environment subject to a network average power constraint. In our investigation we assume a spatially dense layout of nodes and we incorporate channel gain due to path-loss caused by the mobility of nodes. It is assumed that during exchange of control packets, a transmitter node can estimate channel gain and employ adaptive power control, modulation and coding. We consider a *periphery* limited mobility scenario in which nodes are restricted to move in their own local, approximately circular, periphery. For the calculation of the average throughput with path-loss, this kind of a mobility model leads us to compute the probability density function (PDF) of random distance between two nodes inside their circular periphery. Computation of this PDF constitutes a problem in geometric probability and to the best of our knowledge the derivation of PDF of random distance between points in each of two circles has never been investigated before. This is thus the first main contribution of this paper. The second main contribution of this paper is that, to the best of our knowledge, ours is the first attempt to derive a throughput maximizing optimal hop distance in a dense ad hoc network environment with such a restricted mobility model.



II. NETWORK AND MOBILITY MODEL

Consider a dense multi-hop wireless network where the nominal locations of mobile nodes are densely distributed. A contention based distributed channel access mechanism such as the CSMA/CA based DCF is employed by nodes for transmission scheduling. It is assumed that all nodes use the same contention mechanism with identical parameters and every node has packets to be transmitted at all times. For control signalling (such as RTS/CTS in IEEE 802.11), nodes do not employ power control and hence use constant power. We assume a "single cell" situation in which control packets are heard by all nodes in the network and after a node wins a contention, it reserves the channel exclusively for itself. Even if it is possible for a source to transmit packets directly to its far away destination in a single hop (by using high enough transmission power), we assume that the source instead routes packets over shorter hops taking advantage of smaller transmit power consumption in the latter case. It is further assumed that, during control signalling, a transmitter can measure the channel attenuation and apply adaptive power control, coding and modulation on a per transmission basis.

We assume a *periphery* limited mobility scenario in which every node is restricted to move within an approximate circular periphery centered at its nominal location. Inside their confined areas, the movement of nodes is in accordance with *any* arbitrary mobility model that generates a uniform spatial distribution of nodes in steady-state. Figure 1 shows such a scenario. We could approximate the periphery by a square or any other shape but a circle is a more natural choice.

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Fig. 2. Consecutive relay nodes in a route do not overlap

For the sake of clarity, the magnified box shows only nonoverlapping periphery nodes, but in general any two nodes that are moving sufficiently close to each other can have overlapping peripheries, as shown in the left box. We define *periphery hop distance* d as the distance between the centers of the periphery circles of two adjacent relay nodes that constitute a route (see Figure 2). We assume that peripheries of consecutive or adjacent relay nodes constituting a route, do not overlap (Figure 2). To be more clear on this, even though peripheries of any two nodes in general that are moving sufficiently close to each other, can overlap, peripheries of adjacent relay nodes that constitute a route are assumed to be *non-overlapping*. Thus any *feasible* periphery hop distance dsatisfies d > 2a where a is the radius of the circular periphery. This is a fairly practical assumption since for instance in the security guard MANET example that follows, two guards constituting a route need not have their assigned security zones overlapping. This kind of a mobility model can be readily applied to various situations. For example, a MANET formed by security guards who are restricted to move in their assigned security zones during a public event. A similar MANET formed by people across different rooms of a building restricted to move in their rooms. Soldiers in a battlefield moving inside their own units, a group of sensor robots moving in a mine field or nuclear establishment restricted to their confined areas, etc. are other examples. With such a mobility model, our goal is to obtain an optimal periphery hop distance d^* that maximizes an end-to-end flow throughput measure. In the rest of the paper, when we say that the distance between two nodes is d, we always actually mean that the distance between the centers of the periphery circles of the two nodes is d. Our model incorporates only randomly varying pathloss characteristics of channel gain and does not include any multi-path fading or shadowing effects. If the channel gain measured by a transmitter is denoted by h and P(h) denotes the corresponding power control applied, the channel capacity by Shannon's formula is then given by

$$C(h) = W log_2 \left(1 + \frac{P(h)h\alpha}{\sigma^2} \right)$$

where, W is the RF bandwidth, σ^2 is AWGN power and α is a constant depending on the minimum distance between periphery centers of two nodes, 2a.

III. BACKGROUND AND PROBLEM OBJECTIVES

With multi-hop communication between a source and its destination taking place in such a scenario, there is an inherent tradeoff between employing shortest path routing (in terms of number of hops) with high transmission power and many small hops with low transmission power. Shortest path routing (long hop distance routing) with high transmission power provides low end-to-end packet delays but at the same time can easily result in fast depletion of battery energy. Instead, the use of smaller hops reduces power consumption, but increases the total number of packets to be relayed in the network. These arguments clearly illustrate the need for an optimal power control policy combined with an optimal hop distance choice. Such an optimization problem for fixed multi-hop networks has been studied in [1] in which the authors obtain an optimal power control and hop distance that maximizes an end-to-end flow throughput measure in bit-metres/sec, under an average network power constraint. In this paper we use a slightly modified version of the results obtained for fixed networks in [1]. Our network model is similar to the model used in [1] augmented by our mobility model described in Section II. For the sake of clarity and completeness, we briefly re-visit the derivation of results in [1] to deduce a slightly modified version for our use.

Objective function with end-to-end flow throughput: Consider a source and its destination that are distance D apart and engage intermediate relay nodes for multi-hop communication. By the dense node layout assumption, we can always find a multi-hop path between the source and its destination such that the periphery centers of the relay nodes lie on the straight line connecting them. We assume that the periphery hop distances between consecutive relay nodes are all equal to d metres. Only one transmitter can successfully transmit packets at any given time (see Section II). Let $\Theta(d)$ denote the long-term, time average rate of packet transmissions over the entire network. Each packet transmitted in the network is counted in $\Theta(d)$ and when a packet is relayed, it is counted as many times as it is forwarded. We consider the fixed transmission time case [1] along with channel gain due to variable path-loss caused by mobility of nodes. By fixed transmission time it is meant that, upon winning channel access, a node is allowed to transmit only for a fixed amount of time T irrespective of the channel conditions. If power control is applied during a transmission opportunity then the node will be able to transmit only L(h) := C(h)T amounts of data, where h is the channel attenuation and C(h) is the available Shannon capacity as described before. Averaging the data sent during a transmission opportunity over random path-loss, the long-term data transmission rate in the network, obtained by approximate analysis in [2] (also see [3]) would be given by,

$$\Theta(d) = \frac{p_s \int_h L(h) g_H(h) dh}{p_i T_i + p_c T_c + p_s (T_o + T)}$$

where p_i is the probability that the contention period goes idle, p_s is the probability that there is a successful transmission, p_c is the probability that there is a collision, T_i is the average idle period during the contention mechanism, T_o and T_c are fixed overheads associated with a successful transmission and a collision, respectively and $q_H(h)$ is the probability density function (PDF) of random path-loss h.

Now, if we suppose that all source-destination pairs are distance D apart then there are approximately $\frac{D}{d}$ hops for a pair. As discussed before, each end-to-end packet is counted as many times as it is relayed, then with $\Theta(d)$ as the total packet rate of the network, the end-to-end aggregate flow throughput is given by

$$\frac{\Theta(d)}{\frac{D}{d}} = \frac{\Theta(d)}{D}d.$$

However each packet is transmitted across distance D. Thus the long-term end-to-end flow throughput in the system in bit*metres/sec* is given by

$$\frac{\Theta(d)}{\frac{D}{d}} \times D = \Theta(d) \, d.$$

The objective function that is to be maximized can therefore be defined as

$$\phi(P(h), d) \stackrel{\Delta}{=} \Theta(d) \, d,$$

subject to an average network power constraint that will be detailed in the following discussion. Now, denote the random distance separation between consecutive relay nodes by l. Then for a fixed d, the randomly varying path-loss from transmission to transmission is due to the randomly changing quantity l. The variation in power control P(h) from transmission to transmission will thus depend only on l for a fixed d. Maximizing $\phi(P(h), d)$ can therefore be isolated into first maximizing over the (deterministic) function $P(\cdot)$ for a fixed d and then maximizing over d. Consequently we seek to solve the following optimization problem

$$\max_{d} \max_{P(\cdot):E[P(h)] \le \bar{P}} \phi(P(h), d) \quad \text{, or}$$

$$\max_{d} \max_{P(\cdot):E[P(h)] \le \bar{P}} \frac{p_s\left(\int_h L(h) g_H(h) dh\right) d}{p_i T_i + p_c T_c + p_s\left(T_o + T\right)} \tag{1}$$

Consider the 'Channel Left Idle when Bad' case [1] when the channel is left idle for T seconds (and not relinquished for a new contention mechanism cycle to begin) if the power P(h)allocated by the transmitter is 0 for any channel state h. We see that the denominator of the objective function $\phi(P(h), d)$ in Equation 1 does not depend on P(h) and d. Thus our maximization problem of Equation 1 reduces to maximizing $\phi(P(h), d)$ given by

$$B_{\phi} \cdot d \cdot \int_{h} L(h)g_{H}(h) dh = \frac{B_{\phi}WTd}{\ln 2} \int_{h} \ln\left(1 + \frac{P(h)h\alpha}{\sigma^{2}}\right) g_{H}(h) dh$$
(2)

subject to an average network power constraint given by

$$\frac{p_i E_i + p_c E_c + p_s (E_o + \int_h P(h) g_H(h) \, dh \, T)}{p_i T_i + p_c T_c + p_s (T_o + T)} \le \bar{P}$$

where

$$B_{\phi} = \frac{p_s}{p_i T_i + p_c T_c + p_s (T_o + T)}$$

is a constant and E_i , E_c and E_o are average energy overheads due to idle time, collision and transmission respectively. As the denominator of the power constraint, as well, is a constant independent of P(h) and d, the above power constraint can be rewritten as

$$\int_h P(h)g_H(h) \, dh \leq \bar{P}',$$

where $\bar{P}' = (\bar{P}(p_i T_i + p_c T_c + p_s (T_o + T)) - (p_i E_i + p_c E_c + p_c E_c)$ $(p_s E_o))/p_s$ is now different from the previous \bar{P} . This is however a well known optimization problem that has a waterpouring form solution [6] given by

$$P(h) = \left(\frac{B_{\phi}WTd}{\lambda \ln 2} - \frac{\sigma^2}{h\alpha}\right)^+,$$

where λ is obtained from

$$\int_h P(h)g_H(h) \, dh = \bar{P}'.$$

From the above equation, note that such a water-pouring form solution assigns a large transmission power when the channel is good (large h) and no power at all when the channel is bad (small h). For computing λ and hence P(h), we determine the PDF $q_H(h)$ of path-loss in the following section. Once P(h)is known we determine the optimal periphery hop distance d^* in Section V.

IV. OBTAINING THE PATH-LOSS DENSITY

In the network model described before in Section II, we assume a spatially dense layout of nodes with peripheries of arbitrary nodes that move sufficiently close to each other, overlapping. However, according to the feasibility of periphery hop distance assumption d > 2a, the circular peripheries of consecutive or adjacent relay nodes, constituting a route in a multi-hop connection, do not overlap (see Section II for details). For simplicity, let the radius of circular periphery of all nodes be same and equal to a metres. Now, consider two circles C_1 and C_2 of radius *a* centered at (0,0) and (d, 0) with two consecutive relay nodes of a route, moving according to the mobility model described before, one in each one of them (Figure 3). Packets are transmitted between this transmitter-receiver pair after the transmitter wins a transmission opportunity at the end of a contention attempt. At the next transmission opportunity, either only the transmitter or only the receiver or both or none would have moved to another random point(s) in the circle(s). Thus each time a transmission takes place on a transmitter-receiver link, it can be assumed that the end point positions of that link are sampled independently randomly from within their peripheries. There is no correlation between positions in between transmission opportunities. Then, from transmission to transmission, the path-loss between a transmitter-receiver pair will vary randomly owing to the random distance separation phenomenon between the two nodes. In the following part of this section



Fig. 3. Random distance between two consecutive relay nodes moving inside their circular peripheries

we first compute $f_L(l)$ the PDF of random distance between any two random points in two circles and then later compute the PDF of random path-loss $g_H(h)$ from $f_L(l)$.

A. PDF of random distance between two circles

Let the distance between two randomly chosen points p_1 and p_2 in each of the two circles C_1 and C_2 shown in Figure 3 be denoted by l. Then the probability density for this random distance l can be computed as

$$f_L(l) = \frac{\int_{\mathcal{C}_1} d\vec{p_1} \int_{\mathcal{C}_2} \delta(|\vec{p_1} - \vec{p_2}| - l) d\vec{p_2}}{\int_{\mathcal{C}_1} d\vec{p_1} \int_{\mathcal{C}_2} d\vec{p_2}}$$
(3)

where $\delta(\cdot)$ is Dirac's delta function. For a fixed l, the term $\int_{C_2} \delta(|\vec{p_1} - \vec{p_2}| - l) d\vec{p_2}$ in the numerator of the equation above, represents the length of an arc of the circumference of a circle of radius l centered at $\vec{p_1}$, that lies inside C_2 . Referring to the geometry shown in Figure 3, if k is the distance from the center of the circle of radius l, centered at $p_1 = (r \cos \phi, r \sin \phi)$, to the line joining points of its intersection with C_2 , then the length of the arc inside C_2 is given by $2 l \alpha$, where

$$\alpha = \arccos\left(\frac{k}{l}\right).$$

Using polar coordinates and $d\vec{p}_1 = r \, dr \, d\phi$, the entire numerator can thus be written as

$$2l \int_{\mathcal{C}_1} \arccos\left(\frac{k}{l}\right) r \, dr \, d\phi.$$

The denominator is simply the product of the areas of C_1 and C_2 given by $\pi^2 a^4$. For computing k we proceed as follows. Consider the two circles centered at $(r \cos \phi, r \sin \phi)$ and (d, 0) with radii l and a, respectively. Denote the difference between their x coordinates as $e = d - r \cos \phi$, difference between their y coordinates as $f = 0 - r \sin \phi$ and distance between their centers by $p = \sqrt{e^2 + f^2}$. Now in $\triangle ABC$, the cosine formula for triangles gives $a^2 = l^2 + p^2 - 2 l p \cos \alpha$. But we also have $\cos \alpha = \frac{k}{l}$ and this gives the distance between center of first circle and line joining points of their intersection as

$$k = \frac{p^2 + l^2 - a^2}{2p}.$$

Note that l can vary as $d - 2a \le l \le d + 2a$. Now, from Equation 3, the PDF $f_L(l)$ can be written as

$$f_{L}(l) = \frac{2l}{\pi^{2}a^{4}} \iint_{\mathcal{C}_{1}^{o}} r \arccos\left[\frac{d^{2} + r^{2} - 2dr\cos\phi + l^{2} - a^{2}}{2l\sqrt{d^{2} + r^{2} - 2dr\cos\phi}}\right] dr d\phi$$
(4)

where $\mathcal{C}_1{}^o$ is a sub-region of the circle \mathcal{C}_1 given by

$$\cos \phi \ge \frac{d^2 + r^2 - (l+a)^2}{2dr}$$

for $d - 2a \leq l < d$ and

$$\cos\phi \le \frac{d^2 + r^2 - (l-a)^2}{2dr}$$

for $d < l \leq d + 2a$. The sub-regions are derived using the bounds $l-a \leq p$ and $l+a \geq p$ for the two circles to intersect. We have not been able to integrate $f_L(l)$ to obtain a closed form expression and hence we will pursue numerical analysis in Section V.

B. PDF of path-loss as transformation of $f_L(l)$

The path-loss h for a transmission distance l is given by $h = \frac{1}{l^{\eta}}$ where η is the path-loss exponent. Since l is randomly changing due to mobility of nodes, the transmissions encounter random path-loss. For mathematical convenience let h be defined as $h := \left(\frac{d}{l}\right)^{\eta}$, then the path-loss PDF $g_H(h)$ can be computed as

$$g_H(h) = f_L\left(\frac{d}{h^{\frac{1}{\eta}}}\right) \left|\frac{-d}{\eta \cdot h^{1+\frac{1}{\eta}}}\right|$$

From Equation 4, $g_H(h)$ is thus given by

$$g_{H}(h) = \frac{2}{\pi^{2}a^{4}} \times \frac{d^{2}}{\eta \cdot h^{1+\frac{2}{\eta}}} \times \\ \iint_{\mathcal{C}_{1}^{o}} r \arccos\left[\frac{d^{2} + r^{2} - 2 \, d \, r \cos \phi + \frac{d^{2}}{h^{\frac{2}{\eta}}} - a^{2}}{2 \frac{d}{h^{\frac{1}{\eta}}} \sqrt{d^{2} + r^{2} - 2 \, d \, r \cos \phi}}\right] dr d\phi$$
(5)

where C_1^{o} is the transformed region

$$\cos \phi \ge \frac{d^2 + r^2 - \left(\frac{d}{h^{1/\eta}} + a\right)^2}{2dr}$$

for $1 < h \le \left(\frac{d}{d-2a}\right)^\eta$ and
$$\cos \phi \le \frac{d^2 + r^2 - \left(\frac{d}{h^{1/\eta}} - a\right)^2}{2dr}$$

for $\left(\frac{d}{d+2a}\right)^\eta \le h < 1.$



Fig. 4. $\phi(P(h), d)$ for $\overline{P}=0.01$ watts



Fig. 5. $\phi(P(h), d)$ for $\overline{P}=0.05$ watts



Fig. 6. $\phi(P(h), d)$ for $\overline{P}=0.1$ watts

V. OPTIMAL HOP DISTANCE BY NUMERICAL ANALYSIS

Having obtained the PDF of path-loss in the previous section, we now obtain the optimal periphery hop distance d^* with the help of numerical analysis, since we have not been able to symbolically integrate $g_H(h)$ to obtain a closed form expression. With $h = \left(\frac{d}{l}\right)^{\eta}$ defined in the previous section, the water-pouring form optimal power control becomes

$$P(h) = \left(\frac{B_{\phi} W T d}{\lambda \ln 2} - \frac{\sigma^2 d^{\eta}}{h\alpha}\right)^+.$$

The expression for $\phi(P(h), d)$ from Equation 2 then translates to

$$\frac{B_{\phi}WTd}{\ln 2} \int_{h} \ln\left(1 + \frac{P(h)h\alpha}{\sigma^{2}d^{\eta}}\right) g_{H}(h) dh = \frac{B_{\phi}WTd}{\ln 2} \int_{h} \ln\left(\frac{h\alpha B_{\phi}WT}{\sigma^{2}d^{\eta-1}\lambda\ln 2}\right) g_{H}(h) dh$$
(6)

In order to obtain the optimal periphery hop distance d^* for an IEEE 802.11 standard scenario, in Figures 4-6 we plot $\phi(P(h), d)$ against d. In IEEE 802.11, time is divided into discrete standardised time intervals called *slots* (e.g., 1 slot = $20\mu s$ for 802.11b). The various parameters that we consider for our numerical analysis are W = 20MHz (typical channel bandwidth), T = 300 slots (example fixed transmission time), $\sigma^2 = 2.208 \times 10^{-13} watts$ (typical noise power from kTB formula), $\eta = 4$ (typical path-loss exponent for an obstructed path within a building), $\{T_i, T_o, T_c\} = \{1, 90.8, 17\}$ slots (for 802.11b), a = 10m and different values of \overline{P} . Typical values of $p_i = 0.0271$, $p_s = 0.0981$ and $p_c = 0.8748$ are used for a network of 1000 (802.11b) nodes. For computational simplification we assume $p_i E_i + p_c E_c + p_s E_o = 7.14 \times$ $10^{-6} joules$ which is a practical assumption for a network of 1000 nodes.

Figure 4 shows that, the optimal periphery hop distance, for an average network power of P = 0.01 watts, is the distance to the nearest node. In other words, the throughput metric $\phi(P(h), d)$ is maximized while choosing the smallest possible periphery hop distance. However, similar Figures 5 and 6 for $\bar{P} = 0.05 watts$ and $\bar{P} = 0.1 watts$ respectively, show that non-trivial optimal periphery hop distances are obtained and they increase with increasing average network power. An interesting shift of $\phi(P(h), d)$ towards concavity is observed, thus yielding non-trivial optimal periphery hop distances of $d^* = 25m$ and $d^* \approx 29m$, respectively. Note that even though d^* is theoretically obtained from a continuous set of feasible values, due to our dense node layout assumption, we should always be able to find a next hop whose periphery center is at-least approximately, d^* metres from the periphery center of transmitting node.

Figures 7 and 8 show PDF of path-loss $g_H(h)$ and optimal power control P(h) for $\overline{P} = 0.1watts$ and different values of d. In both these figures the path-loss h is given by $h = \left(\frac{d}{l}\right)^{\eta}$. Note that the peaks of the path-loss PDF for different d are attained at values of h < 1. They are not attained at h =1 and hence l = d is not the maximum likelihood random distance between points in two different circles whose centers are separated by distance d. Also observe that the optimal power control P(h) in Figure 8 assigns very little or almost no power for bad channel conditions (small h) and most of the power is assigned for good channel conditions (large h). Note the increase in optimal throughput value with increasing \overline{P} in Figures 4-6.

VI. CONCLUSION

In this paper we have shown that neither shortest path routing (in terms of number of hops; longest hop distance



channel gain due to path-loss 'h' 10 20 periphery hop distance 'd' (metres)

Fig. 7. Path-loss distr. $g_H(h)$ for $\overline{P}=0.1$ watts



Fig. 8. Power control P(h) for $\overline{P}=0.1$ watts

routing), nor smallest hop distance routing may be optimal for a dense mobile ad hoc network in which nodes follow a periphery restricted mobility model. With low average network power, a bit-metres/sec throughput metric may be maximized at the trivial choice of nearest node, with non-overlapping periphery, as the next hop. However, with higher amounts of average network power, we obtain optimal throughput performance at non-trivial periphery hop distances. Apart from this performance behavior, to the best of our knowledge, we have made the first attempt to derive the PDF of random distance between points in two circles.

REFERENCES

- Venkatesh Ramaiyan, Anurag Kumar and Eitan Altman "On the Transport Capacity of a Single Cell, Dense, Multihop WLAN with Fading", In preparation.
- [2] A. Kumar, E. Altman, D. Miorandi and M. Goyal "New insights from a fixed point analysis of single cell IEEE 802.11 WLANs", Proceedings of IEEE Infocom, Miami, USA, March, 2005.
- [3] G. Bianchi, "Performance analysis of the IEEE 802.11 Distributed Coordination Function", IEEE Journal on Selected Areas in Communications, 18(3): 535-547, March 2000.
- [4] Matthias Grossglauser and David N. C. Tse. "Mobility Increases the Capacity of Ad Hoc Wireless Networks", IEEE/ACM Transactions on Networking, Vol. 10, No. 4, August, 2002.
- [5] V. Kawadia and P. R. Kumar. "Power Control and Clustering in Ad Hoc Networks", Proceedings of IEEE Infocom, 2003.
- [6] A. J. Goldsmith and P. Varaiya "Capacity of fading channels with channel side information" IEEE Trans. on Information Theory, Vol. 43, No. 6, pp. 1986-1992, Nov. 1997.

[7] R. Negi and A. Rajeswaran "Capacity of power constrained ad-hoc networks", Proceedings of IEEE Infocom, 2004.