A Markovian queueing system for modeling a smart green base station

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EPEW 2015, Madrid, 31 August 2015

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Outline

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4. Numerical experiments
5. Simplified G-network model

Introduction

Motivation: modeling a autonomous and smart base station for wireless networks.

Autonomous: capable of operating without being connected to the electric grid

Smart: capable of adjusting its energy consumption to its energy level

Modeling elements

For such models, key elements are:

- energy reservoirs with energy sources
- packet flow/tasks consuming energy.

Those elements may be common to other situations: wireless sensors, autonomous robots, smart grid nodes, etc.

Modeling objectives

Principal metrics of interest:

- energy depletion probability
- statistics on time to depletion
- packet loss probability/lost traffic intensity
- statistics on time to overflow
- average queue lengths: packet/energy

at the service of (static) optimization problems for dimensioning:

- minimize delay, loss rate, ...
- maximize efficient throughput, autonomy, ...
- under appropriate constraints: maximal loss rate, minimal traffic served, etc.

Modeling inspiration

Our work shares features with previous papers:

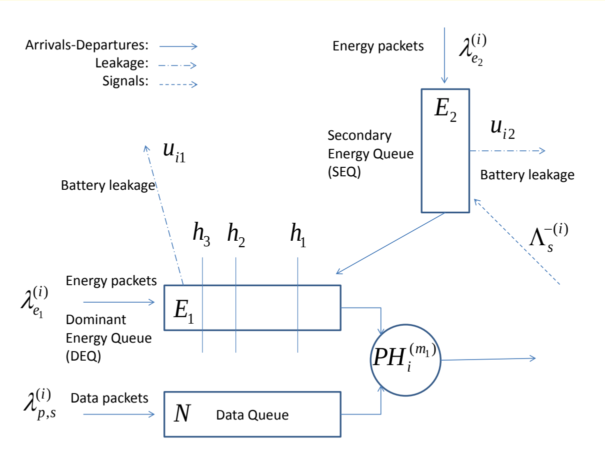
- energy is modeled as discrete units
- energy can be transferred ("energy packets") in response to signals
- an energy queue and a data queue are synchronized
- the introduction of "phases" leads to a QBD structure

Our specific contribution:

- merging these features in a quite generic and flexible model
- introducing smartness: control of operations based on the energy state.

The model: overview

We construct a model with three queues and a random environment



Modeling elements: Queues

Queues:

- Dominant Energy Queue (DEQ):
  - fed by a random enegy source
  - plus energy transfers from the secondary queue
  - depleted by the device's energy consumption
  - and energy leakage
- Secondary Energy Queue (SEQ):
  - fed by a random enegy source
  - depleted by energy transfers to the secondary queue
  - and energy leakage
- Data Packet Queue (DQ):
  - fed by a packet source
  - emptied by packet transmission.

The Data Queue and the Dominant Energy Queue are synchronized

Modeling elements: Environment

Random environment:
A Markov-modulated random environment with generator Q_Y is used to account for:

- changes in the energy supply (wind, sunlight, ...)
- changes in the offered data traffic intensity
- non-Poisson traffic
- etc.

Modeling elements: service times

Service time distribution:
The service time distribution is of phase type of order ν and depends both on the state of the RE and that of the DEQ.

Modeling elements: traffic control

Data traffic control:
The rate of packet arrival to the BS depends on its coverage area. We adopt a multi-threshold scheme in order for the BS to dynamically adjust the coverage area according to the available energy units in the DEQ.
Thresholds:

$$h_0 = 0 < h_1 < h_2 < \dots < h_K < E_1 = h_{K+1}.$$

Given the state i of RE, and if $h_s < m_1 \leq h_{s+1}$ the users' arrival rate equals $\lambda_{p,s}^{(i)}$.

Modeling elements: energy consumption

Energy consumption: Assumptions on energy consumption:

- Packet service may begin only if at least one energy unit is available in the DEQ
- Energy is consumed at the end of each service phase; the distribution depends on the service phase and the environment
- Transmission is cancelled (and packet is lost) if the DEQ depletes during transmission.

Other source of energy depletion: leakage. Depends on the environment (e.g. temperature) and battery level.

Modeling elements: energy transfers

Energy transfers: Energy passes from the SEQ to the DEQ in response to signals.

- Signals are generated at a rate $\Lambda_s^{-(i)}$: depends on the enviroment state i and the threshold level s in the DEQ
- Reception of a signal triggers the movement of k energy units from the SEQ towards the DEQ with probability $q_{ks}^{(m_1,m_2,i)}$

The state process

We have a 5-dimensional process
$X_t = \{Q_p(t), J(t), Q_{e_1}(t), Q_{e_2}(t), Y(t)\}$:
$Q_p(t)$, $J(t)$, $Q_{e_j}(t)$ are queue lengths,
$J(t)$ service process (only when $Q_p > 0$ and $Q_{e_1} > 0$),
$Y(t)$ environment process.

The state space is structured in "levels":

$$l(n) = \{ \text{ states where } Q_p = n\}$$

Cardinals:

$$\begin{aligned} |l(0)| &= M(E_1+1)(E_2+1) \\ |l(n)| &= M(E_2+1)(\nu E_1+1), \quad n \geq 1, \\ \text{State space:} \quad |\widehat{H}| &= M(E_2+1)[N(\nu E_1+1)+(E_1+1)]. \end{aligned}$$

QBD structure

The process has the structure of a homogeneous, finite-state QBD:

$$
Q = \begin{pmatrix}
B_0 & \widetilde{C} & 0 & 0\ldots & 0 & 0 \\
A_{10} & A_1 & C & 0\ldots & 0 & 0 \\
0 & A_{21} & A_1 & C\ldots & 0 & 0 \\
 & \ddots & \ddots & \ddots & & \\
 & & & & & \\
0 & 0 & 0 & \ldots A_{21} & A_1 & C \\
0 & 0 & 0 & \ldots 0 & A_{21} & A_2
\end{pmatrix}.
$$

Matrix C (packet arrivals) is diagonal and non-singular if all arrival rates $\lambda_{p,s}^{(i)}$ are > 0.

Sparcity

QBD matrices are naturally sparse. Constituting blocks are themselves quite sparse:

- Storing the blocks of Q requires space

$$O\left((\nu + M)ME_1^2E_2^2\right)$$

- Storing all Q in sparse form would require space

$$O\left(N(\nu + M)ME_1^2E_2^2\right)$$

Sparsity ratio: if environment generator Q_Y is itself sparse with factor β,

$$\alpha = O((\nu + \beta M)/N\nu M).$$

Solution algorithms

Algorithms usual for QBDs of this class (C invertible):

- Power method (after uniformization): $\pi_k = \pi_{k-1} P$ taking advantage of the sparsity
- "Exact" algebraic methods, among which:
  Recursive solution to the system of Global Balance equations:

$$
\begin{aligned}
\underline{p}_0 B_0 + \underline{p}_1 A_{10} &= 0 \\
\underline{p}_0 \widetilde{C} + \underline{p}_1 A_1 + \underline{p}_2 A_{21} &= 0 \\
\underline{p}_{i-1} C + \underline{p}_i A_1 + \underline{p}_{i+1} A_{21} &= 0 \\
\underline{p}_{N-1} C + \underline{p}_N A_2 &= 0.
\end{aligned}
$$

Solution algorithms

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- "Exact" algebraic methods, among which:
  Recursive solution to the system of Global Balance equations:

$$
\begin{aligned}
\underline{p}_0 &= -\underline{p}_1 A_{10} B_0^{-1} \\
\underline{p}_0 \widetilde{C} + \underline{p}_1 A_1 + \underline{p}_2 A_{21} &= 0 \\
\underline{p}_{i-1} &= (\underline{p}_i A_1 + \underline{p}_{i+1} A_{21}) C^{-1} \\
\underline{p}_{N-1} &= \underline{p}_N A_2 C^{-1}
\end{aligned}
$$

Experimentation

Preliminary experimentation was realized with the following setting:

- Environment: 2 states with generator

$$Q_Y = \begin{pmatrix} -0.01 & 0.01 \\ 0.1 & -0.1 \end{pmatrix}.$$

- Service durations: phase type with parameters $\underline{\tau}^{(1)} = (0.2, 0.8)$, $\underline{\tau}^{(2)} = (0.7, 0.3)$ and the matrices

$$T^{(1)} = \begin{pmatrix} -0.437 & 0.408 \\ 0.426 & -1.718 \end{pmatrix}, \quad T^{(2)} = \begin{pmatrix} -0.640 & 0.568 \\ 0.223 & -1.343 \end{pmatrix}.$$

- Energy transfers: we assumed

$$q_{ks}^{(m_1,m_2,i)} \quad \alpha \quad 2^{ki}\ 1_{\{m_1+m_2 \leq E_1\}}$$

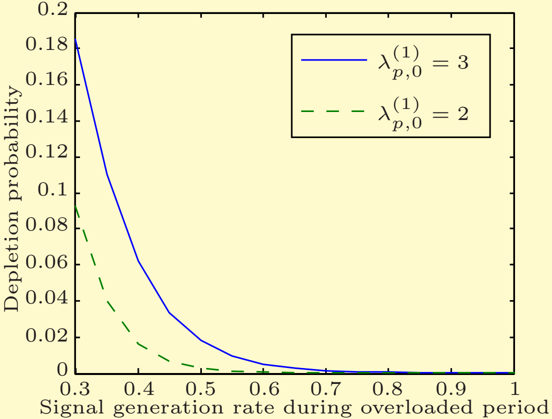
- Energy consumption: we choose $p_{kx}^{(i,m_1)}$ proportional to $(ix)^{-k}$

Other parameters

Overview of system's parameters

$E_1 = 10,\ E_2 = 12$	$M = 2,\ \nu = 2$
$N = 15,$	$h_1 = 8,\ h_2 = 12$
$\Lambda_{p,0} = diag(1.5, 1)$	$\Lambda_{p,1} = diag(2, 1.5)$
$U_1 = diag(0.1, 0.3)$	$U_2 = diag(0.3, 0.4)$
$\Lambda_1^- = diag(0.3, 0.2)$	$\Lambda_2^- = diag(0.2, 0.1)$
$\Lambda_{e_1} = diag(1.5, 1)$	$\Lambda_{e_2} = diag(1.8, 1.2)$

Experiment on the depletion probability



Sensibility of DP w.r.t. signal rate when the RE is in overload state

A G-network model

Consider the following simplifications:

- infinite capacity buffers for all three queues
- no threshold control
- 0 transmission time, consuming one energy unit
- environment-dependent parameters are averaged out using the stationary distribution $\underline{q}_Y$ of the generator Q_Y:

$$\widehat{\lambda}_p = \sum_{i=1}^{M} \lambda_p^{(i)} (\underline{q}_Y)_i .$$

Similarly for energy arrival, signal $\widehat{\lambda}^-$ and leakage $\widehat{u}_i$ rates.

G-network (ctd)

State of this model: (n, m), where

- $n = 0$, means that the BS has neither energy units in the DEQ, nor data packets to transmit,
- $n > 0$, means n data packets but no energy units in DEQ,
- $n < 0$, means $-n$ energy units in DEQ but no data packets.
- $m \geq 0$: number of energy units in SEQ.

Stability condition:

$$\widehat{\lambda}_p < \widehat{\lambda}_{e_1} + \widehat{\lambda}^- q_2, \quad \widehat{\lambda}_{e_2} < \widehat{u}_2 + \widehat{\lambda}^-, \quad \widehat{\lambda}_{e_1} + \widehat{\lambda}^- q_2 < \widehat{\lambda}_p + \widehat{u}_1,$$

where $q_2 = \frac{\widehat{\lambda}_{e_2}}{\widehat{u}_2 + \widehat{\lambda}^-}$, is the probability that the SEQ is not empty.

Simplified model, end

Proposition

If the stability condition holds, the stationary distribution of $\widetilde{X}$ has the product form:

$$p(n,m) \;=\; C\; g(n)\; q_2^m, \qquad m \geq 0,$$

$$g(n) = \begin{cases} 1, & n = 0 \\ q_1^n := \left(\dfrac{\widehat{\lambda}_p}{\widehat{\lambda}_{e_1} + \widehat{\lambda}^- q_2} \right)^n, & n > 0 \\ (\tilde{q}_1)^{-n} := \left(\dfrac{\widehat{\lambda}_{e_1} + \widehat{\lambda}^- q_2}{\widehat{\lambda}_p + \widehat{u}_1} \right)^{-n}, & n < 0 \end{cases}$$

$$C = \frac{(1-q_1)(1-\tilde{q}_1)(1-q_2)}{1 - q_1 \tilde{q}_1}.$$

Conclusion and Outlook

Present contributions:

- A quite generic queueing framework of energy/service interactions, applied to a smart base station
- QBD structure to cope with the state space size
- Simplified product-form instance

Open issues:

- Realistic numerical examples
- Statistics on time to energy depletion
- Convergence to stationarity and environment decomposition

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