On the influence of resequencing on the regularity of service

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Plan

- Introduction
- The resequencing model
- General results
- Markovian analysis
- Performance comparisons

Introduction

Communication networks $\Longrightarrow$ disordering $\Longrightarrow$ resequencing.

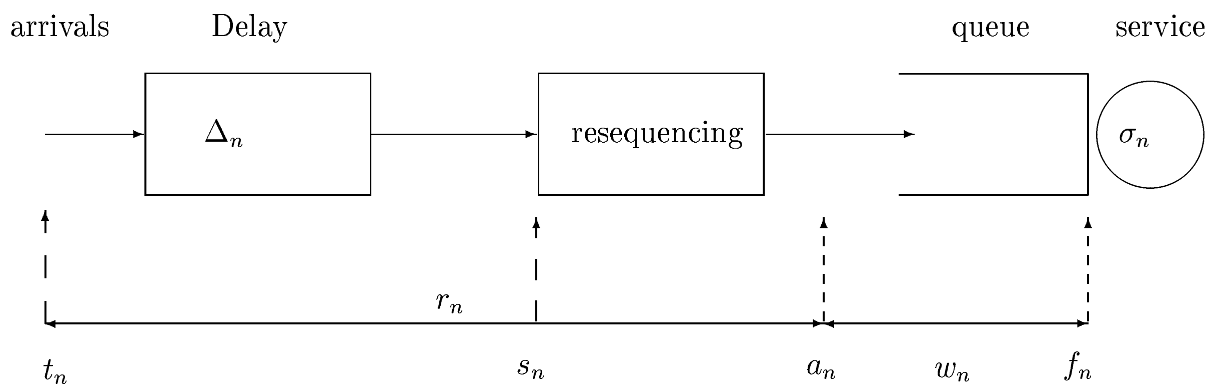
Application Level Framing $\Longrightarrow$ applications may cope with disordered elements of information.

Increased programming complexity $\Longrightarrow$ is it worth the effort?

Intuition says “yes” when *jiter* is the measure.
Jitter is caused by variations in the network delay *plus* resequencing.
Diot and Gagnon confirm this using simulation.

Our contribution: propose a quantitative model to investigate the issue.

The resequencing model



The general resequencing model

Interesting quantities:

- r_n = total delay of customer n, *i.e.* delay plus resequencing time.
- f_n = time at which the service starts.
- w_n = the waiting time in the queue.
- y_n = time spent by the server waiting customer number n.

Jitter

Need one definition of jitter.

A function of the fidelity of the network:

$$\begin{aligned} j_n &= \big((f_{n+1} + \sigma_{n+1}) - (f_n + \sigma_n)\big) - (t_{n+1} - t_n) \\ &= y_{n+1} + \sigma_{n+1} - (t_{n+1} - t_n) \ . \end{aligned}$$

Then, since σ_{n+1} is independent from the rest:

$$\begin{aligned} \mathrm{Var}(j_n) \;=\; & \mathrm{Var}(y_{n+1}) \;+\; \mathrm{Var}(\sigma) \;+\; \mathrm{Var}(t_{n+1} - t_n) \\ & -\; 2\ \mathrm{Cov}(y_{n+1}, t_{n+1} - t_n) \ . \end{aligned}$$

In the particular case where $\sigma_n \equiv \sigma$ and $t_{n+1} = t_n + \delta t$.

$$\mathrm{Var}(j_{n+1}) = \mathrm{Var}(y_{n+1}) \ .$$

Preliminaries

Model analyzed by Baccelli, Gelenbe and Plateau, 1984. Survey of results in Baccelli and Makowski, 1989.

The queueing system is actually a $G/G/\infty \to ./G/1$ network.

The presence of resequencing modifies the arrival process to the second queue.

Results known in some cases

– $M/GI/\infty$ queue: resequencing time distribution (Harrus and Plateau, 1982)

– $M/M/\infty \to ./GI/1$ queue: Laplace transform of the end-to-end delay distribution (BGP, 1984).

Dynamics

Denote

$$\xi_n := t_{n+1} - t_n \qquad \beta_n := \xi_n - \sigma_n \qquad z_n := w_n \ + \ r_n$$

Then:

$$r_{n+1} \ = \ \max\{\Delta_n, r_n - \xi_n\} \qquad \tau_n \ = \ [t_{n+1} - t_n + \delta_n - r_n]^+$$
$$= \ r_{n+1} - r_n + \xi_{n+1}$$
$$z_{n+1} \ = \ \max\{\Delta_n, r_n - \beta_n\} \qquad y_{n+1} \ = \ [w_n + \sigma_n - (r_{n+1} - r_n + \xi_n)]^-$$
$$= \ z_{n+1} - z_n + \beta_{n+1}$$

$$w_{n+1} \ = \ [w_n + \sigma_n - \tau_n]^+$$

Using Loynes' scheme:

$$z_n = \max\left\{ \max_{m=1..n}\{\Delta_m - \sum_{j=m}^{n} \beta_j\}, z_0 - \sum_{j=0..n} \beta_j \right\}$$

Qualitative Results

Stability
Resequencing does not modify stability: if $\{t_{n+1} - t_n\}_n$ and $\{\sigma_n\}_n$ are jointly stationary and ergodic, then

$$\mathbb{E}(t_{n+1} - t_n) \ > \ \mathbb{E}\sigma$$

implies the existence of a stationary regime.

Insensitivity
For all $\{t_{n+1} - t_n\}_n$ and $\{\Delta_n\}_n$, in stationary regime:

$$\mathbb{E}y_n \ = \ 1 - \mathbb{E}\sigma\mathbb{E}(t_{n+1} - t_n) \ .$$

Stochastic comparison
Internal monotonicity:
Assume that $r_0 = 0$ and $z_0 = 0$. Then for all $n \geq 0$:

$$r_{n+1} \geq_{\text{icx}} r_n$$
$$z_{n+1} \geq_{\text{icx}} z_n \; .$$

External monotonicity:
Assume that $\Delta_n \leq_{\text{icx}} \widetilde{\Delta}_n$ for all n. Then:

$$r_n \leq_{\text{icx}} \tilde{r}_n$$
$$w_n \leq_{\text{icx}} \tilde{w}_n$$

Question: under which conditions is it true that

$$y_n \leq_{\text{cx}} \tilde{y}_n?$$

Quantitative results

Assume

– a disordering model of type $D/GI/\infty$ ($t_n = n$);

– Δ integrable;

– a queueing model of type $./D/1$ with $\sigma < 1$

Then:

$$
\begin{aligned}
\mathbb{P}\{r_n \leq x\} &= \mathbb{P}\{r_0 \leq x + n\} \prod_{m=0}^{n-1} \Delta(x+m) \\
\mathbb{P}\{z_n \leq x\} &= \mathbb{P}\{z_0 \leq x + n\beta\} \prod_{m=0}^{n-1} \Delta(x+m\beta) \\
\mathbb{P}\{R \leq x\} &= \prod_{m=0}^{\infty} \Delta(x+m) \\
\mathbb{P}\{Z \leq x\} &= \prod_{m=0}^{\infty} \Delta(x+m\beta) \ ,
\end{aligned}
$$

Computing Var(Y)

In the stationary regime:

$$Y \ = \ [\Delta \ - \ Z \ + \ \beta]^+$$

Therefore for $y > 0$,

$$\mathbb{P}\{Y \geq y\} \ = \ \mathbb{P}\{Z \leq \Delta + \beta - y\}$$

and we know:

$$\mathbb{P}\{Z \leq x\} \ = \ \prod_{m=0}^{\infty} \Delta(x + m\beta) \ .$$

Markov Chains

With resequencing

Assume that the sequences $\{\Delta_n, n \in \mathbb{N}\}$, $\{t_{n+1} - t_n, n \in \mathbb{N}\}$ and $\{\sigma_n, n \in \mathbb{N}\}$ are i.i.d. and mutualy independent. Then, the following processes are Markov chains:

- $\{r_n, n \in \mathbb{N}\}$
- $\{(r_n, w_n), n \in \mathbb{N}\}$
- $\{(r_n, z_n), n \in \mathbb{N}\}$

Assume that the delays Δ_n are bounded by a constant Δ. Then w_n takes a finite number of different values and the Markov chains $\{(r_n, w_n), n \in \mathbb{N}\}$ and $\{(r_n, z_n), n \in \mathbb{N}\}$ are finite.

Without resequencing

Define: $\tilde{r}_n$ as the list of the next relative times of arrival of the delayed customers, just after the n^{th} arrival to the queue.

$\tilde{r}_n = (\tilde{r}_n^1, \tilde{r}_n^2, ..., \tilde{r}_n^{m(n)})$, where $m(n)$ is the number of delayed customers at time n.

For all Δ, $\sharp E_\Delta = 2^{\Delta+1} - 2^{\Delta-1} = 3\ 2^{\Delta-1}$.

The process $\{(\tilde{r}_n, w_n)\}$ is a finite Markov chain.

Distribution of inter-arrivals

With resequencing

$$\begin{aligned} \mathbb{P}(\tau = 0) &= \bar{\epsilon}\,(1 - \bar{\epsilon}^{\Delta}) & \mathbb{P}(\tau = k) &= \epsilon^2\,\bar{\epsilon}^{k+1} \quad 2 \le k \le \Delta \\ \mathbb{P}(\tau = 1) &= \bar{\epsilon}^{\Delta+1} + \epsilon^2 & \mathbb{P}(\tau = \Delta + 1) &= \epsilon\,\bar{\epsilon}^{\Delta}. \end{aligned}$$

$$\mathrm{Var}(\tau) = \frac{\bar{\epsilon}}{\epsilon}(1 - \bar{\epsilon}^{\Delta}(1 + \Delta\epsilon)).$$

Without resequencing

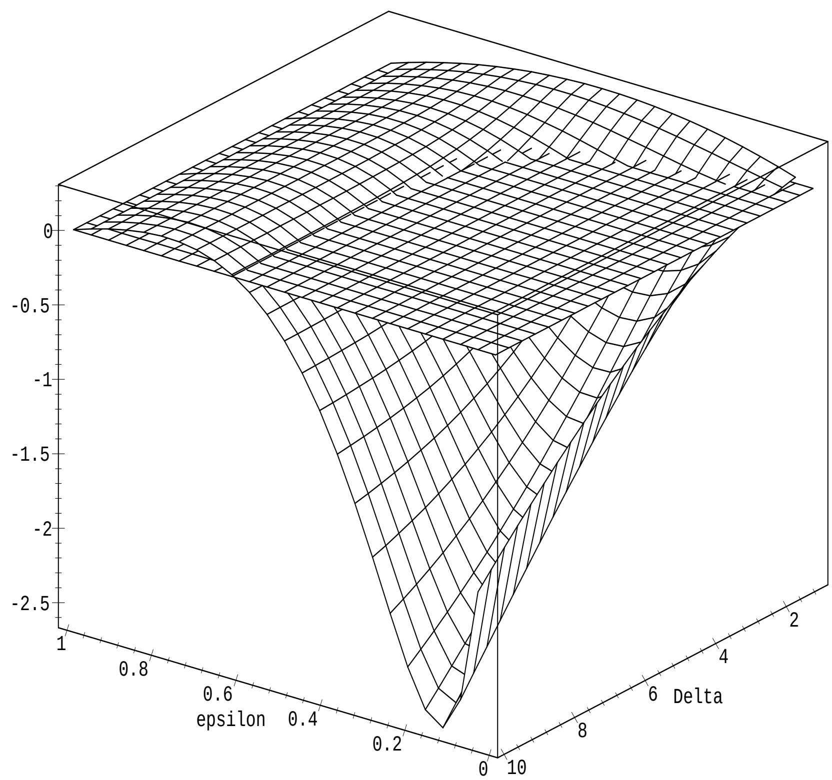
$$\mathrm{P}(\tau = 0) = \epsilon, \quad \mathrm{P}(\tau = k) = \bar{\epsilon}^2 \cdot \epsilon^{k-1} \text{ for } k \ge 1;$$

$$\mathrm{Var}(\tau) = \frac{2\epsilon\bar{\epsilon}\,[1 - (\epsilon\bar{\epsilon})^{\Delta}]}{1 - \epsilon\bar{\epsilon}}.$$

Comparison of arrival processes

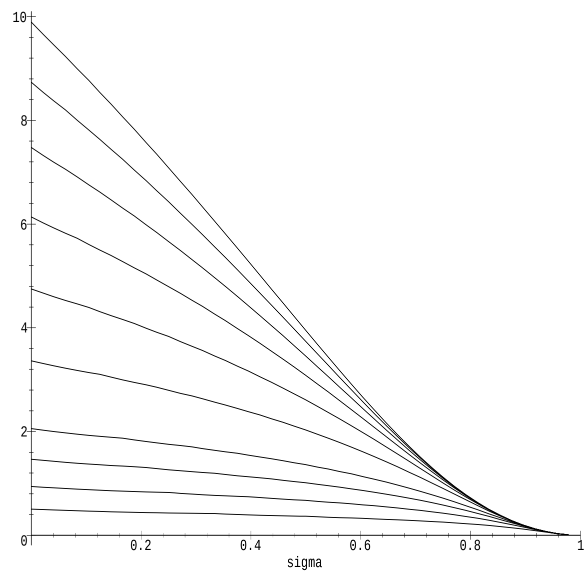
With resequencing: τ_R. Without resequencing: τ_N.

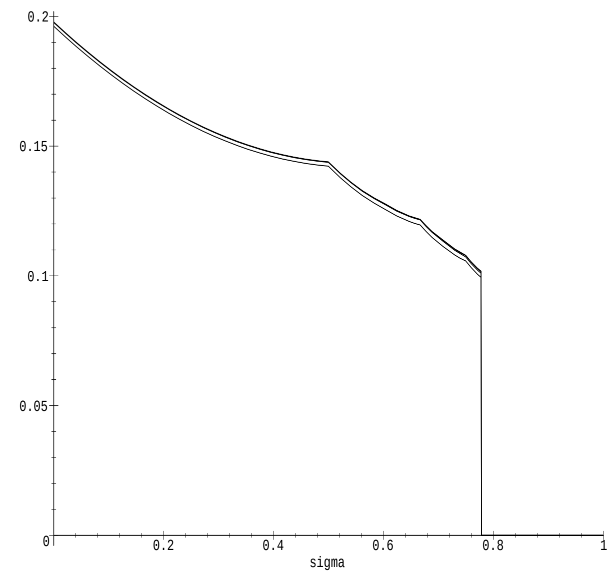
Graph of the difference between variances: $\mathrm{Var}(\tau_N) - \mathrm{Var}(\tau_R)$.



Increased variance: when Δ large and ϵ small.

Comparison of server waiting times





Values of $V_R(\sigma)$ (left) and $V_N(\sigma)$ (right) for different values of Δ, $\epsilon = 0.1$

A bad performance measure

The idle period length distribution:

$$\mathbb{E}I = \frac{1-\sigma}{\epsilon + \bar{\epsilon}^{\left\lfloor \frac{\Delta}{1-\sigma} \right\rfloor + 1}}$$

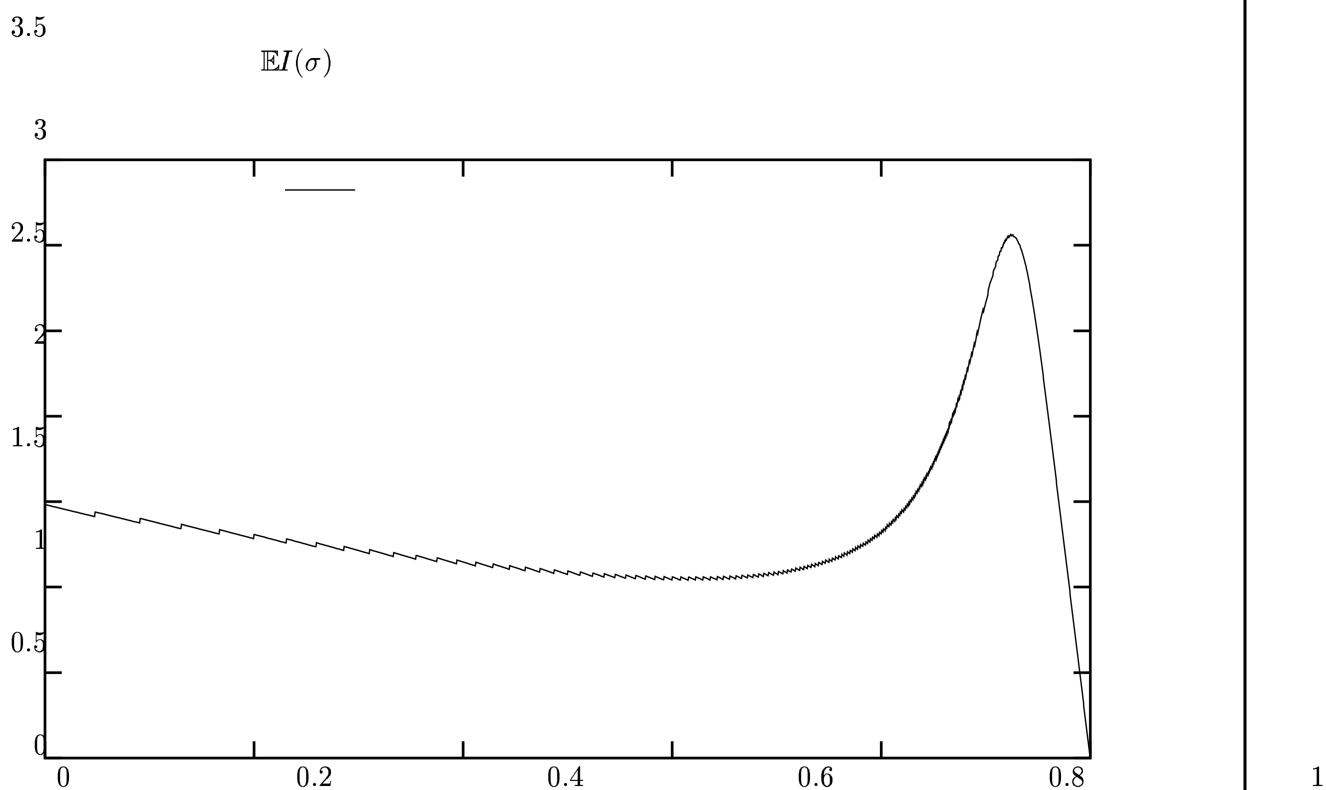


Figure 1: $\mathbb{E}I(\sigma)$ for $\Delta = 20$, $\epsilon = 0.02$

Conclusion

– Intuition is not always founded: resequencing may *de*^{crease} the variance of the arrival process, and jitter;

– Not resequencing is better if $\sigma \leq \sigma^*(\epsilon)$, and $\sigma^*(\epsilon) \simeq 1$ when $\epsilon \to 0$;

– More research with other tractable models: $D/Geom/\infty \to ./D/1$ with and without resequencing;