An Anonymous Sequential Game Approach for Battery State Dependent Power Control

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Informal description

- A large population of mobile terminals competing for a wireless access.
- Each terminal attempts transmission over a sequence of time slots.
- At each attempt, it has to take the decision on the transmission power based on its battery energy state.
- The transmission ends when the battery is empty.
- The aim of each player to maximize his throughput minus the cost of the transmission over whole lifetime of his battery.

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Challenges and mathematical tools I

- Problem has 2 dimensions:
  1. Increase in the transmission power has a crucial impact on the achievable throughputs.
  2. Each mobile has to take into consideration the battery state in the decision taking, which influenced by its past choices.

  A mobile that is interested in maximizing the amount of information during its lifetime, has to balance the two.

  $\Longrightarrow$ Dynamic model (stochastic game)
- In a CDMA type cellular system all mobiles transmit simultaneously to a common base station.
  1. Performance of a mobile determined by the distribution of the actions used by all other mobiles.
  2. Mobiles are indistinguishable.

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The framework that combines the two mentioned above is that of anonymous sequential game

- Introduced by Jovanovic and Rosenthal (1988), used subsequently in some economic models.
- Only for discounted cost.

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Model for individual player I

We associate with each player a Markov Decision Process with the following parameters:

- $S = \{0, 1, \dots, N\}$ is a finite set of states.
- $Q = \{q_1, \dots, q_K\}$ is a finite set of actions.
- P is the transition probability law. Namely, $p(s'|s, q)$ is the probability that the next state is s' given that the actual state is s and the action taken is q.

Model for individual player II

- The state s^i of player i = his battery energy state
- The action of a player = energy level at which he transmits
  The set of actions available to a player in state s is

  $$Q_s = \{q_1, \ldots, q_s^+\} \subset Q$$

  For any two $s_1 < s_2$, $q_{s_1}^+ \leq q_{s_2}^+$.
- The transitions are defined for individual states of the players.
  The probability of staying in state s by a player taking action q is $p(q)$.

  $$p(q) = 1 - \alpha q - \gamma,$$

  where α and γ are some fixed positive coefficients.
  At any given time a mobile whose battery is empty may have it recharged with probability p_{0N}

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Model of interactions

- The global state of the system at time t is

$$X^t = (X_0^t, \ldots, X_N^t),$$

such that $\sum_{n=0}^{N} X_n^t = 1$
X_s^t = fraction of mobiles with battery state s.

- The reward at time t for a user in state s_t playing q_k^t when the vector of proportions of players using different actions is w^t is

$$R^t(q_k^t, s_t; w^t) = \frac{q_k^t}{\sigma^2 + C\sum_{l=1}^{K} q_l^t w_l^t} - \beta q_k^t,$$

C – interference parameter
σ^2 – noise power
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Policies

A stationary policy of a player u is a map from the set of possible individual states S to the set of probability measures over set Q, $P(Q)$.

- We do not consider general (history/time-dependent) policies.
- We assume the game is in a stationary regime
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The equilibrium

The objective of player is to maximize his reward over the whole lifetime of his battery:

$$J(v,u) = E^{p,X^0} \sum_{t=1}^{\tau} R^t(u,v).$$

τ – lifetime of a player (random variable)

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Threshold policies

$$\mathcal{U}_0 = \Bigg\{ u \in \mathcal{U} : \exists s_0 \in S, \exists r \in [0,1],$$

$$u_s = \left\{ \begin{array}{ll} \delta[q_1], & s < s_0 \\ r\delta[q_1] + (1-r)\delta[q_s^+], & s = s_0 \\ \delta[q_s^+], & s > s_0 \end{array} \right\} \Bigg\}.$$

$\delta[q]$ – probability measure concentrated in q.

Equilibrium characterization theorem I

Theorem

The game under consideration always possesses an equilibrium $u \in \mathcal{U}_0$. Moreover:

(i) *This equilibrium is unique in the set $\mathcal{U}_0$.*

(ii) *If*

$$\beta CN > \frac{N\alpha(1-\beta\sigma^2)+N\gamma}{\alpha q_1^+ + \gamma} \quad \text{and} \quad p_{0N} \leq \frac{\alpha(1-\beta\sigma^2)}{\beta CN - \frac{N\alpha(1-\beta\sigma^2)+N\gamma}{\alpha q_1^+ + \gamma}}$$

then $u^+(s) = \delta[q_s^+]$ is the equilibrium.

Equilibrium characterization theorem II

(iii) *If*

$$\beta C q_1 > 1 - \beta\sigma^2 \quad \textit{and} \quad p_{0N} \geq \frac{(\alpha q_1 + \gamma)(1 - \beta\sigma^2)}{N(\beta C q_1 - (1 - \beta\sigma^2))}$$

then $u^-(s) = \delta[q_1]$ *is the equilibrium.*

The proof I

Step 1:

For each policy $u \in \mathcal{U}$ we compute a stationary global state $X(u)$.

Step 2:

We introduce in the set $\mathcal{U}_0$ a linear ordering under which $\mathcal{U}_0$ is homeomorphic with the interval $[1, N]$ (h – the homeomorphism). We show that the global interference

$$A(u) = \sigma^2 + C \sum_{k=1}^{K} w_k(u) q_k$$

is continuous increasing on $\mathcal{U}_0$.

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The proof II

Step 3:

We show that the equilibrium policy is a solution to the equation

$$A(u) = \frac{1}{\beta},$$

which exists and is unique, if A admits values both greater and smaller than $\frac{1}{\beta}$, since A has an intermediate value property.

Step 4:

We compute when $A(u) > \frac{1}{\beta}$ for every $u \in \mathcal{U}_0$ and when $A(u) < \frac{1}{\beta}$ for every $u \in \mathcal{U}_0$. This gives the conditions for u^- or u^+ to be optimal. $\square$

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Theorem

The equilibrium $u^* \in \mathcal{U}_0$ in the game under consideration can be computed using bisection applied to function

$$\phi(a) = A(h^{-1}(a)) - \frac{1}{\beta}$$

on the interval $[1, N]$. The approximation of u^* will be given by $h^{-1}(a^*)$, where a^* is the (approximate) zero of function ϕ. If the zero does not exist then either u^+ (when $\phi < 0$) or u^- (when $\phi > 0$) is the equilibrium.

Simple example with three battery states I

Three possible states of the battery:

- empty (E),
- almost empty (A),
- full (F).

Whenever the player is in state F he has two actions possible:

- To transmit at high power h,
- To transmit at low power l.

If he is in state A, he can only transmit at low power l.

Simple example with three battery states II

The strategy u^- is optimal when

$$\beta Cl > 1 - \beta\sigma^2$$

and

$$p_{EF} \geq \frac{(\alpha l + \gamma)(1 - \beta\sigma^2)}{\beta Cl - (1 - \beta\sigma^2)}.$$

The strategy u^+ is optimal when

$$\beta\sigma^2(\alpha(l+h) + 2\gamma) > (l+h)(C\beta\gamma - \alpha) + 2(C\beta\alpha hl - \gamma)$$

or

$$p_{EF} \leq \frac{(\alpha l + \gamma)(\alpha h + \gamma)(1 - \beta\sigma^2)}{(l+h)(C\beta\gamma - \alpha) + 2(\beta\alpha hl - \gamma) - \beta\sigma^2(\alpha(l+h) + 2\gamma)}$$

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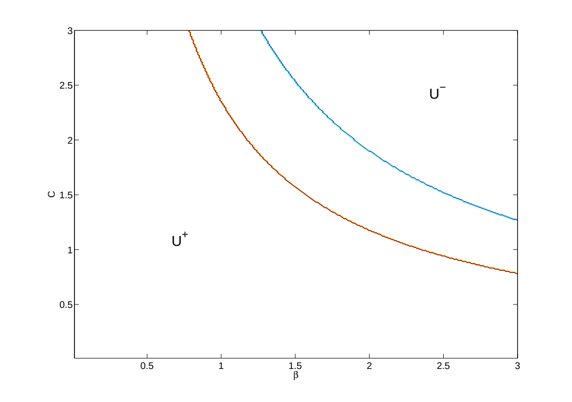
Simple example with three battery states III

The regions where strategies u^+ and u^- are optimal as a function of β and C for $p_{EF} = 0.5$, $\alpha = 0.8$, $\gamma = 0.05$, $h = 1$, $l = 0.5$, $\sigma^2 = 0.00001$.



Simple example with three battery states IV

Equilibrium strategies for the region where neither u^- nor u^+ are optimal:

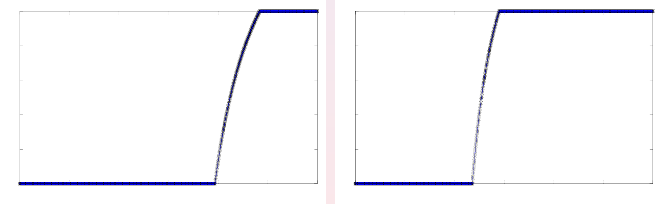
$$u^*(s) = \begin{cases} \delta[l], & s = L \\ r\delta[l] + (1-r)\delta[h], & s = H \end{cases}$$

where

$$\begin{aligned} r &= \frac{Cp_{EF}[2\alpha lh + \gamma(l+h)]}{(l-h)[(1-\beta\sigma^2)\alpha(p_{EF}+\alpha l+\gamma) - Cp_{EF}(2\alpha l+\gamma)]} \\ &- \frac{(1-\beta\sigma^2)[\alpha p_{EF}(l+h) + 2p_{EF}\gamma + (\alpha l+\gamma)(\alpha h+\gamma)]}{(l-h)[(1-\beta\sigma^2)\alpha(p_{EF}+\alpha l+\gamma) - Cp_{EF}(2\alpha l+\gamma)]}. \end{aligned}$$

Simple example with three battery states V

The dependance of r on β for $p_{EF} = 0.5$, $\alpha = 0.8$, $\gamma = 0.05$, $h = 1$, $l = 0.5$, $\sigma^2 = 0.00001$, and for $C = 1.2$ (left) and $C = 2$ (right)



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- Checking whether in our model the dynamics converge for variants of known evolutionary dynamics.
- Finding the algorithms that converge (proving their convergence).
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2. Extension of the model presented here to include different types of users (differing by some battery characteristics like maximal achievable battery energy or probabilities $p(q)$).

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