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Resume of research activities

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My research activities concern the feedback control of nonlinear systems, with a particular interest in the sub-class of "critical systems" and the applications in mechanics. By "critical system" I mean a control system, in its more general form

$$\dot{x} = X(x, u) \tag{1}$$

which is controllable in a reasonable sense (like STLC), but such that the linearized systems at equilibrium points are not asymptotically stabilizable (and thus, not controllable). In this framework, the problem that I (and many other people) try to address, is the design of feedback laws capable of stabilizing "reference trajectories", i.e., curves $t \longmapsto x_r(t)$ in the state space, solutions to (1) for some "reference input" u_r. A specificity of my research is the attention paid to robustness issues. Robustness is the main motivation for using feedback control (by opposition to open-loop control), and the "linear systems community" has devoted much effort to the development of robust control approaches. Robustness issues are still more important for nonlinear systems, and above all for critical systems. An important part of my research has been devoted to the search for feedback control solutions with good performance and robustness properties. Another objective of this research is to achieve a good balance between theoretical developments and applications. From a theoretical standpoint, it is important to develop solutions as general as possible. But it is also important to keep in mind potential applications. In this respect, I've had the chance to work in a team driven by robotic applications. Having said this, my research activities has essentially focused (to simplify) on the control of nonholonomic systems, and the control of underactuated systems.

1 Control of nonholonomic systems

Kinematic equations of nonholonomic mechanical systems are given by driftless control systems of the form

$$\dot{x} = \sum_{i=1}^{m} u_i X_i(x) \tag{2}$$

with x belonging to a n-dimensional manifold M, $X_1, \ldots, X_m$ the system's control vector fields (v.f.) representing admissible directions compatible with the nonholonomic constraints, and $u_1, \ldots, u_m$ the control inputs. The system's nonholonomy is characterized by the fact that $m < n = \dim(x)$. Any point x_0 is an equilibrium point of System (2), associated with the input $u = (u_1, \dots, u_m) = 0$, and the linearized system at any equilibrium is not controllable nor asymptotically stabilizable due to that $m < n$. Thus, all these systems are critical. The celebrated Brockett's theorem (Brockett 83) implies, in addition, that if $X_1(x_0), \dots, X_m(x_0)$ are independent vectors, then x_0 cannot be asymptotically stabilized by smooth pure-state feedback $u(x)$. To achieve asymptotic stabilization of such an equilibrium point, other types of feedback laws have to be used. As shown in (Samson 90) for a wheeled robot example, and generalized in (Coron 92) for arbitrary controllable systems of the form (2), smooth time-varying (time-periodic) feedback is a possible solution to achieve asymptotic stabilization. At the end of the 1990s, I have worked a lot on this type of feedback, and more specifically on the sub-class of (degree-zero) homogeneous feedbacks which allow to achieve exponential convergence rate.

1.1 Exponential stabilization of equilibrium points by time-varying homogeneous feedback

Homogeneity of vector fields (w.r.t. families of dilations) is a concept that has been used for a long time in nonlinear control theory. It is an essential ingredient of controllability analyses (Sussmann 83; Sussmann 87; Kawski 90a), but it is also useful for asymptotic stabilization studies (Hermes 91; Kawski 90b). For example, given an autonomous system $\dot{x} = X(x)+R(x)$ with an equilibrium point at x_0, the asymptotic stability of x_0 is equivalent to its *exponential* stability[1] whenever X is a vector field "homogeneous of degree zero" (w.r.t. a family of dilations), and R is homogeneous of higher-order. This generalizes the classical "linear-like" case $\dot{x} = Ax + o(x)$, i.e., $X(x) = Ax, R(x) = o(x)$. Understandably, this is an interesting property. (M'Closkey 93) is one of the first paper in which time-varying homogeneous feedback is used to exponentially stabilize the pose (i.e. position and orientation) of a nonholonomic mobile robot. Previous methods such as (Samson 90; Pomet 92; Teel 92), based on the use of *smooth* time-varying feedback, did not allow to achieve exponential convergence rates. In (Morin 99a), we have proposed a method for the design of asymptotically stabilizing continuous and homogeneous feedback laws, which applies to *any* controllable system (2). The asymptotic (and thus exponential) stability is only local in the general case, but it is global when the closed-loop system is itself homogeneous. This method makes use of the open-loop control algorithm developed in (Sussmann 91; Liu 97), and in fact it shows how the open-loop control there obtained can be transformed into asymptotically stabilizing feedbacks. While this method is general, its complexity increases rapidly with the size of the system (more precisely, with the *length* of the Lie brackets of the control v.f. necessary to generate n independent directions at the considered equilibrium). For this reason, we have also developed other methods for the design of stabilizing homogeneous time-varying feedbacks (Morin 97a; Morin 00), less general, but providing much simpler solutions and better results in practice for standard wheeled vehicles like unicycles or car-like vehicles.

[1] w.r.t. a homogeneous norm.

1.2 Robustness to unmodeled dynamics and hybrid feedback

The homogeneous feedback laws evoked above for the asymptotic stabilization of nonholonomic systems have a disadvantage. They are not "robust w.r.t. unmodeled dynamics". To make this statement more precise, let us consider the following "perturbation" of System (2) :

$$\dot{x} = \sum_{i=1}^{m} u_i(X_i(x) + \varepsilon_i Y_i(x)) \tag{3}$$

with $\varepsilon_1, \ldots, \varepsilon_m$ denoting "small constants", and $Y_1, \ldots, Y_m$ denoting smooth v.f. The terms $\varepsilon_i Y_i(x)$ can represent for example, in the case of a car-like robot, modeling errors in the vehicle's geometry. Now the question is : given a feedback law designed for System (2) to achieve some property, will it still achieve the same property for System (3) with the Y_i's being arbitrary and the ε_i's small enough ? When the property at stake is the asymptotic stability of the equilibrium point, we have shown in (Lizárraga 99) that the answer to this question is no, whatever the used homogeneous feedback law. In practice, this means that there are always some Y_i's such that, for arbitrarily small (but nonzero of course) ε_i's, the system's state will keep on oscillating around the equilibrium point. This is clearly not satisfactory in practice for a mechanical system. This has led us to look for other control solutions, more robust to this type of errors. In (Maini 99), we had shown that for unicycle and car-like vehicles, robustness to unmodeled dynamics could be obtained, but with smooth feedbacks, thus yielding slow convergence (i.e. not exponential). Another approach had been proposed in (Bennani 95), with the additional benefit of exponential convergence rates. It is based on what is commonly called *hybrid feedback*, which consists essentially in iterating open-loop control laws, i.e. $u = u(x(kT), t),\ k \in \mathbb{N},\ t \in [kT, (k+1)T)$, with T an "updating-period", which must not be confused with the sampling period of low-level numerical controllers. In (Morin 99b), we have generalized (and in some sense clarified) the result of (Bennani 95), by providing a control design method of hybrid feedback laws that applies to *any* controllable system (2). We have also proved that these control laws are robust to unmodeled dynamics : whatever the v.f. Y_i, exponential convergence to the equilibrium is preserved for small enough values of the ε_i's.

1.3 Practical stabilization and transverse functions : another perspective

1.3.1 Motivation and theoretical foundations

Although we have devoted a lot of time and energy to the search for robust exponential stabilizers of equilibrium points, we have not been able to obtain completely satisfactory solutions. The hybrid control solutions described in the previous section are robust w.r.t. unmodeled dynamics, and thus better than homogeneous feedbacks in this respect, but they suffer from other robustness problems. For example, variations of the "updating-period" T, even very small, can destroy the property of convergence to the equilibrium, leading once again to oscillations around the equilibrium point. Achieving simultaneously fast (exponential) convergence and robustness seems unrealistic in practice. For this reason (and others that will become clear below), we have been developing a feedback control approach that relies on an objective of *practical* stabilization, instead of the usual *asymptotic* stabilization objective. The foundation of this so-called "transverse function approach" is a non-trivial result that we have published in (Morin 01). It says that the two following properties are equivalent :

1. System (2) is locally controllable around a point x_0, i.e., it satisfies the Lie Algebra Rank Condition at this point.
2. There exists $\bar{n} \in \mathbb{N}$ and a family of smooth functions $(f_\varepsilon)_{\varepsilon>0}$, with $f_\varepsilon : \mathbb{T}^{\bar{n}-m} \longrightarrow B(x_0, \varepsilon)$, such that for *any* $\alpha \in \mathbb{T}^{\bar{n}-m}$, the vectors

$$X_1(f_\varepsilon(\alpha)), \ldots, X_m(f_\varepsilon(\alpha)), \frac{\partial f_\varepsilon}{\partial \alpha_1}(\alpha), \ldots, \frac{\partial f_\varepsilon}{\partial \alpha_{\bar{n}-m}}(\alpha)$$

span the tangent space at $f_\varepsilon(\alpha)$. Here $\mathbb{T}^{\bar{n}-m}$ denotes the torus of dimension $\bar{n}-m$, and $B(x_0, \varepsilon)$ the ball of radius ε centered at x_0. Functions f_ε satisfying this property are called "transverse functions".

The way this result can be used to control System (2) is as follows : instead of trying to stabilize the state x to x_0, one tries to stabilize x to the image set $f_\varepsilon(\mathbb{T}^{\bar{n}-m}) \subset B(x_0, \varepsilon)$. This becomes (at least locally) a trivial task because, around this set, one disposes of n independent degrees of freedom to move "in the direction of the set" : the directions tangent to the set itself, and the control directions. This is in this way that practical stabilization is obtained, with the control parameter ε used to monitor the stabilization precision.

1.3.2 The Lie group framework

The proof that we had given in (Morin 01) was constructive but relatively elaborate. Furthermore the value of $\bar{n}$ provided by this algorithm was not always optimal. Finally, the paper did not provide much insight concerning the applications of the concept of transverse functions to the control design and to applications. All these issues have been clarified in (Morin 03) by considering the framework of invariant systems on a Lie group. To avoid confusion in the notation, I use the standard notation g for an element of a Lie group G, and rewrite System (2) as

$$\dot{g} = \sum_{i=1}^{m} u_i X_i(g) \tag{4}$$

where the X_i's are now assumed to be *left-invariant* vector fields on G, meaning that $dL_{g_1}(g_2)X_i(g_2) = X_i(g_1 g_2)\ \forall i$, where $g_1 g_2$ is the group product of g_1 and g_2, L_{g_1} is the left-translation by g_1, and d stands for the operator of differentiation. For more details on Lie groups see e.g. (Varadarajan 84; Warner 83). Before describing some of the results that we have obtained, let me provide two reasons why this framework is important and not so restrictive as it may first seem.

1. First, there are many nonholonomic systems which can be modeled by System (4). The simplest examples are unicycle-type wheeled robots, much used in robotics. Although they are not systems on Lie groups, other wheeled vehicles like cars, possibly with trailers, can be "semi-globally" modeled by the celebrated chained-form systems

(Murray 91; Sørdalen 93), which are systems on Lie groups. Other robotic systems, like e.g. the rolling sphere (Bicchi 97; Halme 96), are also systems on a Lie group. As a matter of fact, the importance of this concept for robotic applications has long been recognized (Brockett 84; Leonard 95b; Murray 94).

2. Second, from a *local* point of view, the properties of System (2) are essentially determined by the properties of an adequate "homogeneous approximation", i.e., a system

$$\dot{y} = \sum_{i=1}^{m} u_i Y_i(y) \tag{5}$$

where each Y_i is an homogeneous v.f. which approximates X_i. Due to the nilpotence of the Lie algebra generated by the family $\{Y_1, \ldots, Y_m\}$, either System (5) is a system on a Lie group, or it can be "lifted" to a system on a Lie group (see (Morin 03) for more details).

Let me now return to what is shown in (Morin 03). First, we assert that for systems on a Lie group, $\bar{n}$ is the smallest possible value, i.e. $\bar{n} = n$. Then, we provide a formula for the calculation of transverse functions, which can be defined as the group product of $n - m$ functions of one variable, i.e.

$$f_\varepsilon(\alpha) = f_{\varepsilon,n-m}(\alpha_{n-m}) f_{\varepsilon,n-m-1}(\alpha_{n-m-1}) \cdots f_{\varepsilon,1}(\alpha_1)$$

with $\alpha = (\alpha_1, \ldots, \alpha_{n-m})$. Finally, we derive, from a transverse function, feedback laws that yield practical stabilization of *any* reference trajectory g_r, i.e. any smooth curve $t \longmapsto g_r(t) \in G$. The simplicity of such a derivation allows to expose it here. Given a reference trajectory g_r, the "tracking-error" is naturally defined in the Lie group sense as $\tilde{g} = g_r^{-1} g$. Using standard differentiation rules in Lie groups, one obtains

$$\dot{\tilde{g}} = \sum_{i=1}^{m} u_i X_i(\tilde{g}) + P(\tilde{g}, g_r)\dot{g}_r \,, \quad \text{with} \quad P(\tilde{g}, g_r) = -dR_{\tilde{g}}(e) dL_{g_r^{-1}}(g_r)$$

Here R_g is the right-translation by g, e the group's unit element, and L and d are defined as above. Now, let $z = \tilde{g} f_\varepsilon(\alpha)^{-1}$ with f_ε a transverse function defined on $\mathbb{T}^{n-m}$. Then,

$$\dot{z} = dR_{f_\varepsilon(\alpha)^{-1}}(\tilde{g}) dL_z(f_\varepsilon(\alpha)) \left(\sum_{i=1}^{m} u_i X_i(f_\varepsilon(\alpha)) - df_\varepsilon(\alpha)\dot{\alpha} + dL_{z^{-1}}(\tilde{g}) P(\tilde{g}, g_r)\dot{g}_r \right)$$

Due to the invertibility of the operators dL_g and dR_g (for any g), and the property of transversality of f_ε, it is possible (and straightforward) to determine u and $\dot{\alpha}$ so that $\dot{z} = Z(z)$ with Z an arbitrary vector field. Choosing this vector field such that e is an asymptotically stable equilibrium of $\dot{z} = Z(z)$ yields the asymptotic convergence of the tracking error $\tilde{g}$ to the image set $f_\varepsilon(\mathbb{T}^{n-m})$. Among the important properties thus obtained let us mention the following ones.

1. The obtained control laws are smooth. This accounts for much stronger robustness properties than previous methods (trying to achieve exponential stabilization) when the reference trajectory is a fixed point.

2. The upper-bound of the ultimate tracking error, ε, is *independent* of the reference trajectory.
3. *No assumption* is made on g_r (except of course differentiability). This is a very important property for applications. First, the same controller can be used for any reference trajectory. By contrast, it has been shown in (Lizárraga 04a) that achieving *asymptotic* stabilization of arbitrary admissible trajectories with a unique controller is usually not possible for nonholonomic systems (thus yielding to the necessity of hard to analyze switching strategies, when the reference trajectory is not known in advance). Then, the controller can also be used for nonadmissible trajectories (i.e., which do not satisfy the nonholonomic constraints of the system). One can easily imagine how this can be used for motion algorithms in cluttered environments.

1.3.3 Applications

The domain of application of this approach is large. While the above description provides the general framework, further work can be necessary to refine the method and improve the performances, either from a general viewpoint, or in relation with applications. Let me just mention some applications on which we have concentrated so far.

1. Unicycles : this case has been investigated in detail in the Ph.D. thesis of (Artus 05), with very good experimental results on our mobile platform ANIS equipped with a camera for the pose reconstruction. In these experiments, the vehicle was able to track an *omnidirectionnal* moving target based on the measurements provided by the camera. This system has also been studied in the Ph.D. thesis of (Maya-Mendez 07), with the objective of characterizing the robustness of the approach to pose estimation errors.
2. Cars : although this system is not an invariant system on a Lie group, the approach can be easily applied to this case. We have obtained very good simulation results in this case, but currently we do not dispose of an operational vehicle for experiments.
3. Nonholonomic mobile manipulators : this class of systems, composed of a manipulator arm mounted on a nonholonomic platform, has been studied in the Ph.D. thesis of (Fruchard 05). Based on the transverse function approach, we have developed two methods for the coordinated control of these systems. One of them has been published in (Fruchard 06).
4. Rolling spheres : this example presents very original features w.r.t. standard wheeled robots. We have recently started to work on this system (Morin 08), and plan to pursue this study.

2 Control of underactuated systems

The class of underactuated systems contains a large number of examples with diverse properties, and the difficulties associated with their control can be quite different. From a mechanical viewpoint, one can roughly define under-actuation as follows : the number of independent force and torque inputs is strictly smaller that the dimension of the velocity space. From a control viewpoint, it is important in the first place to distinguish between *critical* underactuated systems, and non-critical ones.

2.1 Control of Euler-Poincaré equations

Critical underactuated systems are especially difficult to control because, contrary to nonholonomic systems, their main nonlinearities are at the dynamical level. Therefore, one has to study them at this level, in the framework of systems with drift. For nonlinear systems with drift there remains many open questions like, to begin with, the complete characterization of controllability. In mechanics, an important class of systems can be modeled by the so-called " Euler-Poincaré equations" (Marsden 99), given by :

$$\left\{ \begin{array}{rcl} \dot{g} & = & \displaystyle\sum_{i=1}^{n} v_i X_i(g) \\ \dot{v} & = & Q(v) + \displaystyle\sum_{i=1}^{m} u_i b_i \end{array} \right. \tag{6}$$

Like in Eq. (4), g denotes the element of a n-dimensional Lie group G, the X_i's are left-invariant (and independent) vector fields on G, and $v = (v_1, \ldots, v_n)$ is the velocity vector. At the dynamical level, Q denotes a vector-valued quadratic form in v, the b_i's are independent "control directions", and the u_i's are the control inputs, with $m < n$ due to the system's underactuation. System (6) represents the dynamics of rigid bodies, in the absence of external forces (e.g. gravity), like underactuated spacecrafts, "hovercrafts", blimps, underwater vehicles, etc. Even for this well delimited and structured class, fundamental questions remain open despite many research efforts (Bullo 05). Concerning the control of these systems, methods based on "periodic forcing" (i.e. periodic control inputs), possibly iterated to yield hybrid feedback in the sense of Section 1.2, have been proposed (Leonard 95a; Bullo 00; Lizárraga 04b). My work on the stabilization of this class of systems concerns the two following problems : asymptotic stabilization of fixed points, and practical stabilization of reference trajectories based on the transverse function approach.

2.1.1 Asymptotic stabilization of fixed points

Like nonholonomic systems, critical underactuated systems are very difficult to control around fixed points, and even more because of the system's drift. At the beginning of the 1990's no solution had been proposed for these systems to asymptotically stabilize fixed points. The underactuated spacecraft (rotation dynamics of a rigid body with two torque control) is the archetype of this class of system. In (Morin 95), we have proposed a smooth time-varying feedback to asymptotically stabilize this system at a desired orientation. Latter, we have proposed other solutions for this system (Morin 97b) and other underactuated systems (M'Closkey 98), based on the use of homogeneous feedback. Like for nonholonomic systems, this allows in theory to achieve exponential convergence rates. But the same robustness problems occur in this case, suggesting that asymptotic stabilization is also a too strong objective in practice for these systems.

2.1.2 Practical stabilization of reference trajectories based on the transverse function approach

The transverse function approach can also be applied to underactuated systems, with the objective of stabilizing reference trajectories in the configuration space, i.e. smooth curves $t \longmapsto g_r(t) \in G$. Like in the case of nonholonomic systems, it allows to stabilize g in a neighborhood of the reference trajectory g_r, with the size of this neighborhood determined by the parameter ε of the transverse function. We have published two results on this topic (Morin 05; Morin 06), with the latter generalizing the former. An alternative approach, also based on the transverse function approach, has been proposed in (Lizárraga 05). In the case of three dimensional Lie groups, our method is completely general in the sense that it can be applied to *any* controllable system in the classical (STLC) sense. This includes the 3-d second-order chained system, the underactuated PPR manipulator, the planar rigid body (also called "hovercraft" or "slider"), and the underactuated spacecraft. We also show in (Morin 06) that the method applies to the rigid body in $SE(3)$ with mass matrix (allowing to model added mass effects) with only three control inputs (one force and two torques). This is to my knowledge the most challenging example of mechanical system in the class of Euler-Poincaré equations. From a theoretical viewpoint, it would be very satisfactory to have a method applicable to *any* controllable system (6). But we know that this is a very difficult problem since, to begin with, a necessary and sufficient controllability condition (in term of the system's control Lie algebra) is not yet available for these systems (Lewis 97).

2.2 Control of noncritical underactuated systems

While many mechanical systems are underactuated, they are not all critical. This is due to external forces, like gravity, which can modify the system's dynamics. While the transverse function approach can also be applied in this case, simpler control methods can usually be used to design stabilizing feedback laws. Recently, we have proposed a control method for what we call "thrust-propelled vehicles" (Hua 07; Hua 08), i.e., noncritical vehicles with one thrust force in a body-fixed direction and full torque actuation. This is a typical actuation structure for aircrafts, VTOL vehicles (i.e., Vertical Take-Off and Landing), etc. Besides the large spectrum of possible applications, one of the motivations of this study is related to robustness issues (but in a sense different from the one evoked in Section 1.2 since we are here interested in noncritical systems). Indeed, there has been an increasing interest in recent years in small or light vehicles, like VTOL's (Hamel 02; Lipera 01; Pflimlin 07) or airships (Azinheira 06). These systems can be very sensitive to wind-induced perturbations and can operate in a very wide range of angles of attack. It is thus unrealistic to dispose of precise models. Therefore, it is most important to dispose of very robust controllers for these systems. In this respect, the method that we propose in (Hua 07; Hua 08) present original features like the compensation of constant bias (related e.g. to modeling errors or wind-induced perturbations) via nonlinear integral-type control terms, and (proven) large domains of stability. This property is achieved by a combination of factors, like for example by exploiting the dissipativity of aerodynamic forces (this is of course not a new idea, but by contrast with other methods we also show how these forces can help to stabilize *non-stationary* reference trajectories). Simulations on a realistic model of a small VTOL (the "HoverEye" of the french company Bertin Technologies) have shown very good robustness properties of these controllers, and we plan to experiment this method on the physical system.

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