Projection model

Christopher Mei

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Model

The projection model used in the toolbox is an extension of the model proposed by Geyer and Barreto [2; 1]. We choose the standard convention that the z axis is the optical axis of the sensor and points *outwards* and that the mirror axis points in the same direction (this is different to Barreto's model : $\xi \rightarrow -\xi$).

The projection of 3D points can be done in the following steps (the values for ξ and γ are detailed in Table 1):

1. world points in the mirror frame are projected onto the unit sphere,

$$(\boldsymbol{\mathcal{X}})_{\mathcal{F}_m} \rightarrow (\boldsymbol{\mathcal{X}}_s)_{\mathcal{F}_m} = \frac{\boldsymbol{\mathcal{X}}}{\|\boldsymbol{\mathcal{X}}\|} = (X_s, Y_s, Z_s)$$

2. the points are then changed to a new reference frame centered in $\boldsymbol{\mathcal{C}}_p = (0, 0, \xi)$,

$$(\boldsymbol{\mathcal{X}}_s)_{\mathcal{F}_m} \rightarrow (\boldsymbol{\mathcal{X}}_s)_{\mathcal{F}_p} = (X_s, Y_s, Z_s - \xi)$$

3. they are then projected onto the normalised image plane,

$$\mathbf{m}_u = (\frac{X_s}{Z_s - \xi}, \frac{Y_s}{Z_s - \xi}, 1) = \hbar(\boldsymbol{\mathcal{X}}_s)$$

4. we then add radial and tangential distortion,

$$\mathbf{m}_d = \mathbf{m}_u + D(\mathbf{m}_u, V)$$

5. the final projection involves a generalised camera projection matrix $\mathbf{K}$ (with γ the generalized focal length, (u_0, v_0) the principal point, s the skew and r the aspect ratio)

$$\mathbf{p} = \mathbf{K}\mathbf{m} = \begin{bmatrix} \gamma & \gamma s & u_0 \\ 0 & \gamma r & v_0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{m} = k(\mathbf{m})$$

In the case of a catadioptric sensor (with a mirror), we only consider points with $Z_s < \xi$. In the case of a dioptric sensor (only a lens), we only consider points with $Z_s > \xi$. (This takes care of keeping the axis in the standard direction.)

The distortion function adds a greater flexibility to misalignement. It also takes care of the distortion introduced by the use of telecentric lenses with paracatadioptric sensors. The generalised camera model is a change in conception from Barreto's model : we consider the camera and mirror as a unique sensor. γ includes the camera focal length and a parameter depending on the mirror shape.

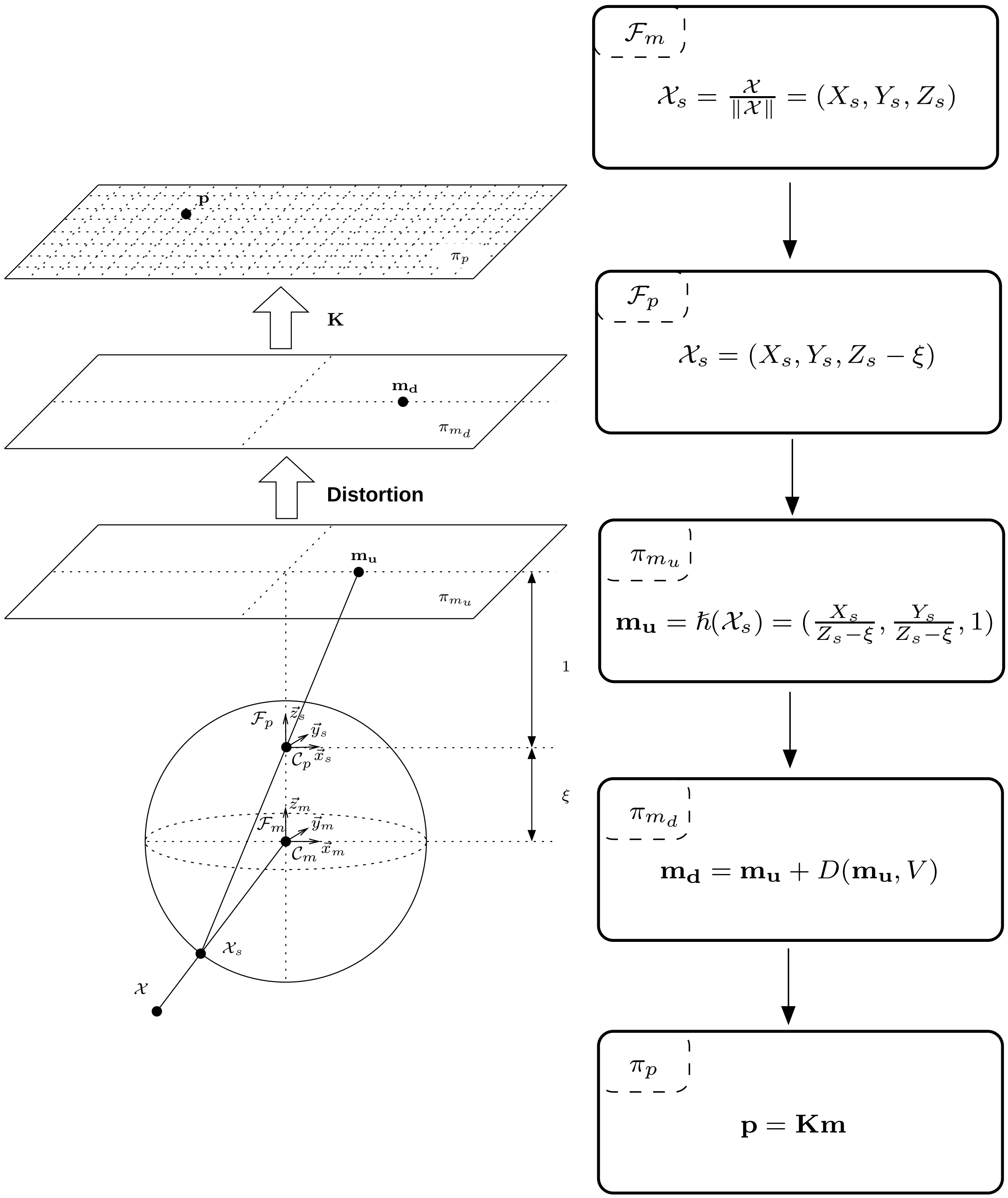

Figure 1: Full projection model

Table 1: Unified model parameters

	ξ	γ
Parabola	1	$-2pf$
Hyperbola	$\frac{df}{\sqrt{d^2+4p^2}}$	$\frac{-2pf}{\sqrt{d^2+4p^2}}$
Ellipse	$\frac{df}{\sqrt{d^2+4p^2}}$	$\frac{2pf}{\sqrt{d^2+4p^2}}$
Planar	0	-f
Perspective	0	f
d: distance between focal points $4p$: latus rectum		

Why choose this model ?

This model presents a compromise between a very general model which can be difficult to calibrate and a model that does not take into account important factors such as misalignment and optical distortion. It is based on the realistic assumption of small errors compared to the ideal theoretical model.

References

[1] Joao P. Barreto and Helder Araujo. Issues on the geometry of central catadioptric image formation. In *CVPR*, volume 2, pages 422–427, 2001.

[2] C. Geyer and K. Daniilidis. A unifying theory for central panoramic systems and practical implications. In *ECCV*, pages 445–461, 2000.