Projection model

Christopher Mei

August 9, 2006

Model

The projection model used in the toolbox is an extension of the model proposed by Geyer and Barreto [2; 1]. We choose the unusual convention that the z axis points *towards* the camera but *outwards* from the mirror or lens (Figure 1).

The projection of 3D points can be done in the following steps (Figure 2, the relationship between (ξ, γ) and the mirror values are detailed in Table 1):

1. world points in the mirror frame are projected onto the unit sphere,

$$(\boldsymbol{\mathcal{X}})_{\mathcal{F}_m} \rightarrow (\boldsymbol{\mathcal{X}}_s)_{\mathcal{F}_m} = \frac{\boldsymbol{\mathcal{X}}}{\|\boldsymbol{\mathcal{X}}\|} = (X_s, Y_s, Z_s)$$

2. the points are then changed to a new reference frame centered in $\boldsymbol{\mathcal{C}}_p = (0, 0, \xi)$,

$$(\boldsymbol{\mathcal{X}}_s)_{\mathcal{F}_m} \rightarrow (\boldsymbol{\mathcal{X}}_s)_{\mathcal{F}_p} = (X_s, Y_s, Z_s + \xi)$$

3. they are then projected onto the normalised image plane,

$$\mathbf{m}_u = (\frac{X_s}{Z_s + \xi}, \frac{Y_s}{Z_s + \xi}, 1) = \hbar(\boldsymbol{\mathcal{X}}_s)$$

4. we then add radial and tangential distortion,

$$\mathbf{m}_d = \mathbf{m}_u + D(\mathbf{m}_u, V)$$

5. the final projection involves a generalised camera projection matrix $\mathbf{K}$ (with γ the generalized focal length, (u_0, v_0) the principal point, s the skew and r the aspect ratio)

$$\mathbf{p} = \mathbf{K}\mathbf{m} = \begin{bmatrix} \gamma & \gamma s & u_0 \\ 0 & \gamma r & v_0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{m} = k(\mathbf{m})$$

$\hbar$ lifts a the point from π_{m_u} to the sphere :

$$\hbar^{-1}(\mathbf{m}_u) = \begin{bmatrix} \frac{\xi + \sqrt{1 + (1-\xi^2)(x^2+y^2)}}{x^2+y^2+1} x \\ \frac{\xi + \sqrt{1 + (1-\xi^2)(x^2+y^2)}}{x^2+y^2+1} y \\ \frac{\xi + \sqrt{1 + (1-\xi^2)(x^2+y^2)}}{x^2+y^2+1} - \xi \end{bmatrix} \tag{1}$$

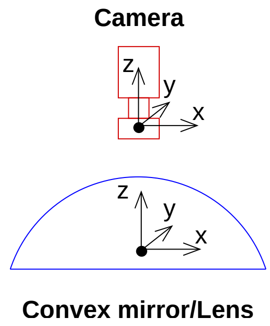

Figure 1: Axis

The distortion function adds a greater flexibility to misalignment. It also models the distortion introduced by the use of telecentric lenses with paracatadioptric sensors. The generalised camera model means we consider the camera and mirror as a unique sensor. γ includes the camera focal length and a parameter that depends on the mirror shape.

Table 1: Unified model parameters

	ξ	γ
Parabola	1	$-2pf$
Hyperbola	$\frac{df}{\sqrt{d^2+4p^2}}$	$\frac{-2pf}{\sqrt{d^2+4p^2}}$
Ellipse	$\frac{df}{\sqrt{d^2+4p^2}}$	$\frac{2pf}{\sqrt{d^2+4p^2}}$
Planar	0	-f
Perspective	0	f
d: distance between focal points $4p$: latus rectum		

Why choose this model ?

This model presents a compromise between a very general model which can be difficult to calibrate and a model that does not take into account important factors such as misalignment and optical distortion. It is based on the realistic assumption of small errors compared to the ideal theoretical model.

References

[1] Joao P. Barreto and Helder Araujo. Issues on the geometry of central catadioptric image formation. In *CVPR*, volume 2, pages 422–427, 2001.

[2] C. Geyer and K. Daniilidis. A unifying theory for central panoramic systems and practical implications. In *ECCV*, pages 445–461, 2000.

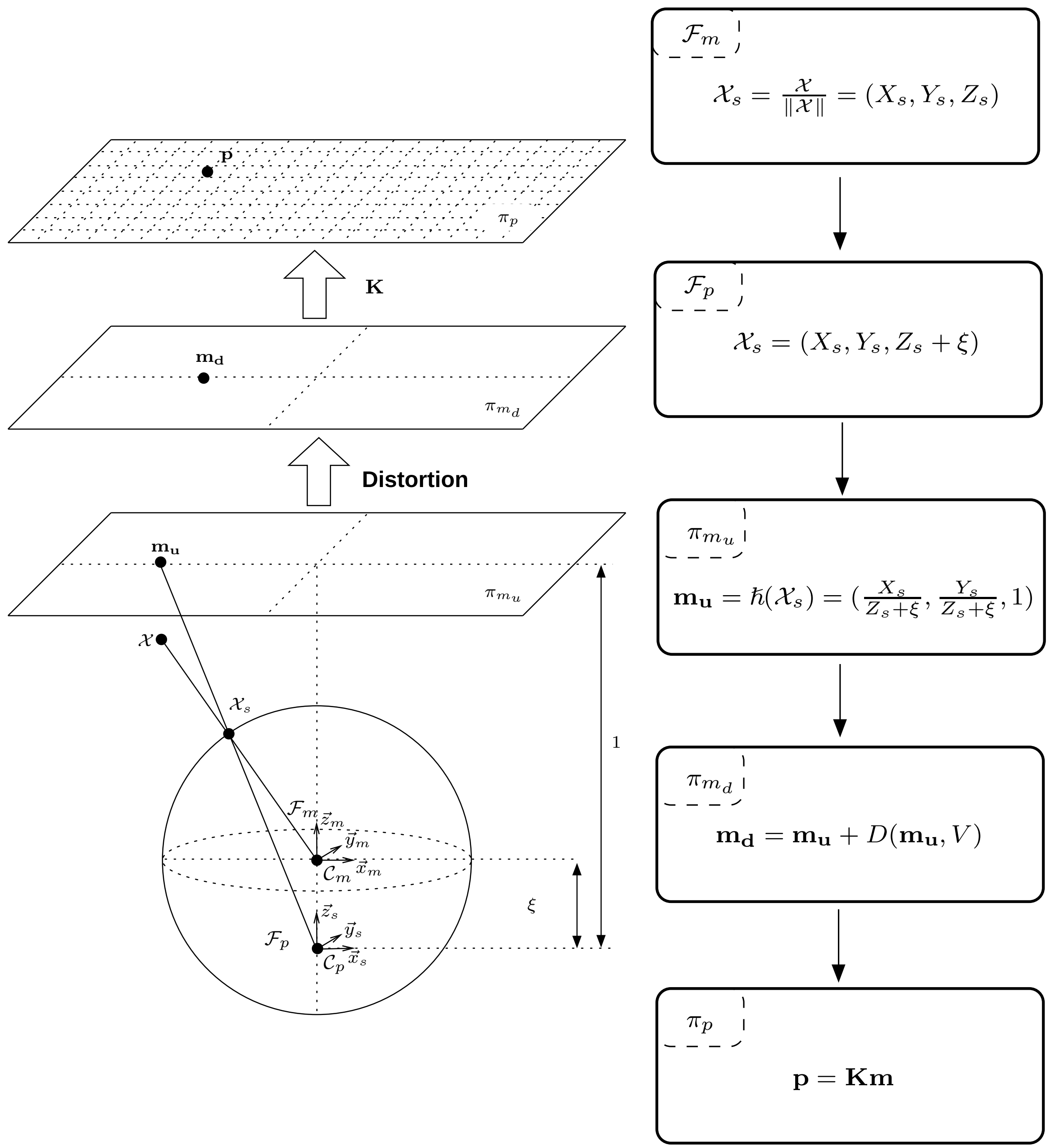

Figure 2: Full projection model