Numerical methods for complex flows in porous media

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- Stella Krell (MCF)
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Permanent researchers

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Postdoc

- Walid Kheriji (10/2012-10/2014)
  Domain decomposition in reservoir simulations

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- Yumeng Zhang with Andra since 09/2012
  Coupling of compositional two phase porous media flow with single phase free flow
- Mayya Groza with Total since 09/2013
  Hybrid dimensional Darcy flow in fractured porous media with continuous pressure
- Julian Hennicker with GdF Suez since 06/2014
  Hybrid dimensional Darcy flow in fractured porous media with discontinuous pressure

Internships

- Riad Sanchez with Brgm 03-09/2014
  Vapor liquid thermal flow in fractured porous media
- Samira Amraoui 07-09/2014
  Muscl finite volume scheme on polyhedral meshes

Current Research Topics

- Finite Volume discretizations of porous media flow models
    - Schemes adapted to polyhedral meshes and anisotropic heterogeneous media: VAG, HFV, MPFA
    - Convergence analysis: gradient scheme framework
    - Benchmarking with prototype codes
    - Range of difficulties: discontinuous hydrodynamical laws, fractured media, compositional and thermal flows
- Coupling of liquid gas Darcy flow with gas free flow
- Domain decomposition algorithms in reservoir simulations
- Code CoMPaSS: Computing Parallel Architecture to Speed up Simulations
    - Ditributed polyhedral meshes with one layer of cells of overlap
    - FV schemes with d.o.f. at nodes, cells, faces
    - Connection with ParMetis, Petsc, Paraview
    - Current model implemented: VAG scheme for two phase Darcy flows
    - In project: geothermal model in fractured porous media

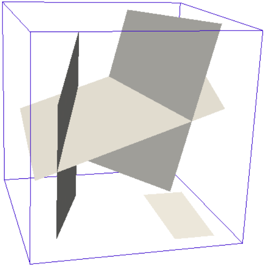
Outline:

- Continuous model
- Vertex Approximate Gradient discretization
  - Non-conservative version
  - Conservative version
- Numerical results and comparison with standard CVFE approach

The model briefly:

- Two phases $\alpha \in \{1, 2\}$ incompressibles and immiscibles
- Pressure - pressure formulation
- Two media $\upsilon \in \{m, f\}$

  3D porous matrix $\Omega \setminus \overline{\Gamma}$
  2D fracture network Γ
- Pressure is continuous at matrix-fracture interface



Some bibliography:

- Single phase: V. Martin *et al.* '05 *(Model)*, Ph. Angot *et al.* '09 *(Convergence + existence)*
- Two phase: J. Jaffré *et al.* '11 *(Model)*, current work *(Convergence + existence)* inspired by R. Eymard *et al.* '13 *(Convergence without fractures)*

Pressure - pressure formulation

Each phase α has its own pressure u^α, saturation S^α and flux $\mathbf{q}^\alpha$.

Mass balance for phase α:

$$\begin{array}{lclcll} \phi_m \partial_t S_m^\alpha & + & \mathrm{div}(\mathbf{q}_m^\alpha) & = & 0 & \text{on } \Omega \setminus \overline{\Gamma}, \\ \phi_f \partial_t S_f^\alpha & + & \mathrm{div}_\tau(\mathbf{q}_f^\alpha) & = & [\![\mathbf{q}_m^\alpha \cdot \mathbf{n}_\Gamma]\!] & \text{on } \Gamma \end{array}$$

Darcy law for phase α:

$$\begin{array}{lcl} \mathbf{q}_m^\alpha & = & -k_m^\alpha(x, S_m^\alpha) \Lambda_m \nabla u^\alpha \\ \mathbf{q}_f^\alpha & = & -k_f^\alpha(x, S_f^\alpha) \Lambda_f \nabla_\tau \gamma u^\alpha \end{array}$$

Saturation condition for media υ: $\sum_{\alpha \in \{1,2\}} S_\upsilon^\alpha = 1$

Capillary pressure law for media υ:

$$S_m^\alpha = S_m^\alpha(\boldsymbol{x}, u^1 - u^2) \text{ and } S_f^\alpha = S_f^\alpha(\boldsymbol{x}, \gamma u^1 - \gamma u^2)$$

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Remark: $\phi_f(x), \Lambda_f(x)$ are proportional to the fracture thickness $d_f(x)$.

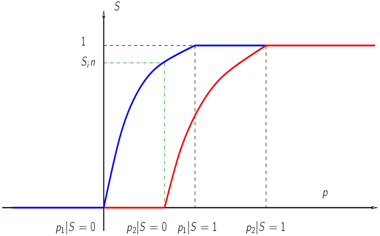
Letting $d_f \to 0$ one formally recovers "a non-fractured model"

Assumptions on data

- Relative mobilities are uniformly bounded from below

$$k^{\alpha}(\boldsymbol{x}, S) \geq k^{\alpha}_{min} > 0$$

$$\Rightarrow \|\nabla u^{\alpha}\|_{L^2(\Omega\times(0,T))} < C.$$



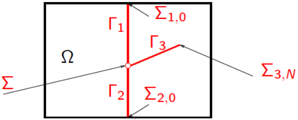
- $S^{\alpha}(\boldsymbol{x}, p) \in [0,1]$ is defined by rocktype:
  - $S(\boldsymbol{x}, p) = S_j(p)$ a.e. in Ω_j, $\quad \overline{\Omega} = \bigcup_{j\in J} \overline{\Omega_j}$
  - S_j is non-decreasing and Lipschitz continuous.

- Similar assumptions on fracture properties.

We define the functional spaces

$$V = \{u \in H^1(\Omega),\ \gamma u \in H^1(\Gamma)\}$$

$$V^0 = \{u \in H_0^1(\Omega),\ \gamma u \in H_{\Sigma_0}^1(\Gamma)\}$$



Weak formulation:

Find $u^1, u^2 \in L^2(0,T;V^0)$ s.t. for each $\alpha \in \{1,2\}$ and for all $\varphi \in L^2(0,T;V^0)$

$$\int_\Omega \phi_m S_{m,0}^\alpha \varphi_0 d\mathbf{x} - \int_0^T \int_\Omega \phi_m S_m^\alpha \partial_t \varphi d\mathbf{x} dt + \int_0^T \int_\Omega k_m^\alpha(S_m^\alpha)\Lambda_m \nabla u^\alpha \cdot \nabla \varphi d\mathbf{x} dt$$

$$+ \int_\Gamma \phi_f S_{f,0}^\alpha \varphi_0 d\tau_f - \int_0^T \int_\Gamma \phi_f S_f^\alpha \partial_t \varphi d\tau_f dt + \int_0^T \int_\Gamma k_f^\alpha(S_f^\alpha)\Lambda_f \nabla_\tau \gamma u^\alpha \cdot \nabla_\tau \gamma \varphi d\tau_f dt = 0$$

Remark: Pressure is *continuous* saturation is not.

Numerical challenges

Diffusion:

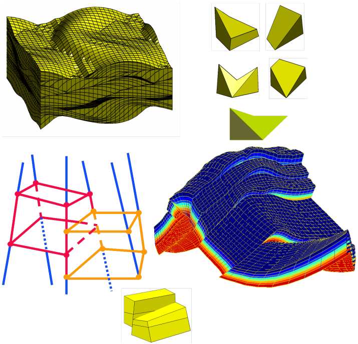
- Complex meshes
    - Polyhedral cell
    - Non planar faces
- Heterogeneous anisotropic media

Convection:

- Convection dominated problems
- Highly contrasted of the flow rates

Discontinuous capillary pressure curves:

- Saturation jumps



Diffusion:

- Complex meshes
  - Polyhedral cell
  - Non planar faces
- Heterogeneous anisotropic media

- *Conforming nodal discretization for u^α*
  using tetrahedral sub-mesh

Convection:

- Convection dominated problems
- Highly contrasted of the flow rates

- *Conservative fluxes and upwinding*
- *Flexible mass lumping approach*

Discontinuous capillary pressure curves:

- Saturation jumps

- *Piecewise constant discretization for S^α*

Semi-discrete problem

Given an the initial capillary pressure P^0, find $u^{1,n}, u^{2,n} \in V^0$, $n = 1, \cdots, N$, such that for $\alpha \in \{1,2\}$ and for all $v \in V^0$

$$\int_\Omega \phi_m \frac{S_m^{\alpha,n} - S_m^{\alpha,n-1}}{\Delta t^n} v d\boldsymbol{x} + \int_\Omega k_m^\alpha(S_m^{\alpha,n}) \Lambda_m \nabla u^{\alpha,n} \cdot \nabla v d\boldsymbol{x}$$

$$+ \int_\Gamma \phi_f \frac{S_f^{\alpha,n} - S_f^{\alpha,n-1}}{\Delta t^n} v d\boldsymbol{x} + \int_\Gamma k_f^\alpha(S_{\mathcal{D}_f}^{\alpha,n}) \Lambda_f \nabla_\tau \gamma u^{\alpha,n} \cdot \nabla_\tau \gamma v d\tau_f = 0$$

with

$$S_m^{\alpha,n}(\boldsymbol{x}) = S_m^\alpha(\boldsymbol{x}, P^n), \qquad S_f^{\alpha,n}(\boldsymbol{x}) = S_f^\alpha(\boldsymbol{x}, P^n)$$

and

$$P^n = u^{1,n} - u^{2,n}.$$

Next step: Let $X_\mathcal{D}$ be a space of d.o.f., we will define

$$\pi_{\mathcal{D}_m} : X_\mathcal{D} \to L^2(\Omega) \qquad \nabla_{\mathcal{D}_m} : X_\mathcal{D} \to L^2(\Omega)^d$$

$$\pi_{\mathcal{D}_f} : X_\mathcal{D} \to L^2(\Gamma) \qquad \nabla_{\mathcal{D}_f} : X_\mathcal{D} \to L^2(\Gamma)^{d-1}$$

Discrete gradient reconstruction

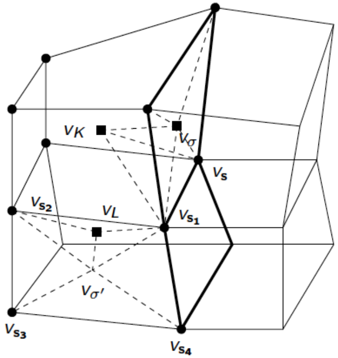
A finite dimensional subspace of V is constructed using P_1 finite elements.

Degrees of freedom $X_{\mathcal{D}}$:

- Cell unknowns $v_K, K \in \mathcal{M}$;
- Nodes unknowns $v_s, s \in \mathcal{V}$;
- Fracture faces unknowns $v_\sigma, \sigma \in \mathcal{F}_\Gamma$;
- At the non-fracture faces $\sigma \in \mathcal{F} \setminus \mathcal{F}_\Gamma$ the unknowns are interpolated :

$$\mathbf{x}_\sigma = \sum_{s \in \mathcal{V}_\sigma} \beta_{\sigma,s} \mathbf{x}_s, \quad v_\sigma = \sum_{s \in \mathcal{V}_\sigma} \beta_{\sigma,s} v_s.$$

We define the mapping $\pi_{\mathcal{T}}$ from $X_{\mathcal{D}}$ to V



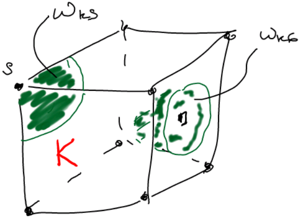
$$\pi_{\mathcal{T}} v = \sum_{K \in \mathcal{M}} \eta_K(\mathbf{x}) v_K + \sum_{\sigma \in \mathcal{F}_\Gamma} \eta_\sigma(\mathbf{x}) v_\sigma + \sum_{s \in \mathcal{V}} \left(\eta_s(\mathbf{x}) + \sum_{\sigma \in \mathcal{F}_s} \beta_{\sigma,s} \eta_\sigma(\mathbf{x}) \right) v_s$$

- Cell unknowns are eliminated algebraically.

$$\nabla_{\mathcal{D}_m} v = \nabla \pi_{\mathcal{T}} v$$
$$\nabla_{\mathcal{D}_f} v = \nabla_\tau \gamma \pi_{\mathcal{T}} v$$

Cells $K \in \mathcal{M}$ partitioning

$$K = \omega_K \cup \bigcup_{s \in \mathcal{V}_K \cap \mathcal{V}_{int}} \omega_{K,s} \cup \bigcup_{\sigma \in \mathcal{F}_K \cap \mathcal{F}_\Gamma} \omega_{K,\sigma}.$$

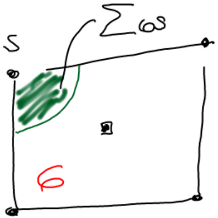


Piecewise constant projector

$$\pi_{\mathcal{D}_m} v(\mathbf{x}) = \begin{cases} v_K, & \mathbf{x} \in \omega_K, K \in \mathcal{M}, \\ v_s, & \mathbf{x} \in \omega_{K,s}, s \in \mathcal{V}_K \cap \mathcal{V}_{int}, K \in \mathcal{M}, \\ v_\sigma, & \mathbf{x} \in \omega_{K,\sigma}, \sigma \in \mathcal{F}_K \cap \mathcal{F}_\Gamma, K \in \mathcal{M}. \end{cases}$$

Fracture faces $\sigma \in \mathcal{F}_\Gamma$ partitioning

$$\sigma = \Sigma_\sigma \cup \bigcup_{s \in \mathcal{V}_\sigma \cap \mathcal{V}_{int}} \Sigma_{K,s}$$



Piecewise constant projector

$$\pi_f v(\mathbf{x}) = \begin{cases} v_\sigma, & \mathbf{x} \in \Sigma_\sigma, \sigma \in \mathcal{F}_\Gamma, \\ v_s, & \mathbf{x} \in \Sigma_{\sigma,s}, s \in \mathcal{V}_\sigma \cap \mathcal{V}_{int}, \sigma \in \mathcal{F}_\Gamma. \end{cases}$$

Assume that data is constant by cell and fracture face

$\Rightarrow$ No need to define the control volumes explicitly.

Matrix - Matrix volume distribution:

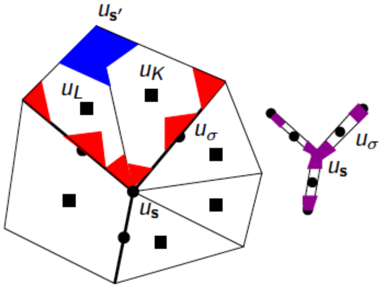
- $m_{K,s'} = \alpha_{K,s'}|K|$

Matrix - Fracture volume distribution:

- $m_{K,s} = \alpha_{K,s}|K|$
- $m_{K,\sigma} = \alpha_{K,\sigma}|K|$

Fracture - Fracture volume distribution:

- $m_{\sigma,s} = \alpha_{\sigma,s}|\sigma|$



$$|K| = m_K + \sum_{s\in\mathcal{V}_K} m_{K,s} + \sum_{\sigma\in\mathcal{F}_K\cap\mathcal{F}_\Gamma} m_{K,\sigma} \qquad \text{and} \qquad |\sigma| = m_\sigma + \sum_{s\in\mathcal{V}_\sigma} m_{\sigma,s}.$$

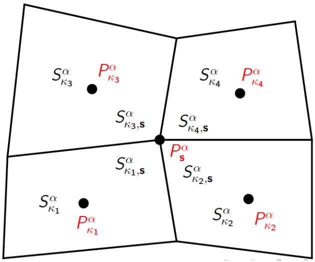
- The choice of $m_{K,s}, m_{\sigma,\sigma}$ (fracture d.o.f.) has a large impact
- There is the good choice

Discrete problem I : convergent scheme

Given an approximation $P^0 \in X_{\mathcal{D}}$ of the initial capillary pressure, find $u^{1,n}, u^{2,n} \in X^0_{\mathcal{D}}$, $n = 1, \cdots, N$, such that for $\alpha \in \{1, 2\}$ and for all $v \in X^0_{\mathcal{D}}$

$$\int_\Omega \phi_m \frac{S^{\alpha,n}_{\mathcal{D}_m} - S^{\alpha,n-1}_{\mathcal{D}_m}}{\Delta t^n} \pi_{\mathcal{D}_m} v d\mathbf{x} + \int_\Omega k^\alpha_m(S^{\alpha,n}_{\mathcal{D}_m}) \Lambda_m \nabla \pi_{\mathcal{T}} u^{\alpha,n} \cdot \nabla \pi_{\mathcal{T}} v d\mathbf{x}$$

$$+ \int_\Gamma \phi_f \frac{S^{\alpha,n}_{\mathcal{D}_f} - S^{\alpha,n-1}_{\mathcal{D}_f}}{\Delta t^n} \pi_{\mathcal{D}_f} v d\mathbf{x} + \int_\Gamma k^\alpha_f(S^{\alpha,n}_{\mathcal{D}_f}) \Lambda_f \nabla_\tau \gamma \pi_{\mathcal{T}} u^{\alpha,n} \cdot \nabla_\tau \gamma \pi_{\mathcal{T}} v d\tau_f = 0,$$



with $P^n = u^{1,n} - u^{2,n}$ and

$$S^{\alpha,n}_{\mathcal{D}_m}(\mathbf{x}) = S^\alpha_m(\mathbf{x}, \pi_{\mathcal{D}_m} P^n)$$

$$S^{\alpha,n}_{\mathcal{D}_f}(\mathbf{x}) = S^\alpha_f(\mathbf{x}, \pi_{\mathcal{D}_f} P^n).$$

Saturation at node s:

$$\left(S^\alpha_{K,s}\right)_{K \in \mathcal{M}_s}, \left(S^\alpha_{\sigma,s}\right)_{\sigma \in \mathcal{F}_s \cap \mathcal{F}_\Gamma}$$

The evolution terms can be written as follows

$$\int_K \phi_m \frac{S_{\mathcal{D}_m}^{\alpha,n} - S_{\mathcal{D}_m}^{\alpha,n-1}}{\Delta t^n} \pi_{\mathcal{D}_m} v d\boldsymbol{x} = \phi_K m_K \frac{S_K^\alpha(p_K^n) - S_K^\alpha(p_K^{n-1})}{\Delta t^n} v_K$$
$$+ \sum_{s \in \mathcal{V}_K} \phi_K m_{K,s} \frac{S_K^\alpha(p_s^n) - S_K^\alpha(p_s^{n-1})}{\Delta t^n} v_s$$
$$+ \sum_{\sigma \in \mathcal{F}_K \cap \mathcal{F}_\Gamma} \phi_K m_{K,\sigma} \frac{S_K^\alpha(p_\sigma^n) - S_K^\alpha(p_\sigma^{n-1})}{\Delta t^n} v_\sigma$$

and

$$\int_\sigma \phi_f \frac{S_{\mathcal{D}_f}^{\alpha,n} - S_{\mathcal{D}_f}^{\alpha,n-1}}{\Delta t^n} \pi_{\mathcal{D}_f} v d\tau_f = \phi_\sigma m_\sigma \frac{S_\sigma^\alpha(p_\sigma^n) - S_\sigma^\alpha(p_\sigma^{n-1})}{\Delta t^n} v_\sigma$$
$$+ \sum_{s \in \mathcal{V}_\sigma} \phi_\sigma m_{\sigma,s} \frac{S_\sigma^\alpha(p_s^n) - S_\sigma^\alpha(p_s^{n-1})}{\Delta t^n} v_s,$$

Main theoretical result

The following convergence result holds independently of nodal volume distribution

- The approximate phase pressure converge weakly to a weak solution $(\overline{u}^1, \overline{u}^2)$
  - $\pi_{\mathcal{T}^{(k)}} u^{\alpha,(k)} \rightharpoonup \overline{u}^\alpha$ in $L^2(\Omega \times (0, T))$,
  - $\gamma \pi_{\mathcal{T}^{(k)}} u^{\alpha,(k)} \rightharpoonup \gamma \overline{u}^\alpha$ in $L^2(\Gamma \times (0, T))$.

- The approximate saturation converges strongly
  - $S^{\alpha,(k)}_{\mathcal{D}^{(k)}_{\boldsymbol{m}}} \to S^\alpha_{\boldsymbol{m}}(\overline{u}^1 - \overline{u}^2)$ in $L^2(\Omega \times (0, T))$,
  - $S^{\alpha,(k)}_{\mathcal{D}^{(k)}_{\boldsymbol{f}}} \to S^\alpha_{\boldsymbol{f}}(\gamma \overline{u}^1 - \gamma \overline{u}^2)$ in $L^2(\Omega \times (0, T))$

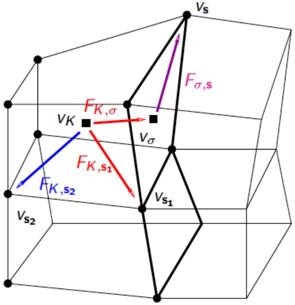
Let $(\eta_K)_{K\in\mathcal{M}}$, $(\eta_\sigma)_{\sigma\in\mathcal{F}_\Gamma}$, $(\eta_s)_{s\in\mathcal{V}}$ be a FE nodal basis taking into account the interpolated interface unknowns

- For each cell K and $\nu \in \mathcal{V}_K \cup (\mathcal{F}_K \cap \mathcal{F}_\Gamma)$

$$F_{K,\nu}(u) = \int_K -\Lambda_m(\mathbf{x})\nabla\pi_{\mathcal{T}}u \cdot \nabla\eta_\nu d\mathbf{x}$$

- For each fracture face σ and $s \in \mathcal{V}_\sigma$

$$F_{\sigma,s}(u) = \int_\sigma -\Lambda_f(\mathbf{x})\nabla\gamma\pi_{\mathcal{T}}u \cdot \nabla\gamma\eta_s d\tau_f$$



Discrete problem II : VAG discretization with upwinding

Mass conservation for phase α:

$$\begin{cases} m_K \phi_K \dfrac{S_K^\alpha(p_K^n) - S_K^\alpha(p_K^{n-1})}{\Delta t^n} + \displaystyle\sum_{\nu \in \mathcal{V}_K \cup (\mathcal{F}_\Gamma \cap \mathcal{F}_K)} k_K^\alpha(S_{K,\nu,up}^\alpha) F_{K,\nu}^\alpha(u^n) = 0, & K \in \mathcal{M}, \\ m_\sigma \phi_\sigma \dfrac{S_\sigma^\alpha(p_\sigma^n) - S_\sigma^\alpha(p_\sigma^{n-1})}{\Delta t^n} + \displaystyle\sum_{\nu \in \mathcal{V}_\sigma \cup \mathcal{M}_\sigma} k_\sigma^\alpha(S_{\sigma,\nu,up}^\alpha) F_{\sigma,\nu}^\alpha(u^n) = 0, & \sigma \in \mathcal{F}_\Gamma, \\ \displaystyle\sum_{\nu \in \mathcal{M}_s \cup (\mathcal{F}_s \cap \mathcal{F}_\Gamma)} m_{\nu,s} \phi_\nu \dfrac{S_\nu^\alpha(p_s^n) - S_\nu^\alpha(p_s^{n-1})}{\Delta t^n} & \\ \qquad - \displaystyle\sum_{\nu \in \mathcal{M}_s \cup (\mathcal{F}_s \cap \mathcal{F}_\Gamma)} k_\nu^\alpha(S_{\nu,s,up}^\alpha) F_{\nu,s}^\alpha(u^n) = 0, & s \in \mathcal{V} \setminus \partial\Omega. \end{cases}$$

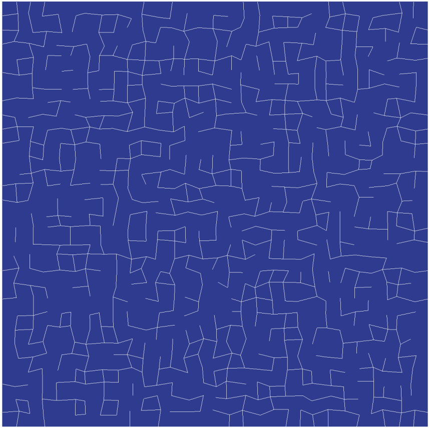
The upwinding is defined by

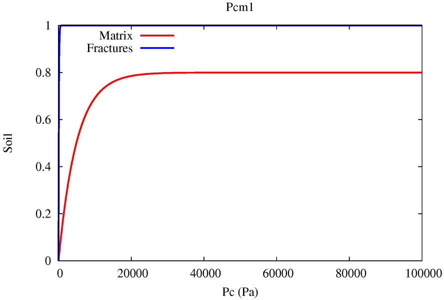
$$\begin{cases} S_{\xi,\nu,up}^{\alpha,n} = S_\xi^{\alpha,n} \text{ if } F_{\xi,\nu}(u^{\alpha,n}) \geq 0, \\ S_{\xi,\nu,up}^{\alpha,n} = S_{\xi,\nu}^{\alpha,n} \text{ if } F_{\xi,\nu}(u^{\alpha,n}) < 0. \end{cases}$$

Oil migration in a 2D random fracture network

We consider an oil migration in 2D porous media connected to the

- Large random network - about 1000 fractures.
- Permeability contrast Λ_f / Λ_m is set to $10^2, 10^4$ or 10^5.





Ratio $\Lambda_f/\Lambda_m = 10^2$

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Ratio $\Lambda_f/\Lambda_m = 10^5$

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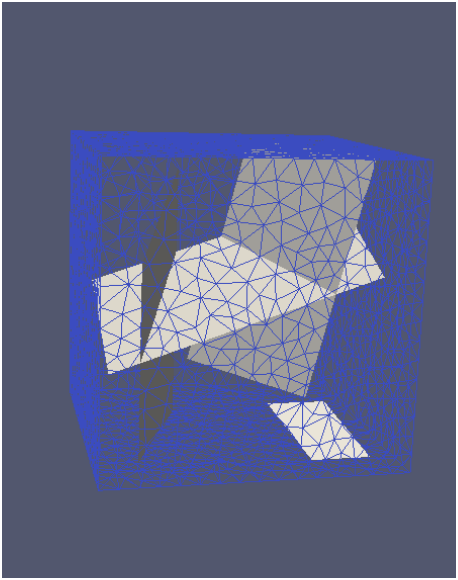
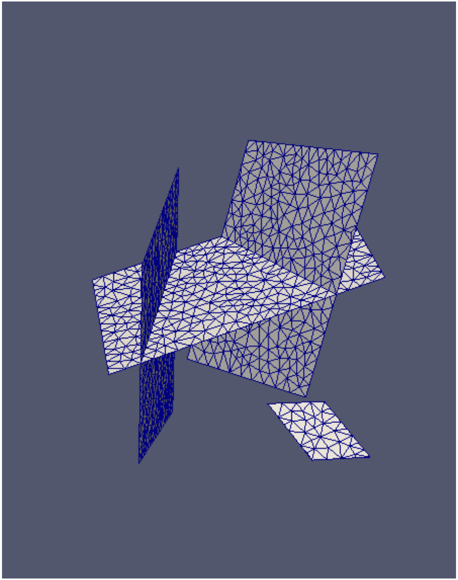
Oil migration in a 3D heterogeneous basin, $\Lambda_m/\Lambda_f = 10^5$

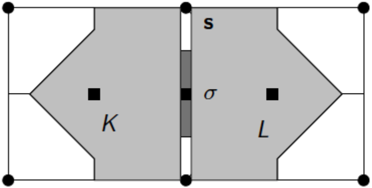




Let for all $K \in \mathcal{M}$

$$\alpha_K \approx \frac{1}{card(\mathcal{V}_K) + card(\mathcal{F}_K \cap \mathcal{F}_\Gamma) + 1}$$

VAG-1

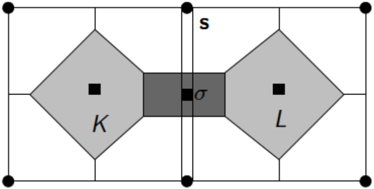


$\alpha_{K,s} = \alpha_K$ for all $s \in \mathcal{V}_K \setminus \mathcal{V}_\Gamma$

$\alpha_{K,s} = 0$ for all $s \in \mathcal{V}_K \cap \mathcal{V}_\Gamma$

$\alpha_{K,\sigma} = 0$ for all $\sigma \in \mathcal{F}_\Gamma$

VAG-2



$\alpha_{K,s} = \alpha_K$ for all $s \in \mathcal{V}_K$

$\alpha_{K,\sigma} = \alpha_K$ for all $\sigma \in \mathcal{F}_K \cap \mathcal{F}_\Gamma$

Oil migration in a 3D heterogeneous basin

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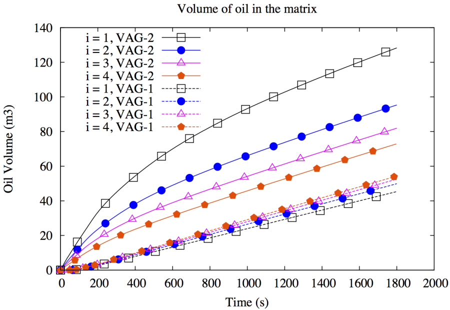
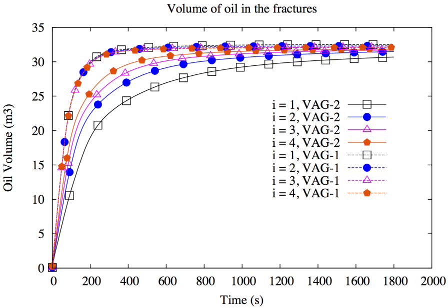
Comparison of VAG-1 and VAG-2: Convergence





Volume of oil in the matrix, volume of oil in the network

Comparison of VAG-1 and VAG-2: Numerical behavior

Discretization properties:

Nb_{cells}	Nb_{nodes}	Nb_{FracF}	linear system d.o.f.
47 670	8 348	1 678	9 278
253 945	41 043	6 655	46 283
837 487	132 778	16 497	147 148
3 076 262	483 786	42 966	523 453

Numerical behavior:

Volumes	$N_{\Delta t}$	N_{Chop}	N_{Newton}	N_{GMRes}	CPU (s)
VAG-1	384	6	2.20	10.05	588
VAG-1	390	10	3.08	15.11	5 898
VAG-1	415	21	4.02	15.93	31 806
VAG-1	784	30	3.37	16.75	209 485
VAG-2	373	0	1.87	6.94	482
VAG-2	373	0	2.42	13.05	4 452
VAG-2	375	1	3.02	14.56	21 645
VAG-2	747	13	2.92	16.55	172 946

Perspectives and work in progress

Modeling

- Fractures as barriers (discontinuous pressure models)
- Geothermal flows
- Compositional flows

Numerical analysis

- Comparison with HFV discretization
- Extend the analysis to Gradient Scheme framework
- Higher order scheme for convection: implicit fractures - explicit matrix

Outlooks

- Go parallel
- Go multiscale

Thank you for your attention!

Relations to other methods

Why VAG is not P_1 FEM:

- Use of interpolated face unknowns (generalized polyhedrons)
- Algebraic elimination of cell unknowns
- Mass lumping

Why VAG is not CVFE:

- Multipoint fluxes, between cells and nodes only (no fluxes between nodes)
- Flexible mass lumping (no mixing of heterogeneities)