Experiments with a Multi-Language Approach to Implementing Multiscale Hybrid-Mixed Methods

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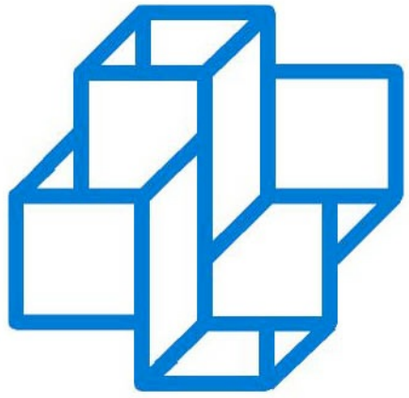

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The MHM method in a (shameful) nutshell...

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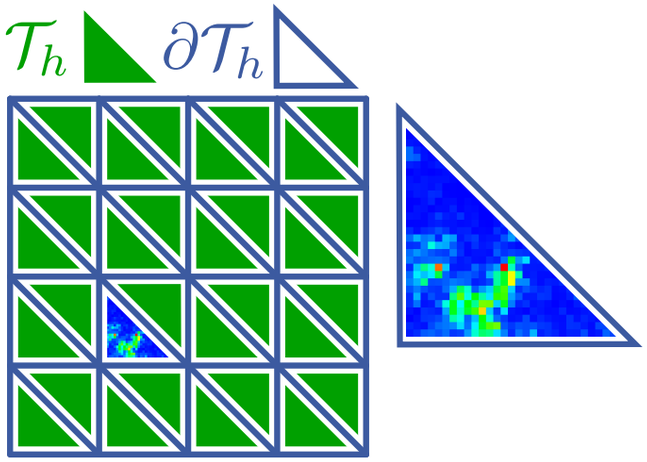
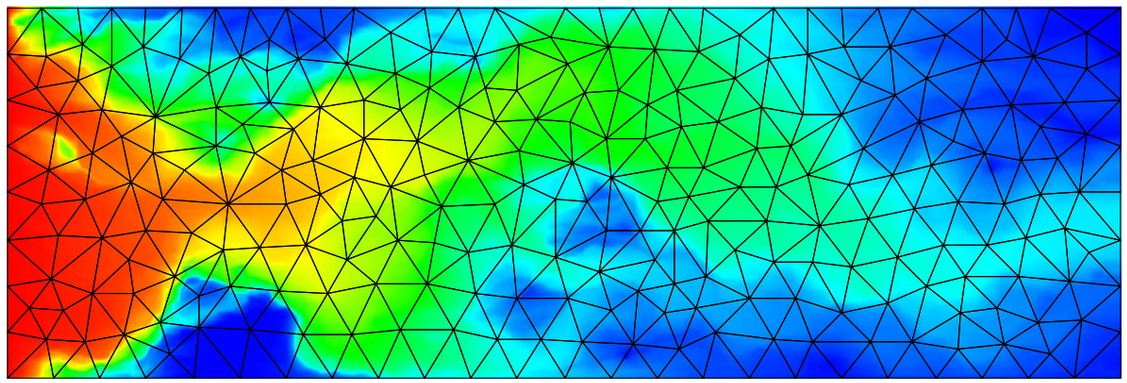


Find $\lambda_h \in \Lambda_h$ such that

$$(u^{\lambda_h}, \mu_h)_{\partial\mathcal{T}_h} = -(u^f, \mu_h)_{\partial\mathcal{T}_h}$$

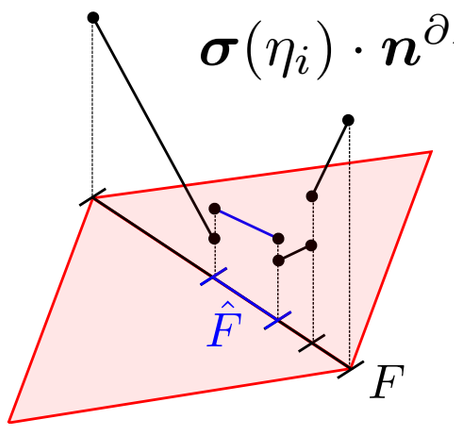
for all $\mu_h \in \Lambda_h$



$$u_h = u^{\lambda_h} + u^f = \sum_{i=1}^{N_h} c_i \eta_i + u^f$$

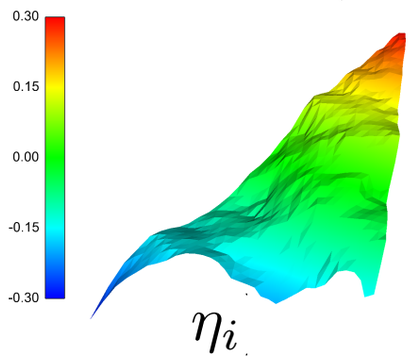
Find η_i such that

$$\left.\begin{aligned} \mathcal{L}\eta_i &= 0, \text{ in } K \\ \boldsymbol{\sigma}(\eta_i)\cdot \boldsymbol{n}^{\partial K} &= \psi_i, \text{ on } \partial K \end{aligned}\right\}$$



Find u^f such that

$$\left.\begin{aligned} \mathcal{L}u^f &= f, \text{ in } K \\ \boldsymbol{\sigma}(u^f)\cdot \boldsymbol{n}^{\partial K} &= 0, \text{ on } \partial K \end{aligned}\right\}$$



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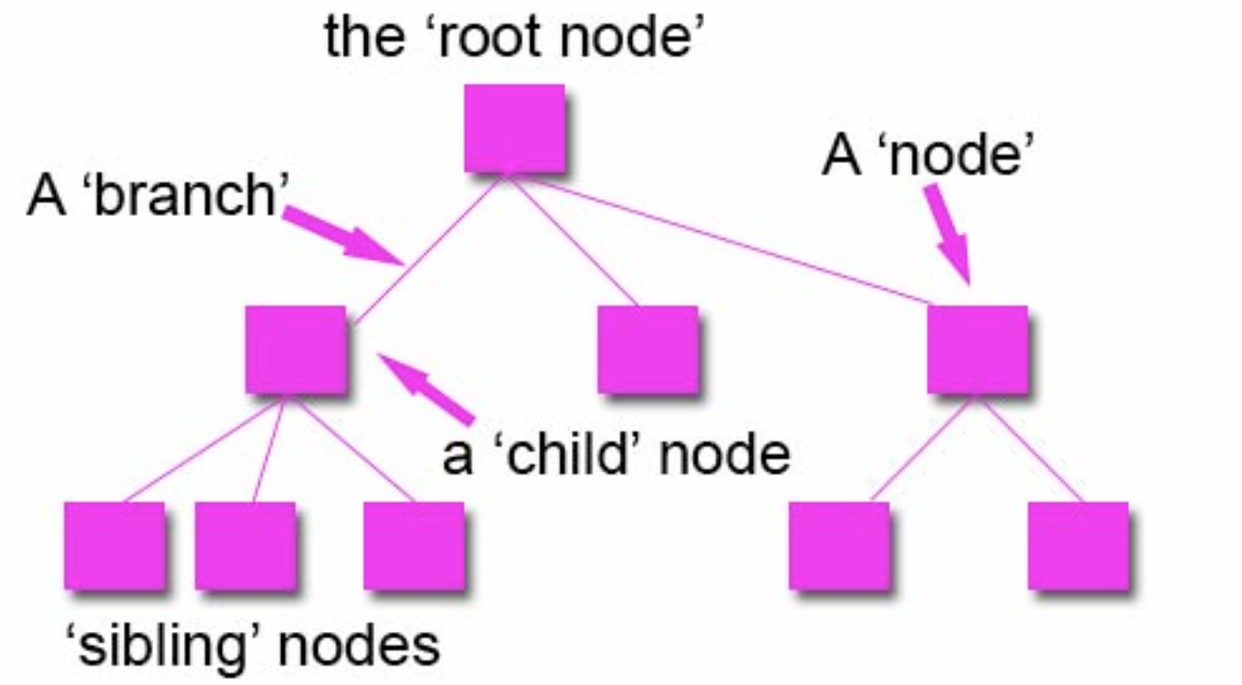
Difficulties

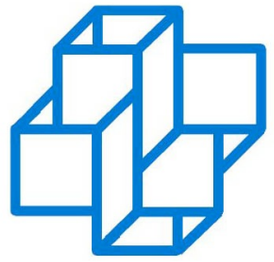
- Remembering SPiNMe (Lua+NeoPZ…)
- Implementing the specific family of MHM methods
  - NeoPZ's architectural drift
- Implementation of a new FEM library specifically crafted for exploring the loosely-coupled strategy of MHM methods to solve global and local problems



PARTS OF A TREE DATA STRUCTURE

Source: www.teach-ict.com

Key design principles



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- Multi-language approach:
  - Erlang for distribution, communication and fault tolerance

  - C++ for number crunching

  - (In the future) Lua for rapid prototyping and high-level configuration

Key design principles

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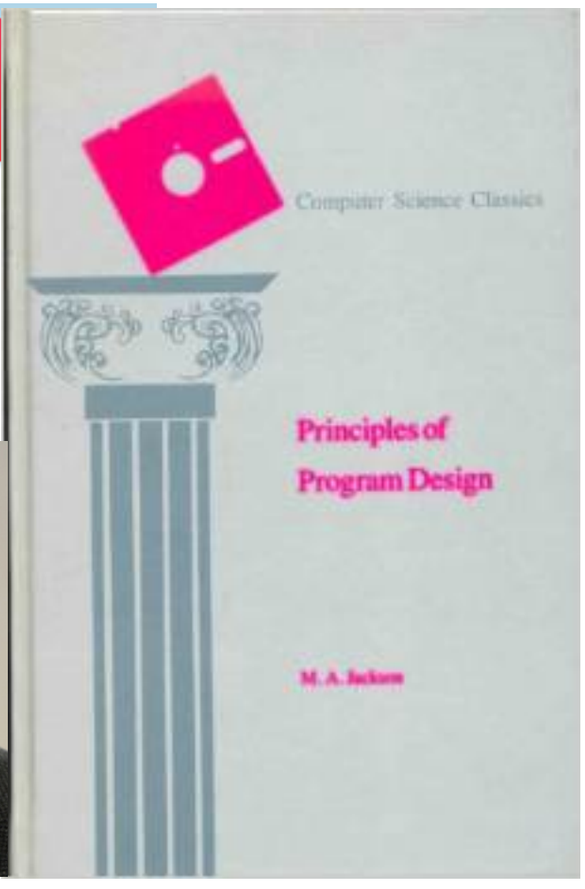
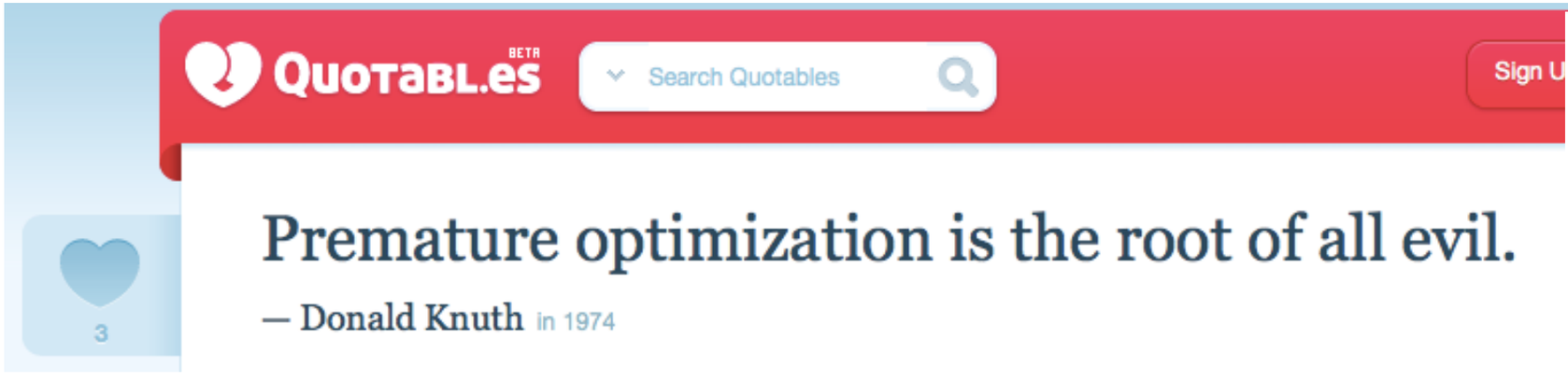
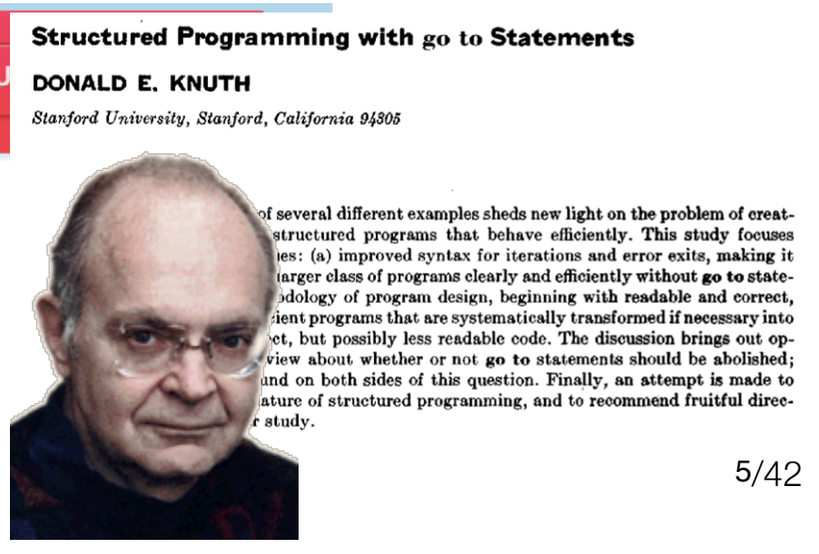
https://quotabl.es



http://mcs.open.ac.uk/mj665/

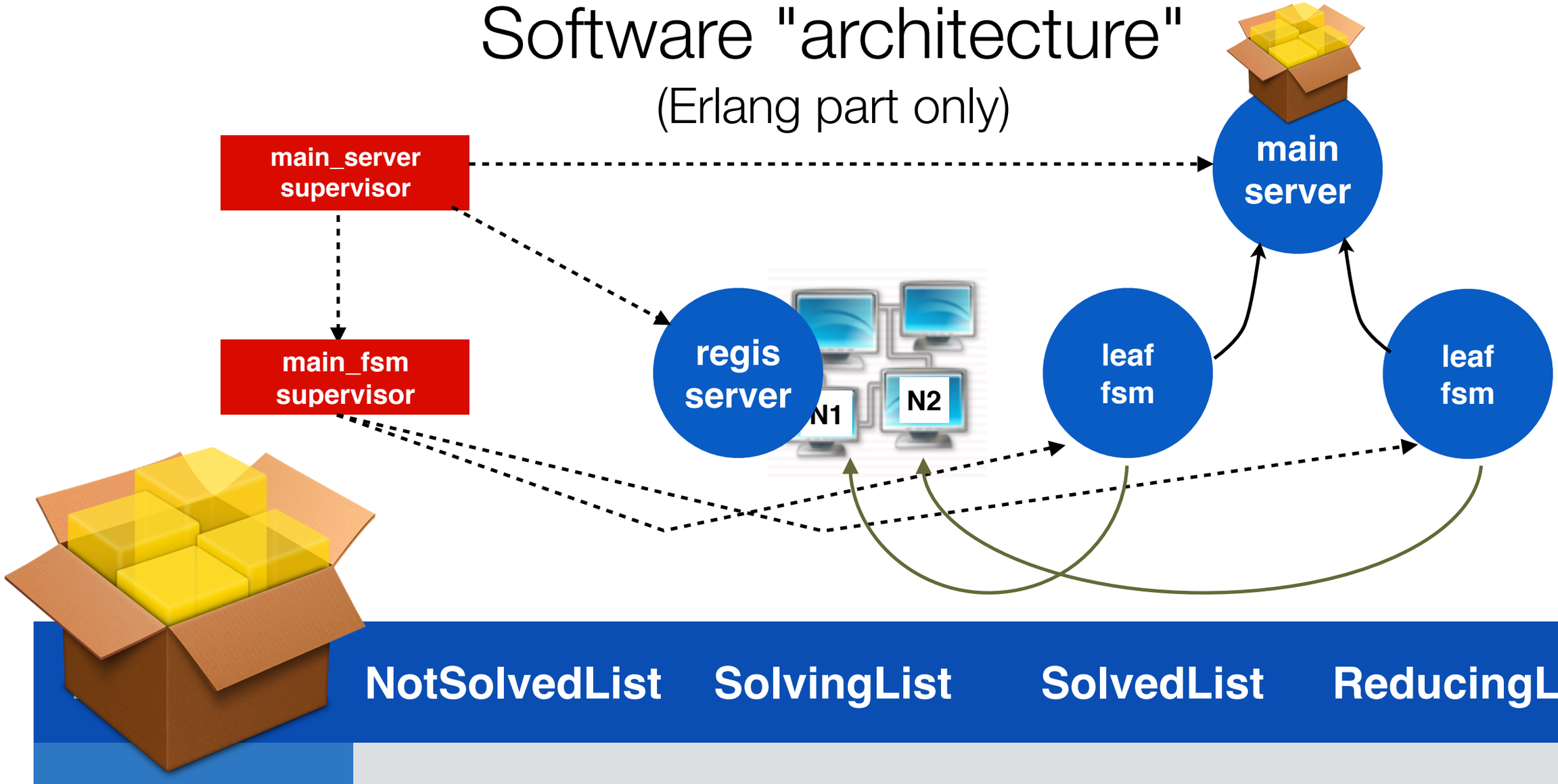


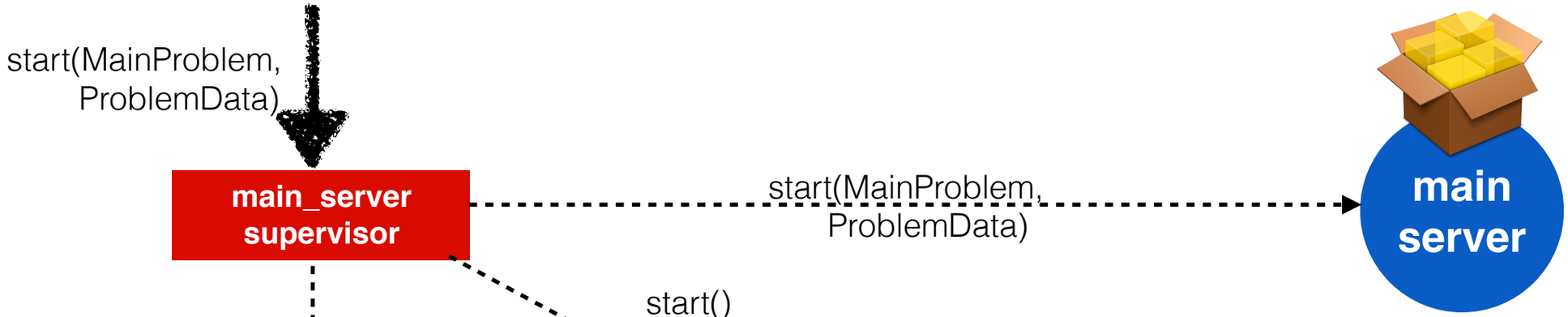


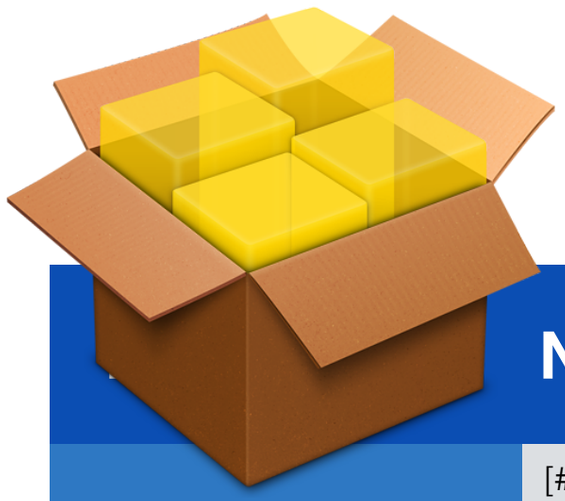
http://www-cs-faculty.stanford.edu/~uno/

Software "architecture"

(Erlang part only)







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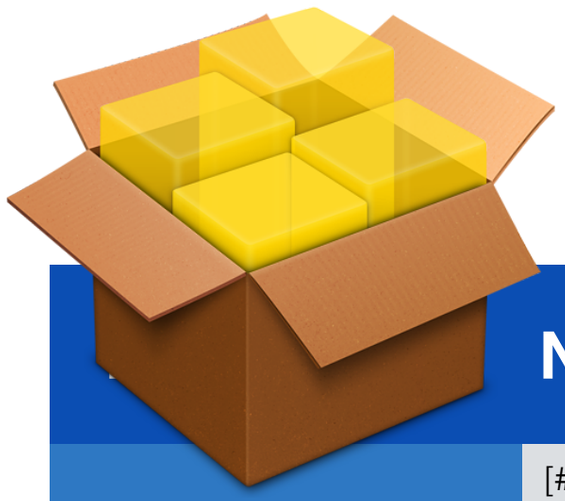

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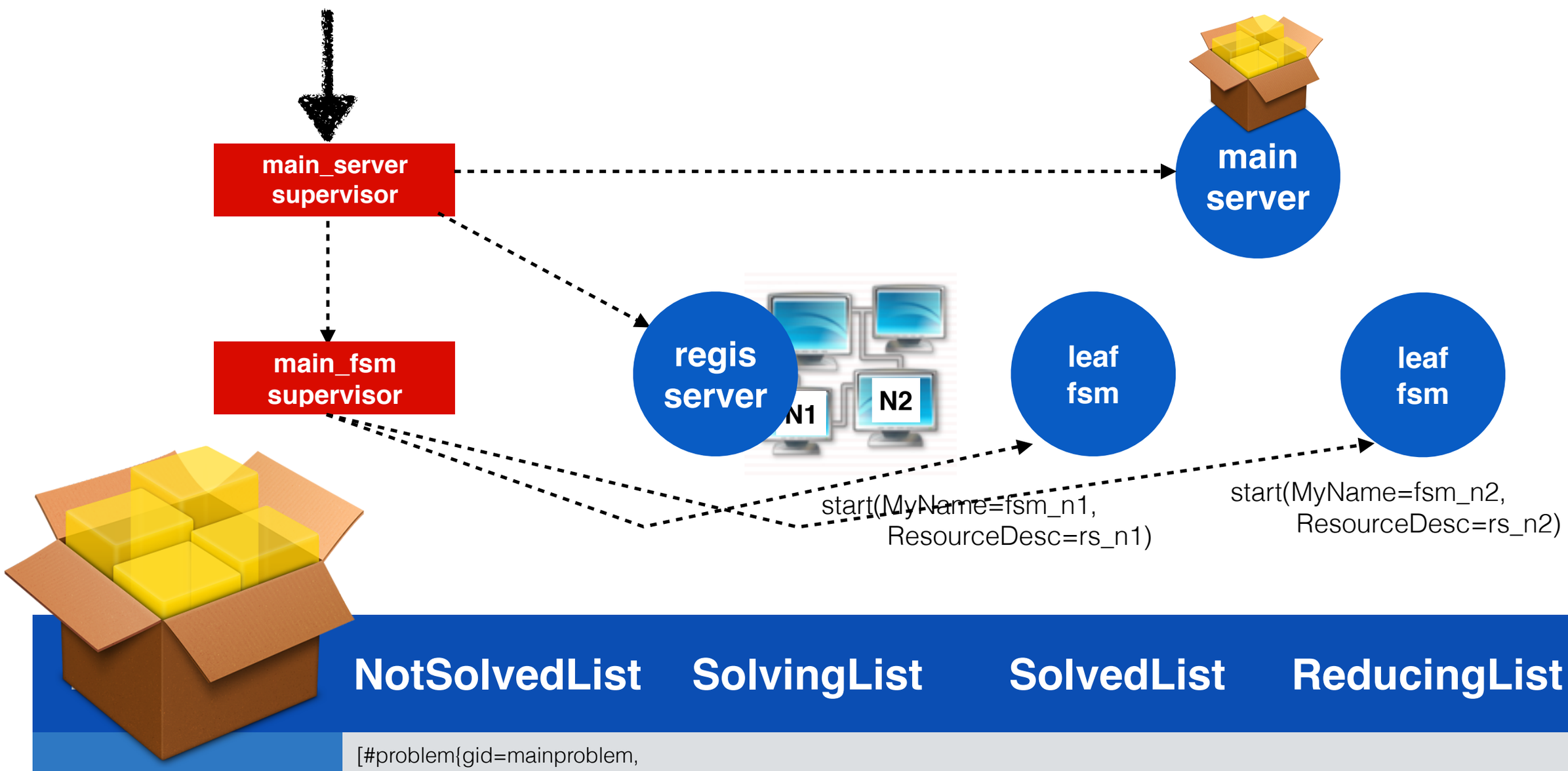

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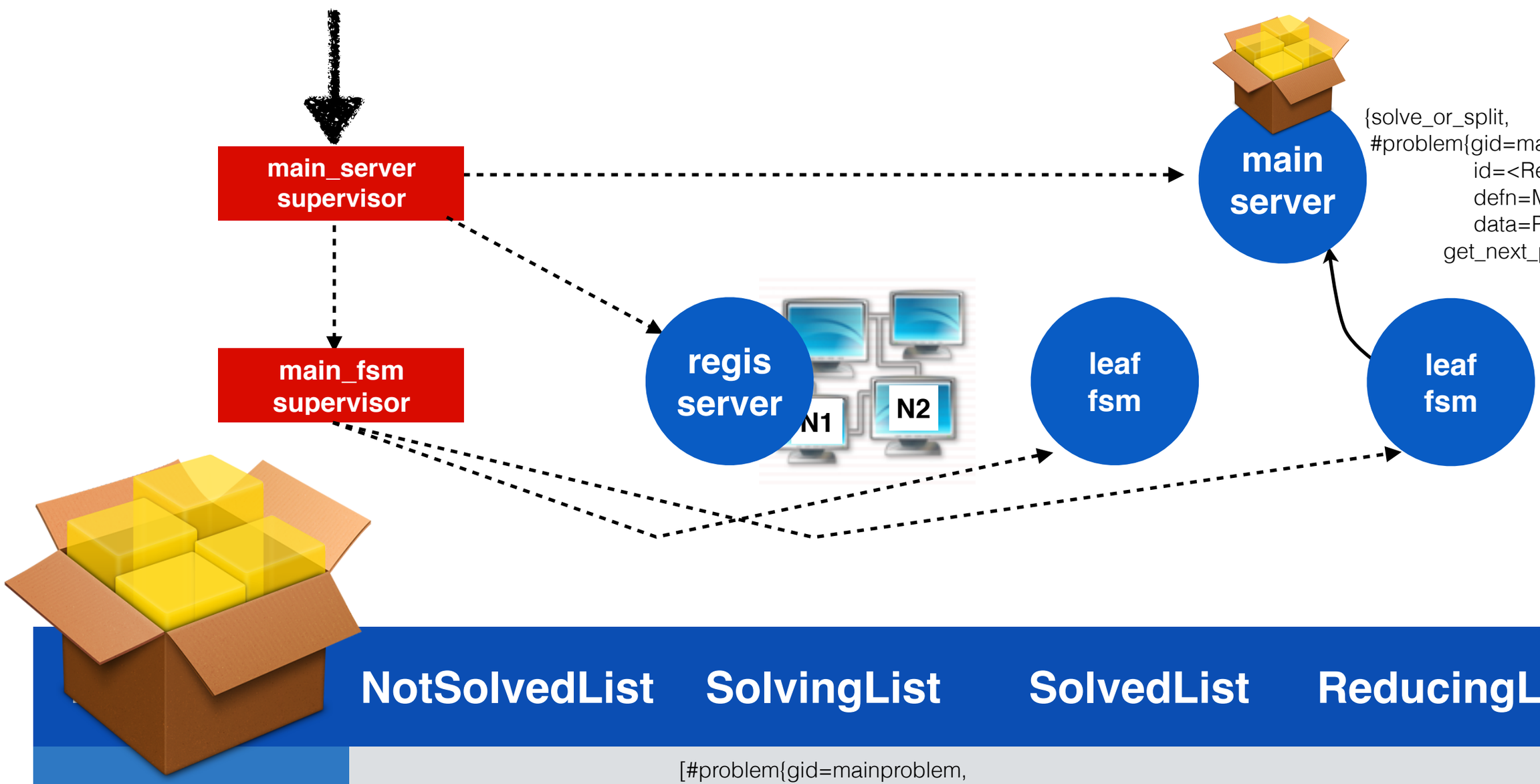

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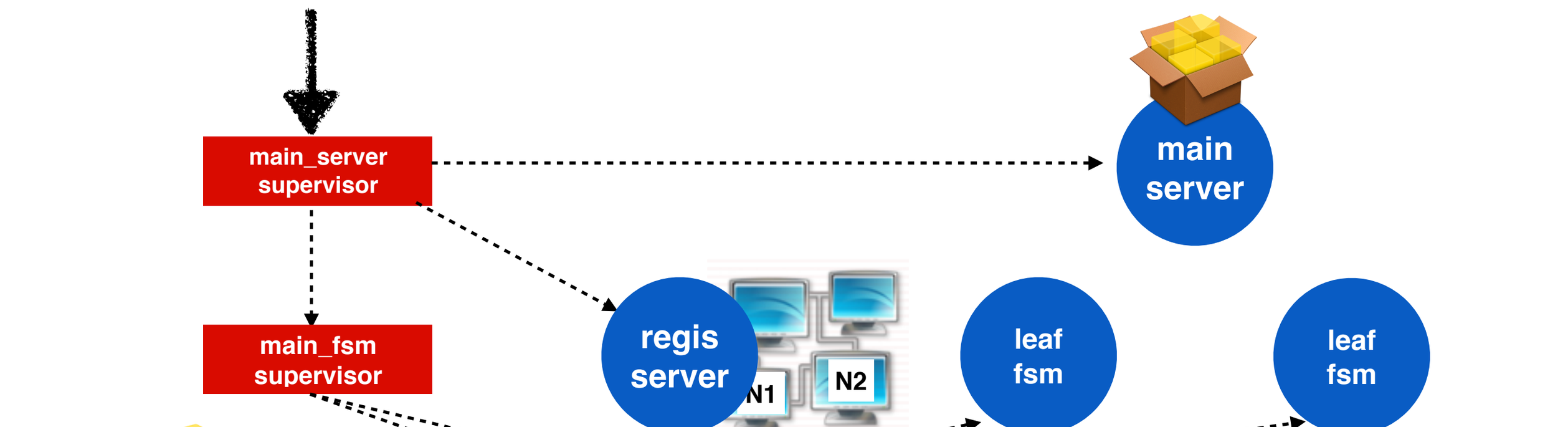

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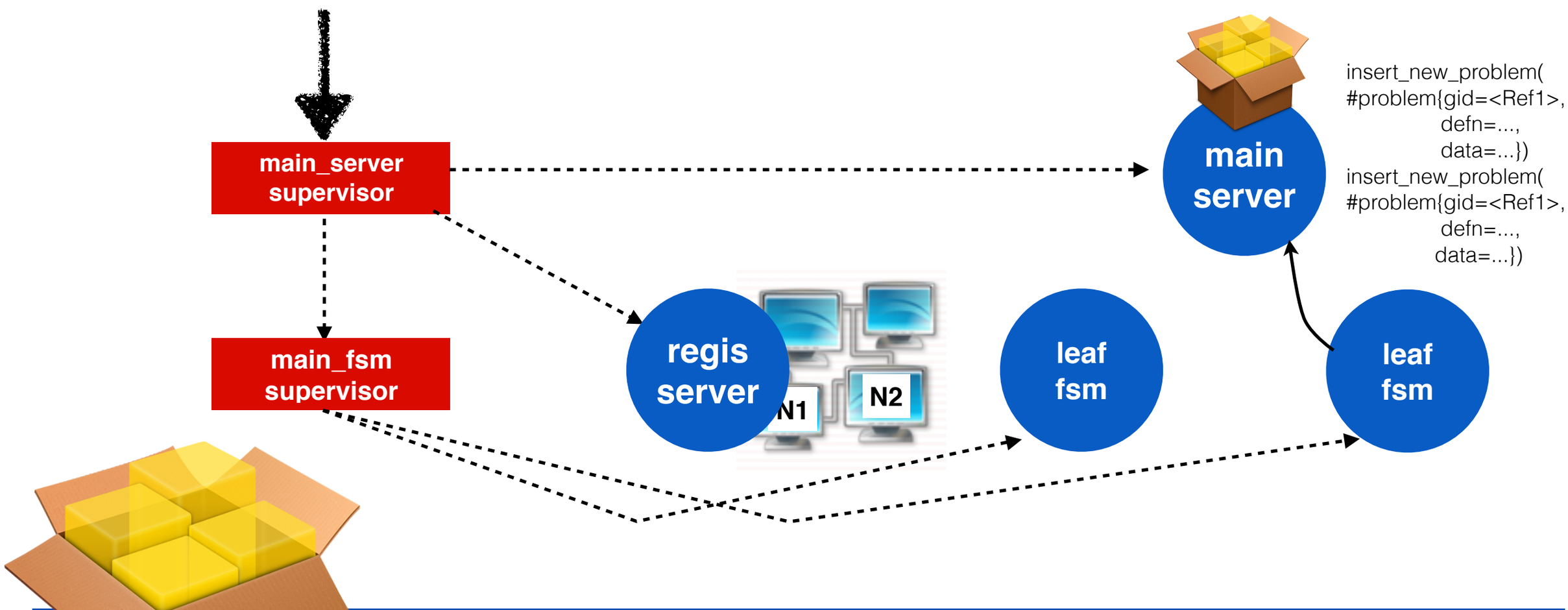

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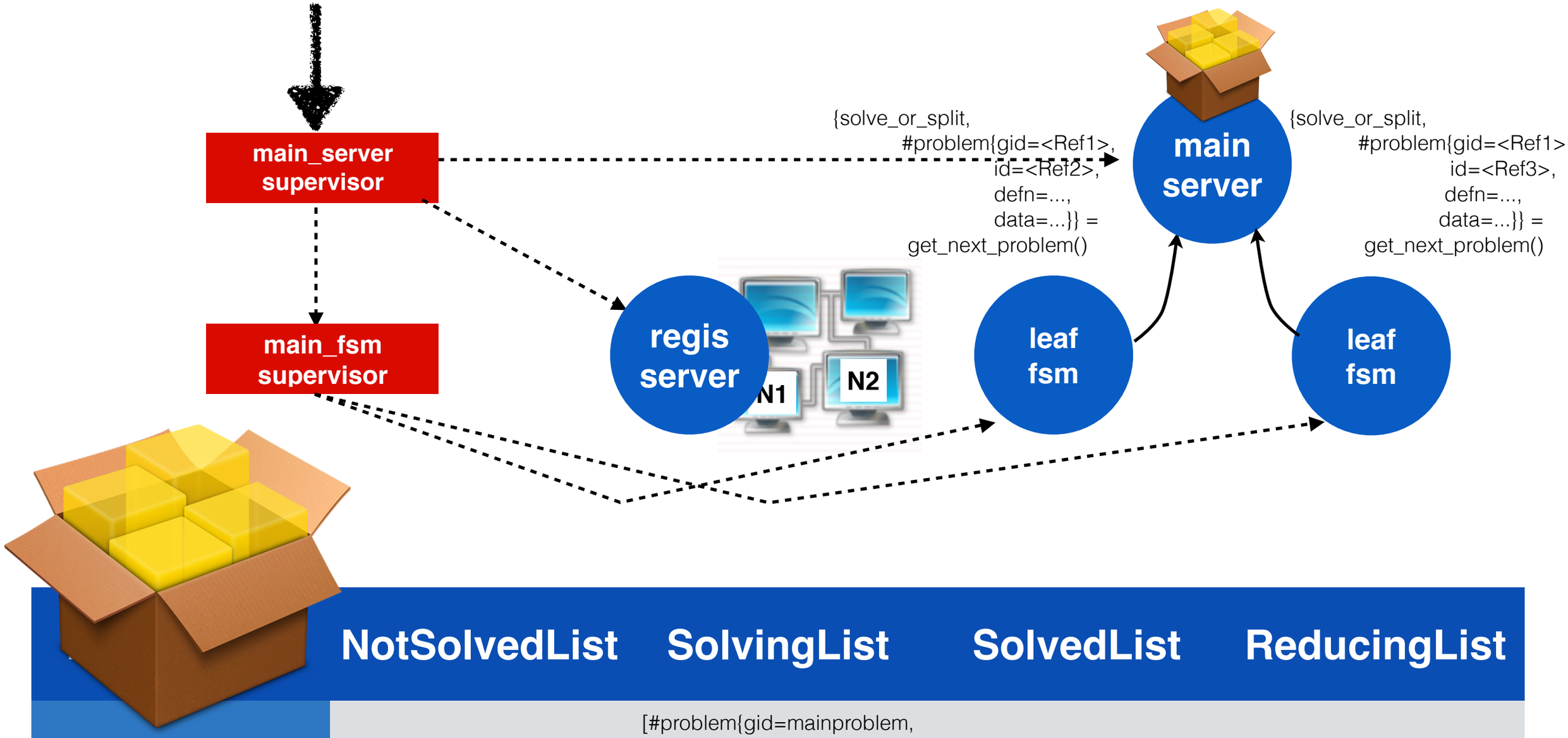

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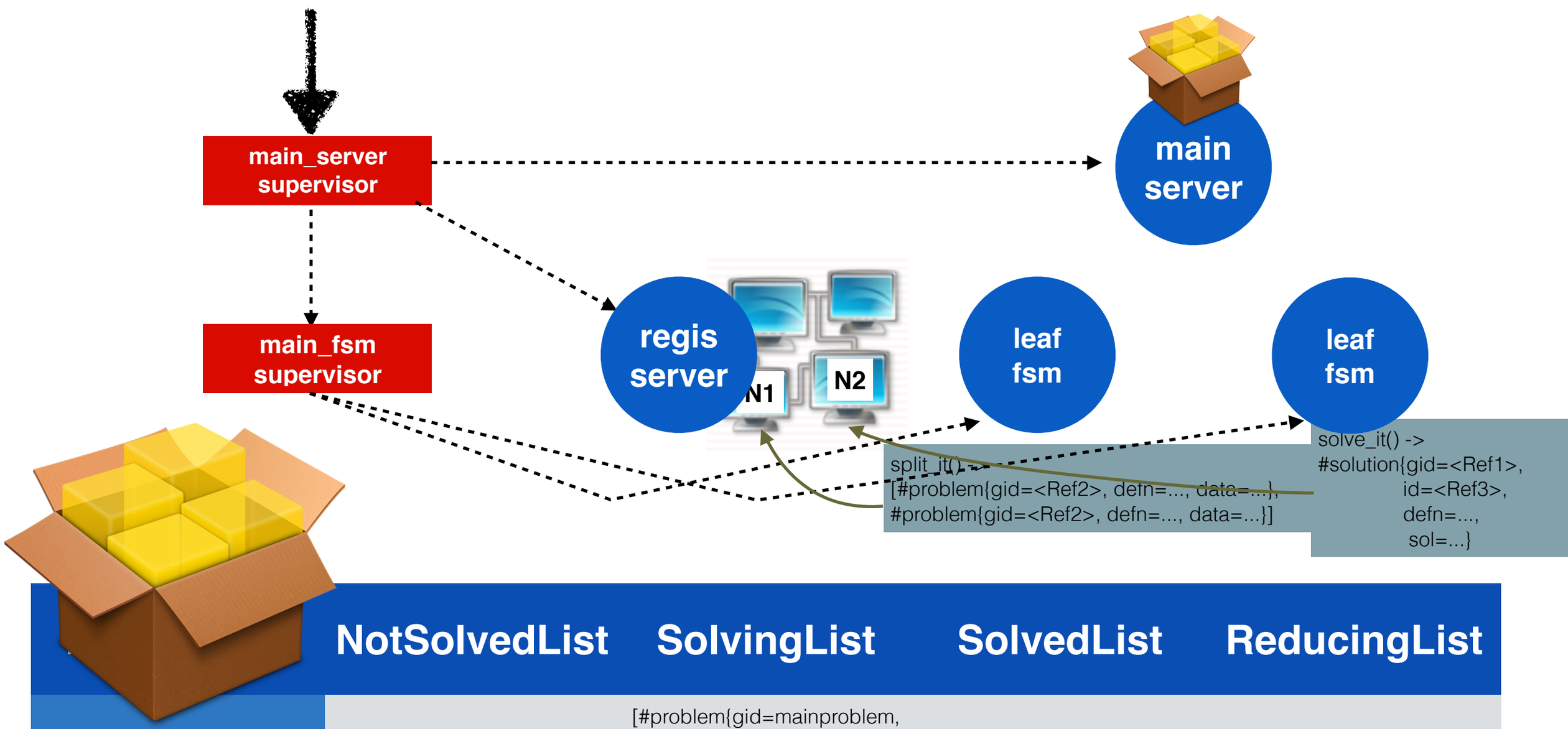

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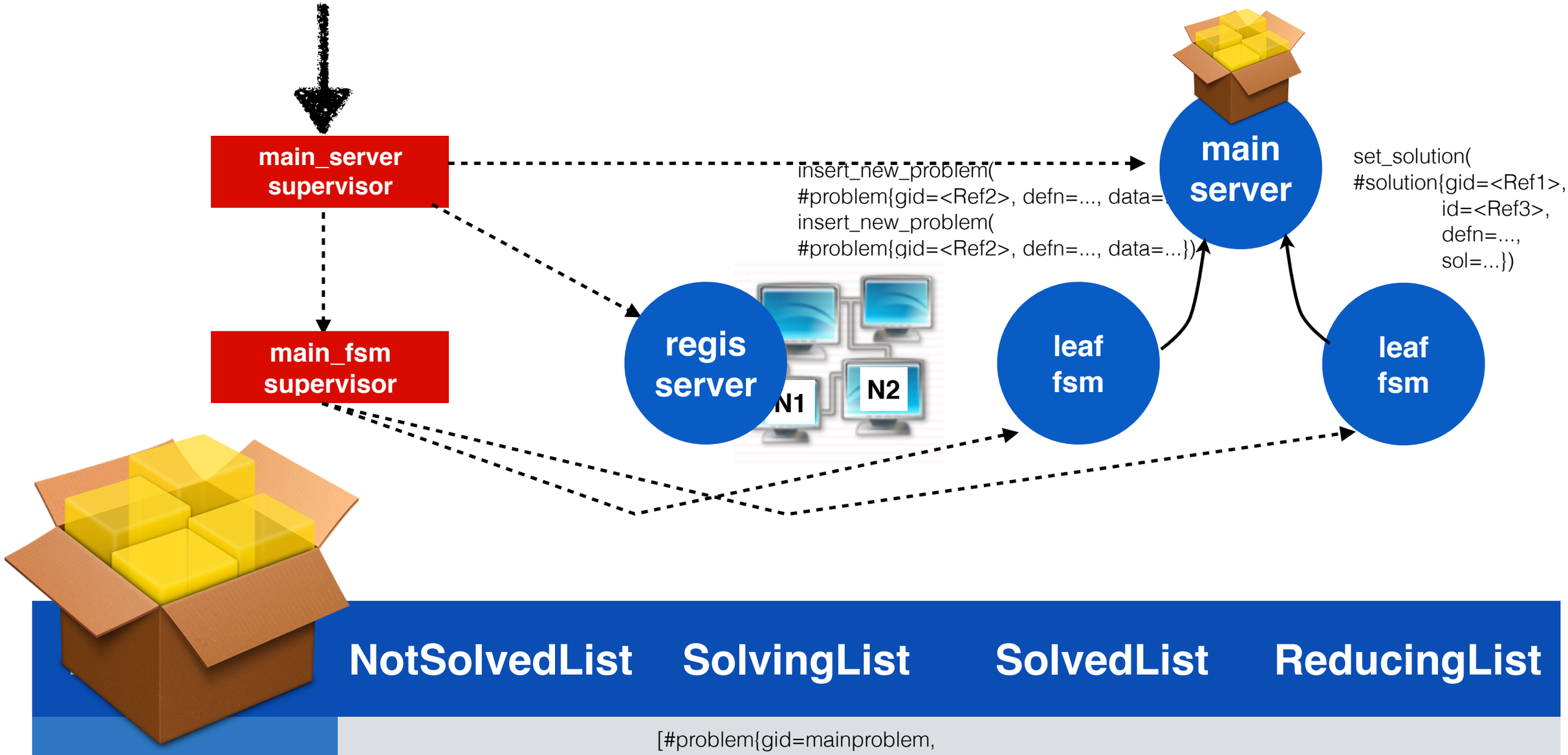

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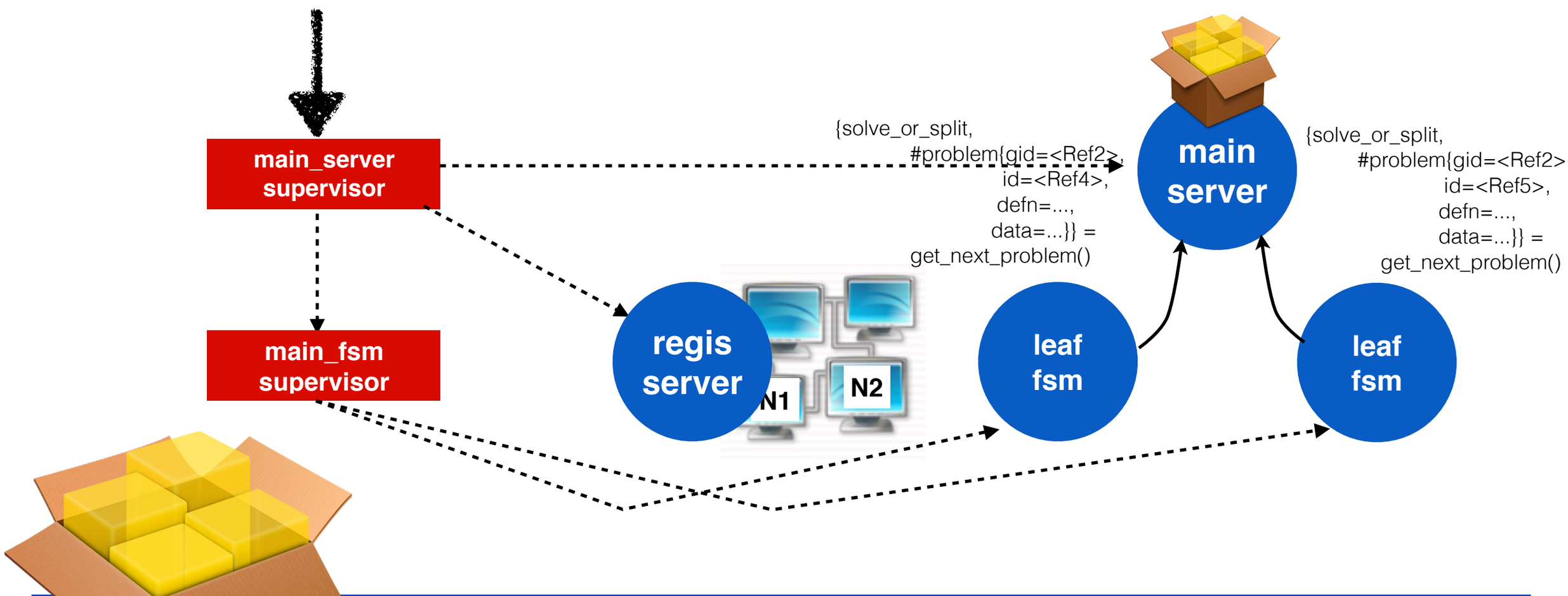

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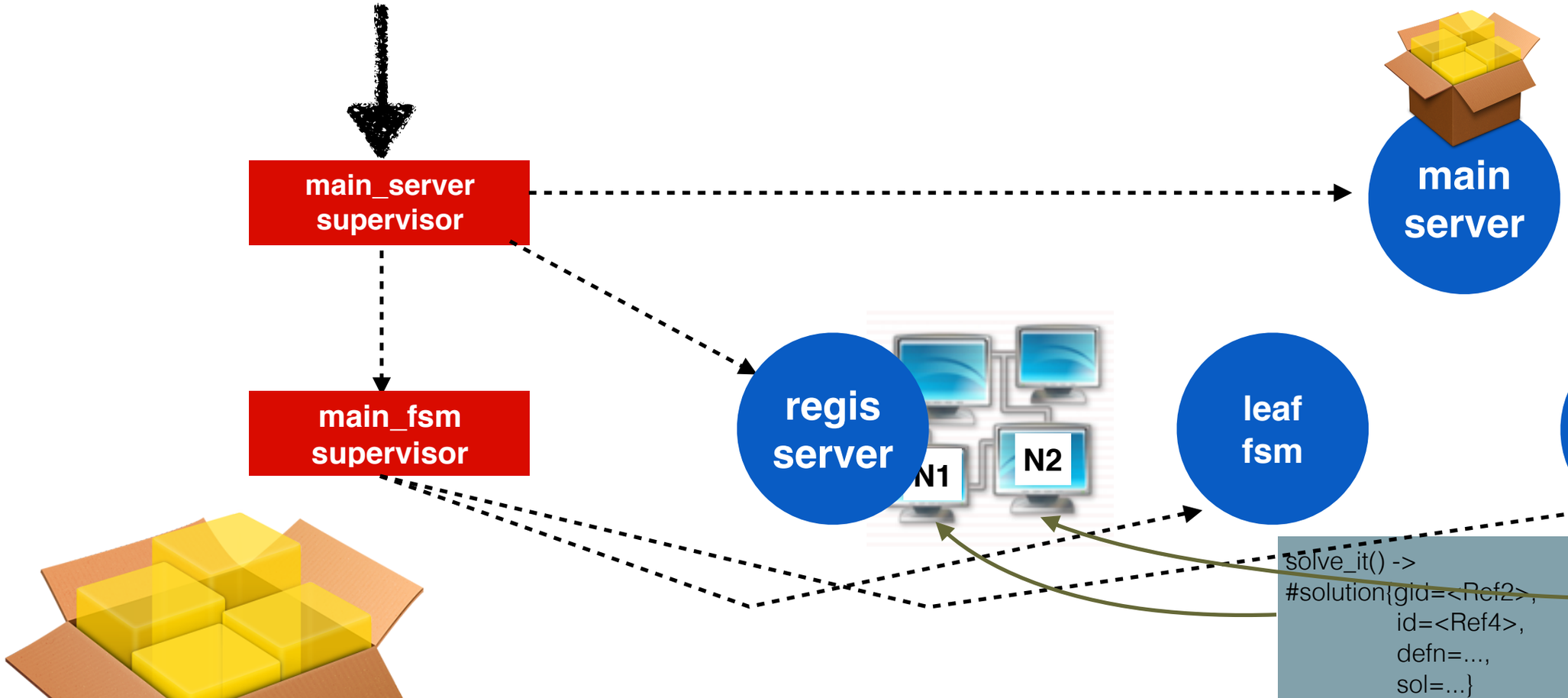

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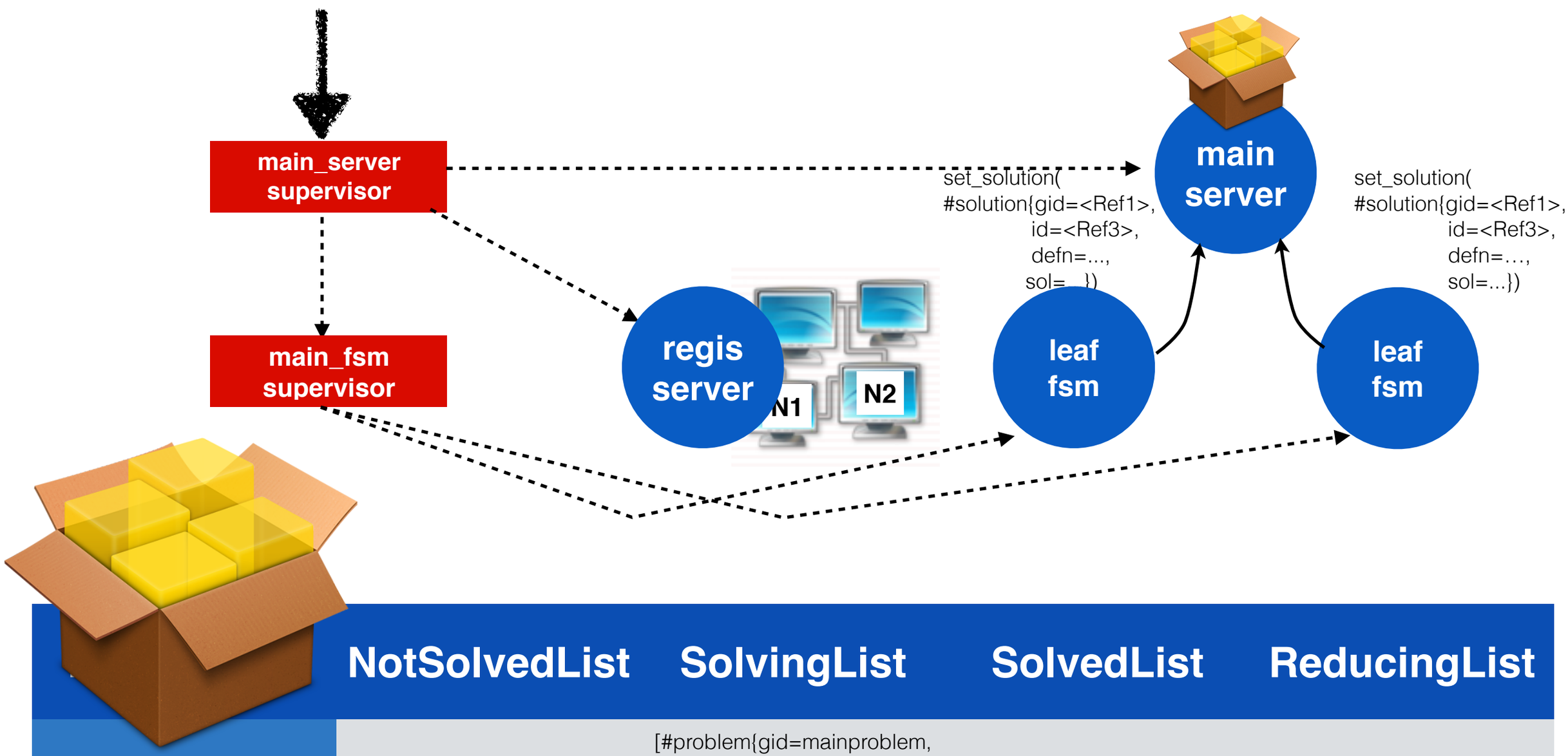

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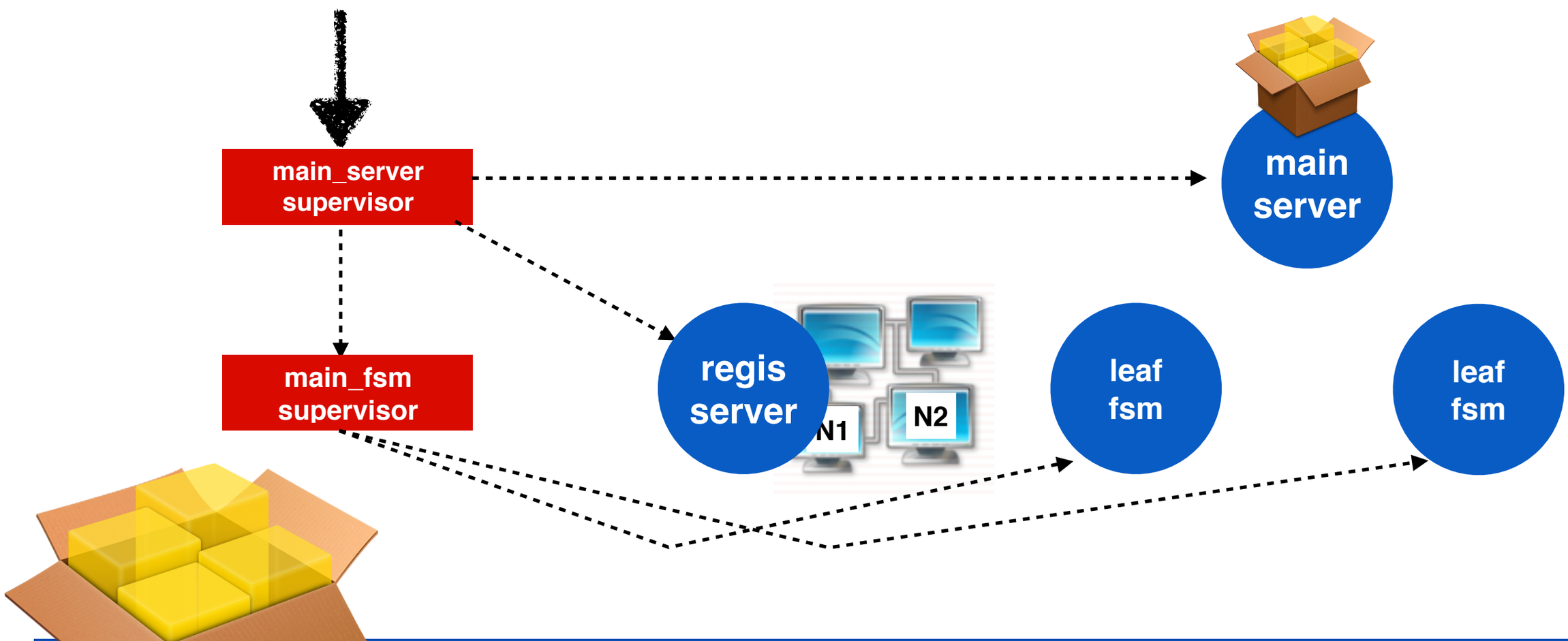

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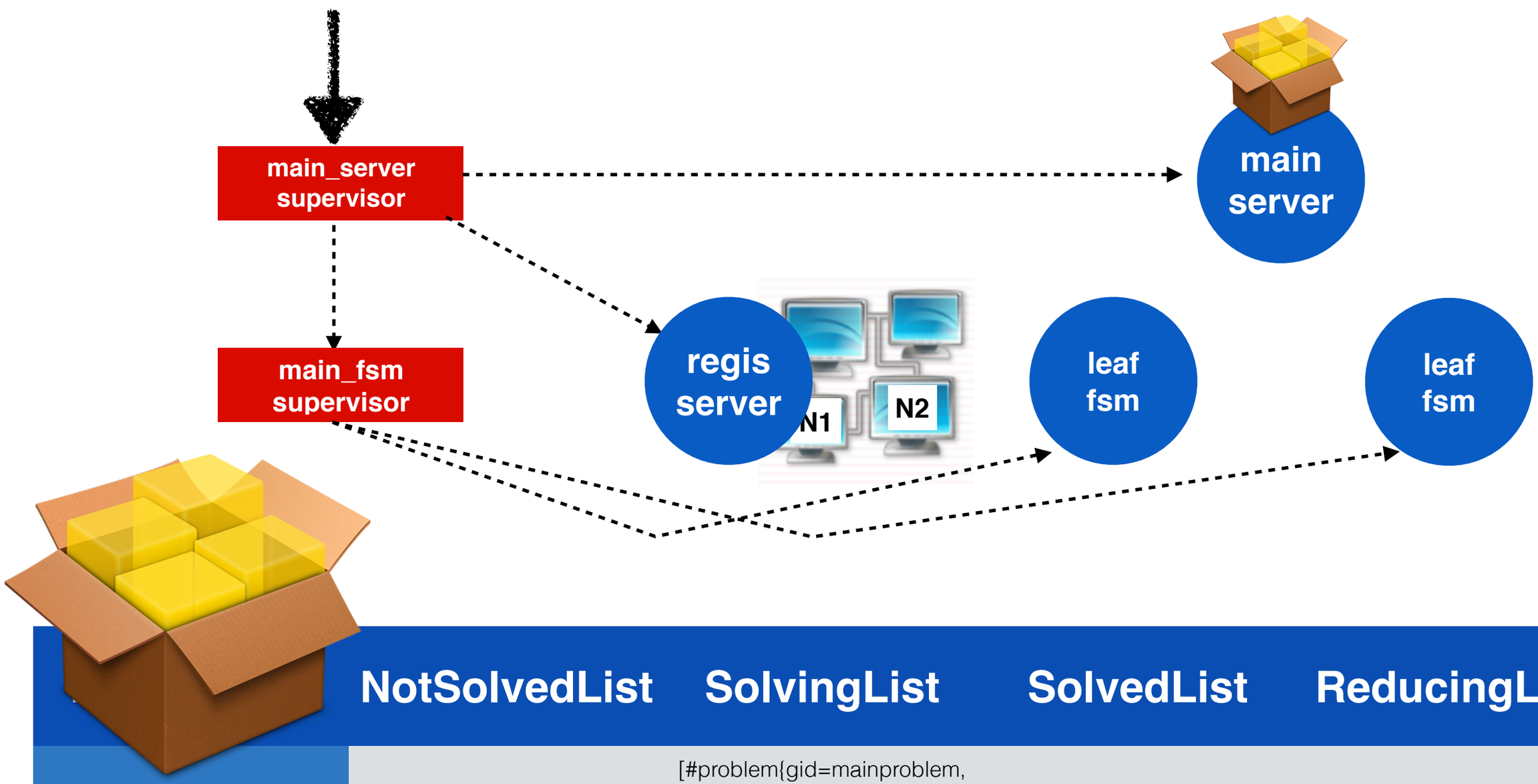
There's no more items in NotSolvedList,
so let's start reducing some problems!!!



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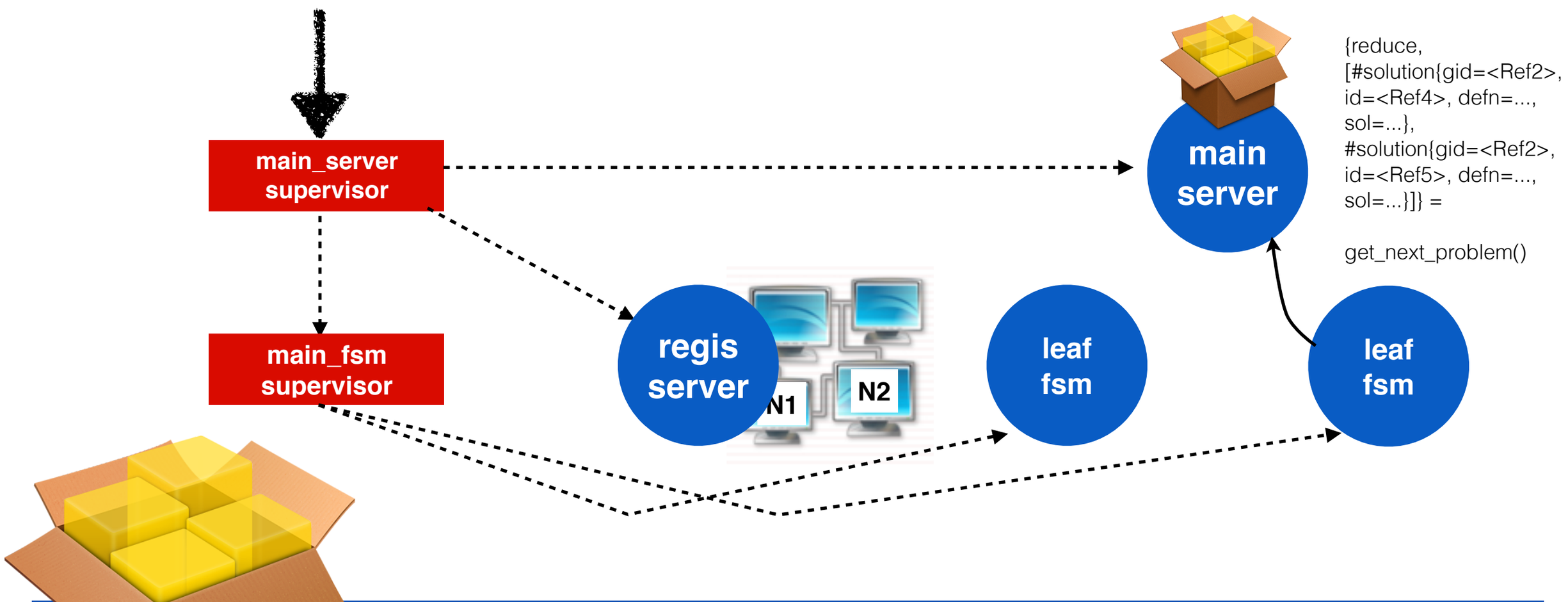
<Ref2>	[]	[]	[#solution{gid=<Ref2>, id=<Ref4>, defn=.., sol=..}, #solution{gid=<Ref2>, id=<Ref5>, defn=.., sol=..}]	[]

But note that only <Ref2> can be reduced, since <Ref1> still has problems being solved!!!



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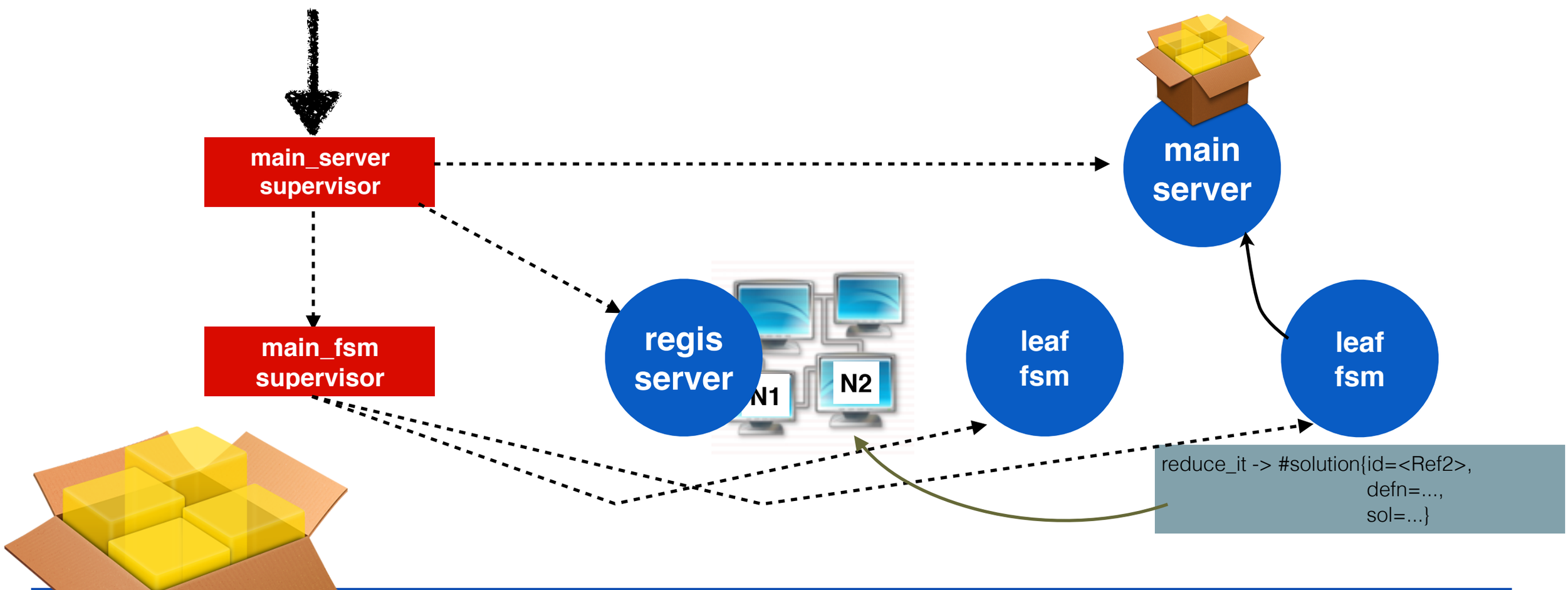

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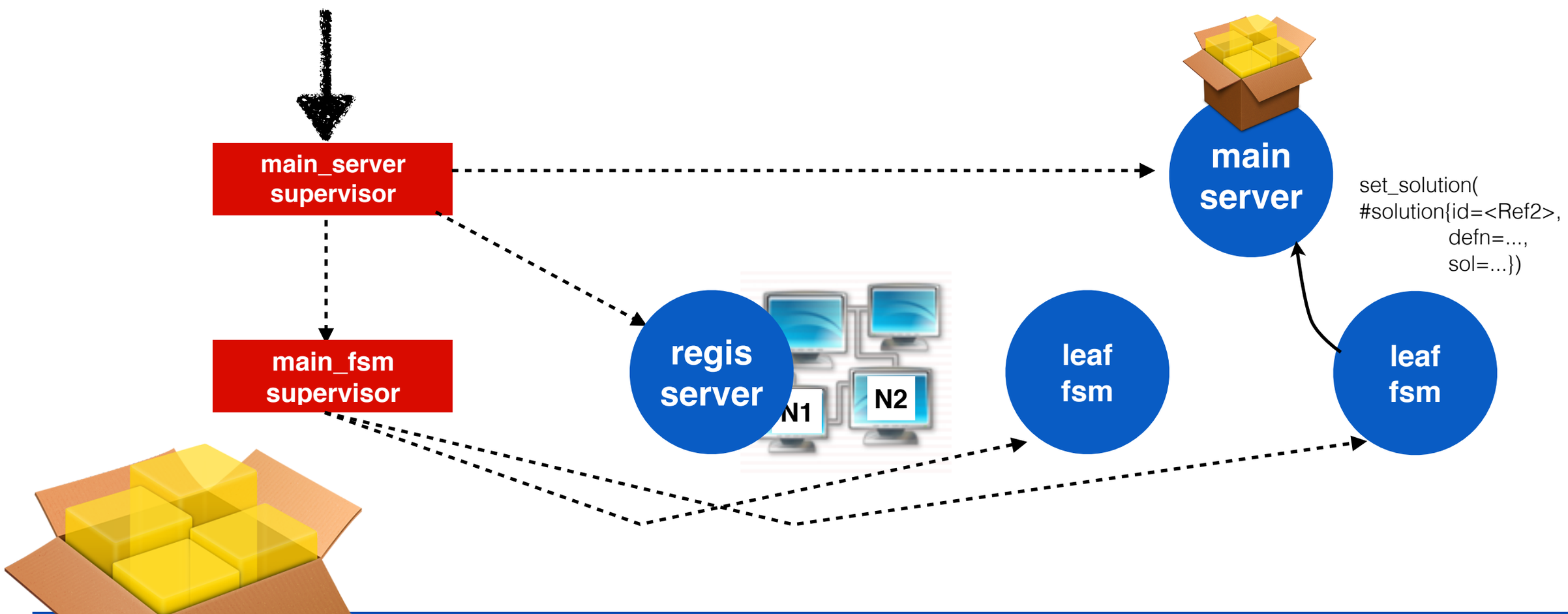

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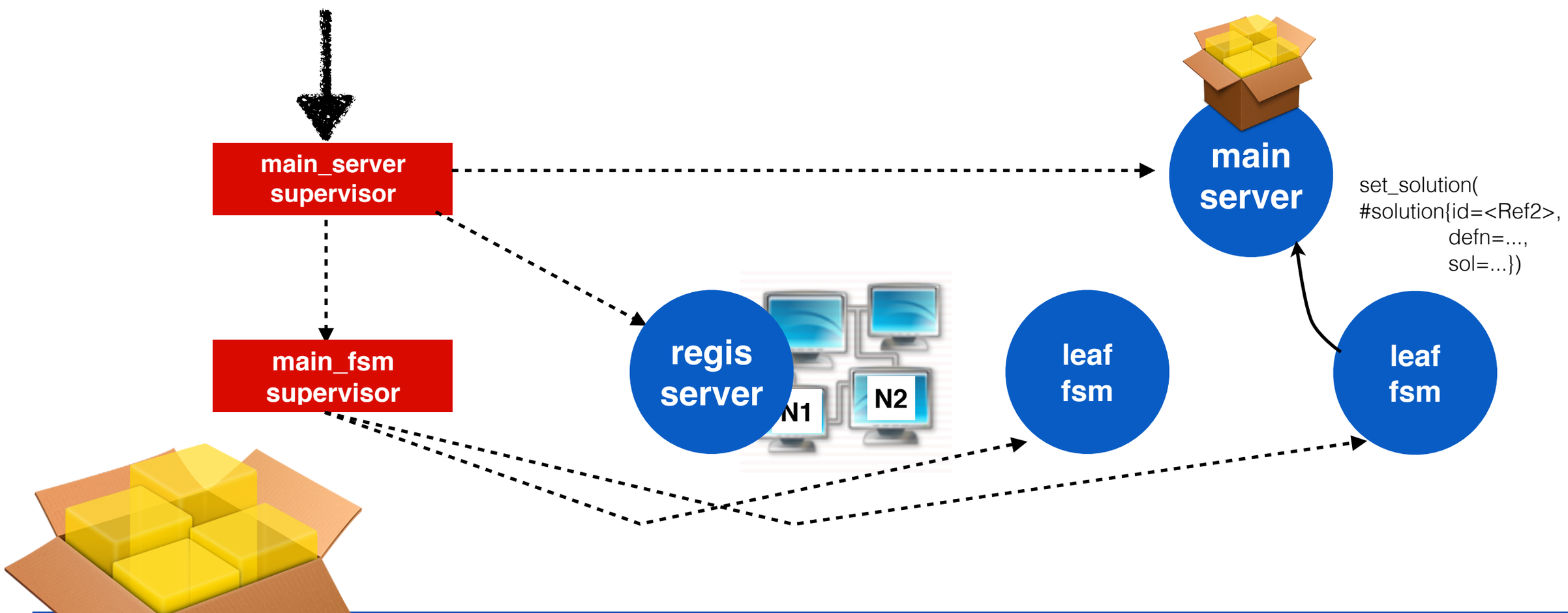
<Ref2>	[]	[]	[]	[#solution{gid=<Ref2>, id=<Ref4>, defn=.., sol=..}, #solution{gid=<Ref2>, id=<Ref5>, defn=.., sol=..}]

Note that there's no gid in this reduced solution, which indicates to main_server that it's a reduction!!!



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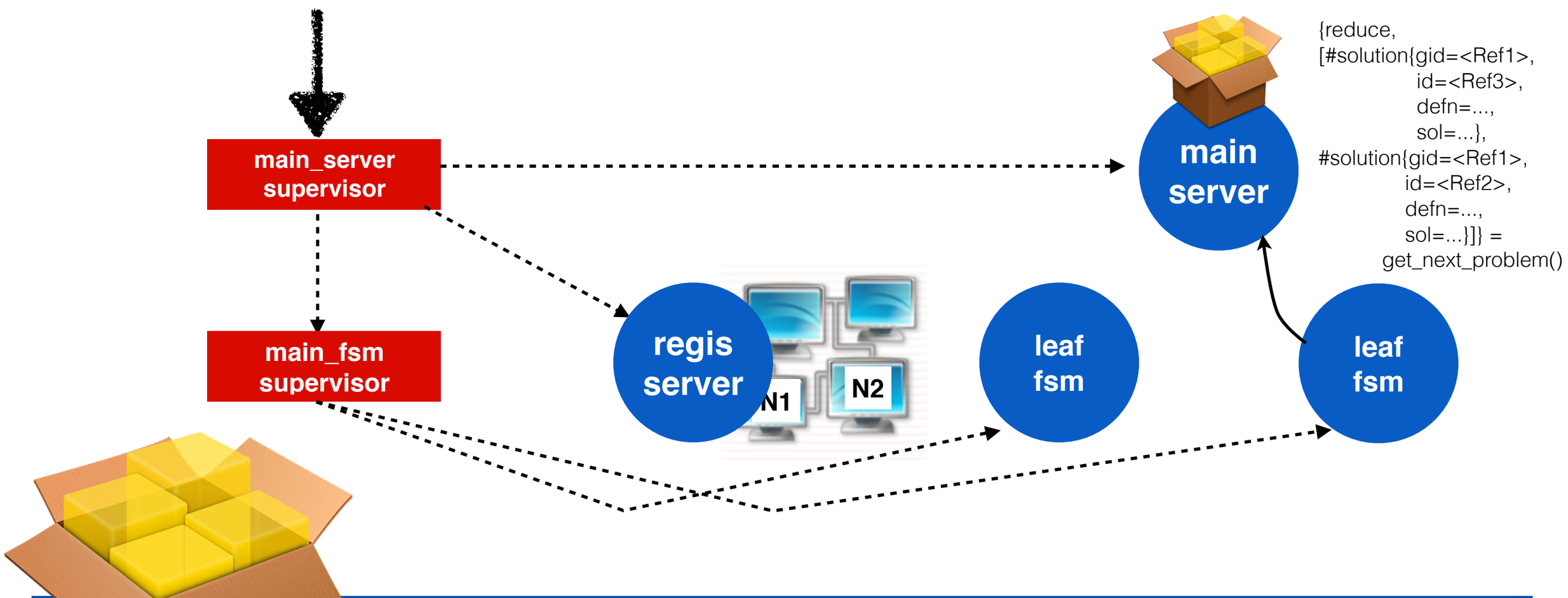

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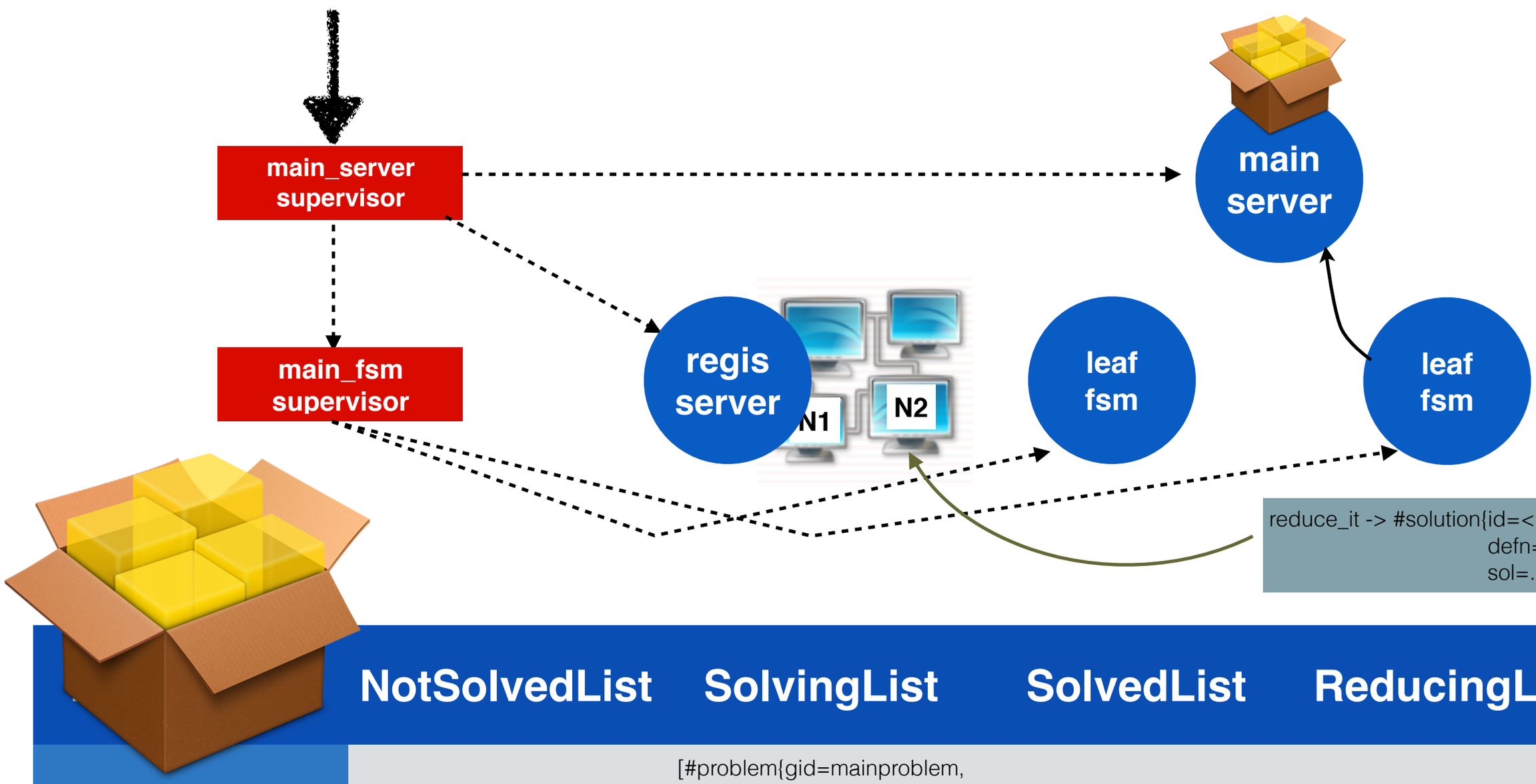

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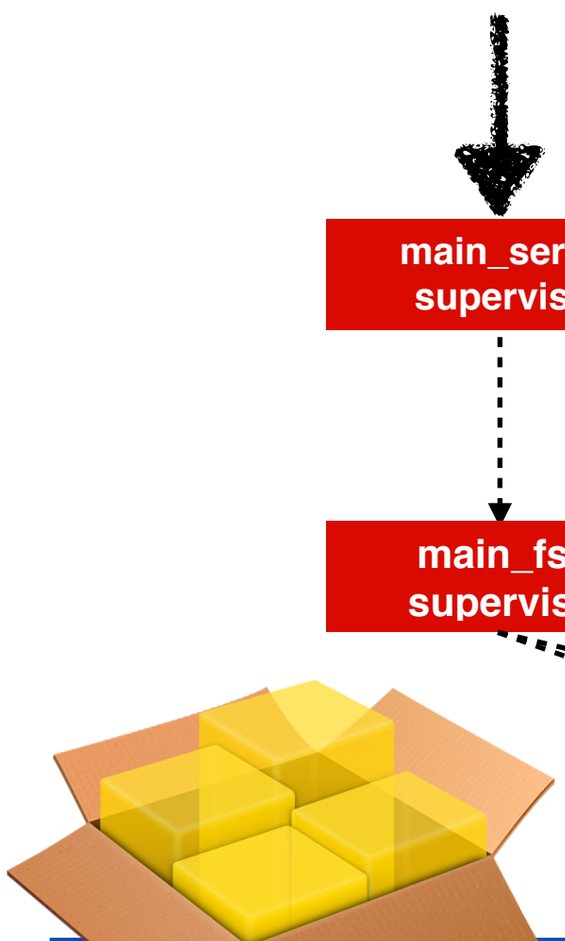

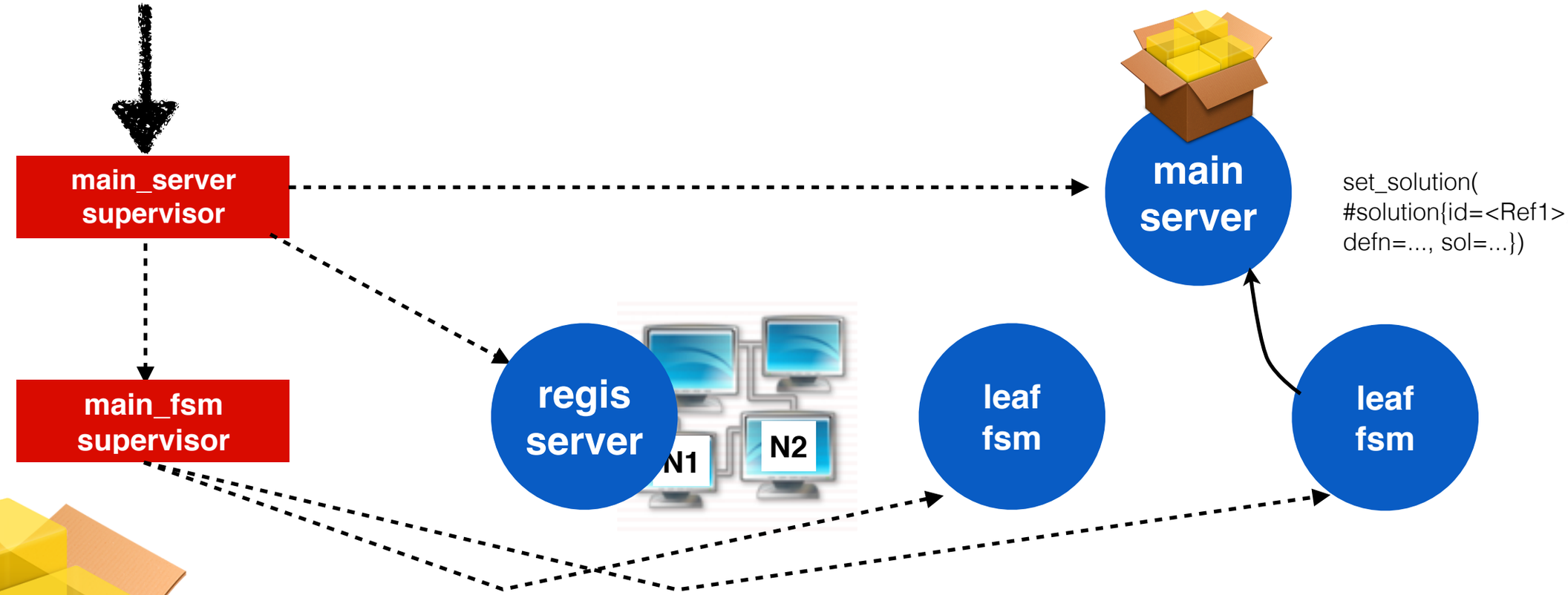

	NotSolvedList	SolvingList	SolvedList	ReducingList
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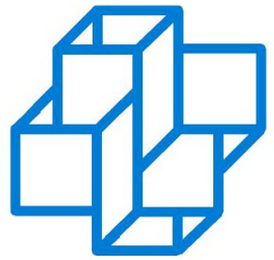
END!!!!

Preliminary tests



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- Testing problem — Darcy (fluid flow through a porous medium)

$$\textit{Find } p = p(\mathbf{x}) \text{ such that}$$

$$\begin{cases} \operatorname{div}(-K\nabla p) = f \ \text{ in } \Omega \\ p = 0 \text{ in } \Gamma \end{cases}$$

K is a positive definite tensor

$K(\mathbf{x})$ and $f(\mathbf{x})$ may have multi-scale characteristics

Preliminary tests

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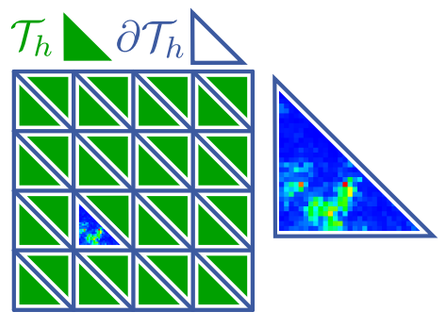
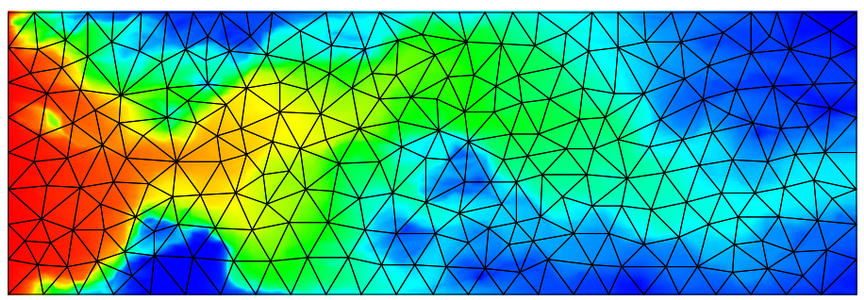
- In MHM (2D):

Find $(p_0, \lambda_h) \in V_0 \times \Lambda_h$ ***such that***

$$\begin{aligned} (\lambda_h, q_0)_{\partial\mathcal{T}_h} &= (f, q_0)_{\mathcal{T}_h} \\ (p_0, \mu_h)_{\partial\mathcal{T}_h} + (p^{\lambda_h}, \mu_h)_{\partial\mathcal{T}_h} &= -(p^f, \mu_h)_{\partial\mathcal{T}_h} \end{aligned}$$

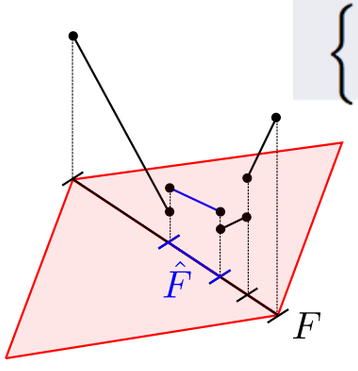




$$p^{\lambda_h} = \sum_{i=1}^{N_h} c_i \eta_i$$

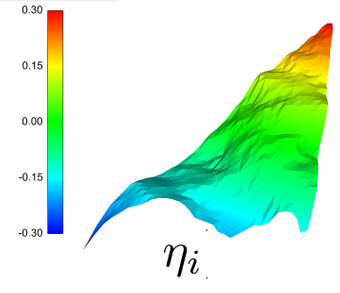
Find $\eta_i \in V_0^{\perp}(K)$ ***such that***

$$\begin{cases} \mathcal{L}\eta_i &= (\mathbf{n}\cdot\mathbf{n}^K)c^{\psi_i} \\ -\mathcal{K}\nabla\eta_i\cdot\mathbf{n}^K &= (\mathbf{n}\cdot\mathbf{n}^K)\psi_i \end{cases}$$



Find $p^f \in V_0^{\perp}(K)$ ***such that***

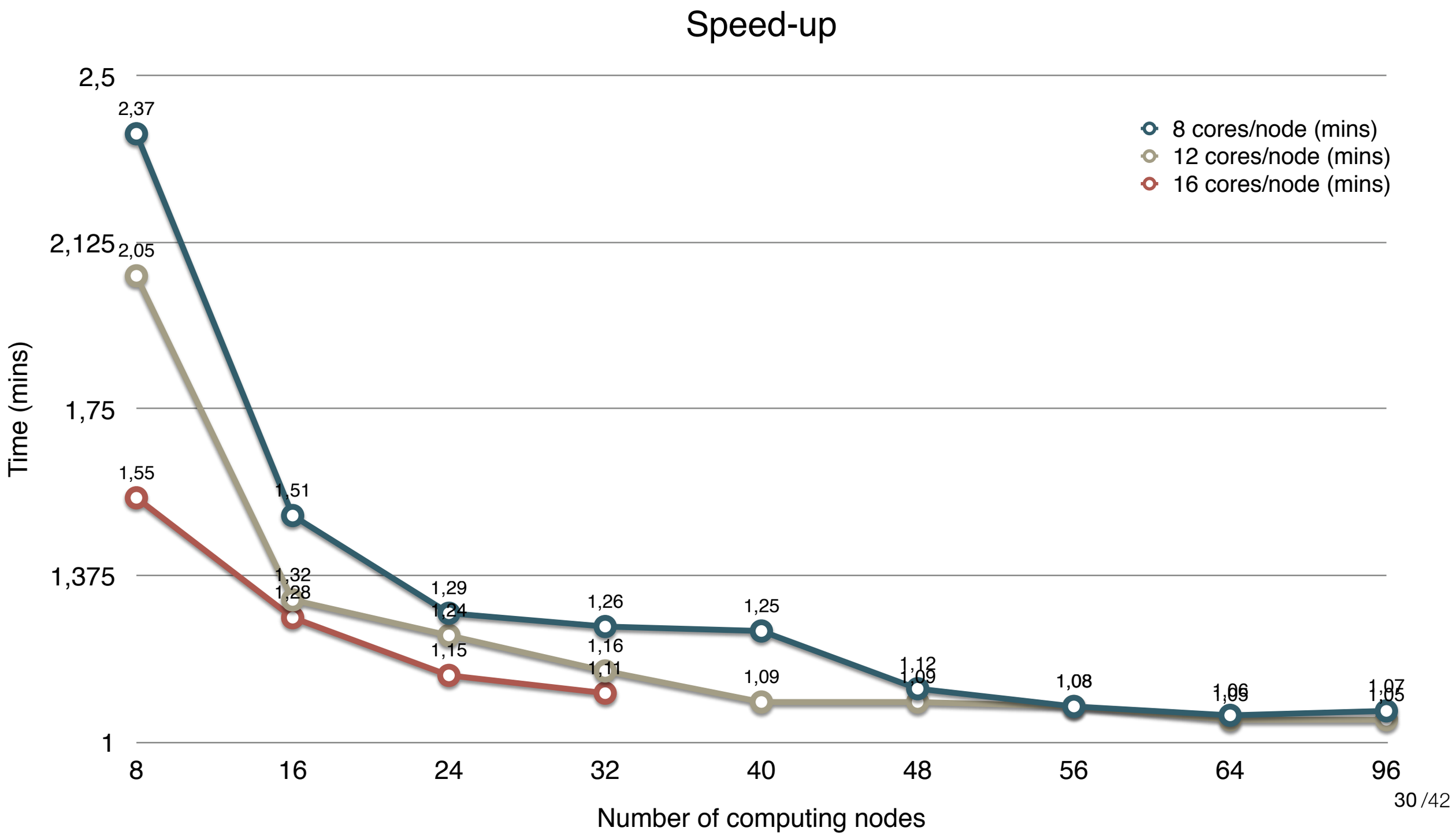
$$\begin{cases} \mathcal{L}p^f &= f^{\perp} \\ -\mathcal{K}\nabla p^f\cdot\mathbf{n}^K &= 0 \end{cases}$$



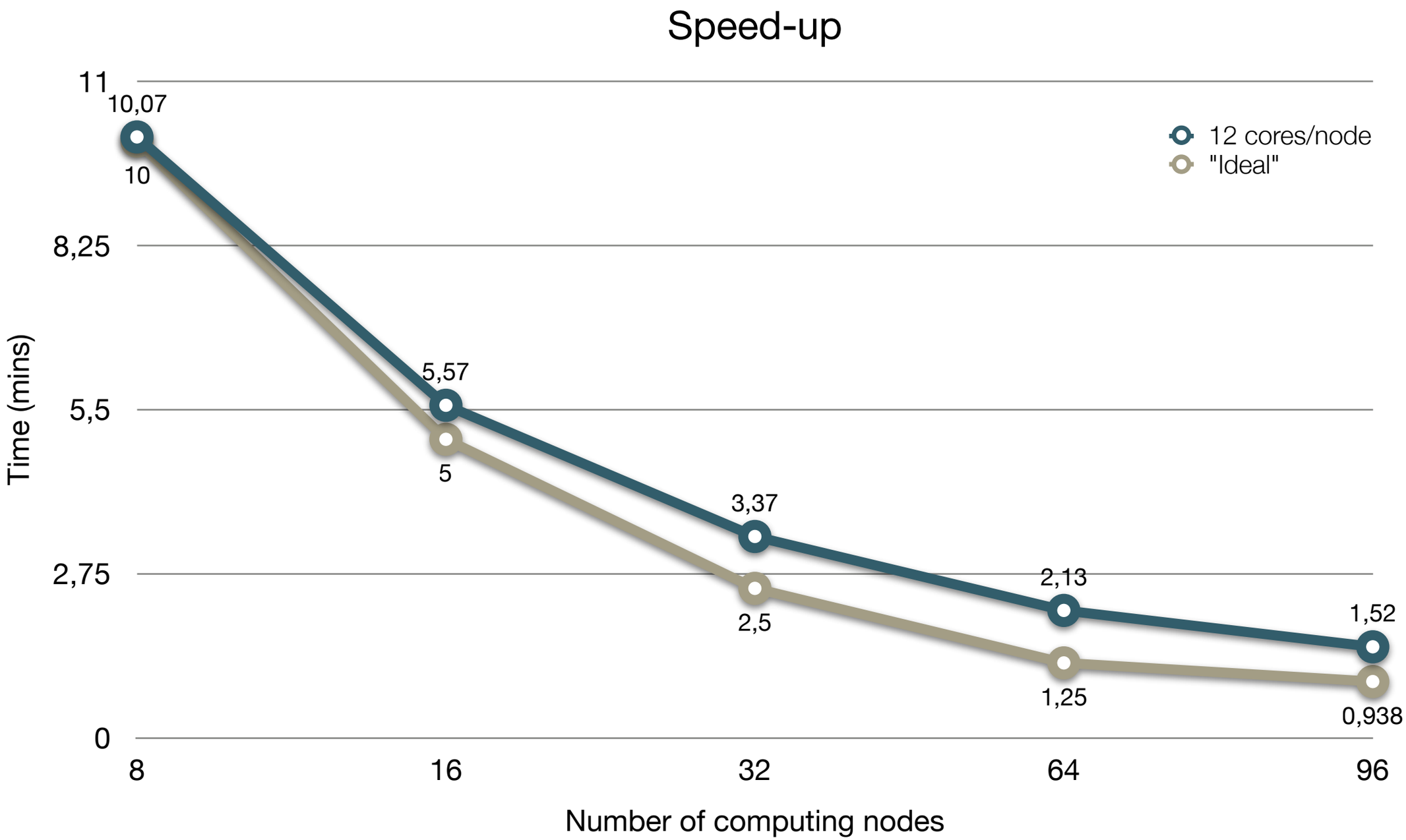
Total number of integration points:

$$\underbrace{G * 3 * L}_{\text{within 1st level elements}} + \underbrace{G * 2 * 3 * \sqrt{L}}_{\text{on 1st level faces}}$$

4,194,304 (G=1,024 x L=4,096) elements



16,777,216 (G=1,024 x L=16,384) elements



1,073,741,824 elements

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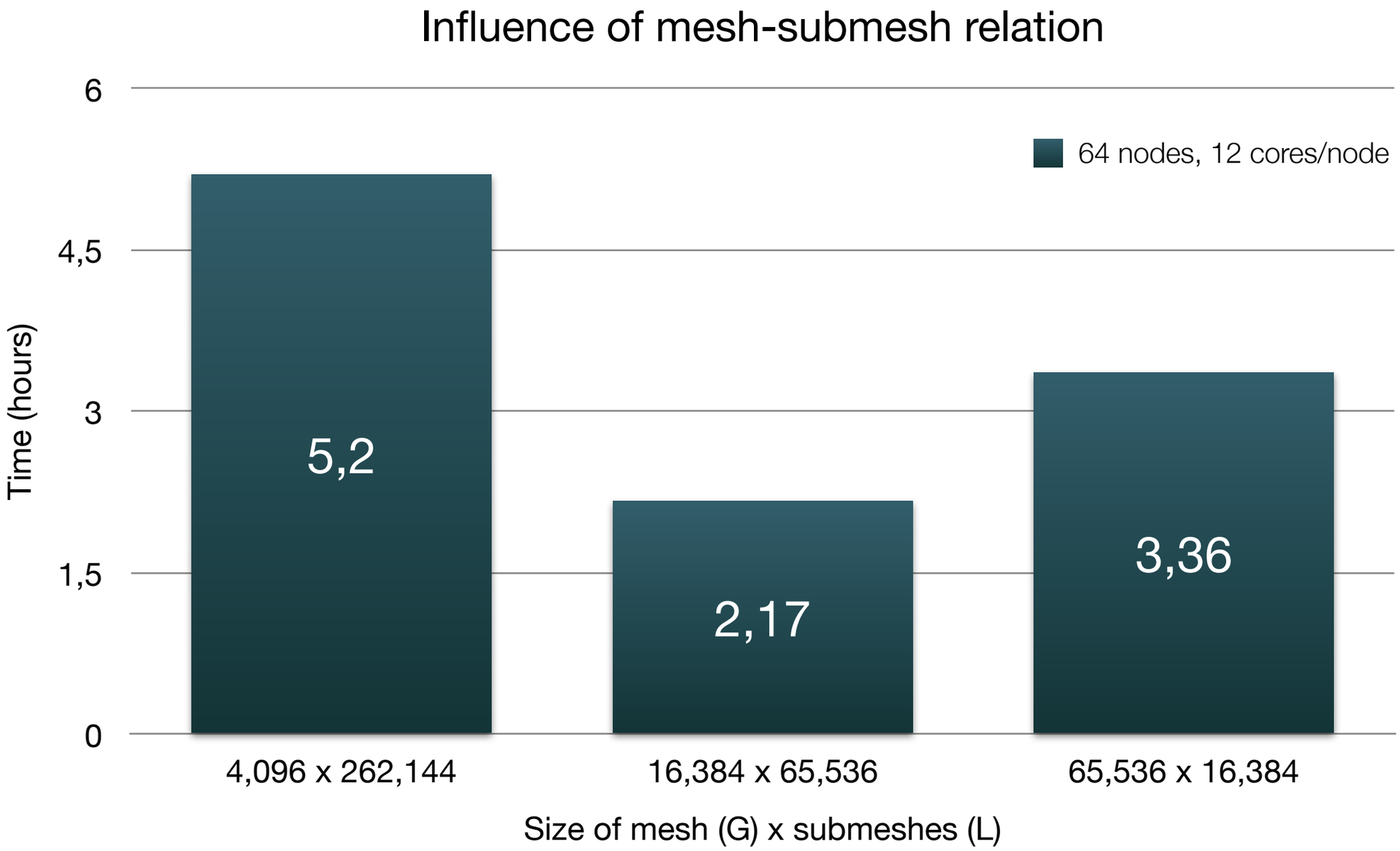

1,073,741,824 elements

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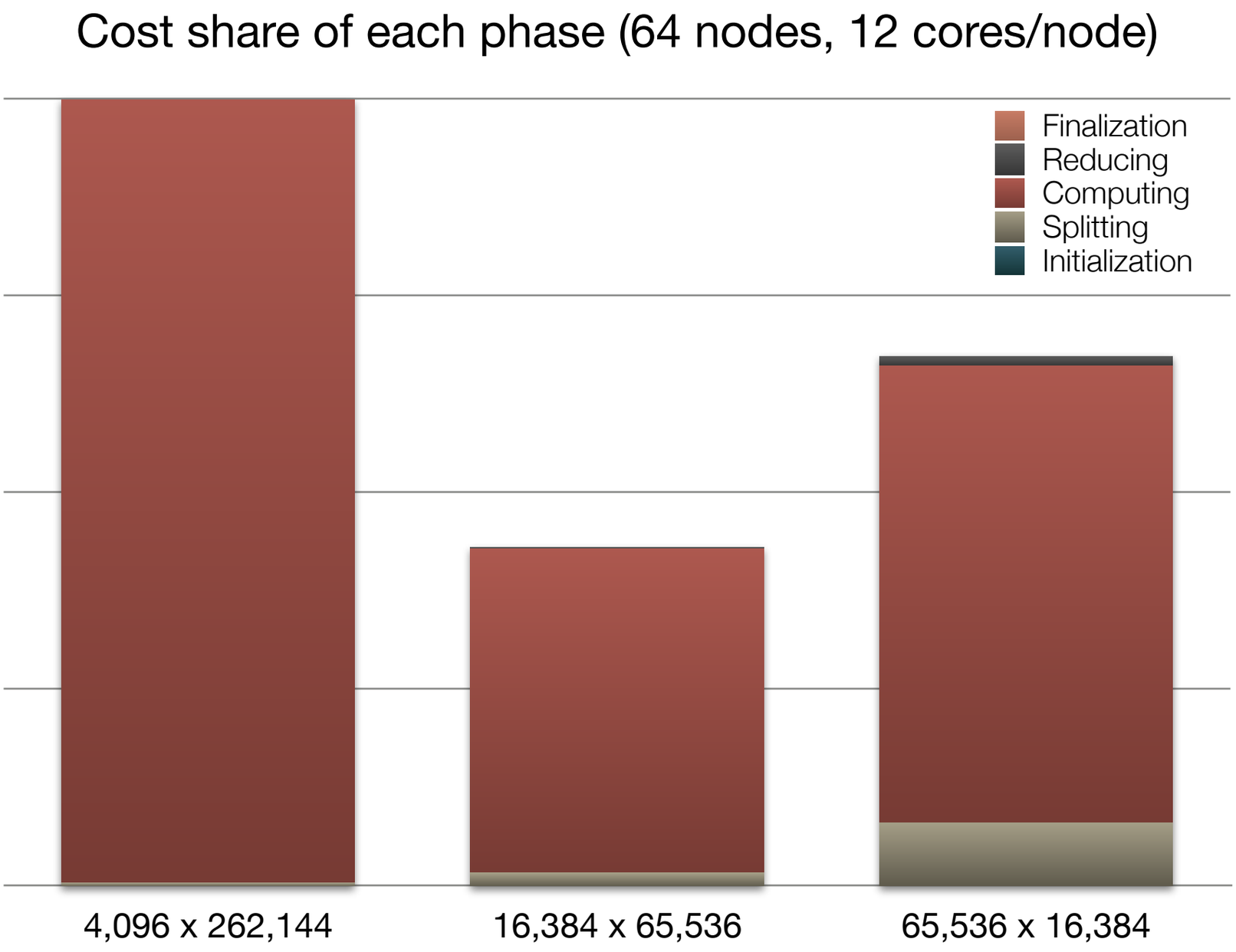

1,073,741,824 elements

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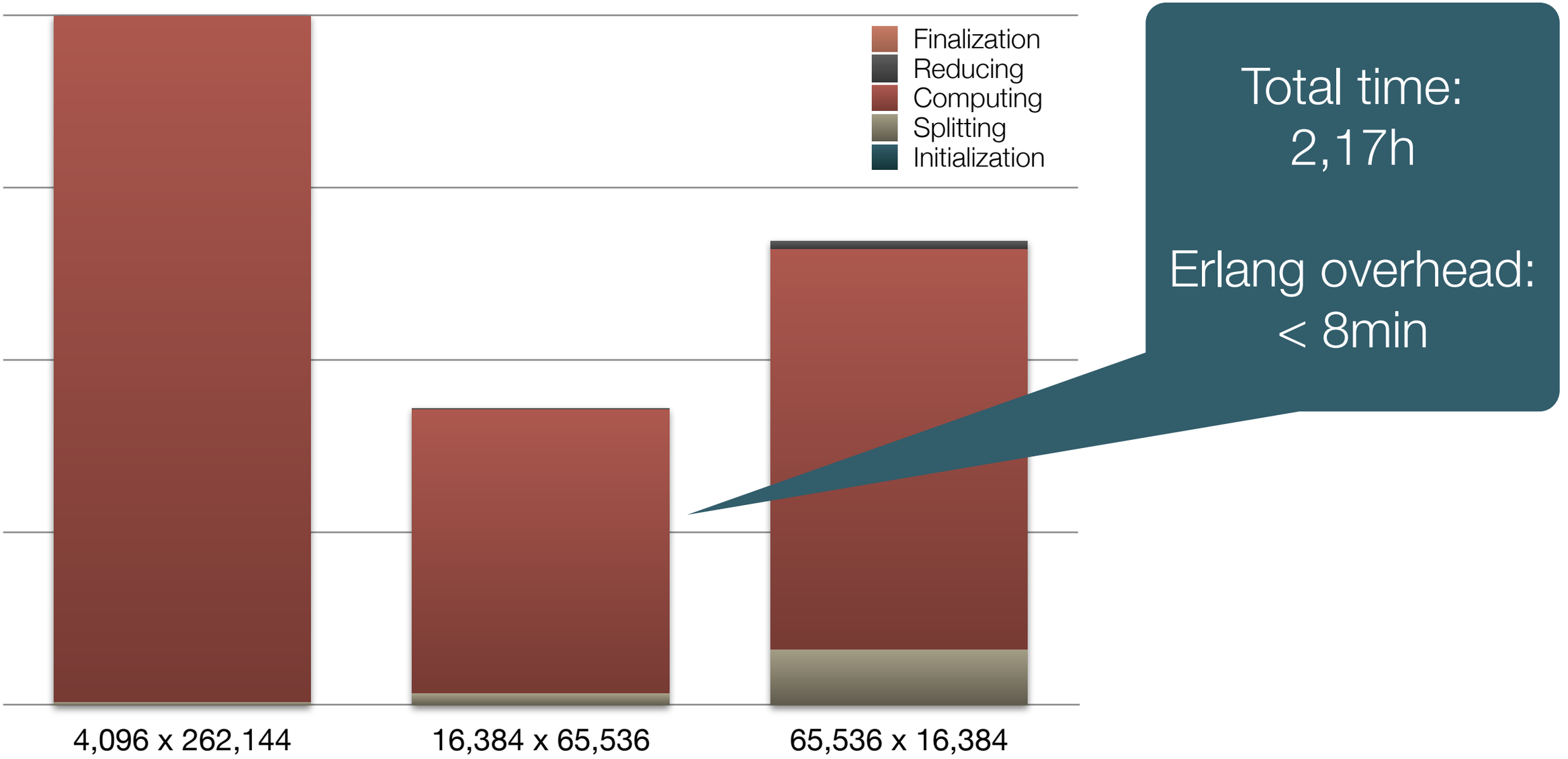

1,073,741,824 elements

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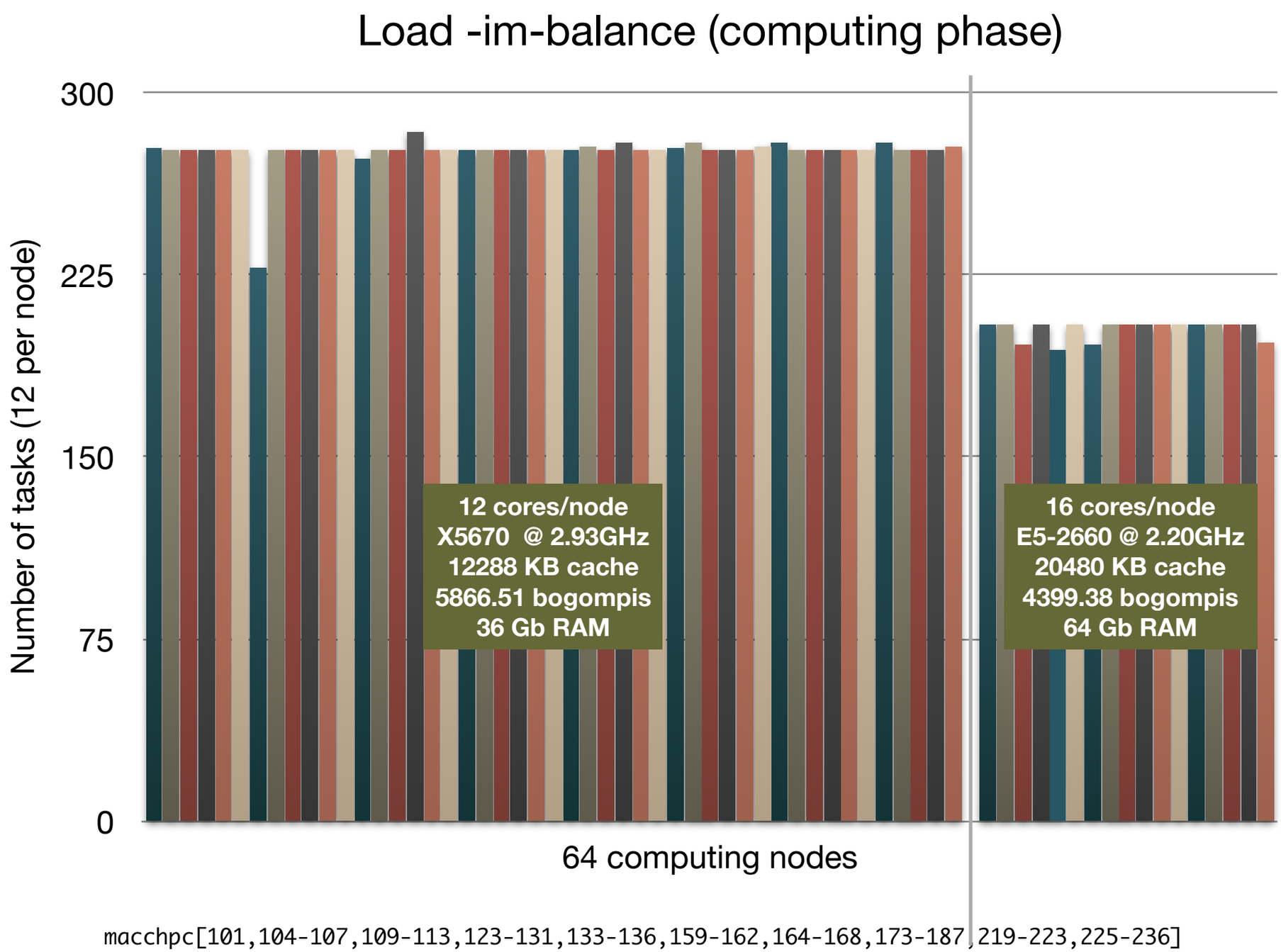

1,073,741,824 elements

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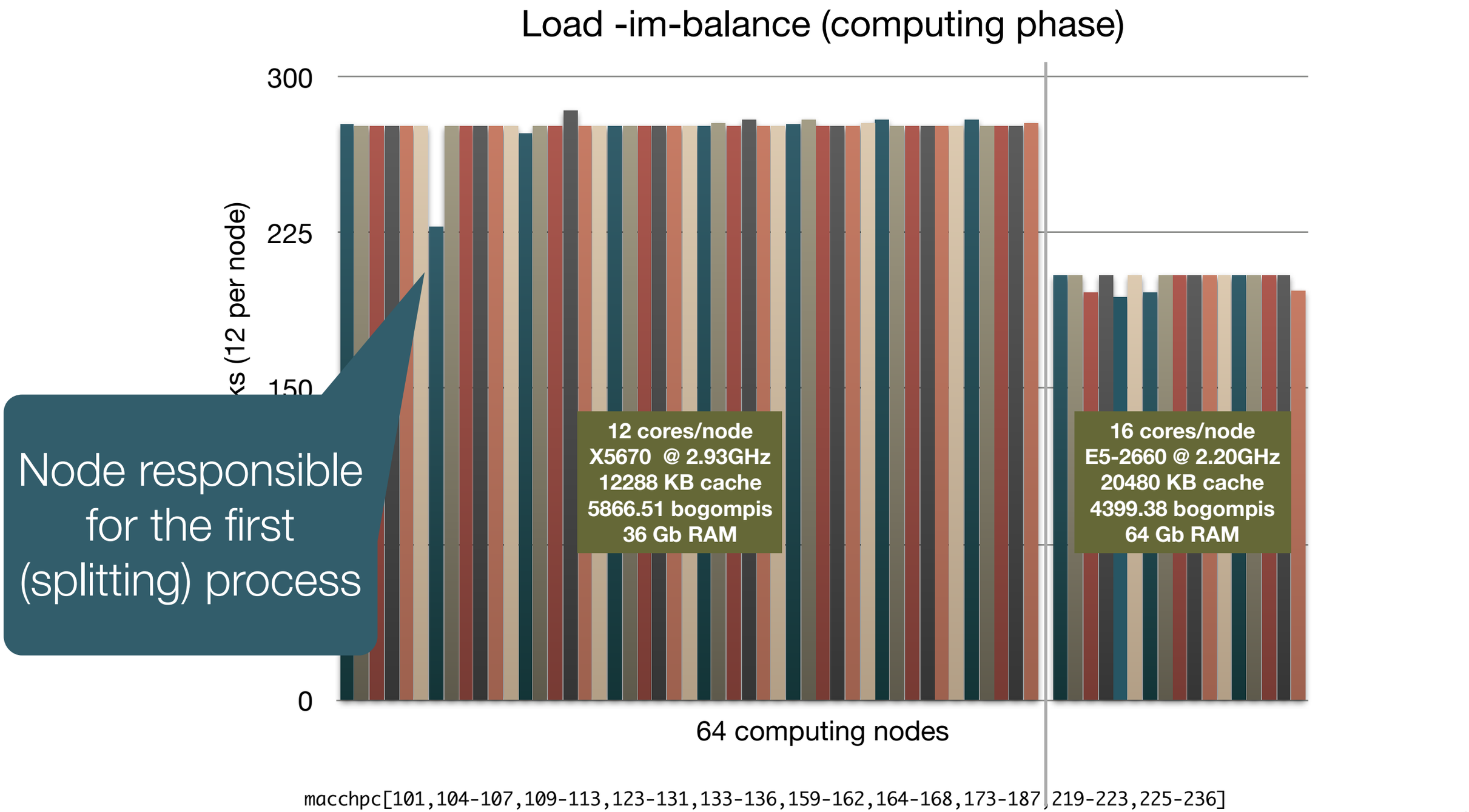

1,073,741,824 elements

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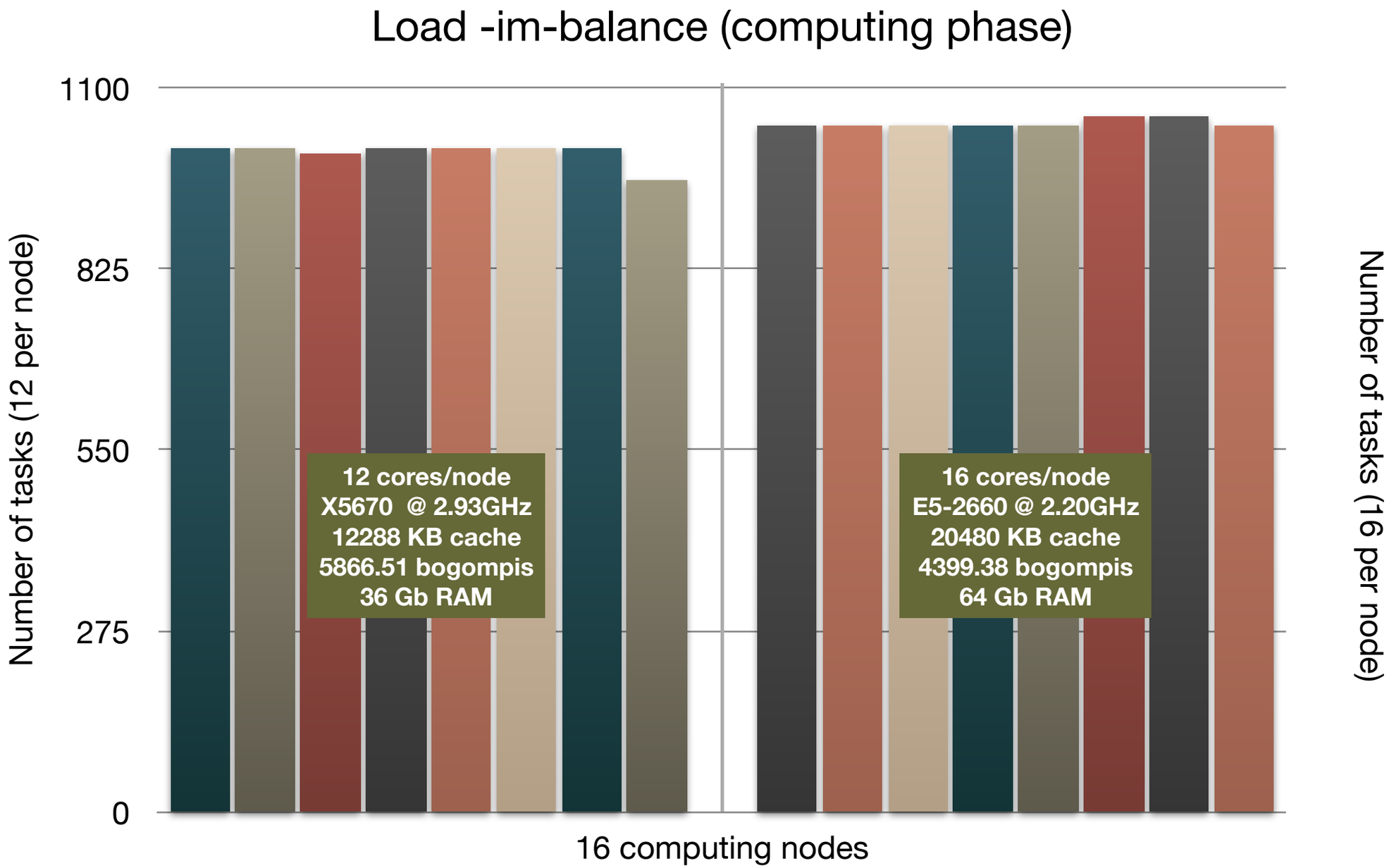

Fault tolerance: let it crash!

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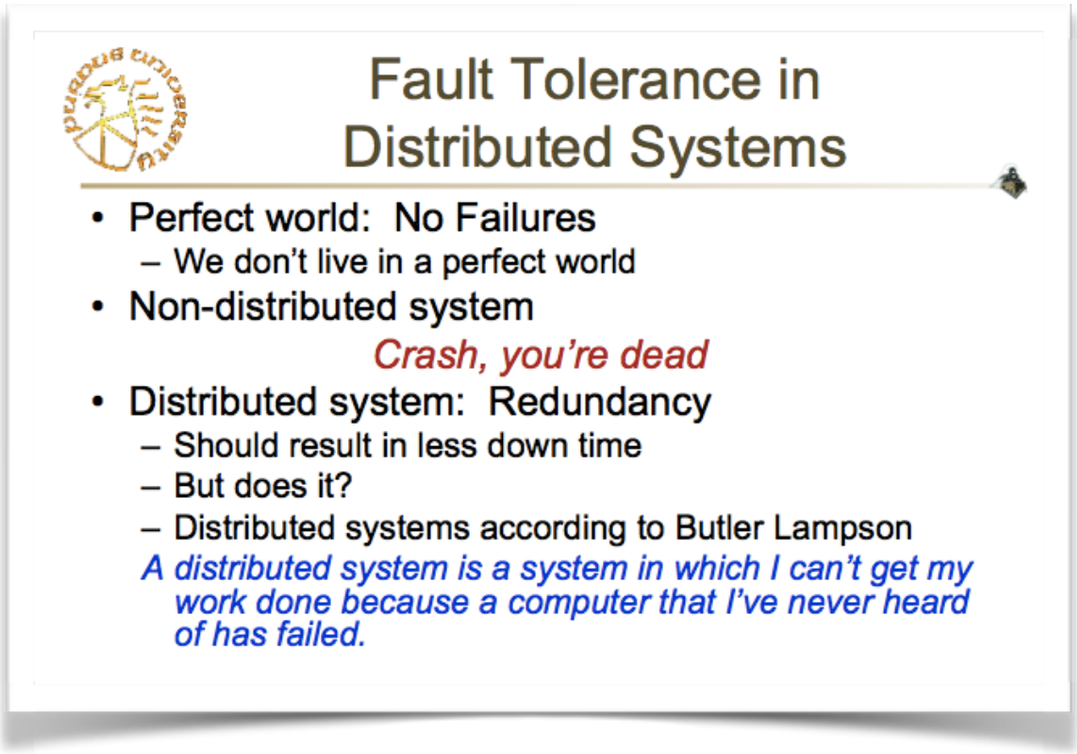
- At the worker nodes:
  - Network partitions and node failures are considered permanent
  - Related “being solved”/ “being reduced" task is rolled back to the “not solved list”/ “solved list"
- At the master node (main_server):
  - Network partitions and node failures are considered transient
  - All tasks in “being solved list”/ “being reduced" are rolled back to the “not solved list”/ “solved list"
  - Backup master node does the failover



Source: https://www.cs.purdue.edu/homes/clifton/cs603/FailureModel.ppt

To-do… (Erlang part)

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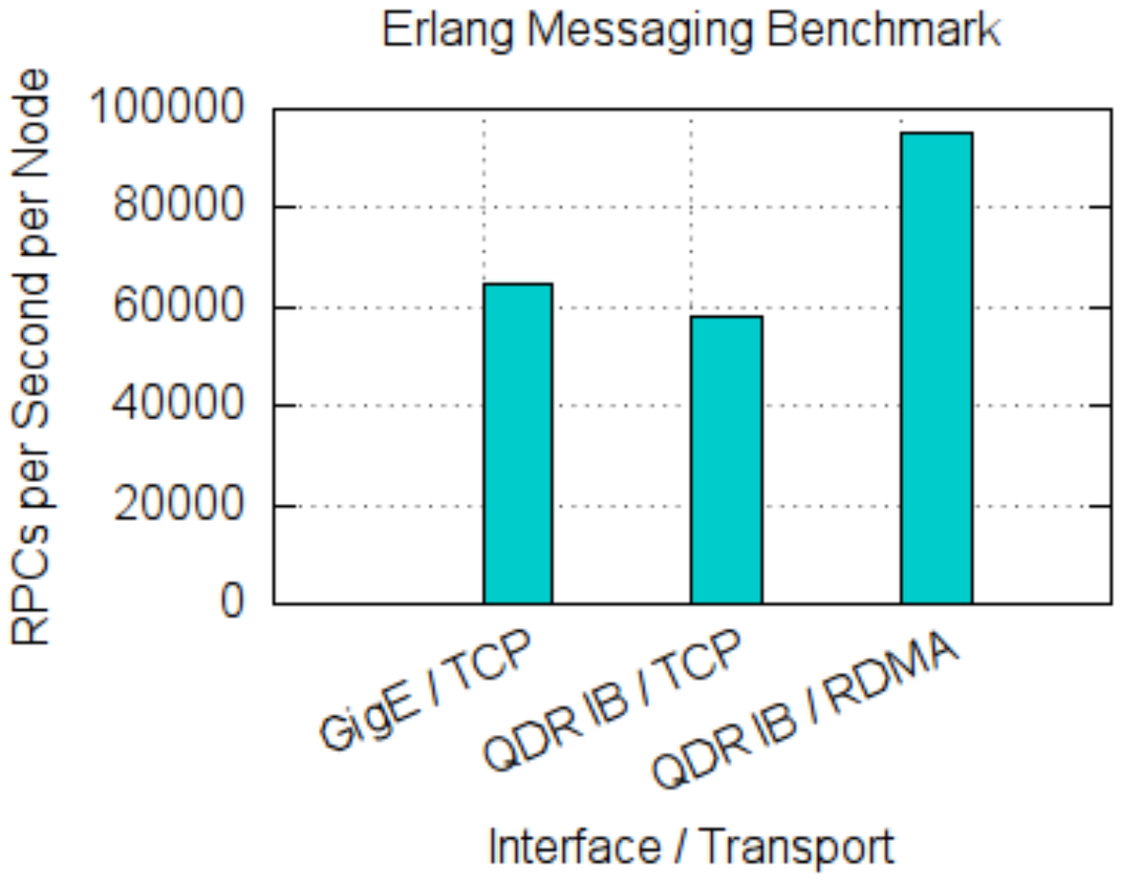
- Employ native RDMA protocol over Infiniband (https://github.com/MrStaticVoid/rdma_dist)

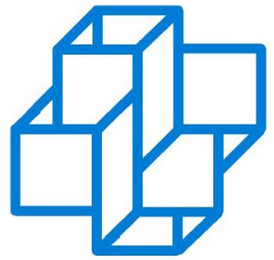
- Adopted message formats are too memory greedy (and unnecessarily wordy)

    - Related with slide 36…

- Do better cleanup of worker nodes in case of network partitioning



To-do… (C++ part)



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- Unexpected results for l>0 (degree of lagrange multiplier polynomials)

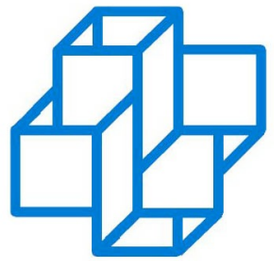
- Unexpected results for k > 5 (degree of polynomials at the subelement level)

- Basic optimizations at the processing phase

- Hotspots at the splitting (I/O intensive) and reducing (computing/memory intensive) phases

- Develop auto-mesher for 3D problems

To-do… (Erlang*C++ part)



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- Give support to transient problems
- Make overall configuration more flexible (a lá SPiNMe)
- Provide better (if any!) visualization output
- Consider other execution environments (e.g. clouds)

Thank you!!!
Merci!!!

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