Multi-level preconditioning strategies for the stabilized finite element formulation of incompressible viscous flow coupled to advective-diffusive transport

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Contents

Why is it important?

1. Problems involving incompressible viscous flow coupled with advective-diffusive transport are used to model a wide range of phenomena of great interest in science and engineering. With the introduction of stabilization techniques, finite elements become an important tool in Computational Fluid Dynamics (CFD).
2. The High Performance Computer Center (NACAD, in Portuguese) has been developing the EdgeCFD software (Ms Renato Elias) with several techniques, algorithms and methods have been explored/incorporated in search of efficient solution comprising the most diverse architectures of high performance processing, and the search for more and more realistic and detailed solutions requires three-dimensional and transient coupled solutions and fine grids, imposing a considerable increase of computational costs and numerical difficulties.

Motivation

1. Thus, the purpose of this research is to accelerate EdgeCFD by using a multi-level preconditioner able to accelerate the iterative driver present in EdgerCFD. Moreover, the preconditioner must enable the efficient use of the edge-based data structure, minimizing the computational effort and providing EdgeCFD global process with more speed and accuracy.

The EdgeCFD software

- Brief description of EdgeCFD: Software for the solution of large-scale problems involving incompressible flow coupled or not to scalar transport (temperature, concentration, ...;
- It uses the stabilized finite element method ($SUPG/PSPG$, $RBVMS$) as a method of discretization;

The EdgeCFD software

- It uses: the Inexact-Newton method, $P - GMRES$ as linear solver, adaptive timestep control by means of PID controller, dynamic desactivation for transport problems, and other advanced numerical techniques.
- Hybrid parallelism (message passing;threading).
- Implementation in Fortran90 gives flexibility in using the software in diverse platforms.

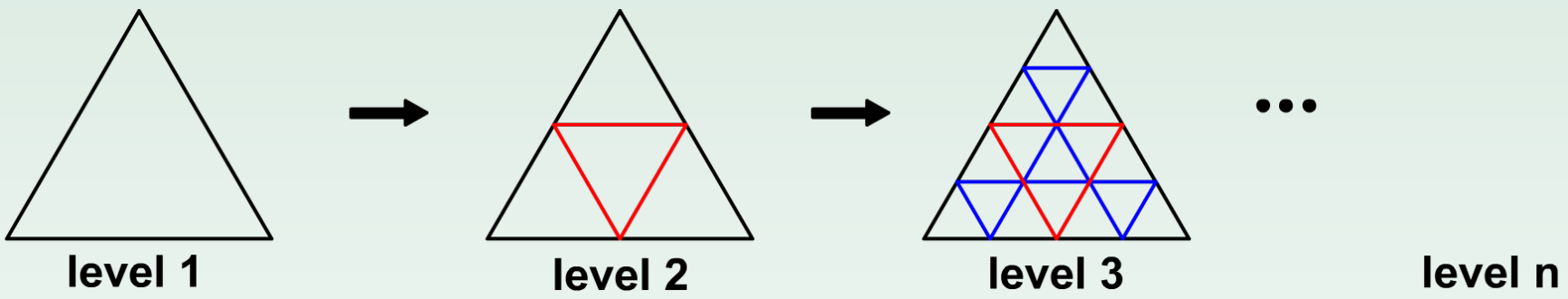
The problem - What we have

- The development of algorithms and methods that efficiently use high performance computing resources is essential to allow for solutions with more and more realistic (refined) detail levels;
- Several techniques, algorithms and methods have been explored/incorporated in search of an efficient solution comprising the most diverse architectures of high performance computing. But this is not enough ...

The problem

- So we decided to invest in the construction of a multi-level preconditioner, able to exploit more efficiently: (1) mesh multiplication and (2) edge-based data structure present in EdgeCFD. This is our problem.

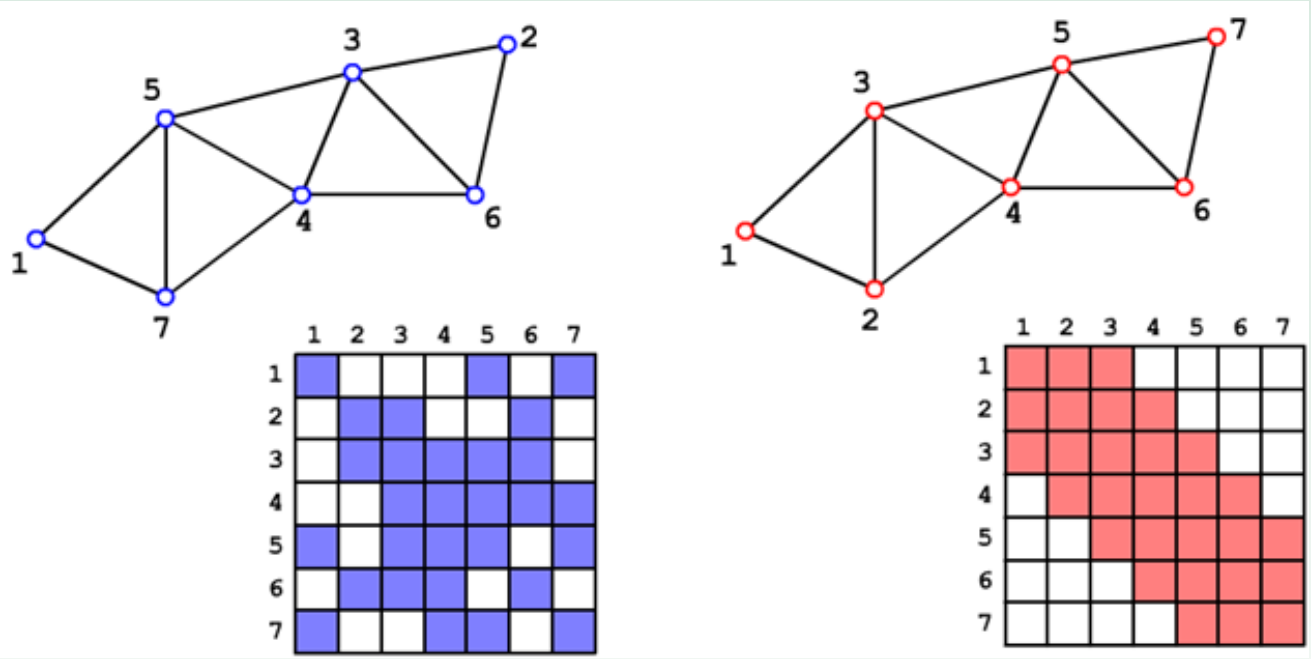
Mesh multiplication



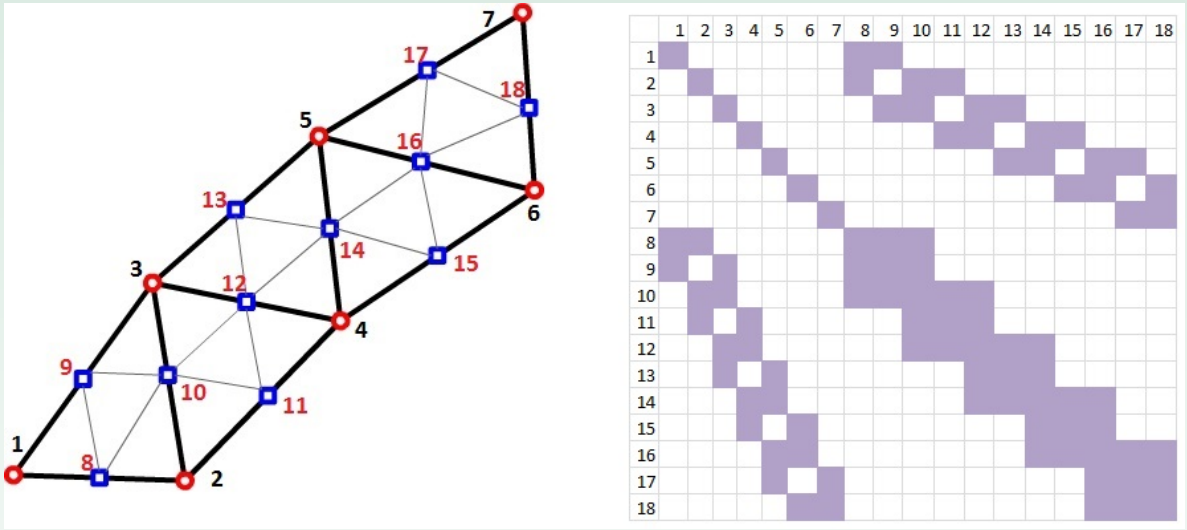
Mesh multiplication

In the edge-based structure, each mesh corresponds to a symmetric sparse matrix built as follows: each line stores information about one vertex, line 1 - vertex 1, line 2 - vertex 2 and so on. Each line i has the indexes j of the vertices that are connected to vertex i, the other elements of this line equal zero. As the index is produced through the enumeration of the mesh, each enumeration corresponds to a matrix and, in particular, we are interested in the matrix structure, which thus depends directly on mesh ordering.

Reordering and improving the matrix structure



Mesh multiplication and matrix structure



Preconditioning strategies (possibilities)

- First step: A comprehensive literature review has been carried out (see main items at the end) about multilevel preconditioners and it seemed appropriate that we start our research with the following two alternatives
- Repartitioning and Data Redistribution [Lin, 2011]:
  - Similarities
    - [Lin, 2011] proposed a geometrical multilevel preconditioner for an elliptical problem which seems to be a good idea, since we work with the same kind of problem.
    - His work has shown the importance of the multigrid preconditioner in determining a reduction in the global process.
  - Differences
    - He did not deal with mesh multiplication,
    - He did not work with edge-based data structures.
    - His work is restricted to two-dimensional problems.

Conclusion

- **EdgeCFD software:** Well, in this presentation I attempted to introduce the EdgeCFD software and give a brief description of what it does in solving a great variety of practical engineering problems.
- **Software Limitations:** I also talked about some limitations it has been facing when dealing with problems that require more and more accurate solutions.
- **multigrid preconditioner:** I stated my intention of working, in my research, on a multigrid preconditioner to assist EdgeCFD in the solution of some of these limitations.
- **Possible solution** Lastly, I presented a first step, a possibility, of dealing with the creation of this multigrid preconditioner in my future research.

Bibliografia

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Thank you

Obrigado

Merci