Coupled Hydro-Geomechanical Computational Models of Pre-Salt Reservoris

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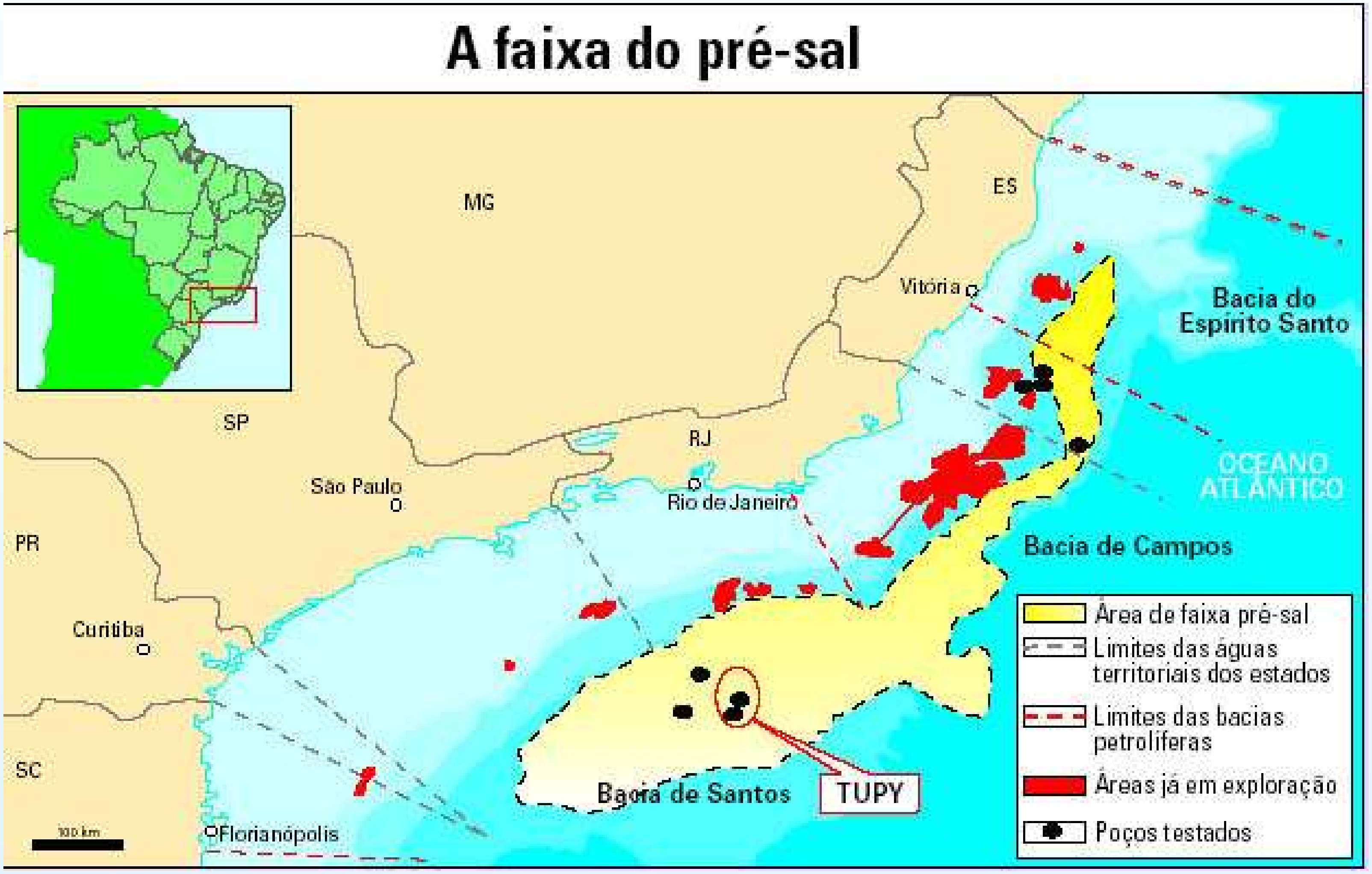
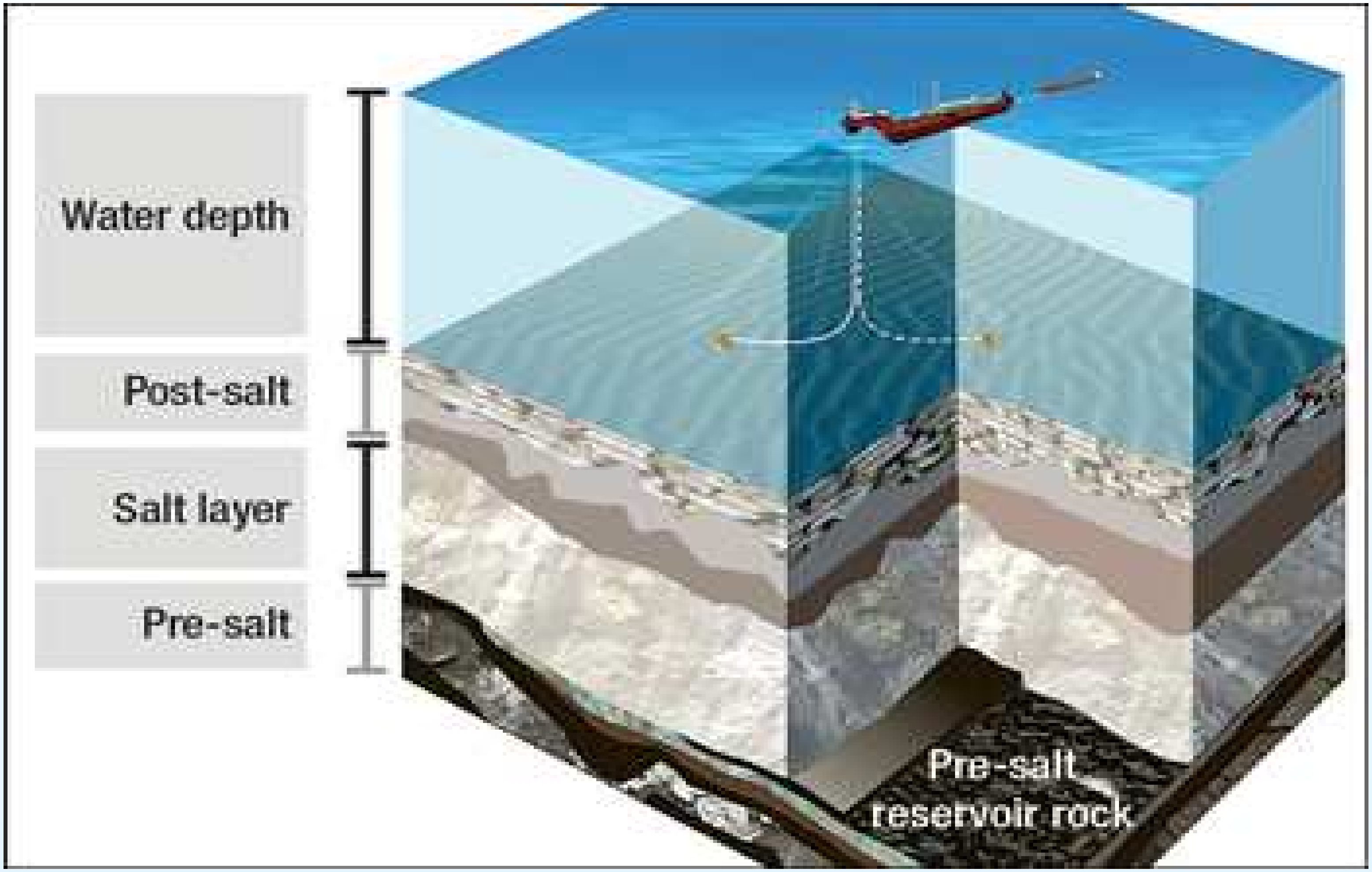
PRE-SALT FORMATIONS



PRE-SALT FORMATIONS



Pre-Salt Microbiolite Carbonate

COMPLEX PHYSiCS

- Multiphase Multicomponent Flow in Deformable Porous Media
- Pay Zone – Carbonate Reservoir (Poroelastic-Poroplastic)
- Non-Pay -Cap - Saline Formation (Poroviscoelastic Creep)
- Heterogeneity, Double Structure – Fractures – Vugs
- Geochemistry – Dissolution/Precipitation Reactions with the Minerals (Calcite)
- Water Weakening - Chemical Degradation

$$CaCO_3 + H^+ \rightarrow C_a^{2+} + HCO_3^-$$

Important Geomechanical Issues

- – Integrity of the Cap Rock
- – Fault Reactivation

Two-Phase Flow in Poroelastic Media

Avoid Compressibility $\beta := \frac{1}{\phi}\frac{d\phi}{dP}, \qquad \frac{\partial \ln \phi}{\partial t} = \beta \frac{\partial P}{\partial t}$

$$\beta\phi\frac{\partial P}{\partial t} - \boldsymbol{\nabla}.(K\boldsymbol{\nabla}P) = F$$

Mass Balance of the Solid (Incompressible at the Pore-Scale)

$$\frac{\partial \phi_s}{\partial t} + \boldsymbol{\nabla} \cdot (\phi_s \mathbf{v}_s) = F_{DISS}$$

$$\phi_s = 1 - \phi, \qquad \mathbf{v}_s = \frac{\partial \mathbf{u}}{\partial t}$$

Mass

$$\frac{\partial (1-\phi)}{\partial t} + \boldsymbol{\nabla} \cdot \left((1-\phi)\frac{\partial \mathbf{u}}{\partial t} \right) = F_{DISS}$$

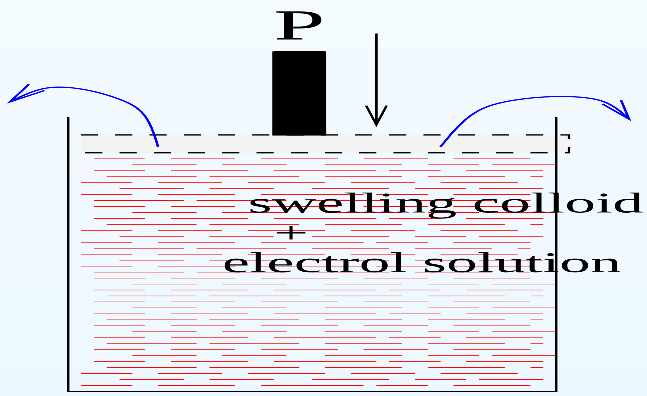
Geomechanics Geochemistry

Equilibrium

Total Stress: $\boldsymbol{\sigma}_T$

$$\boldsymbol{\nabla} \cdot \boldsymbol{\sigma}_T = 0$$

Terzaghi's Principle



$$\boldsymbol{\sigma}_T = \boldsymbol{\sigma} - P\boldsymbol{I}$$

$\boldsymbol{\sigma}$ – Integranular Stress

Constitutive Law

Linear Elastic

$$\boldsymbol{\sigma} = \boldsymbol{C}\boldsymbol{\mathcal{E}}(\mathbf{u})$$

$$\boldsymbol{\mathcal{E}}(\mathbf{u}) = \frac{1}{2}\left(\boldsymbol{\nabla}\mathbf{u} + (\boldsymbol{\nabla}\mathbf{u})^T\right)$$

2 Equations in $\{P, \mathbf{u}\}$

$$\boldsymbol{\nabla} \boldsymbol{.} \boldsymbol{\sigma}_T = 0$$

$$\boldsymbol{\sigma}_T = \boldsymbol{\sigma} - P\boldsymbol{I}$$

$$\boldsymbol{\nabla} \boldsymbol{.} \boldsymbol{\sigma}_T = \boldsymbol{\nabla} \boldsymbol{.} \boldsymbol{\sigma} - \boldsymbol{\nabla} P = \boldsymbol{\nabla} \boldsymbol{.} (\boldsymbol{C}\boldsymbol{\mathcal{E}}(\mathbf{u})) - \boldsymbol{\nabla} P = 0$$

Equilibrium

$$\boldsymbol{\nabla} \boldsymbol{.} (\boldsymbol{C}\boldsymbol{\mathcal{E}}(\mathbf{u})) - \boldsymbol{\nabla} P = 0$$

Two-Phase Flow Heterogeneous in Porelastic Media

Total $\boldsymbol{V}_{Dt} = \boldsymbol{V}_w + \boldsymbol{V}_o$, Assumption: $P_o = P_w = P$

$$\boldsymbol{\nabla}\,.\,(\boldsymbol{C}(\boldsymbol{x},\phi))\boldsymbol{\mathcal{E}}(\mathbf{u})) - \boldsymbol{\nabla} P = 0 \quad \text{in} \quad \Omega$$

$$\boldsymbol{\nabla}\,.\,\frac{\partial \mathbf{u}}{\partial t} + \boldsymbol{\nabla}\,.\,\boldsymbol{V}_{Dt} = 0$$

$$\boldsymbol{V}_{Dt} = -K(\boldsymbol{x},\phi)\lambda_t(S_w)\boldsymbol{\nabla} P$$

$$- - - -$$

$$\frac{\partial \phi}{\partial t} = \boldsymbol{\nabla}\,.\left((1-\phi)\frac{\partial \mathbf{u}}{\partial t}\right) + F_{DISS}$$

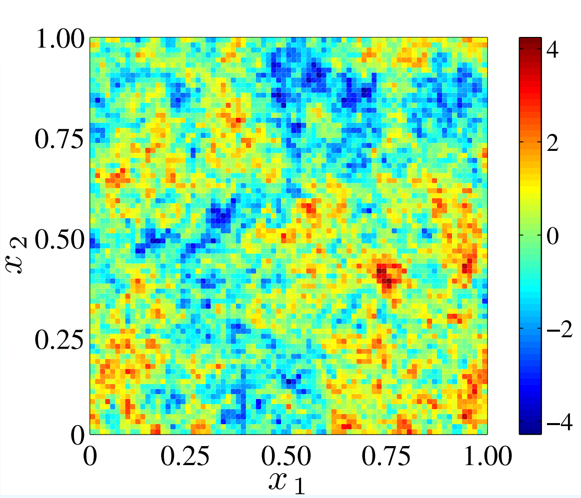
$$\frac{\partial (S_w \phi)}{\partial t} + \boldsymbol{\nabla}\,.\left(S_w \phi \frac{\partial \mathbf{u}}{\partial t}\right) + \boldsymbol{\nabla}\,.\,(\lambda_w(S_w)\boldsymbol{V}_{Dt}) = 0$$

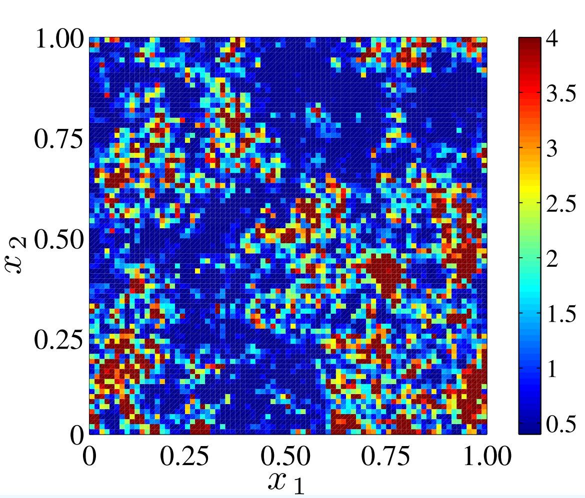
mobilities InItial Porosity $\phi_0(\boldsymbol{x})$ Cross-Correlation

$$\lambda_t(S_w) = \frac{K_{rw}(S_w)}{\mu_w} + \frac{K_{ro}(1-S_w)}{\mu_o}, \qquad \lambda_w(S_w) = \frac{K_{rw}(1-S_w)}{\mu_w \lambda_t}$$

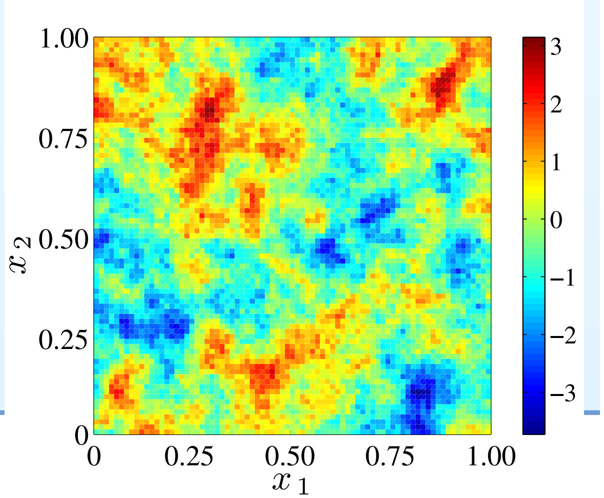
Realizations; $C(\boldsymbol{x}), K(\boldsymbol{x}), \phi_0(\boldsymbol{x})$ Challenge

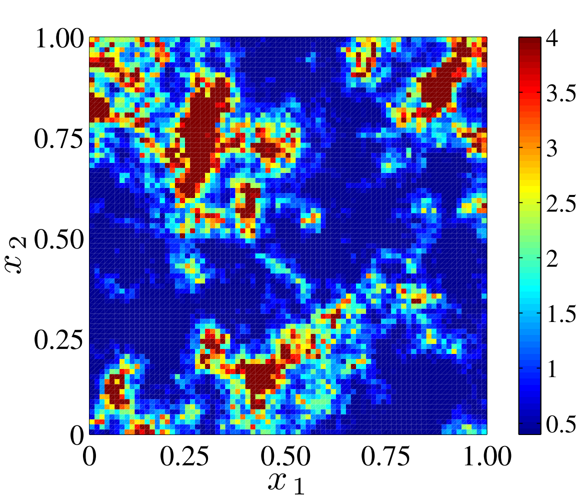
Locally Conservative Numerical Schemes $\rightarrow$ Stat. Moments





(a) Gaussian field and (b) log-normal field.
Power Law $\beta = 0.5$ (up), Exponential:$\lambda = 0.1.\sigma_Y = 1$ (down)





Fully Coupled Formulation:

$$\boldsymbol{\nabla}\,.\,(\boldsymbol{C}(\phi)\boldsymbol{\mathcal{E}}(\mathbf{u})) - \boldsymbol{\nabla} P = 0 \quad \text{in} \quad \Omega$$

$$\boldsymbol{\nabla}\,.\frac{\partial \mathbf{u}}{\partial t} + \boldsymbol{\nabla}\,.\,\boldsymbol{V}_{Dt} = 0$$

$$\boldsymbol{V}_{Dt} = -K(\phi)\lambda_t(S_w)\boldsymbol{\nabla} P$$

$$- - - -$$

$$\frac{\partial \phi}{\partial t} = \boldsymbol{\nabla}\,.\left((1-\phi)\frac{\partial \mathbf{u}}{\partial t}\right) + F_{DISS}$$

$$\frac{\partial (S_w \phi)}{\partial t} + \boldsymbol{\nabla}\,.\left(S_w \phi \frac{\partial \mathbf{u}}{\partial t}\right) + \boldsymbol{\nabla}\,.\,(\lambda_w(S_w)\boldsymbol{V}_{Dt}) = 0$$

Drawbacks

- Inclusion of Adjacent non-pay zones in the analysis
- Larger meshes for Geomechanics
- Impose the same time scale for flow and mechanics
- Computationallly Expensive

Iterative Methods: Drained Split $\beta = 3/(3\lambda + 2\mu)$

$$\beta(\boldsymbol{x})\frac{\partial P^n}{\partial t} + \nabla . \boldsymbol{V}_D^n = F^n(\boldsymbol{x},t) := -\frac{\partial \mathrm{tr}\boldsymbol{\sigma}_T^n}{\partial t}$$

$$\boldsymbol{V}_D^n = -K(x)\lambda_t(S^{n-1})\nabla P^n$$

Include the Cap Rock: $\boldsymbol{\sigma}_T = -P\boldsymbol{I} + \boldsymbol{\sigma}_e$

$$\nabla . \boldsymbol{\sigma}_e^n = -\nabla P^n$$

$$\boldsymbol{\sigma}_e^n = \lambda(\boldsymbol{x})\nabla.\mathbf{u}^n\boldsymbol{I} + 2\mu(\boldsymbol{x})\boldsymbol{\mathcal{E}}(\mathbf{u}^n) \quad \text{in} \quad \Omega$$

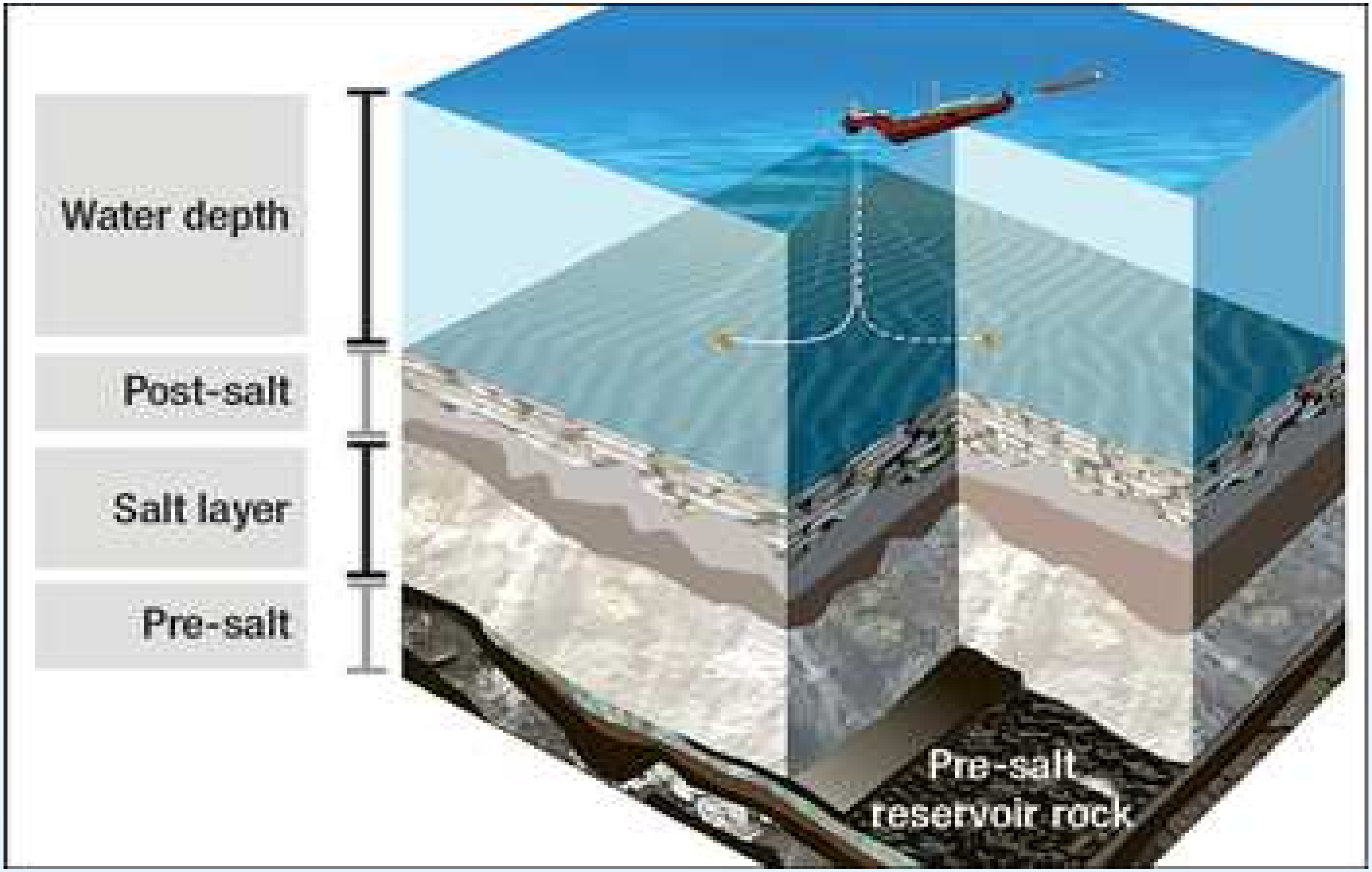
$$\boldsymbol{\sigma}_e^n\boldsymbol{n} = \boldsymbol{h} \quad \text{on} \quad \Gamma_1, \qquad \mathbf{u} = 0 \qquad \text{on} \quad \Gamma_2$$

Creep – Norton Power Law: Von Mises Stress $\boldsymbol{\sigma}_V$

$$\boldsymbol{\sigma}_e = \boldsymbol{C}(\boldsymbol{\mathcal{E}}(\mathbf{u}) - \boldsymbol{\mathcal{E}}_{visc})$$

$$\frac{\partial \boldsymbol{\mathcal{E}}_{visc}}{\partial t} = \boldsymbol{\mathcal{E}}_0(\left(\frac{\boldsymbol{\sigma}_V}{\boldsymbol{\sigma}_0}\right)^n \exp\left(\frac{Q}{R}\left(\frac{1}{T_o} - \frac{1}{T}\right)\right)$$

Discretization of the Geomechanics Subsystem



Pre-Salt Microbiolite Carbonate

Explicit Coupled Methods: One Way Coupling

$$\beta(\boldsymbol{x})\frac{\partial P^n}{\partial t} + \boldsymbol{\nabla}.\boldsymbol{V}_D^n = F^{n-1}(\boldsymbol{x},t) := -\frac{\partial \mathrm{tr}\boldsymbol{\sigma}_T^{n-1}}{\partial t}$$
$$\boldsymbol{V}_D^n = -K(x)\lambda_t(S^{n-1})\boldsymbol{\nabla} P^n$$

Include the Cap Rock: $\mathrm{tr}\boldsymbol{\sigma}_T = -3P + (3\lambda + 2\mu)\boldsymbol{\nabla}.\mathbf{u}$

$$\boldsymbol{\nabla}.\boldsymbol{\sigma}^n = -\boldsymbol{\nabla} P^{n-1}$$
$$\boldsymbol{\sigma}^n = \lambda(\boldsymbol{x})\boldsymbol{\nabla}.\mathbf{u}^n\boldsymbol{I} + 2\mu(\boldsymbol{x})\boldsymbol{\mathcal{E}}(\mathbf{u}^n) \quad \text{in} \quad \Omega$$

One Way Coupling

$$\beta(\boldsymbol{x})\frac{\partial P^n}{\partial t} + \boldsymbol{\nabla}.\boldsymbol{V}_D^n = 0$$
$$\boldsymbol{V}_D^n = -K(x)\lambda_t(S^{n-1})\boldsymbol{\nabla} P^n$$

- – Different Numerical Schemes, Solvers, Grids

Porosity Computations: Mass Balance Solid Phase

$$\frac{d\phi}{dt} = (1-\phi)\boldsymbol{\nabla}\,\boldsymbol{.}\left(\frac{\partial \mathbf{u}}{\partial t}\right) + F_{DISS}$$

Decomposition – Triple Effect

$$\phi^n = \phi_1^n + \phi_2^n$$

Heterogeneity and Geomechanical Coupling

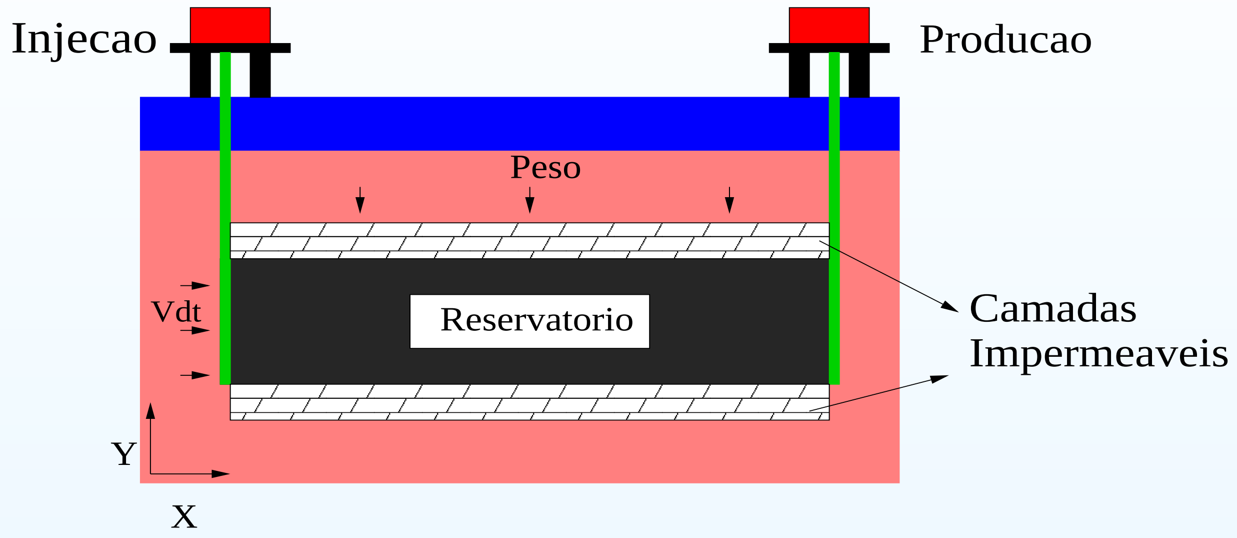
$$\phi_1^n = 1 - (1 - \phi_1^0(\boldsymbol{x}))\exp(-\boldsymbol{\nabla}\,\boldsymbol{.}\,\boldsymbol{U}^n)$$

Porosity Variations – Dissolution-Precipitation Reactions

$$\frac{d\phi_2^n}{dt} = F_{DISS}$$

Constitutive Law: $F_{DISS} =$ Chemical Kinetics

Staggered Algorithm



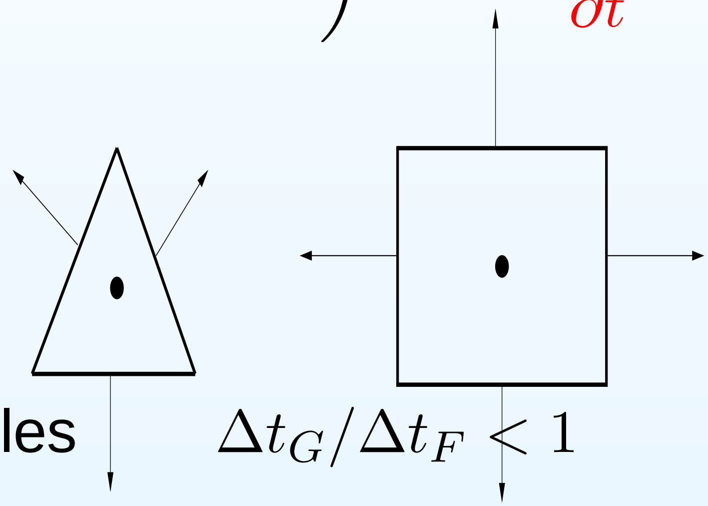
- – HYDRODYNAMICS: $\{P, \boldsymbol{V}_D\}$
- – POROMECHANICS: $P \rightarrow\rightarrow \{\mathbf{u}, \boldsymbol{\sigma}, \boldsymbol{\nabla}.\mathbf{u}\}$: CAPROCK

- - MASS BALANCE SOLID PHASE $\{\boldsymbol{\nabla}.\mathbf{u}, F_{DISS}\} \rightarrow\rightarrow \phi(\boldsymbol{x}, t)$
- - TRANSPORT $\{\boldsymbol{V}_D, \phi\} \rightarrow\rightarrow S_w$

HYDRODYNAMICS: Mixed Formulation

$$(K^{-1}\boldsymbol{V}_D, \mathbf{v}) - (P, \boldsymbol{\nabla}\boldsymbol{.}\mathbf{v}) + (P, \mathbf{v})_\Gamma = 0 \quad \forall \mathbf{v} \in \boldsymbol{U}$$

$$\left(\beta \frac{\partial P}{\partial t}, q\right) + \left(\boldsymbol{\nabla}\boldsymbol{.}\boldsymbol{V}_D, q\right) = -\left(\frac{\partial \mathrm{tr}\boldsymbol{\sigma}_T}{\partial t}, q\right) \quad \forall q \in V$$



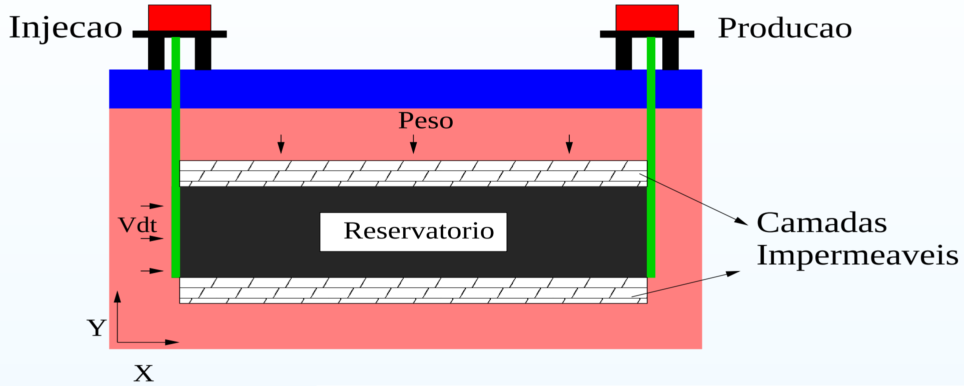
Different Time-Scales $\Delta t_G / \Delta t_F < 1$

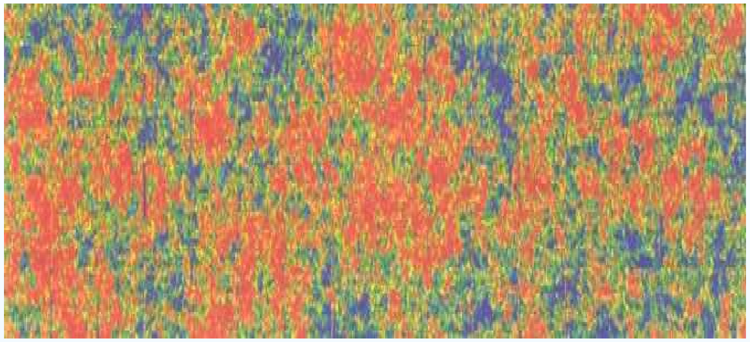
$$\beta P^m = -\Delta t_F \boldsymbol{\nabla}\boldsymbol{.}\boldsymbol{V}_D^m + \beta P^{m-1} - (\Delta t_F/\Delta t_G)(\mathrm{tr}\boldsymbol{\sigma}_T^k - \mathrm{tr}\boldsymbol{\sigma}_T^{k-1})$$

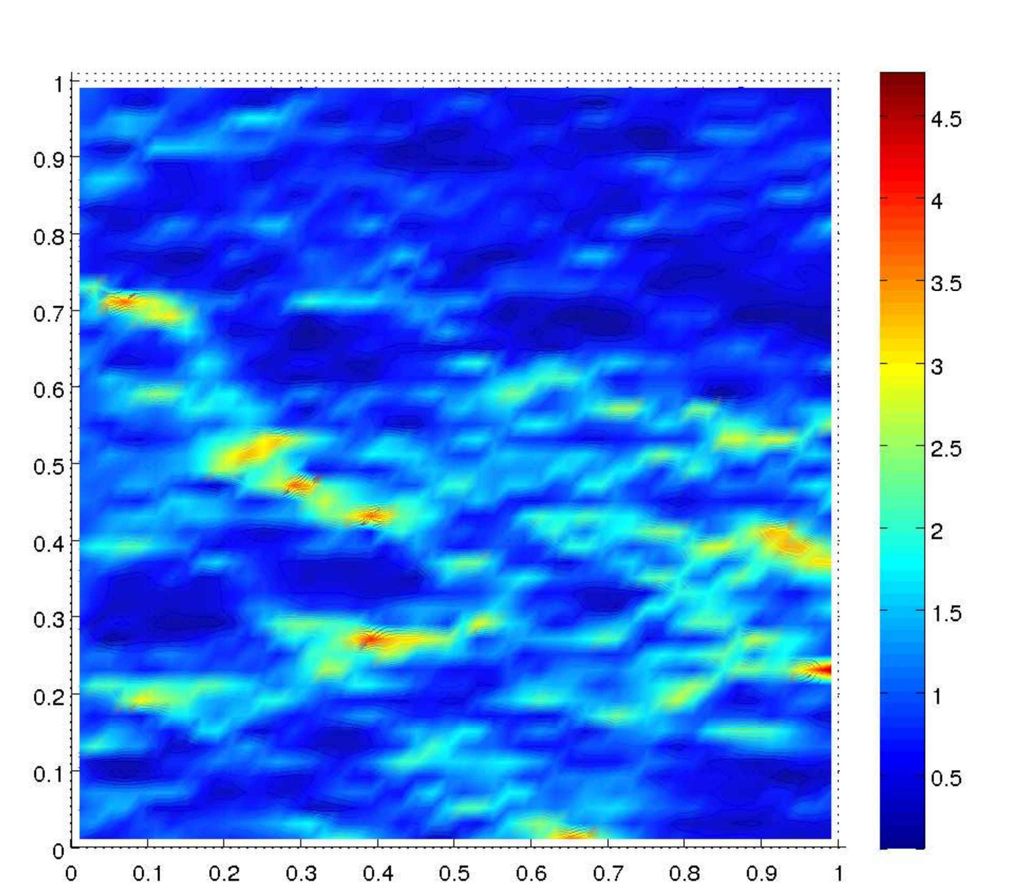
Static Condensation:

$$(K^{-1}\boldsymbol{V}_D^m, \mathbf{v}_h) + \Delta t(\beta^{-1}\boldsymbol{\nabla}\boldsymbol{.}\boldsymbol{V}_D^m, \boldsymbol{\nabla}\boldsymbol{.}\mathbf{v}_h) = (P_h^{m-1} + F_h^k, \boldsymbol{\nabla}\boldsymbol{.}\mathbf{v}_h) - (P_D, \mathbf{v}_h)_\Gamma$$

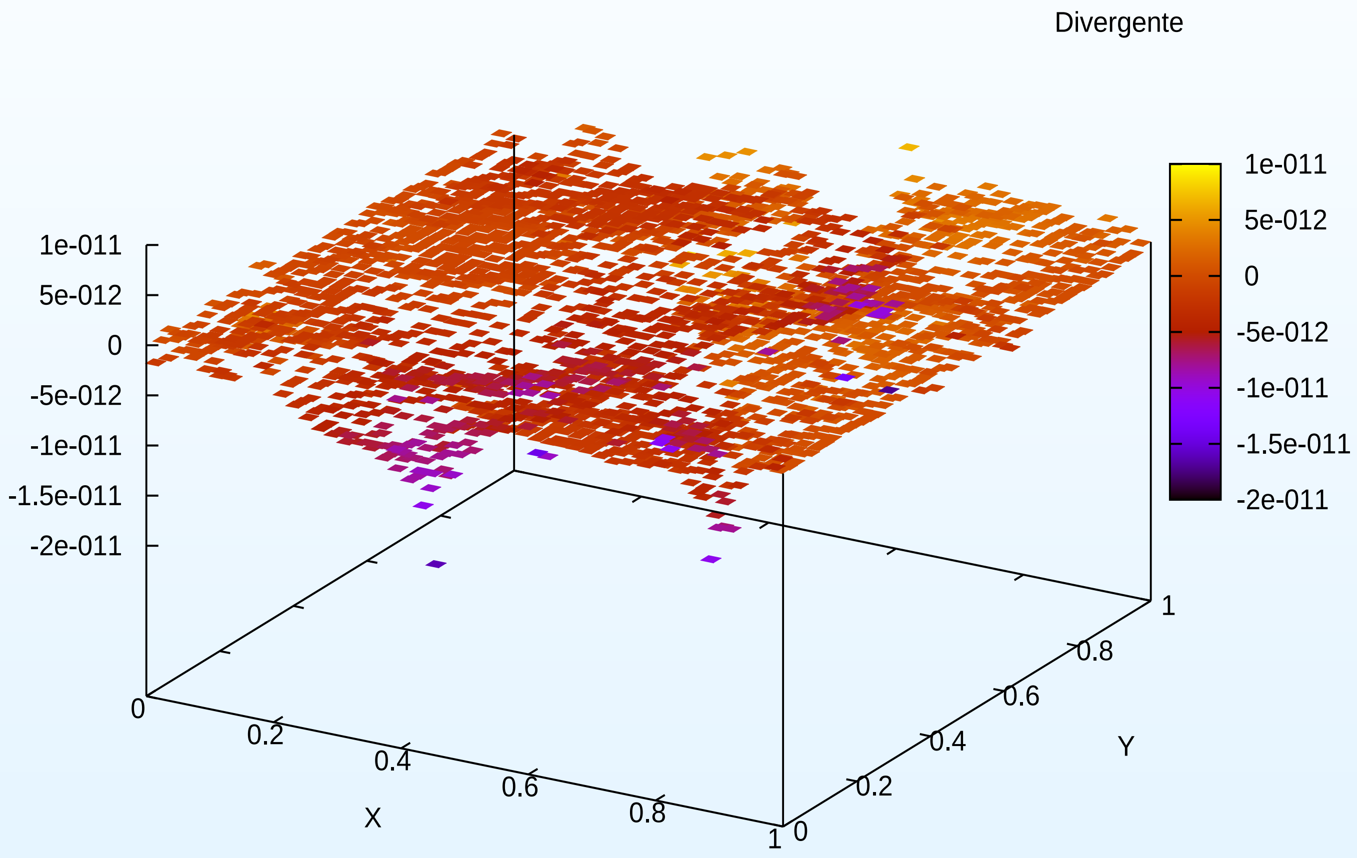
Horizontal Darcy Velocity: Realization K



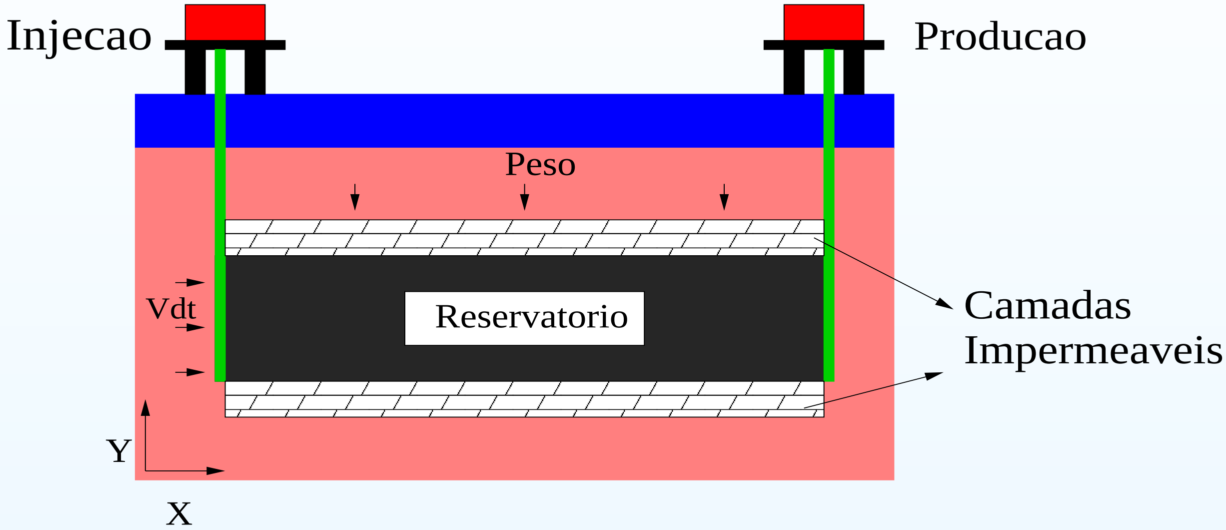




Local Source of Mass



Staggered Algorithm



- – HYDRODYNAMICS: $\{P, \boldsymbol{V}_D\}$
- – POROMECHANICS: $P \rightarrow\rightarrow \{\mathbf{u}, \boldsymbol{\sigma}, \boldsymbol{\nabla}.\mathbf{u}\}$: CAPROCK

- - MASS BALANCE SOLID PHASE $\{\boldsymbol{\nabla}.\mathbf{u}, F_{DISS}\} \rightarrow\rightarrow \phi(\boldsymbol{x}, t)$
- - TRANSPORT $\{\boldsymbol{V}_D, \phi\} \rightarrow\rightarrow S_w$

Poromechanics – Mixed Methods

Effective Stress Decomposition

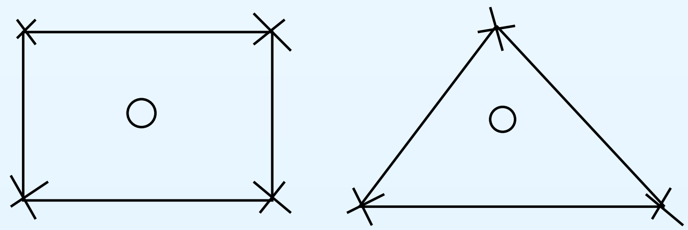
$$\boldsymbol{\sigma} = -P_S \boldsymbol{I} + 2\mu\boldsymbol{\mathcal{E}}(\boldsymbol{U}), \qquad P_S := -\lambda \text{div}\boldsymbol{U}$$

Poroelasticity Find $\{\mathbf{u}, P_S\}$ such that

$$\mu\triangle\boldsymbol{U} - \boldsymbol{\nabla} P_S = F = \boldsymbol{\nabla} P$$

$$P_S = -\lambda\boldsymbol{\nabla}.\boldsymbol{U}$$

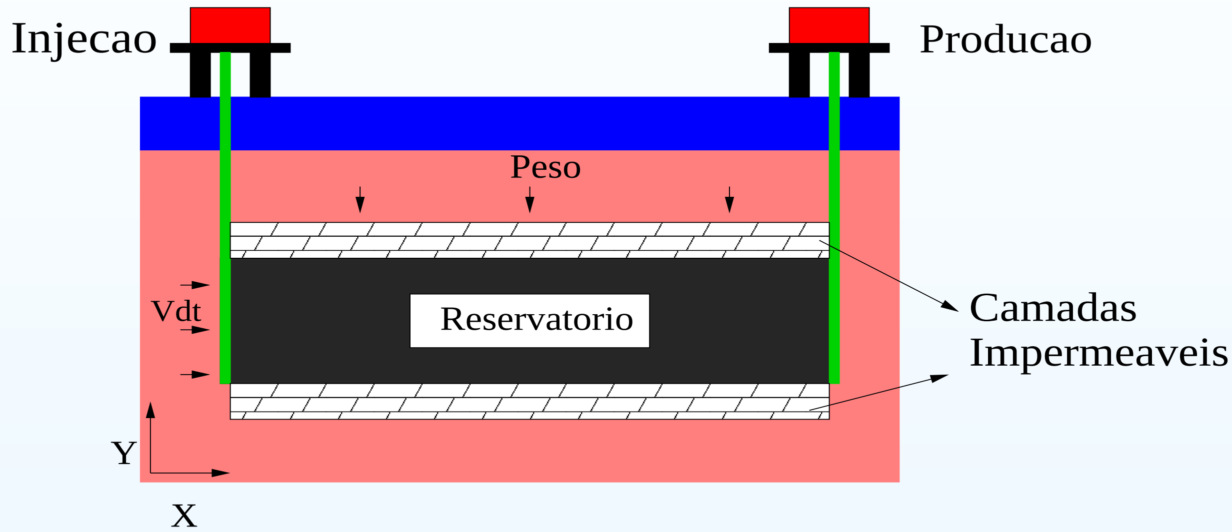
Displacement – Linear: Pressure Constant



Porosity – Geomechanical Contribution

$$\phi_1^n - 1 = -(1 - \phi_1^0(\boldsymbol{x}))\exp(-\boldsymbol{\nabla}\,.\,\boldsymbol{U}^n) = -(1 - \phi_1^0(\boldsymbol{x}))\exp(-P_s/\lambda)$$

Staggered Algorithm



- – HYDRODYNAMICS: $\{P, \boldsymbol{V}_D\}$
- – POROMECHANICS: $P \longrightarrow \{\mathbf{u}, \boldsymbol{\sigma}, \boldsymbol{\nabla}.\mathbf{u}\}$: CAPROCK

- - MASS BALANCE SOLID PHASE $\{\boldsymbol{\nabla}.\mathbf{u}, F_{DISS}\} \longrightarrow \phi(\boldsymbol{x}, t)$
- - TRANSPORT $\{\boldsymbol{V}_D, \phi\} \longrightarrow S_w$

Hyperbolic Problem. Operator Spliting

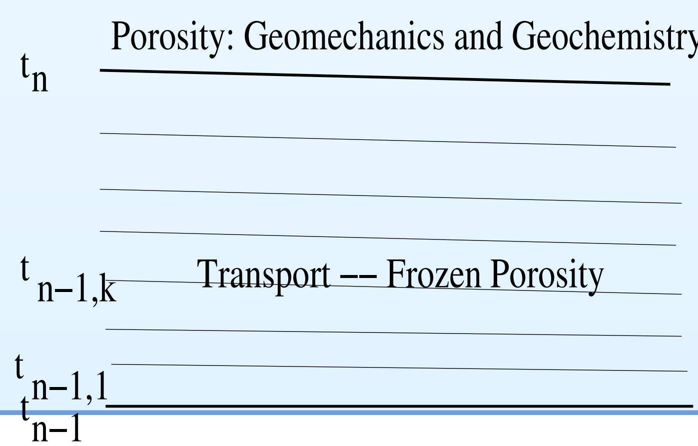
$$\frac{\partial(S_w\phi)}{\partial t} + \nabla\cdot\left(S_w\phi\frac{\partial \mathbf{u}}{\partial t}\right) + \nabla\cdot(\lambda_w(S_w)\boldsymbol{V}_{Dt}) = 0$$

Predictor:

$$\phi(\boldsymbol{x},t)\frac{\partial S_w^*}{\partial t} + \nabla\cdot\left(S_w^*\phi\frac{\partial \mathbf{u}}{\partial t}\right) + \nabla\cdot(\lambda_w(S_w^*)\boldsymbol{V}_{Dt}) = 0$$

Corrector: Initial data $S_w(\cdot,t^n) = S_w^{*n+1}$

$$\phi\frac{\partial S_w}{\partial t} = -S_w\frac{\partial \phi}{\partial t} \longrightarrow\longrightarrow \phi^{n+1}S_w^{n+1} = \phi^n S_w^{*n+1}$$



Locally Conservative Scheme for the Hyperbolic Eq.

- Higher Order Finite Volume:(Kurganov and Tadmor)
- Non-Oscillatory TVD Central Scheme

Integration of the Conservation Law $\phi =$ constant

$$\int_{t^n}^{t^{n+1}} \int_K \left(\frac{\partial \phi S}{\partial t} + \boldsymbol{\nabla} \boldsymbol{.} \boldsymbol{F}(S, \boldsymbol{V}_D, \phi, \mathbf{u}) \right) dx dt = 0$$

Averaged Saturation and Fluxes

$$\phi_j \overline{S}_j^n = \frac{1}{H} \int_{x_{j-1/2}}^{x_{j+1/2}} \phi S(x, t^n) dx, \quad \boldsymbol{F}_{j-1/2}^n = \int_{t^n}^{t^{n+1}} \boldsymbol{F}(S(x_{j-1/2}, t)) dt$$

Discrete 1D Equation: Frozen Porosity: Numerical Fluxes

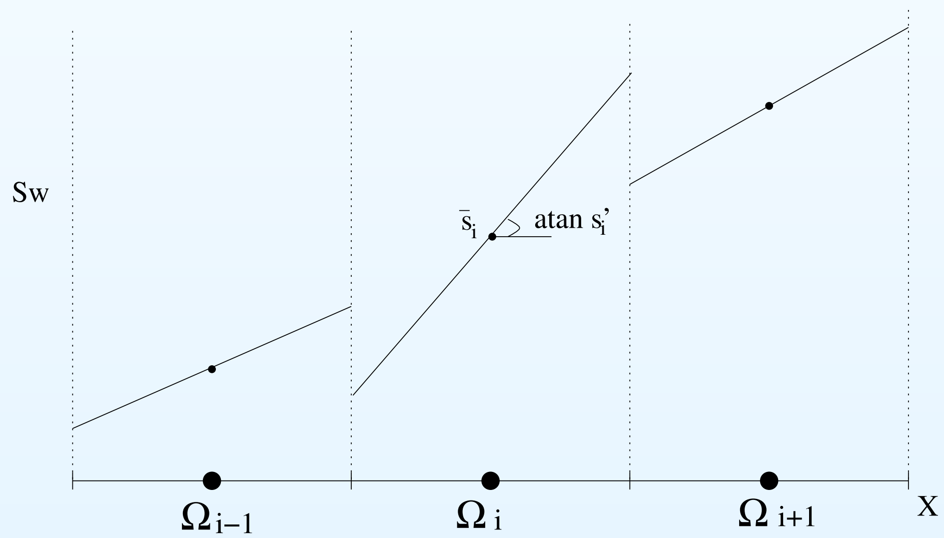
$$\overline{S}_j^{n+1} = \overline{S}_j^n - \frac{\Delta t}{\phi_j H} (\boldsymbol{F}_{j+1/2}^n - \boldsymbol{F}_{j+1/2}^n)$$

REA Algorithm (Reconstruct-Evolve-Average)

- Reconstruction: Piecewise Linear (Quadratic Convergence)

$$S_i(x,t_n) = \overline{S}_i(t_n) + (x - x_i) S_i'(t_n)$$

Derivative Slope Limiters (Min Mod Function)

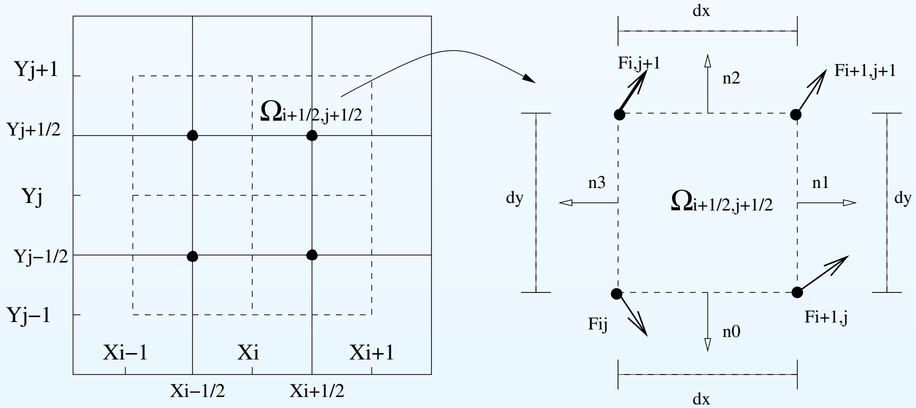


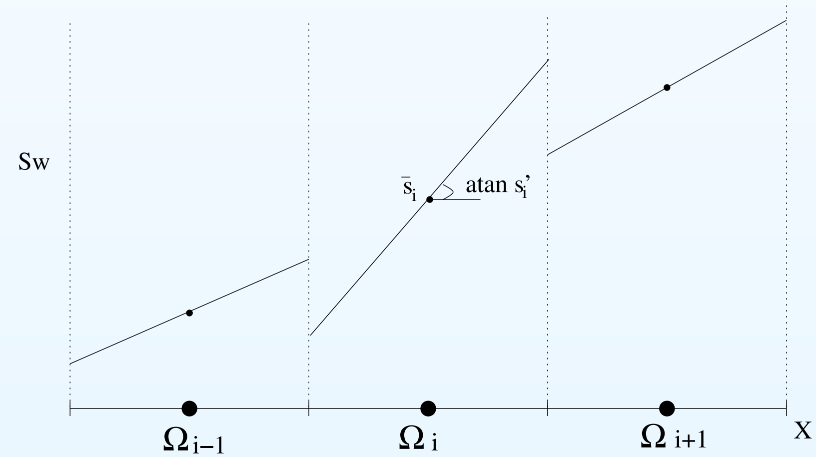
$$S_i' = \text{MinMod}\left(\frac{\overline{S}_i - \overline{S}_{i-1}}{h}, \frac{\overline{S}_{i+1} - \overline{S}_{i-1}}{2h}, \frac{\overline{S}_{i+1} - \overline{S}_i}{h}\right)$$

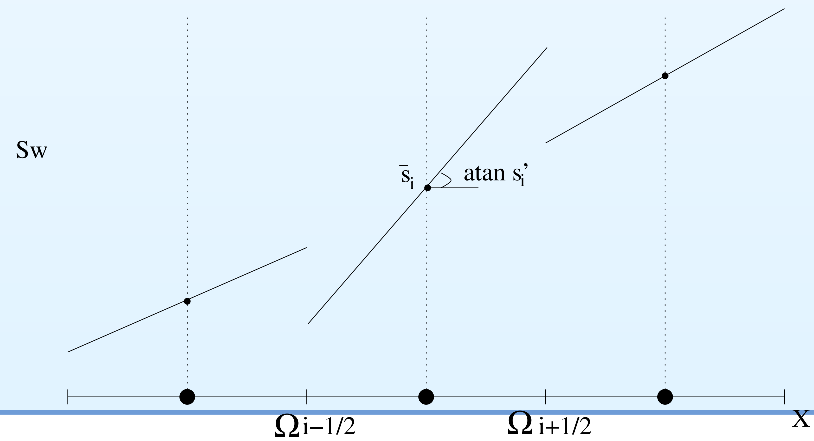
Evolution – Constrained by the CFL condition

$$\overline{S}_j^{n+1} = \overline{S}_j^n - \frac{\Delta t_{CFL}}{H_{RiemannFan}} \left(\boldsymbol{F}^n_{j+1/2} - \boldsymbol{F}^n_{j+1/2} \right)$$

Dual Mesh Avoidance of Solving the Riemann Problems:





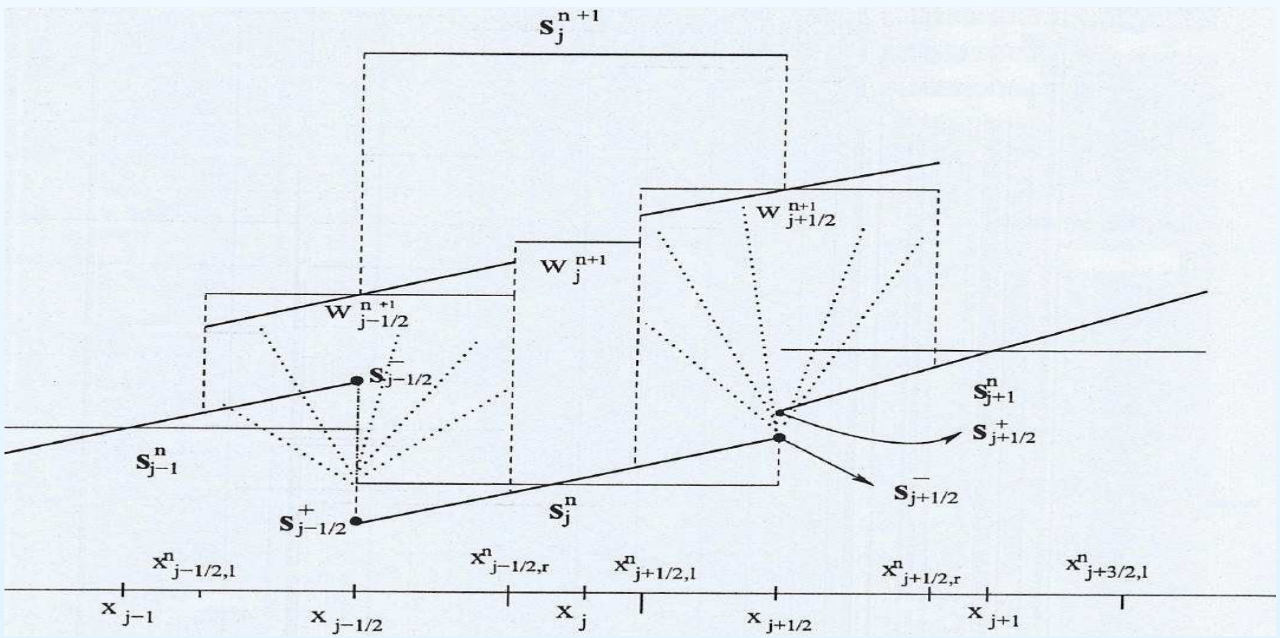


Dual Mesh = Size of the Riemann Fan

Wave Speed $\quad H_{RiemannFan} = 2a^n_{j+1/2}\Delta t_{CFL}$

$$a^n_{j+1/2} = \text{max}\left(|f'(S^+_{j+1/2})|, |f'(S^+_{j+1/2})|\right) \leq \frac{H}{\Delta t_{CFL}} \to\to \text{LAX F, NT}$$

with $\quad S^+_{j/1/2} = \overline{S}^n_{j+1} - \frac{H}{2}(S_x)^n_{j+1}, \qquad S^-_{j/1/2} = \overline{S}^n_j + \frac{H}{2}(S_x)^n_{j+1},$



SEMI-DISCRETE FORMULATION

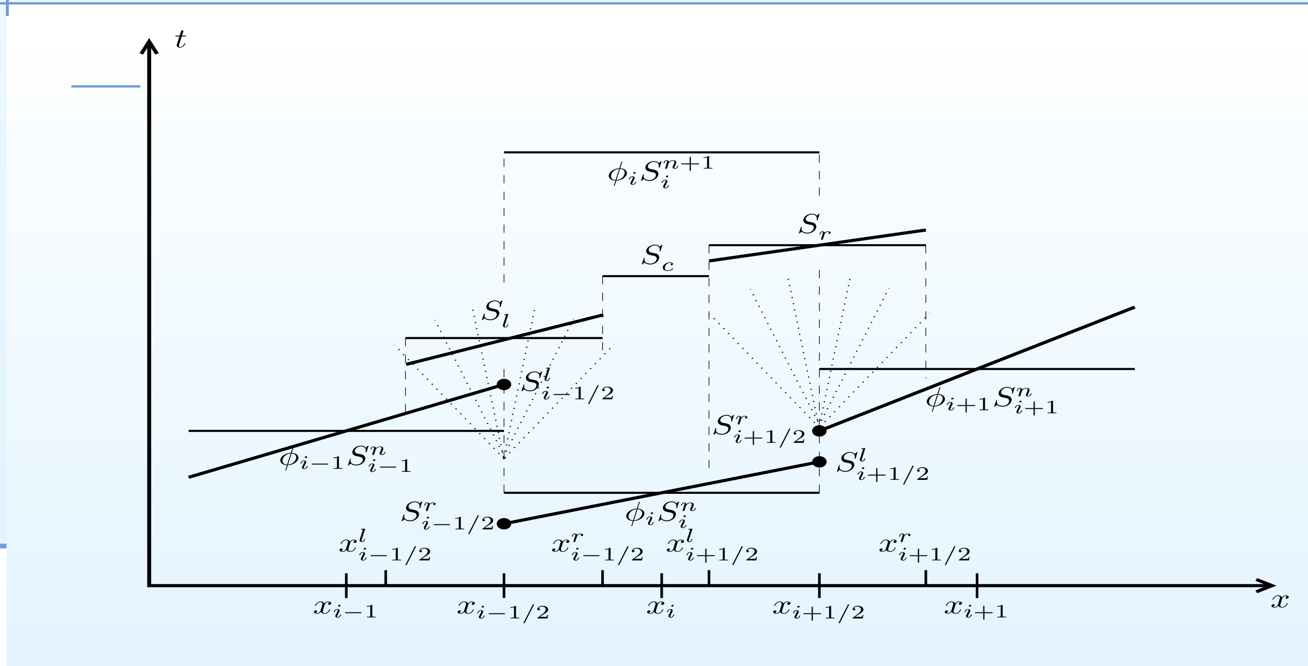
$$\frac{d}{dt}\overline{S_j}(t) = (\boldsymbol{F}^n_{j+1/2} - \boldsymbol{F}^n_{j+1/2})$$

Numerical Flux: $a_{j+1/2} = \max(|f(S^+_{j+1/2}|, |f(S^-_{j+1/2}|))$

$$\boldsymbol{F}^n_{j-1/2} = -(\phi_{i+1} + \phi_i)^{-1})(f(S^+_{j-1/2}) + \phi_{i-1}\phi_i{}^{-1} f(S^-_{j-1/2})$$

$$+\phi_{i-1} a_{j-1/2}(S^+_{j-1/2} - S^-_{j-1/2}))$$

with $S^+_{j+1/2} = \overline{S}^n_{j+1} - \frac{h}{2}(S_x)^n_{j+1}$, $\quad S^-_{j+1/2} = \overline{S}^n_{j+1} - \frac{h}{2}(S_x)^n_j$



Staggered Algorithm

Hydro $$\beta(\boldsymbol{x})\frac{\partial P^n}{\partial t} + \boldsymbol{\nabla}.\boldsymbol{V}_D^n = F^n(\boldsymbol{x},t) := -\frac{\partial \mathrm{tr}\boldsymbol{\sigma}_T^n}{\partial t}$$

GEo. $$\boldsymbol{\nabla}.\boldsymbol{\sigma} = -\boldsymbol{\nabla} P$$

$$\boldsymbol{\sigma} = \lambda(\boldsymbol{x})\boldsymbol{\nabla}.\mathbf{u}\boldsymbol{I} + 2\mu(\boldsymbol{x})\boldsymbol{\mathcal{E}}(\mathbf{u}) \quad \text{in} \quad \Omega$$

Porosity Post-Processing

$$\phi(\boldsymbol{x},t) = 1 - (1 - \phi_1^0(\boldsymbol{x})\exp(-\boldsymbol{\nabla}.\mathbf{u}) + \int_0^t F_{DISS}$$

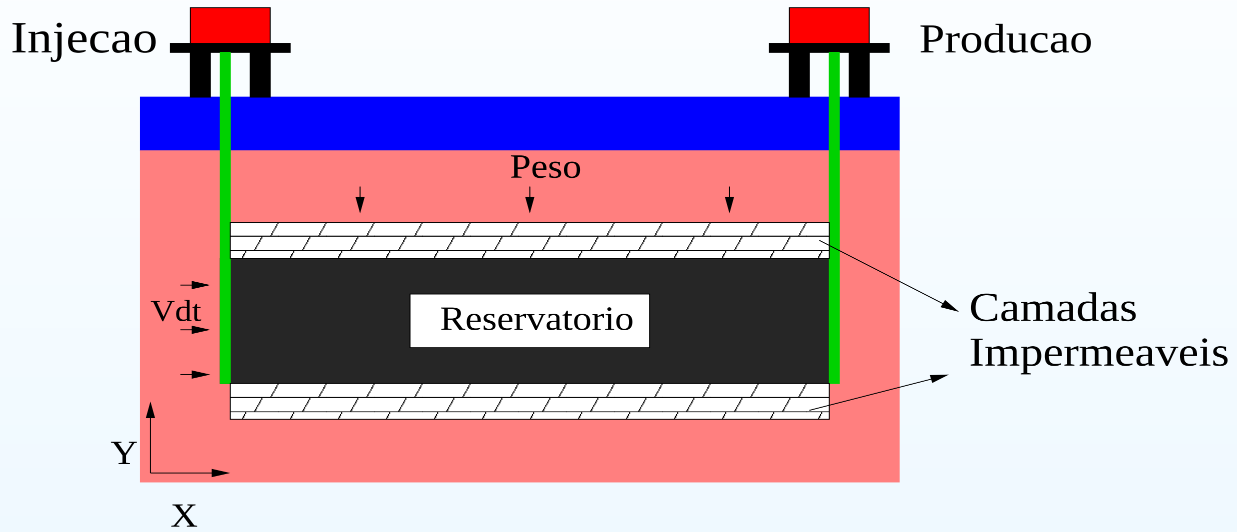
Predictor Step of the Transport: Micro Time-Steps

$$\phi\frac{\partial S_w}{\partial t} + \boldsymbol{\nabla}.\left(S_w\phi\frac{\partial \mathbf{u}}{\partial t}\right) + \boldsymbol{\nabla}.(\lambda_w(S_w)\boldsymbol{V}_{Dt}) = 0$$

Corrector Step of Transport: Initial data $S_w(\cdot,t^n) = S_w^{*n+1}$

$$\phi\partial S_w/\partial t = -S_w\partial\phi/\partial t \longrightarrow \phi^{n+1}S_w^{n+1} = \phi^n S_w^{*n+1}$$

Numerical Simualtion



Two Aims:

- Comparison between one-way coupling and iterative coupling:
- Incorporation of the creep of the cap: rock salt halite

Comparison: One-Way and Iterative Couplings

$$\beta(\boldsymbol{x})\frac{\partial P^n}{\partial t} + \boldsymbol{\nabla}.\boldsymbol{V}_D^n = F^n(\boldsymbol{x},t) := -\frac{\partial \text{tr}\boldsymbol{\sigma}_T^n}{\partial t}$$

$$\boldsymbol{V}_D^n = -K(x)\lambda_t(S^{n-1})\boldsymbol{\nabla}P^n$$

Include the Cap Rock: $\text{tr}\boldsymbol{\sigma}_T = -3P + (3\lambda + 2\mu)\boldsymbol{\nabla}.\mathbf{u}$

$$\boldsymbol{\nabla}.\boldsymbol{\sigma}^n = -\boldsymbol{\nabla}P^n$$

$$\boldsymbol{\sigma}^n = \lambda(\boldsymbol{x})\boldsymbol{\nabla}.\mathbf{u}^n\boldsymbol{I} + 2\mu(\boldsymbol{x})\boldsymbol{\mathcal{E}}(\mathbf{u}^n) \quad \text{in} \quad \Omega$$

One Way Coupling

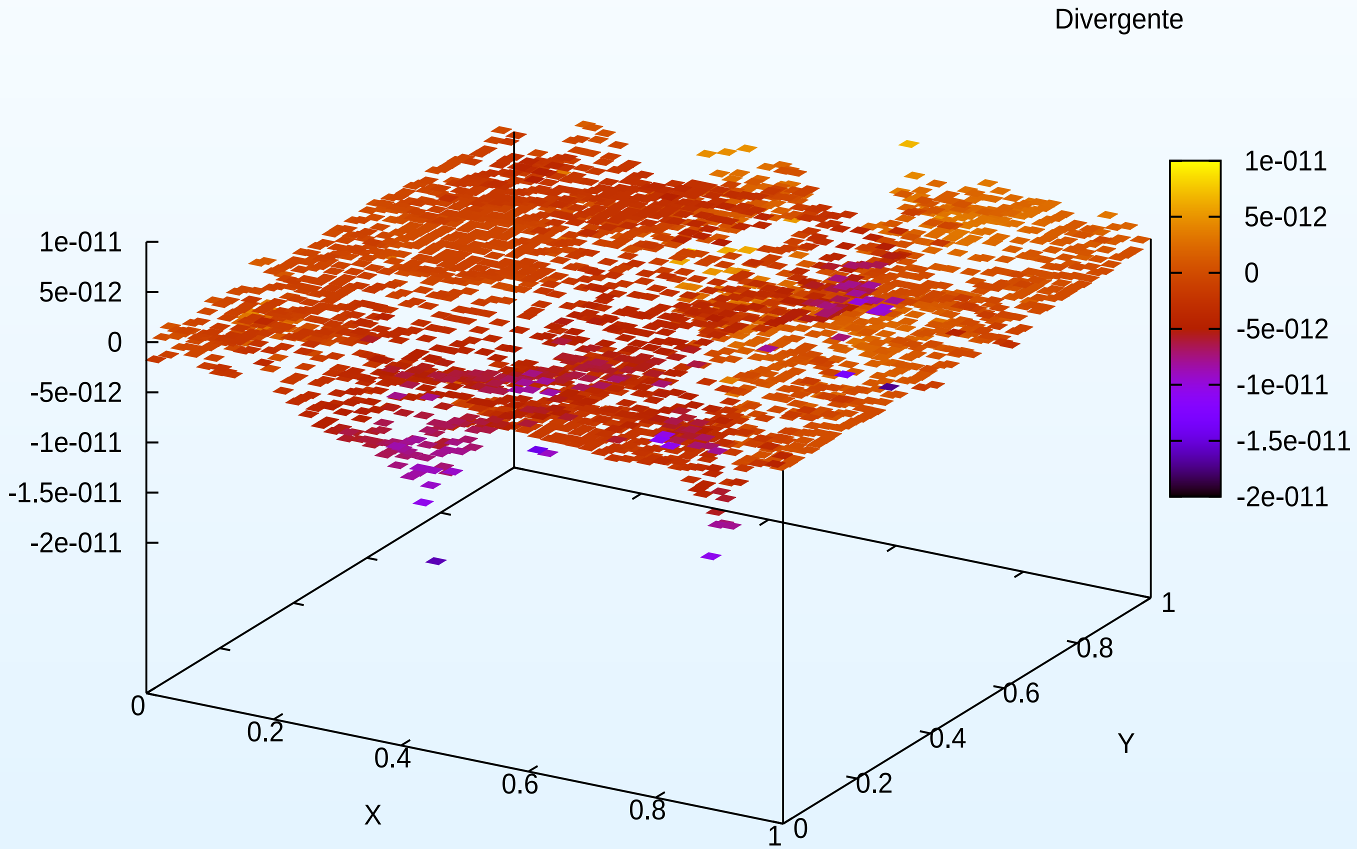
$$\beta(\boldsymbol{x})\frac{\partial P^n}{\partial t} + \boldsymbol{\nabla}.\boldsymbol{V}_D^n = F^n(\boldsymbol{x},t) = 0$$

$$\boldsymbol{V}_D^n = -K(x)\lambda_t(S^{n-1})\boldsymbol{\nabla}P^n$$

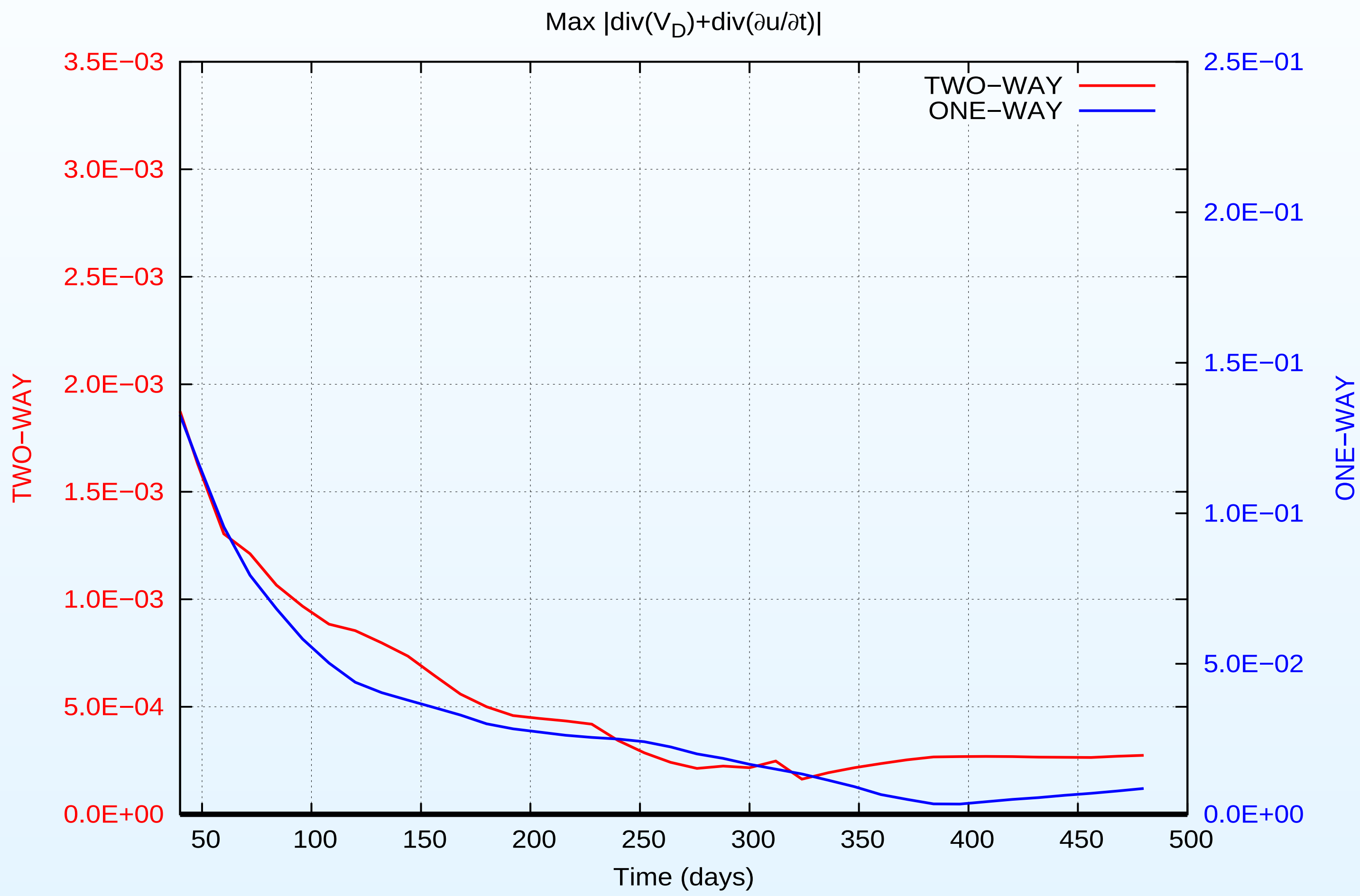
Comparison: One-Way and Iterative Couplings

- Local Source of Mass of the Mixture:

$$RES = \nabla . \partial \mathbf{u} / \partial t + \nabla . \boldsymbol{V}_{Dt}$$



Evolution Local Mass Source – Maximum



Comparison: One-Way and Iterative Couplings

$$\beta(\boldsymbol{x})\frac{\partial P^n}{\partial t} + \boldsymbol{\nabla}.\boldsymbol{V}_D^n = F^n(\boldsymbol{x},t) := -\frac{\partial \mathrm{tr}\boldsymbol{\sigma}_T^n}{\partial t}$$

$$\boldsymbol{V}_D^n = -K(x)\lambda_t(S^{n-1})\boldsymbol{\nabla} P^n$$

Geomechanics: $\mathrm{tr}\boldsymbol{\sigma}_T = -3P + (3\lambda + 2\mu)\boldsymbol{\nabla}.\mathbf{u}$

$$\boldsymbol{\nabla}.\boldsymbol{\sigma}_T^n = \boldsymbol{\nabla}.\boldsymbol{\sigma}^n - \boldsymbol{\nabla} P^n = 0$$

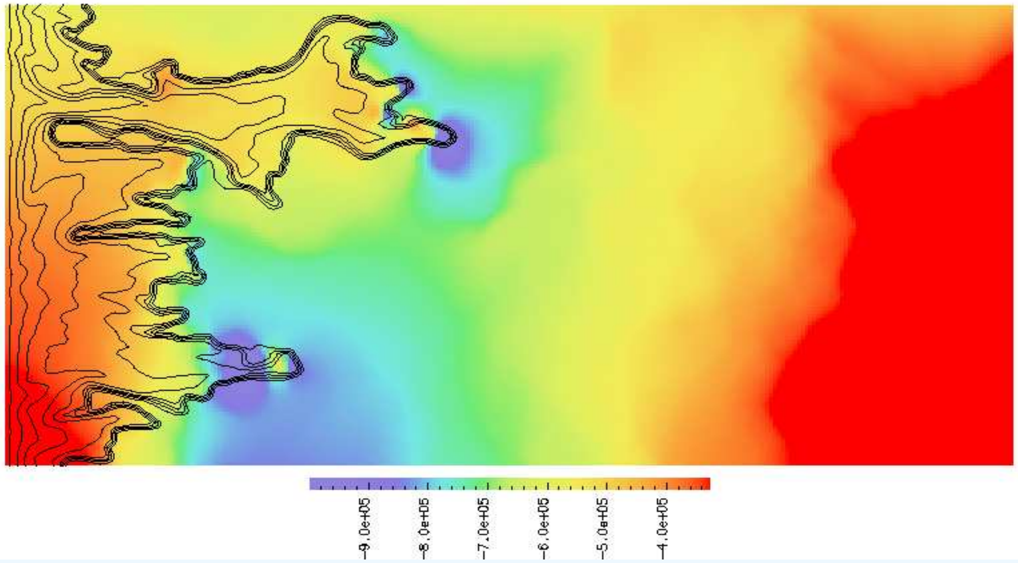
$$\boldsymbol{\sigma}^n = \lambda(\boldsymbol{x})\boldsymbol{\nabla}.\mathbf{u}^n\boldsymbol{I} + 2\mu(\boldsymbol{x})\boldsymbol{\mathcal{E}}(\mathbf{u}^n) \quad \text{in} \quad \Omega$$

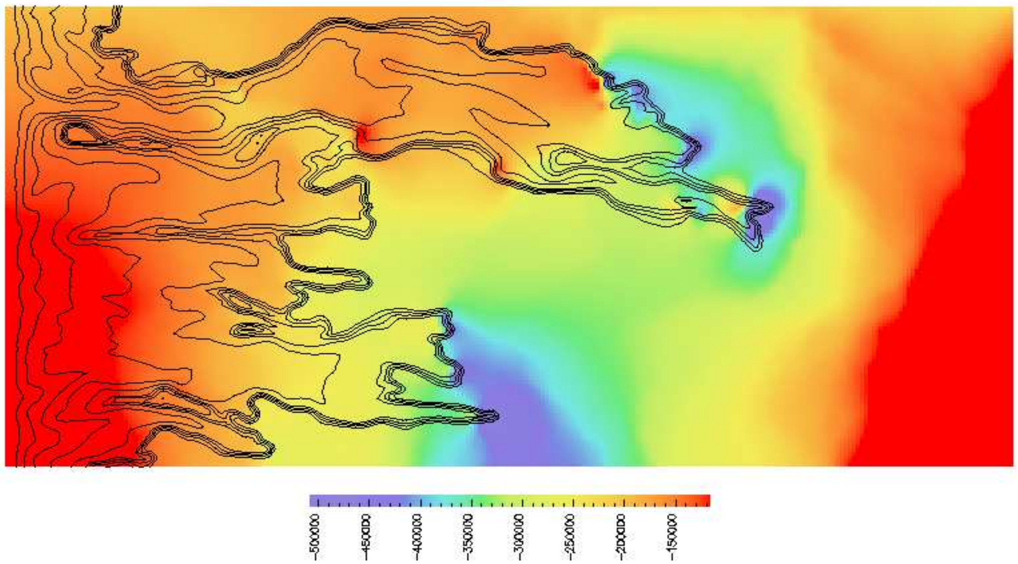
One Way Coupling

$$\beta(\boldsymbol{x})\frac{\partial P^n}{\partial t} + \boldsymbol{\nabla}.\boldsymbol{V}_D^n = F^n(\boldsymbol{x},t) = 0$$

$$\boldsymbol{V}_D^n = -K(x)\lambda_t(S^{n-1})\boldsymbol{\nabla} P^n$$

Time-Derivative Total Stress





Two-Phase Flow

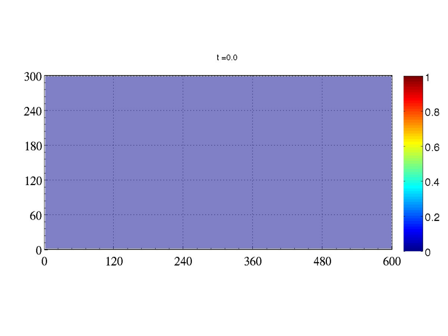
Model Problem:

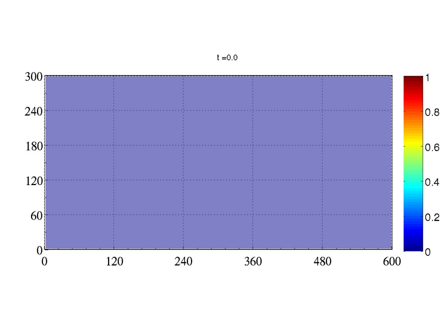
Two-Phase Flow in a Poroelastic Reservoir

- Heterogeneous Permeability and Young Modulus
- Homogeneous Initial Porosity

Two-Phase Flow

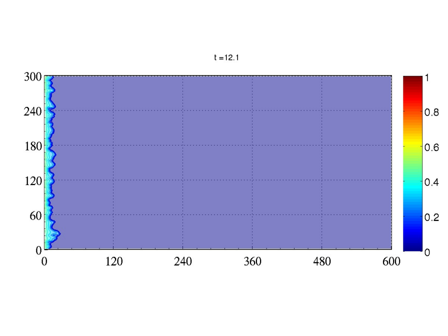
One-way | Two-Way Iterative

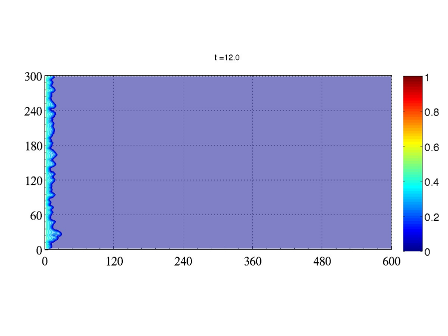




Two-Phase Flow

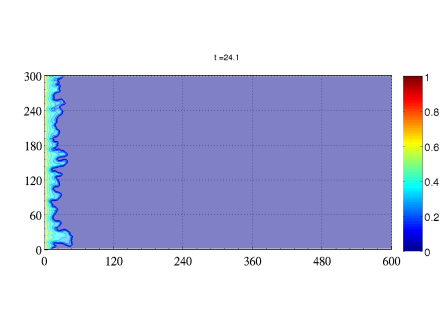
One-way | Two-Way Iterative

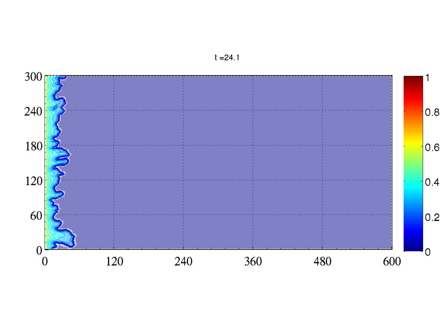




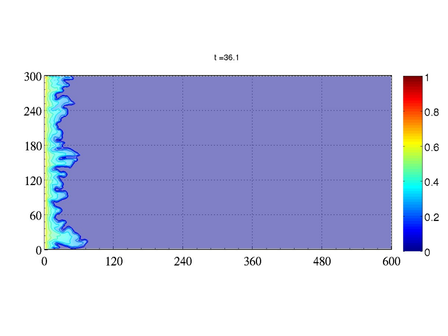
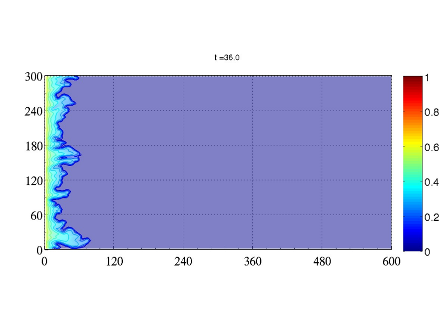
Two-Phase Flow

One-way	Two-Way Iterative



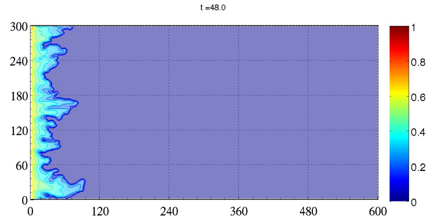


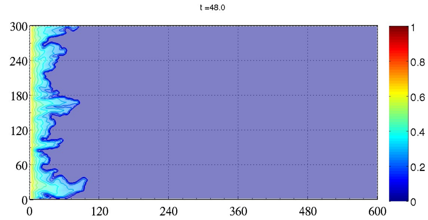
Two-Phase Flow

One-way	Two-Way Iterative
	

Two-Phase Flow

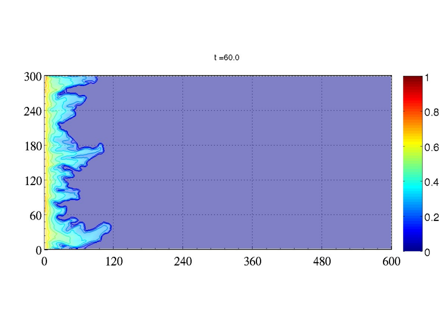
One-way | Two-Way Iterative

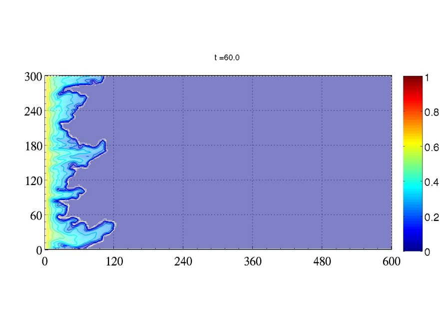




Two-Phase Flow

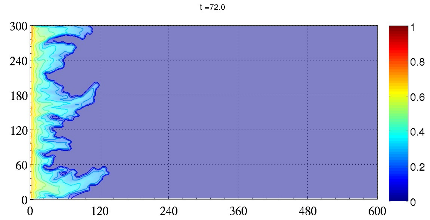
One-way	Two-Way Iterative

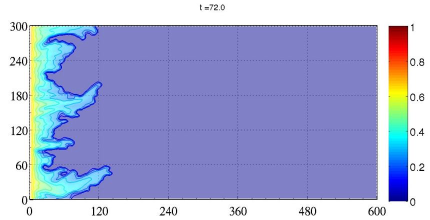




Two-Phase Flow

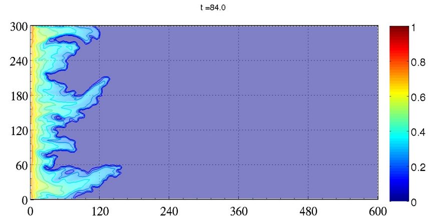
One-way	Two-Way Iterative

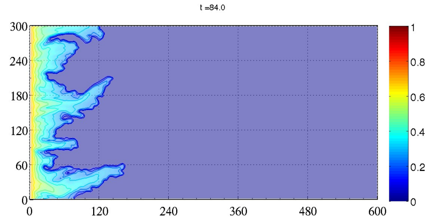




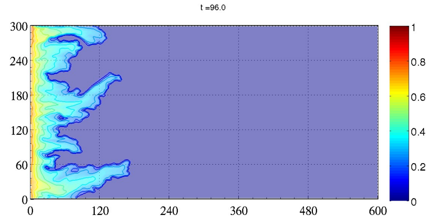
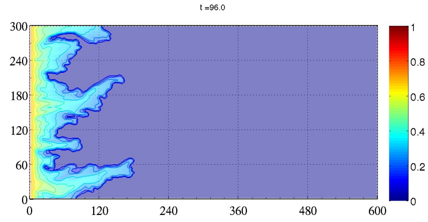
Two-Phase Flow

One-way	Two-Way Iterative



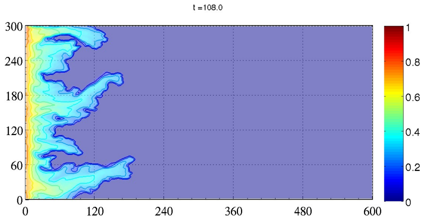


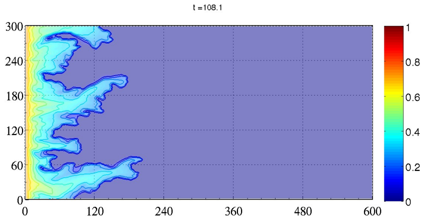
Two-Phase Flow

One-way	Two-Way Iterative
	

Two-Phase Flow

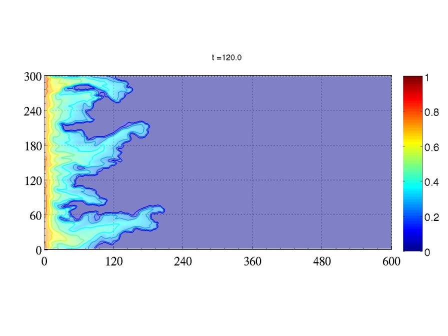
One-way	Two-Way Iterative

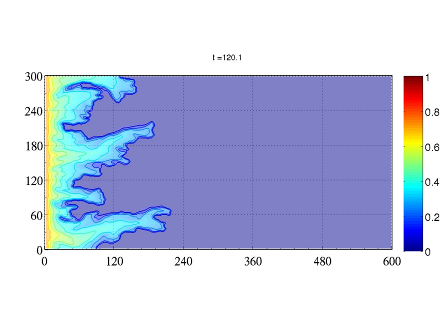




Two-Phase Flow

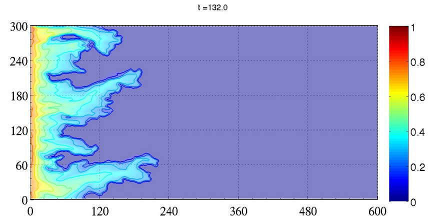
One-way	Two-Way Iterative

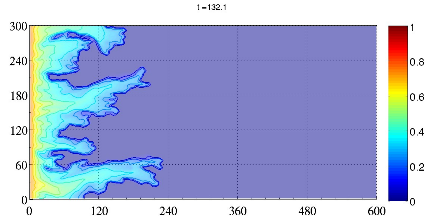




Two-Phase Flow

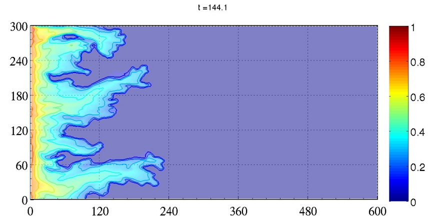
One-way	Two-Way Iterative

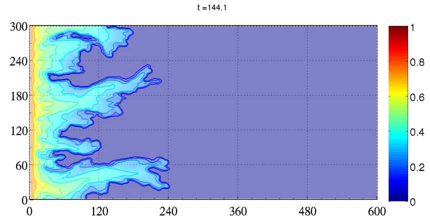




Two-Phase Flow

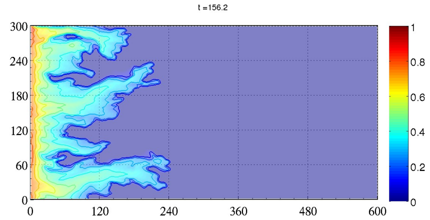
One-way | Two-Way Iterative

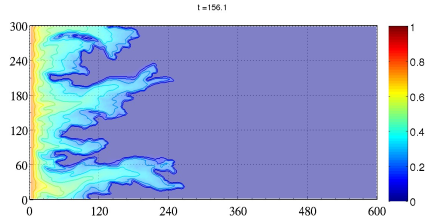




Two-Phase Flow

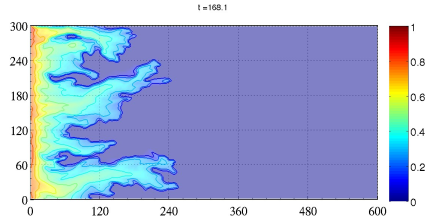
One-way	Two-Way Iterative

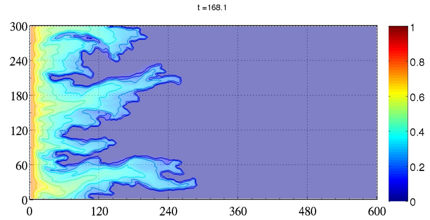




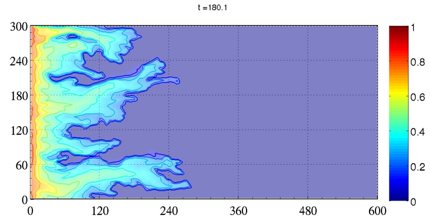
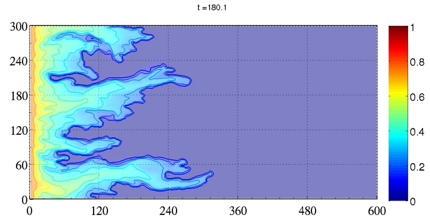
Two-Phase Flow

One-way	Two-Way Iterative



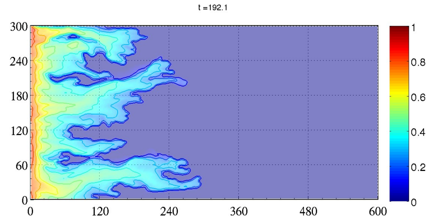


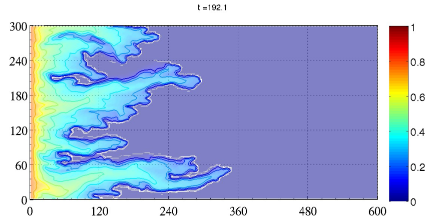
Two-Phase Flow

One-way	Two-Way Iterative
	

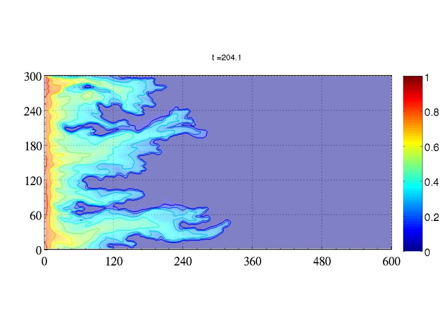
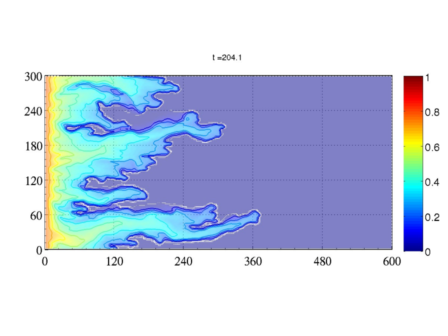
Two-Phase Flow

One-way | Two-Way Iterative



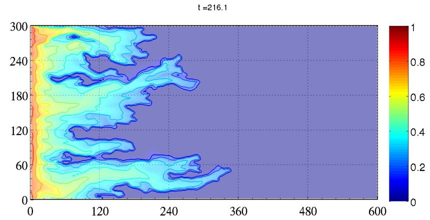


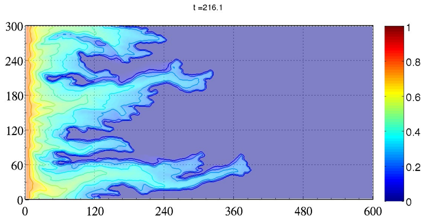
Two-Phase Flow

One-way	Two-Way Iterative
	

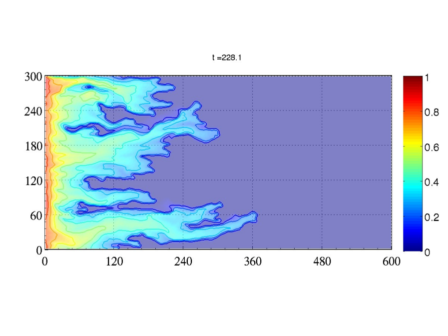
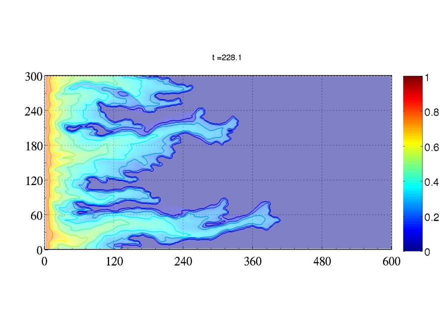
Two-Phase Flow

One-way | Two-Way Iterative



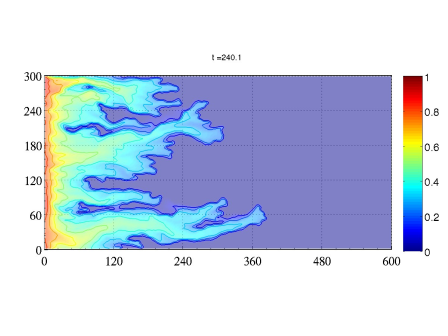


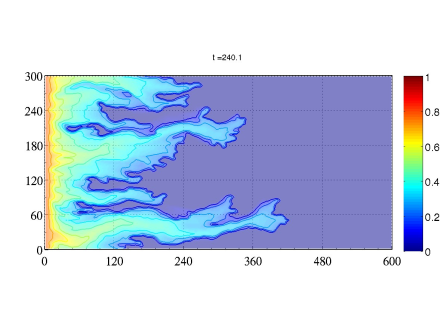
Two-Phase Flow

One-way	Two-Way Iterative
	

Two-Phase Flow

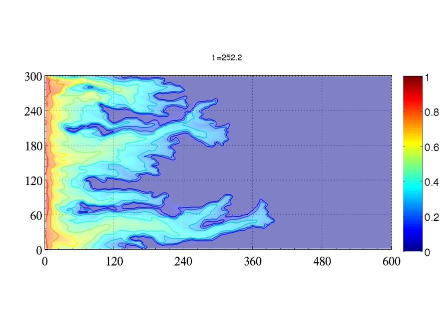
One-way	Two-Way Iterative

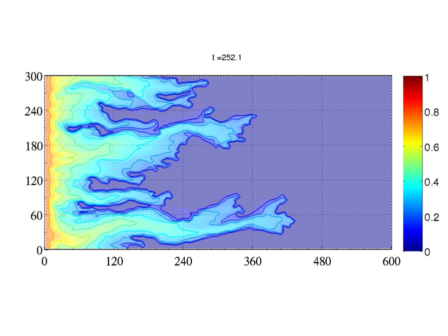




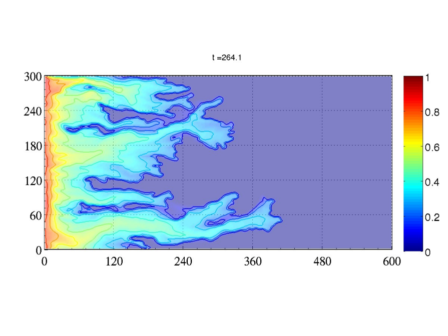
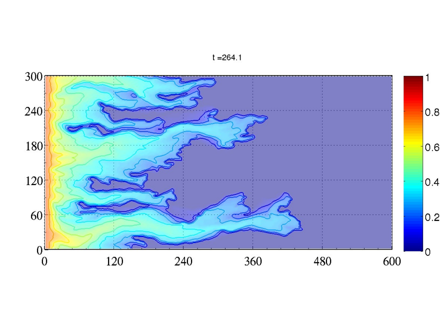
Two-Phase Flow

One-way	Two-Way Iterative

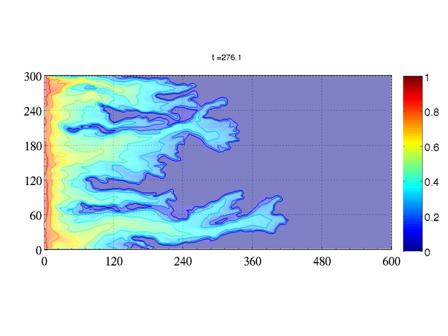
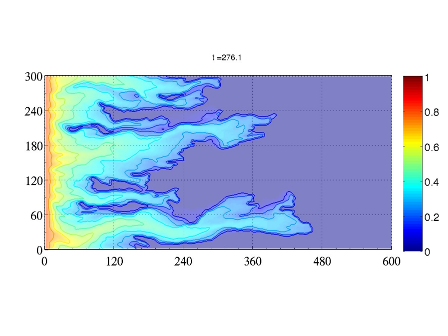




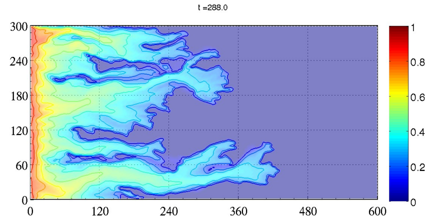
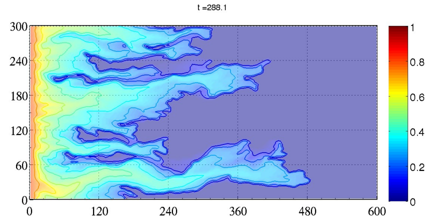
Two-Phase Flow

One-way	Two-Way Iterative
	

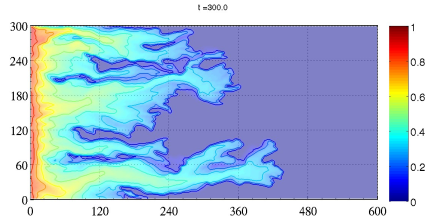
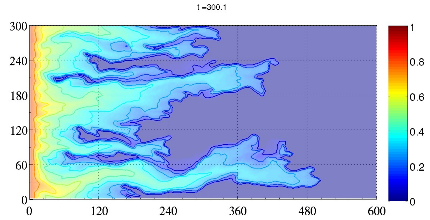
Two-Phase Flow

One-way	Two-Way Iterative
	

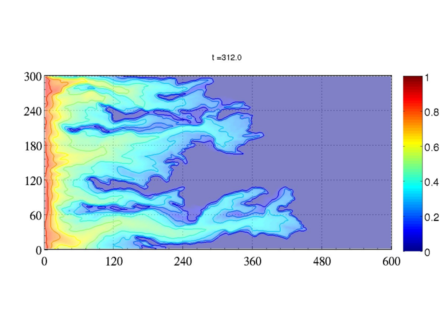
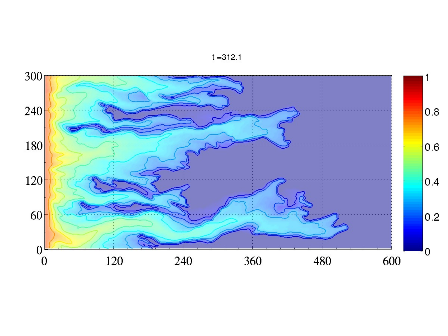
Two-Phase Flow

One-way	Two-Way Iterative
	

Two-Phase Flow

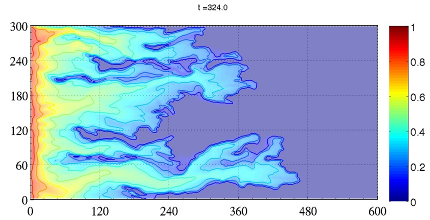
One-way	Two-Way Iterative
	

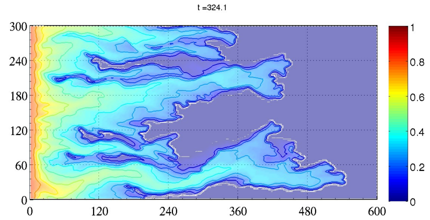
Two-Phase Flow

One-way	Two-Way Iterative
	

Two-Phase Flow

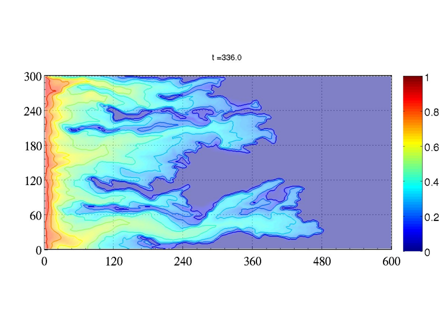
One-way | Two-Way Iterative

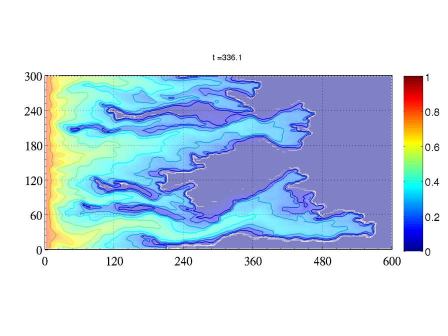




Two-Phase Flow

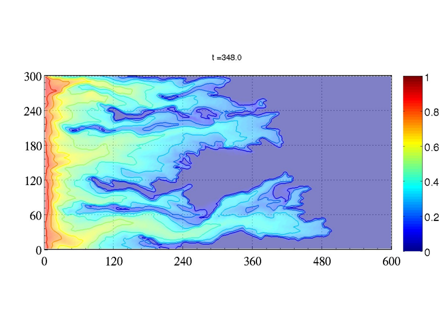
One-way	Two-Way Iterative

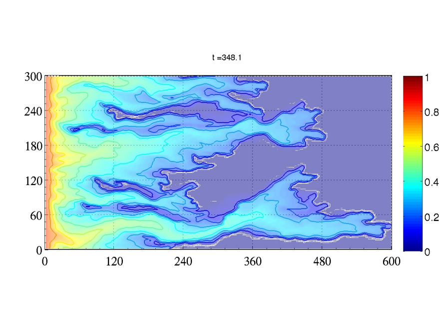




Two-Phase Flow

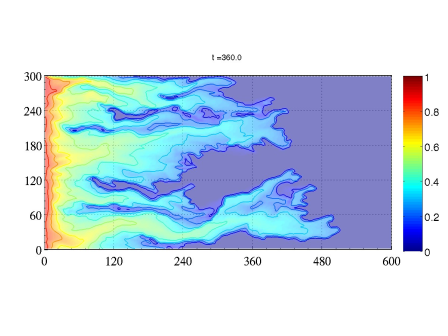
One-way	Two-Way Iterative

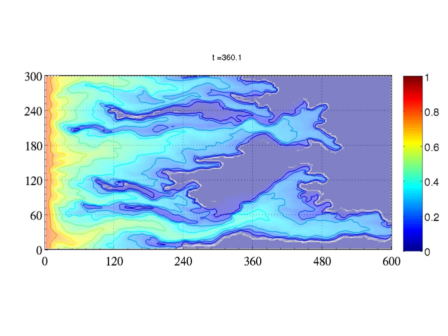




Two-Phase Flow

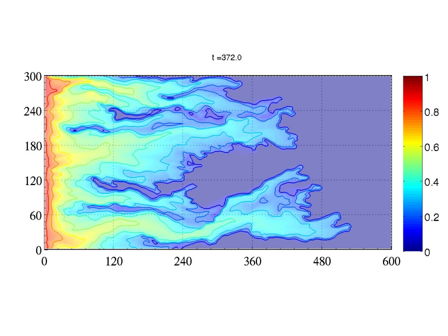
One-way	Two-Way Iterative

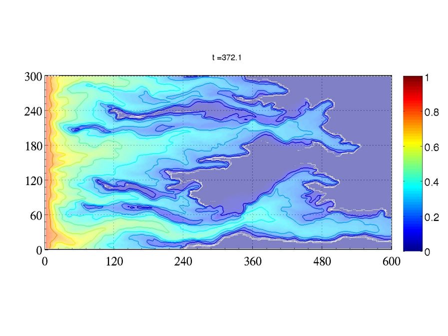




Two-Phase Flow

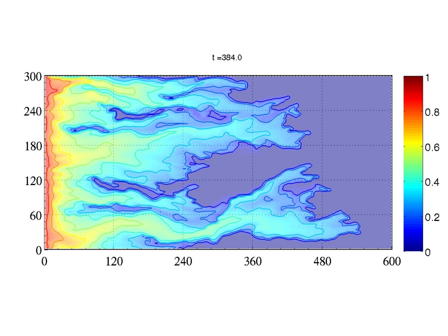
One-way	Two-Way Iterative

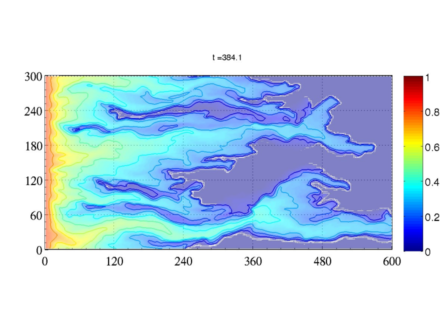




Two-Phase Flow

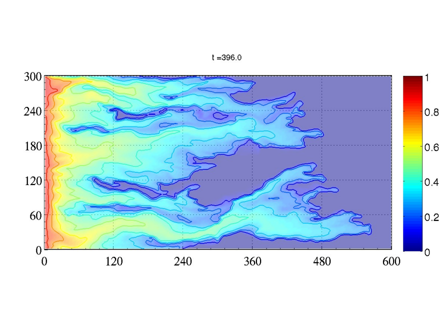
One-way | Two-Way Iterative



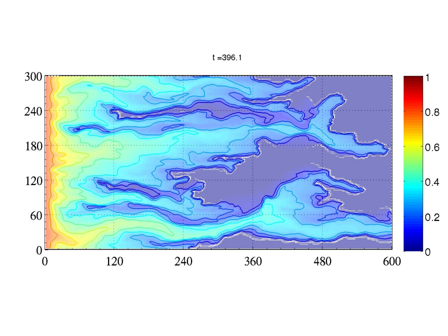


Two-Phase Flow

One-way

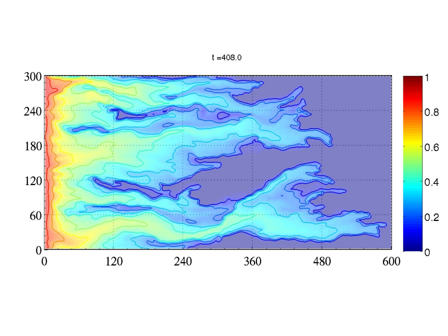


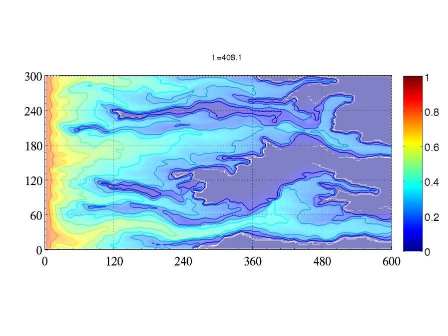
Two-Way Iterative



Two-Phase Flow

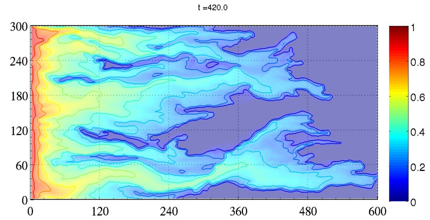
One-way	Two-Way Iterative

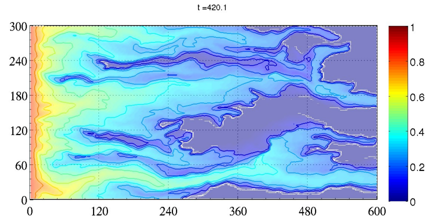




Two-Phase Flow

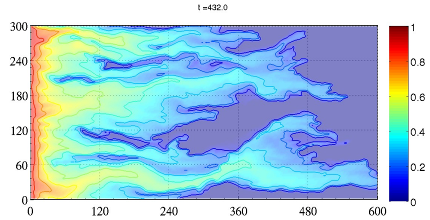
One-way	Two-Way Iterative

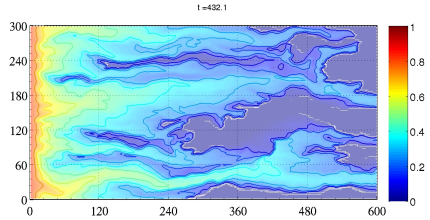




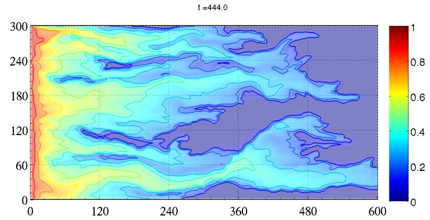
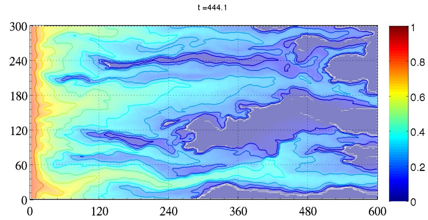
Two-Phase Flow

One-way | Two-Way Iterative



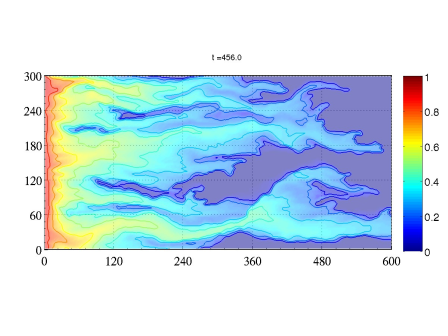


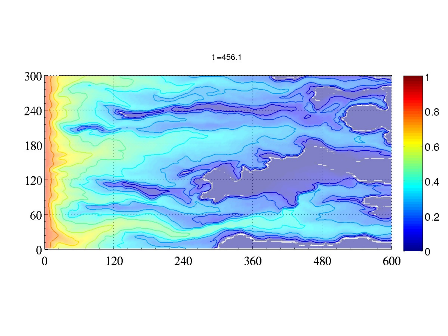
Two-Phase Flow

One-way	Two-Way Iterative
	

Two-Phase Flow

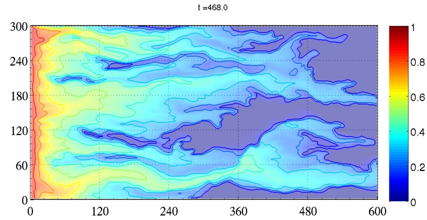
One-way	Two-Way Iterative

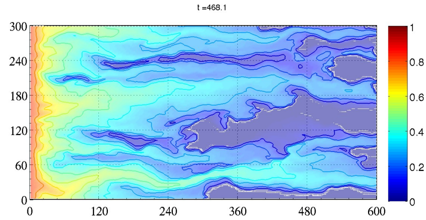




Two-Phase Flow

One-way	Two-Way Iterative

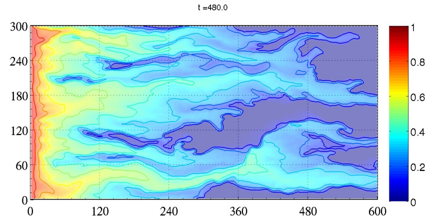


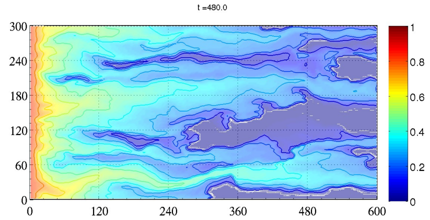


Two-Phase Flow

One-way	Two-Way Iterative

play

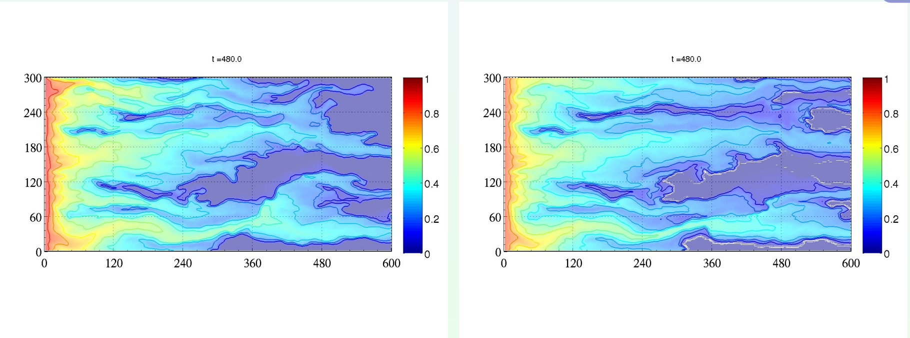




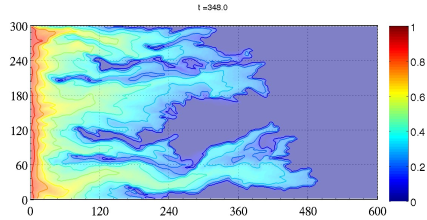
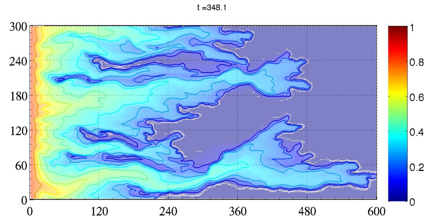
Two-Phase Flow

One-way | Two-Way Iterative

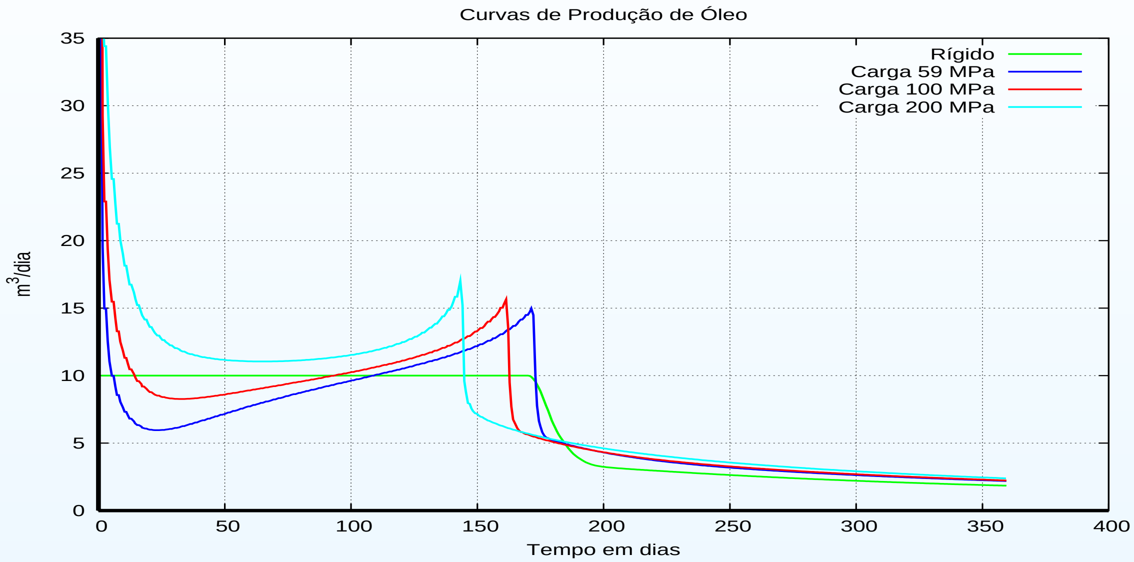
▸ play

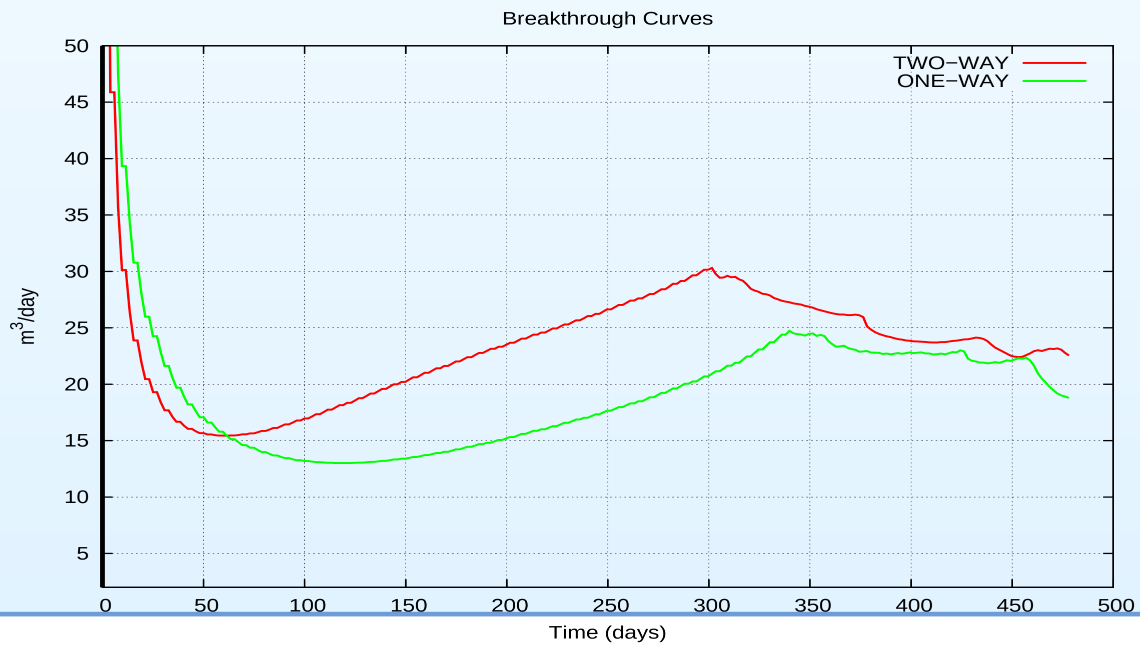


Two-Phase Flow

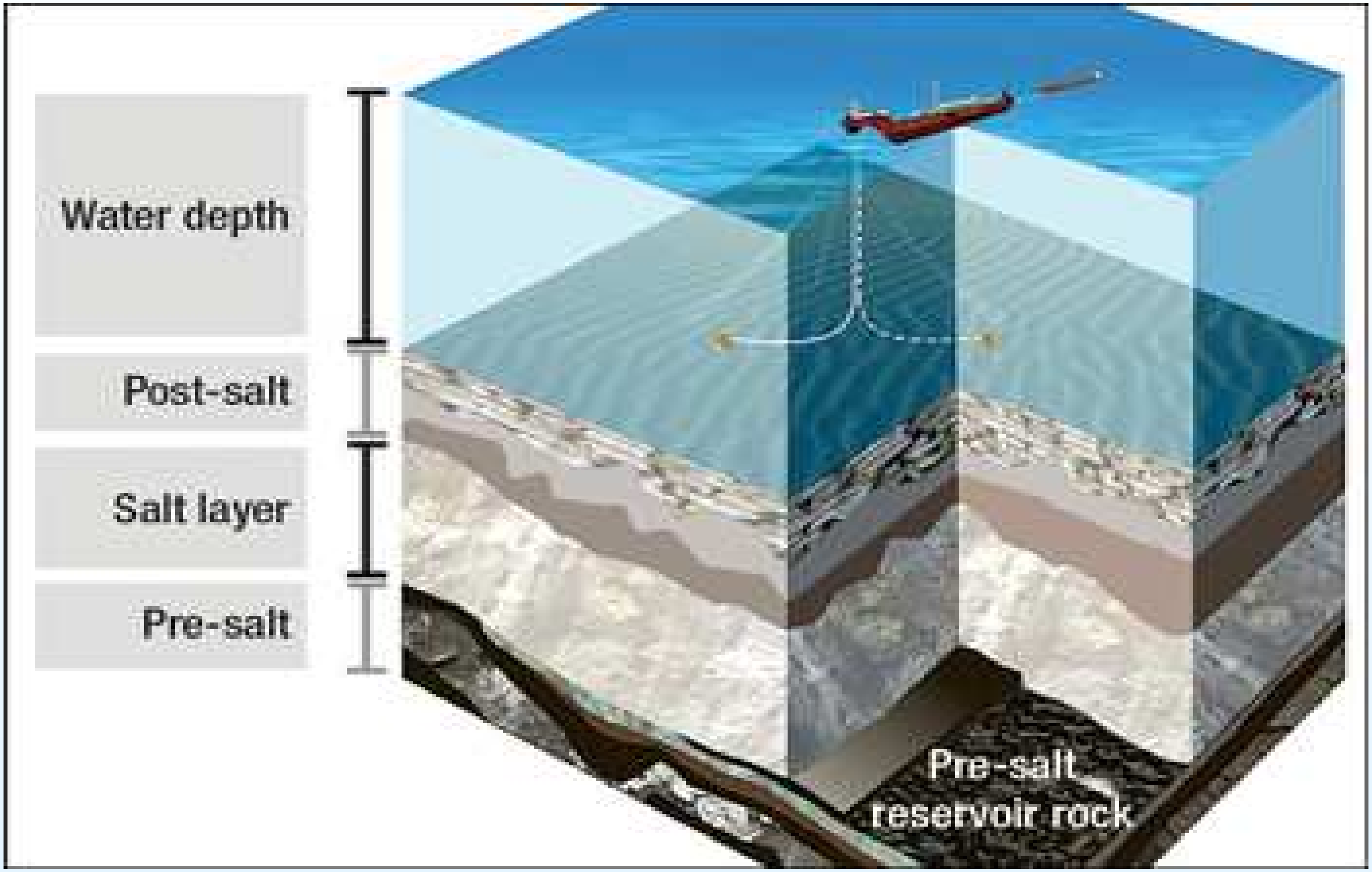
One-way	Two-Way Iterative
	

Breakthrough Curves: $E = 4GPA, Q = 10m^3/d$





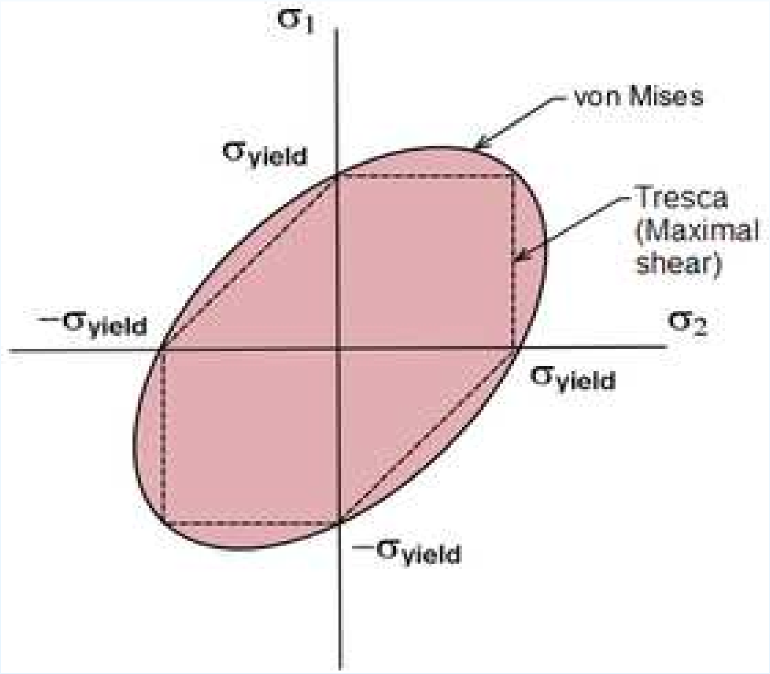
Inclusion of the Cap Rock – Saline Formation



Pre-Salt Microbiolite Carbonate

Von Mises Stress – Distortion Energy

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \boldsymbol{\sigma}^{dev}, \qquad \boldsymbol{\sigma}_V = \sqrt{\frac{3}{2}\sigma_{ij}^{dev}\sigma_{ji}^{dev}}$$



Viscoelastic Model for the Rock Salt

$\boldsymbol{\sigma}_V$ – Energy that triggers creep in the saline cap rock

Inclusion of the Cap Rock

$$\boldsymbol{\nabla} \boldsymbol{.} \boldsymbol{\sigma}_e^n = -\boldsymbol{\nabla} P^n$$

$$\boldsymbol{\sigma}_e^n = \lambda(\boldsymbol{x})\boldsymbol{\nabla}\boldsymbol{.}\mathbf{u}^n \boldsymbol{I} + 2\mu(\boldsymbol{x})\boldsymbol{\mathcal{E}}(\mathbf{u}^n) \quad \text{in} \quad \Omega$$

$$\boldsymbol{\sigma}_e^n \boldsymbol{n} = \boldsymbol{h} \quad \text{on} \quad \Gamma_1, \qquad \mathbf{u} = 0 \qquad \text{on} \quad \Gamma_2$$

Creep – Norton Power Law: Von Mises Stress $\boldsymbol{\sigma}_V$

$$\boldsymbol{\sigma}_e = \boldsymbol{C}(\boldsymbol{\mathcal{E}}(\mathbf{u}) - \boldsymbol{\mathcal{E}}_{visc})$$

$$\frac{\partial \boldsymbol{\mathcal{E}}_{visc}}{\partial t} = \boldsymbol{\mathcal{E}}_0(\left(\frac{\boldsymbol{\sigma}_V}{\boldsymbol{\sigma}_0}\right)^n \exp\left(\frac{Q}{R}\left(\frac{1}{T_o} - \frac{1}{T}\right)\right)$$

Von Mises Stress – Distortion Energy:

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \boldsymbol{\sigma}^{dev}, \qquad \boldsymbol{\sigma}_V = \sqrt{\frac{3}{2}\sigma_{ij}^{dev}\sigma_{ji}^{dev}}$$

Creep Problem: B-bar Method Hughes (1980)

Equilibrium: Unknowns $\{\mathbf{u}, \boldsymbol{\mathcal{E}}, \boldsymbol{\sigma}\}$ $(a,b) = \int_\Omega a\, b\, d\Omega$

$$\left(\boldsymbol{\mathcal{E}}(\boldsymbol{w}), \boldsymbol{\sigma}\right) = \int_{\Gamma_N} \boldsymbol{\sigma} \boldsymbol{n} \boldsymbol{w} \qquad \forall \boldsymbol{w} \in W^u$$

Constitutive Law

$$\left(\boldsymbol{\psi}, \boldsymbol{\sigma} - \boldsymbol{C}(\boldsymbol{\mathcal{E}}(\mathbf{u}) - \boldsymbol{\mathcal{E}}_{visc})\right) = 0 \qquad \forall \boldsymbol{\psi} \in W^\sigma$$

$$\frac{\partial \boldsymbol{\mathcal{E}}_{visc}}{\partial t} = C(T)\boldsymbol{\mathcal{E}}_0(\left(\frac{\boldsymbol{\sigma}_V}{\boldsymbol{\sigma}_0}\right)^n$$

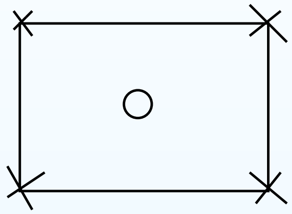
Compatitbility Conditions: Small strains

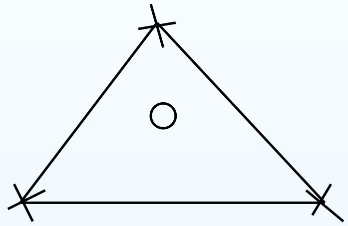
$$(\boldsymbol{\tau}, \boldsymbol{\mathcal{E}} - \boldsymbol{\nabla}^s \mathbf{u}) = 0 \qquad \forall \boldsymbol{\tau} \in W^\sigma$$

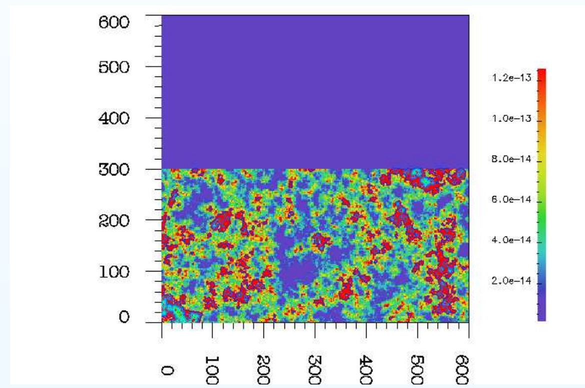
with $\boldsymbol{\nabla}^s \mathbf{u} = \frac{1}{2}(\boldsymbol{\nabla}\mathbf{u} + (\boldsymbol{\nabla}\mathbf{u})^T)$

Static Condensation: $\{\boldsymbol{\sigma}, \boldsymbol{\mathcal{E}}, \mathbf{u}\}$

- Displacement Piecewise Linear:
- Effective Stress and Strain Piecewiese Constants







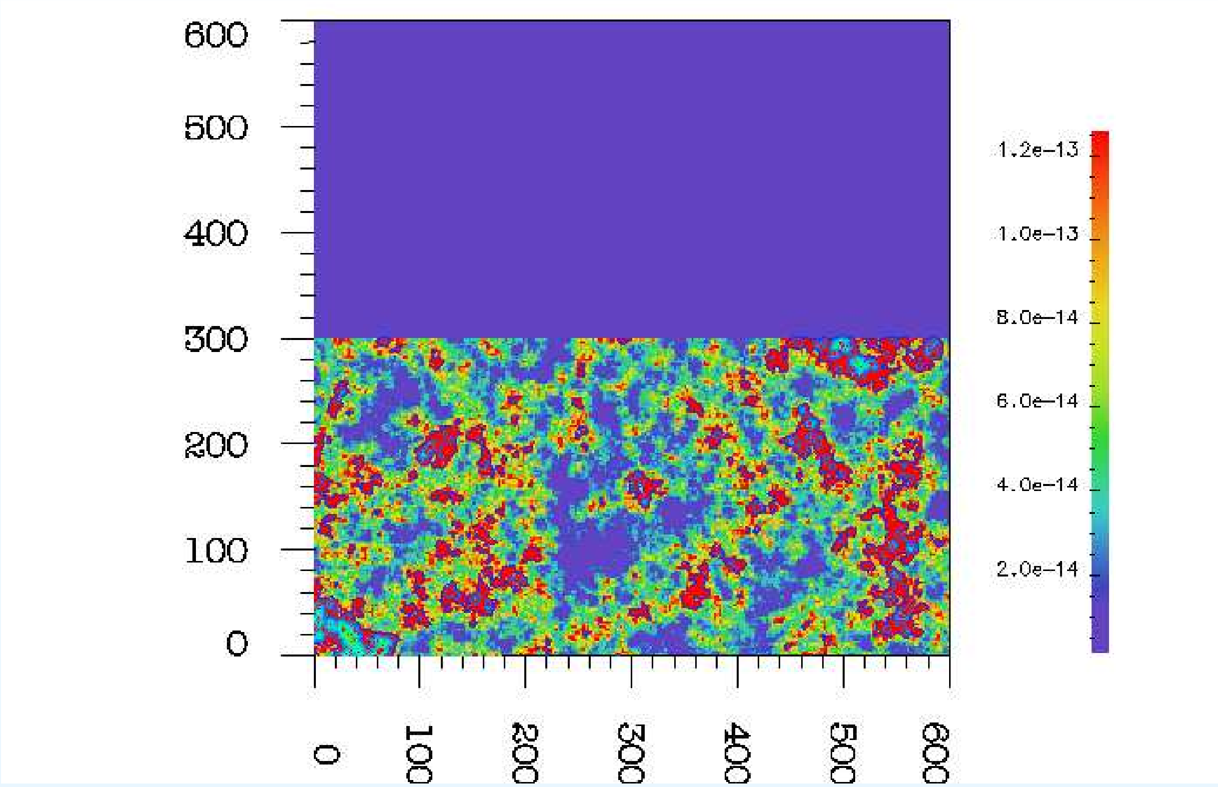
– Tangent Matrix - Newton Method

$$\left(\overline{\boldsymbol{\nabla}^s \boldsymbol{w}_h}\, C(\boldsymbol{\sigma}_h)\, \overline{\boldsymbol{\nabla}^s \mathbf{u}_h^n}\right) = \int_{\Gamma_N} \boldsymbol{\sigma n w} \qquad \forall \boldsymbol{w} \in W^u$$

$$\left(\overline{\boldsymbol{\nabla}^s \boldsymbol{w}_h}\, C\, \overline{\boldsymbol{\nabla}^s \mathbf{u}_h^n}\right) = \left(p_h^n, \boldsymbol{\nabla} \cdot \boldsymbol{w}_h\right)$$

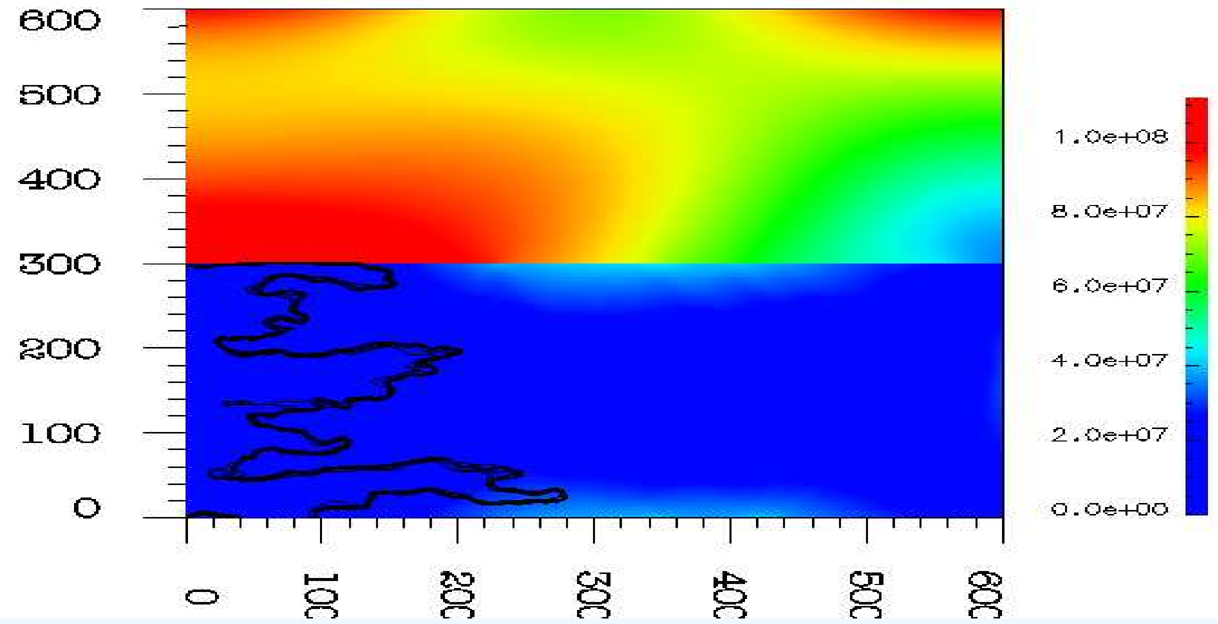
Mean over each Element: $\overline{\boldsymbol{\nabla}^s \mathbf{u}_h} : \frac{1}{\Omega_e} \int_{\Omega_e} \boldsymbol{\nabla}^s \mathbf{u}_h \mathrm{d}\Omega$

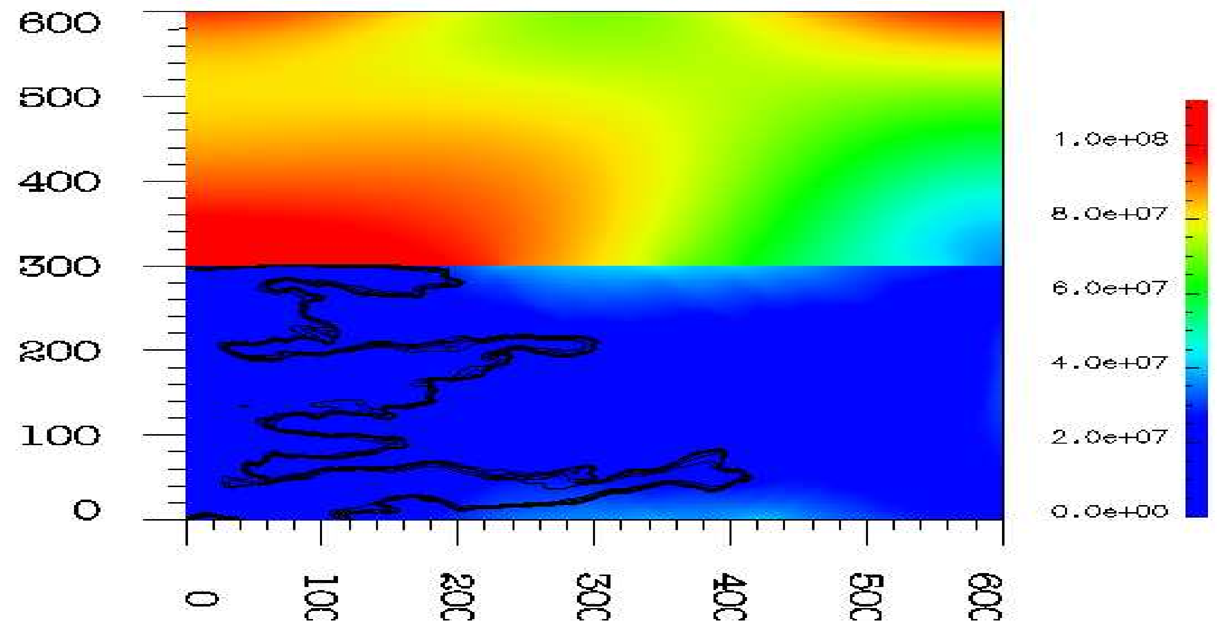
Inclusion of the Cap Rock - Permeability



Permeability field

Von - Mises Stress – Elastic Cap Rock

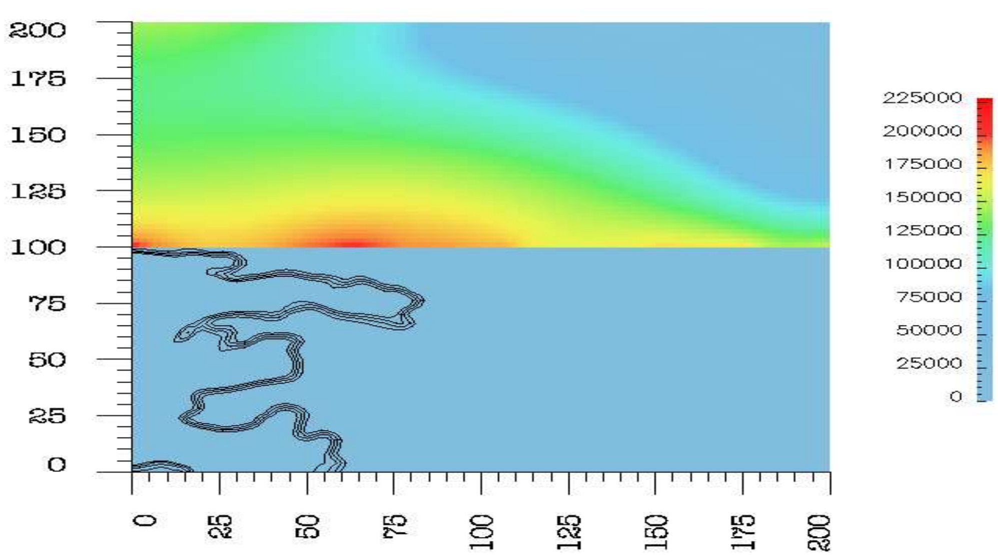




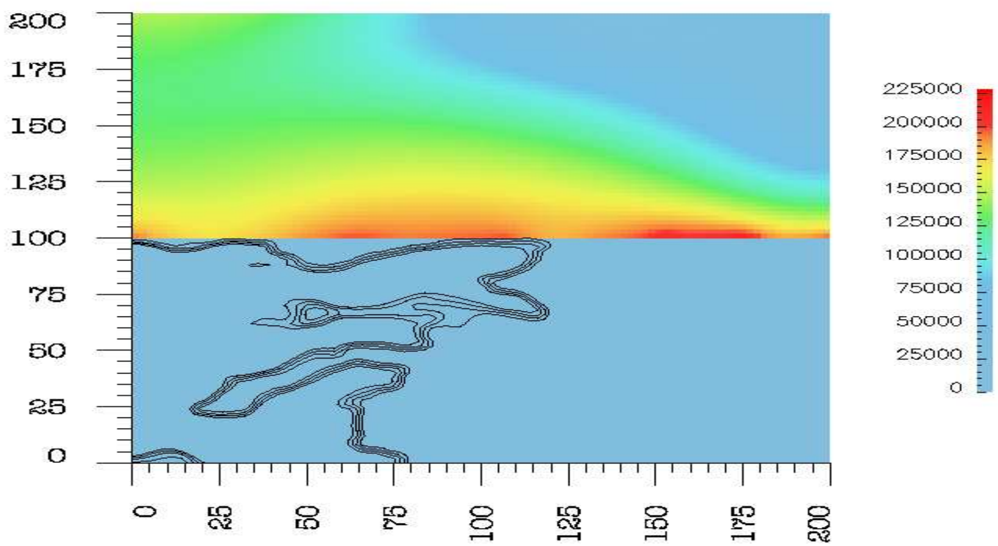
Von - Mises Stress – Creep - Cap Rock

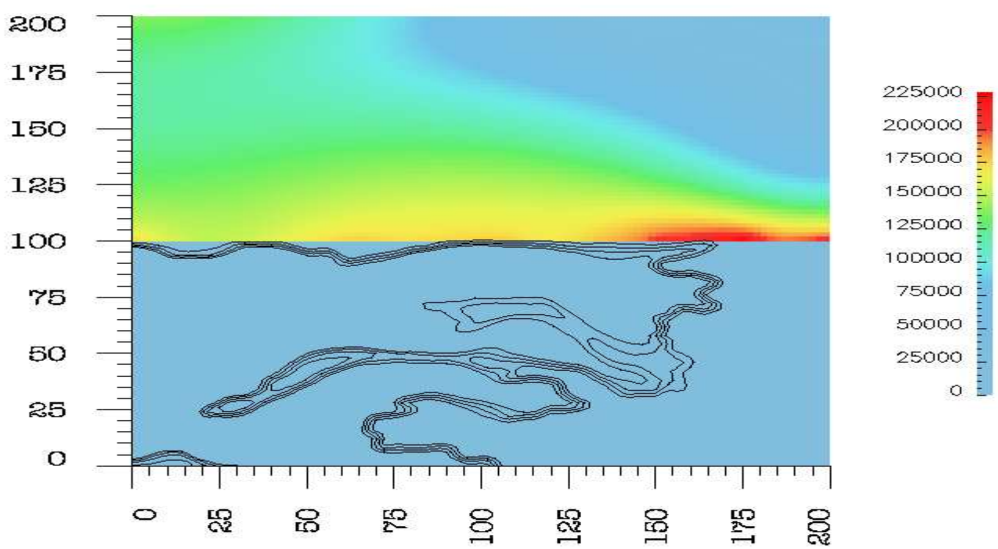
$$\boldsymbol{\sigma}_e = \boldsymbol{C}(\boldsymbol{\mathcal{E}}(\mathbf{u}) - \boldsymbol{\mathcal{E}}_{visc}), \qquad \frac{\partial \boldsymbol{\mathcal{E}}_{visc}}{\partial t} = \boldsymbol{\mathcal{E}}_0 \left(\frac{\boldsymbol{\sigma}_V}{\boldsymbol{\sigma}_0} \right)^n$$

Isothermal Creep Problem: Maia da Costa and Poiate Jr (2008)
Power law parameter n=5
Young Modulus E = 1.0x1010 Pa
Reference Stress $\boldsymbol{\sigma}_0 = 1.0x10^6 Pa$
Time step Δt= 3.6 days



Von - Mises Stress – Creep - Cap Rock

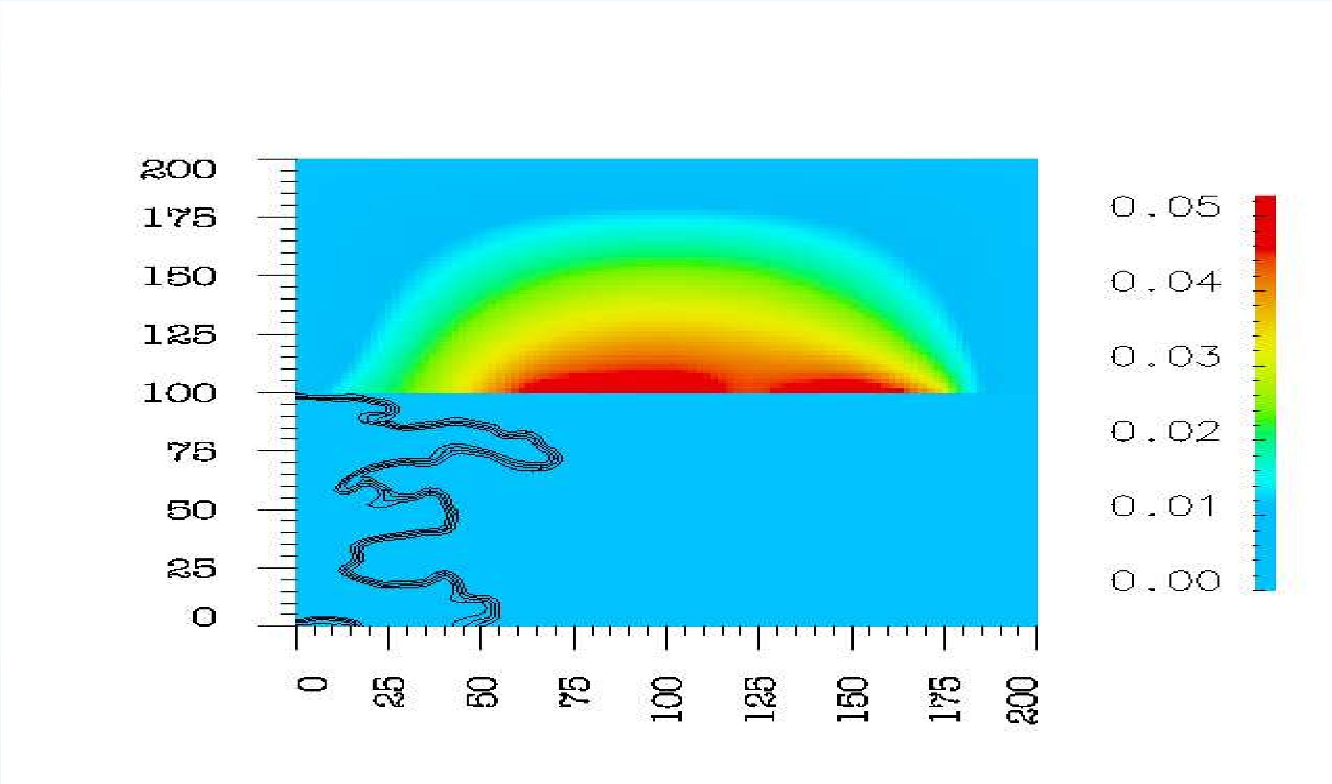




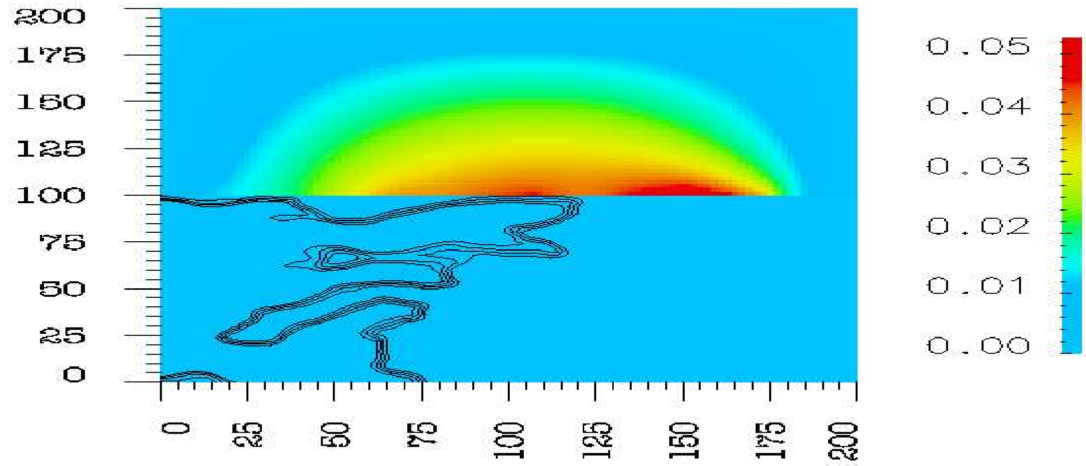
Rate of Viscous Deviatoric Strain – Cap Rock

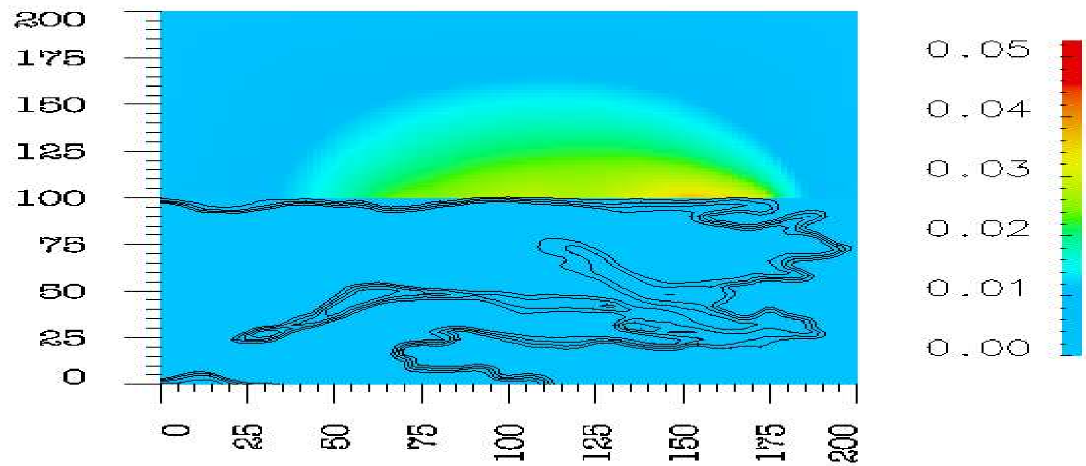
$$\boldsymbol{\sigma} = \boldsymbol{C}(\boldsymbol{\mathcal{E}}(\mathbf{u}) - \boldsymbol{\mathcal{E}}_{visc})$$

$$\frac{\partial \boldsymbol{\mathcal{E}}_{visc}}{\partial t} = C(T)\boldsymbol{\mathcal{E}}_0(\left(\frac{\boldsymbol{\sigma}_V}{\boldsymbol{\sigma}_0}\right)^n$$



Rate of Viscous Deviatoric Strain – Cap Rock





CONCLUDING REMARKS

- – ONE-WAY GEOMECHANICAL COUPLING MAY PROVIDE UNREALISTIC PREDICTIONS IN STRONGLY HETEROGENEOUS MEDIA
- – FEASIBILITY OF INCORPORATING THE VISCOELASTIC GEOMECHANICAL BEHAVIOR OF THE ROCK SALT (CAP ROCK)

FUTURE WORK: 2012

- – GEOMCHANICS: DEEPER INSIGHTS INTO THE SALINE CAP ROCK: INCLUDE PLASTICITY WITH CAP FOR THE CARBONATED ROCK: FAULT REACTIVATION (Collaboration with UPPE)
- – GEOCHEMISTRY
- – EXTENSION TO CARBON DIOXIDE SEQUESTRATION:
- Inclusion of buoyancy effects: Compositional Models: Flash (Water + CO_2: Dissolution/precipitation reactions with the solid minerals: Chemical Damage (Collaboration with UPPE)
- HPC 3D SIMULATIONS
- DATA ASSIMILATION: MARKOV-CHAIN MONTE CARLO: KALMAN FILTER

Characterization of Permeability in Reservoir Models - Markov chain Monte Carlo Method

Equipe LNCC[1]

[1]National Laboratory for Scientific Computing

REDE SIGER
Rede Temática de Simulação e Gerenciamento de Reservatórios

Rio de Janeiro, RJ, August 7, 2012

Outline

2. Introduction
   - Motivation

Outline

- Stochastic Equations

Outline

3 Numerical Results
- Exponential case

Part I

Introduction

Motivation

- Natural porous media:
  - Petroleum reservoirs
  - Aquifers
- Spatial heterogeneity of the rock properties:
  - porosity
  - permeability

Consequence:

Uncertainty in rock properties $\Rightarrow$ uncertainty in reservoir performance or contaminant transport forecasting

Motivation

Accurate prediction × realistic geological model

- **Geological model**
  - static data
    - ✓ well, log and core data
    - ✓ seismic data

The uncertainty can be reduced by introduction of additional data in subsurface modeling

  - dynamic data
    - ✓ well test data
    - ✓ historical pressure data
    - ✓ fractional flow rate
    - ✓ tracer concentration at the sensors

Motivation

- Given the dynamic data F, measured with some precision, we would like to sample k.
- From Bayes' theorem

$$P\left(k|F\right) \propto P\left(F|k\right)P\left(k\right)$$

  - $P\left(k\right)$ is the prior information
  - $P\left(F|k\right)$ is the likelihood
- The objective is the proper sampling from the distribution $\pi(k) = P\left(k|F\right)$

Markov chain Monte Carlo methods (MCMC)

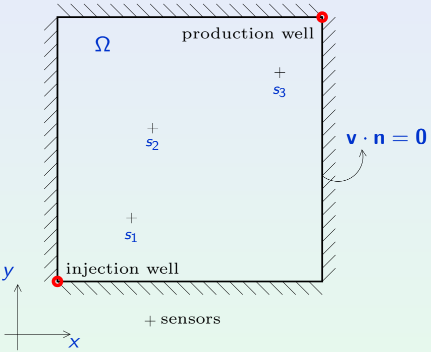
Part II

Stochastic Flow Model

Stochastic Equations

Two-Phase Flow

- Boundary conditions:



- Initial condition:

$$s(\mathbf{x}, 0) = s_{rw}$$

Stochastic Equations

Two-Phase Flow

- $s \mapsto$ water saturation
- $t \mapsto$ dimensionless time PVI

$$\text{PVI} = \int_0^T \left(\frac{\int_{\partial\Omega_{well}} \mathbf{v}_n ds}{V_p} \right) dt$$

  - T is the time for injection
  - V_p is the total pore-volume of the reservoir
- We have observed the sensors until 0.125 PVI

Sampling problem

- Given the tracer concentration $s_{ref}(\mathbf{x}_j, t_k)$ at the sensor j located in $\mathbf{x}_j$ and at time t_k, we set

$$M(s_{ref}) = \{s_{ref}(\mathbf{x}_j, t_k),\ j = 1, 2, 3,\ n = 1, \ldots, 400\} \tag{1}$$

- From Bayes' theorem

$$\pi(k) = P\left(k|M(s_{ref})\right) \propto P\left(M(s_{ref})|k\right) P\left(k\right) \tag{2}$$

- Likelihood

$$P\left(M(s_{ref})|k\right) = \exp\left(-\frac{\sum_{k=1}^{400}\sum_{j=1}^{3}\left[s(\mathbf{x}_j, t_k) - s_{ref}(\mathbf{x}_j, t_k)\right]^2}{2\sigma^2}\right) \tag{3}$$

Two-Stage MCMC [Efendiev et al.(2005)Efendiev, Datta-gupta, Ginting, Ma, ar Christen and Fox(2005)]

Algorithm (Preconditioned MCMC)

Step 1 At k_n generate k' from $q(k'|k_n)$ (proposal distribution)

Step 2 Take the real proposal as

$$k = \begin{cases} k' & \text{with probability } g(k_n, k') \\ k_n & \text{with probability } 1 - g(k_n, k') \end{cases}$$

$$g(k_n, k') = \min\left(1, \frac{q(k_n|k')\,\pi^*(k')}{q(k'|k_n)\,\pi^*(k_n)}\right),$$

Step 3 Accept k with probability

$$\rho(k_n, k) = \min\left(1, \frac{\pi(k)\pi^*(k_n)}{\pi(k_n)\pi^*(k)}\right)$$

take, $k_{n+1} = k$ with probability $\rho(k_n, k)$ and $k_{n+1} = k_n$ with probability $1 - \rho(k_n, k)$

Part III

Numerical Results

Exponential covariance function:

$$\mathcal{C}_Y(\mathbf{x}, \mathbf{y}) = \sigma^2 \exp\left(-\frac{|\mathbf{x} - \mathbf{y}|}{L}\right),$$

with $L = 0.2$ and $\sigma^2 = 1.0$

Numerical Results

Exponential case

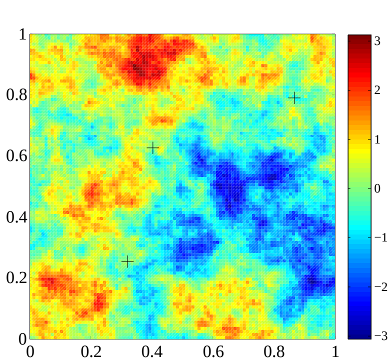


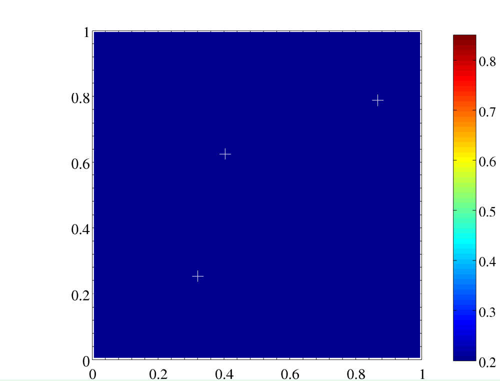
Figure 1: Reference Log-permeability field. $L = 0.2$ and $\sigma^2 = 1.0$.

Two-Phase Flow

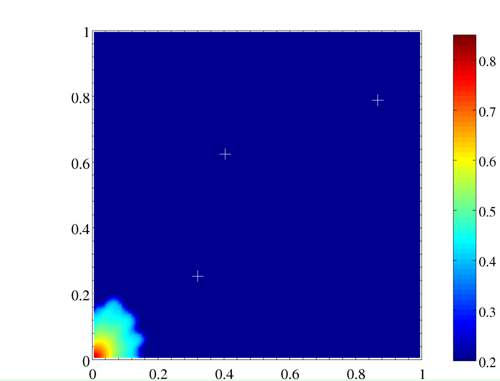
Model Problem:

Two-Phase Flow

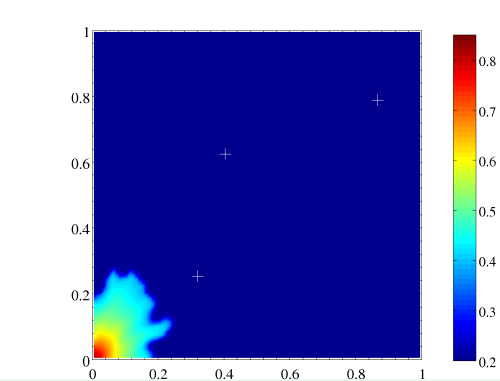
Two-Phase Flow



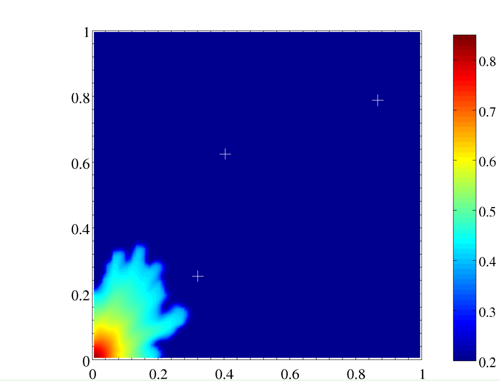
Two-Phase Flow



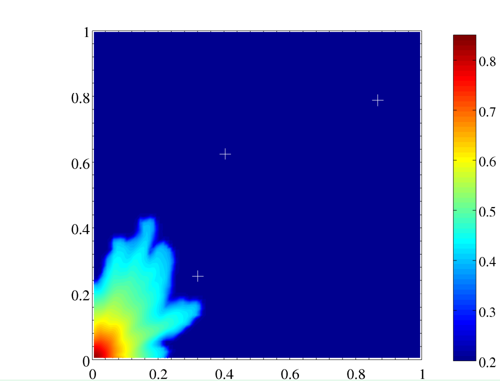
Two-Phase Flow



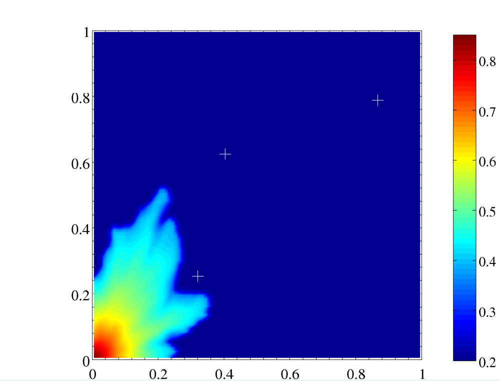
Two-Phase Flow



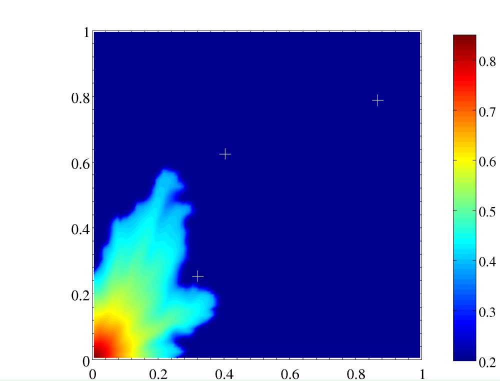
Two-Phase Flow



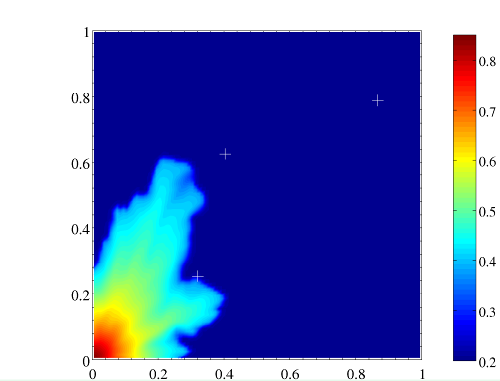
Two-Phase Flow



Two-Phase Flow



Two-Phase Flow



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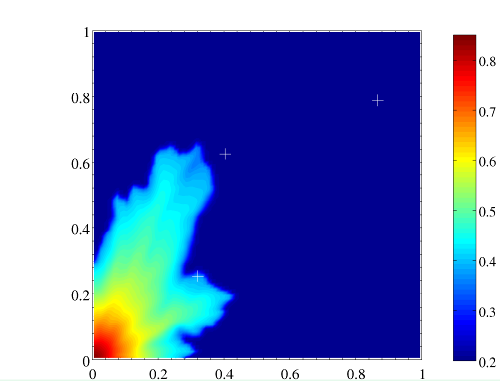

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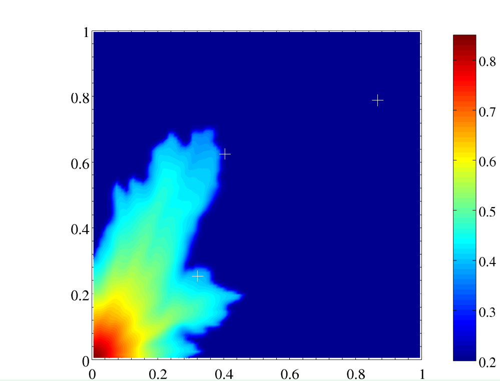

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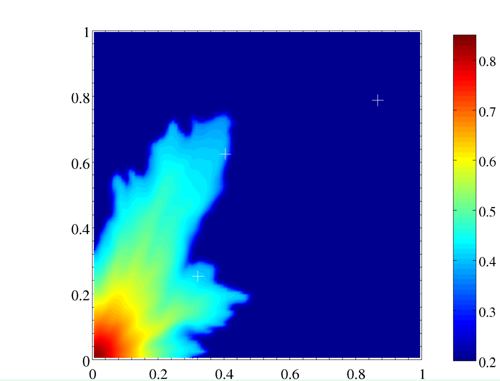

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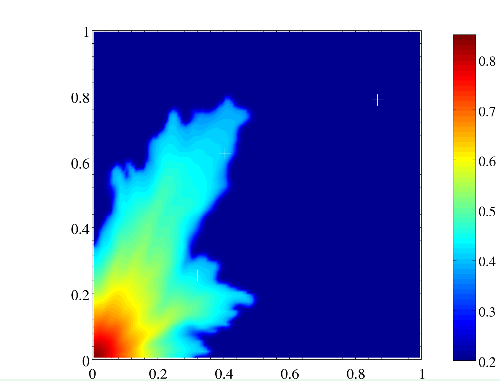

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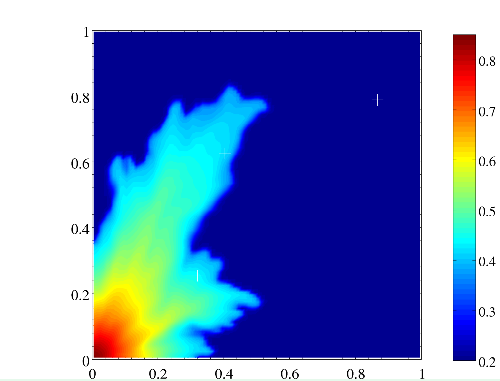

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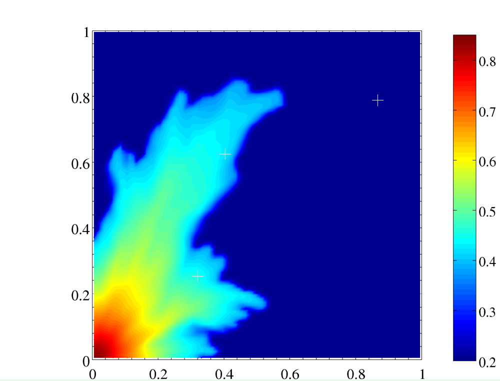

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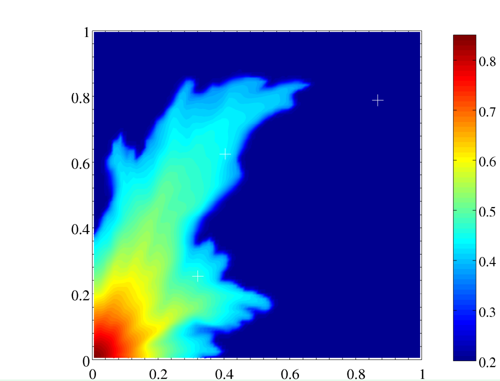

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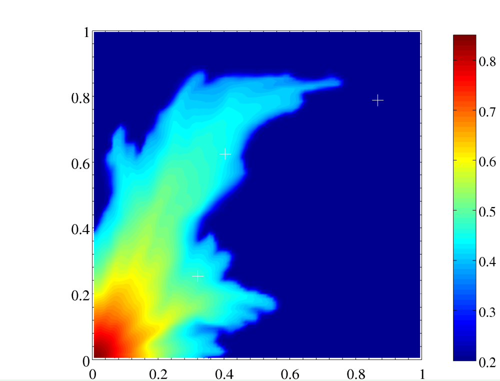

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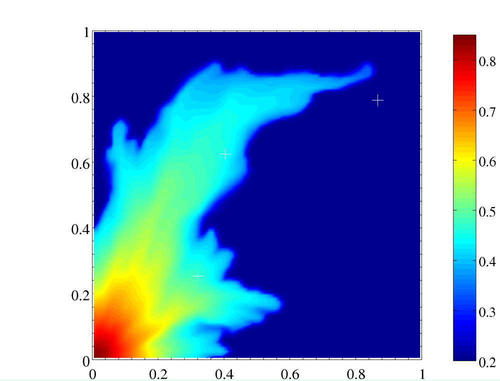

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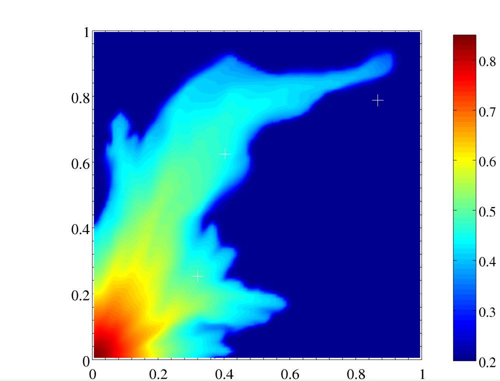

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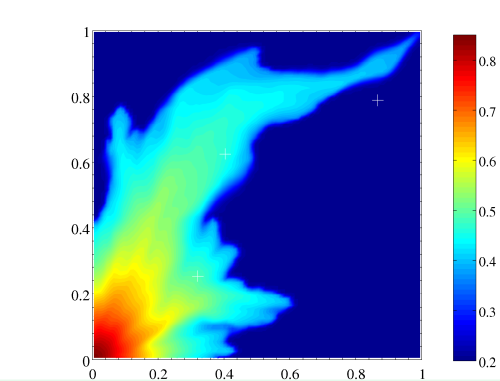

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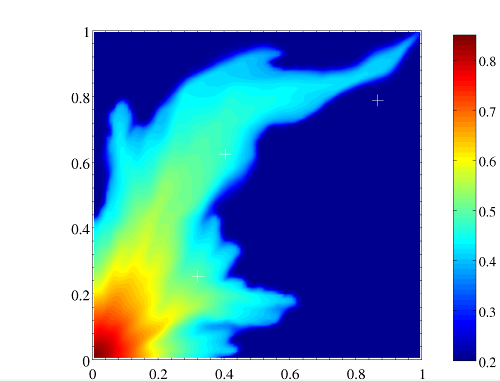

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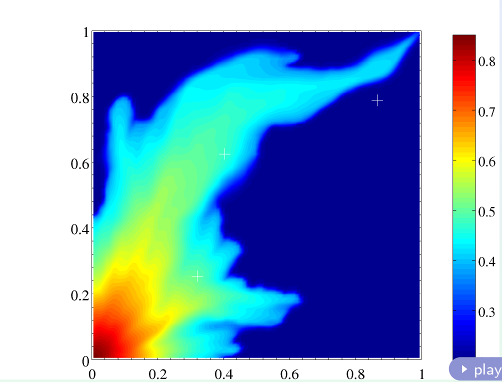
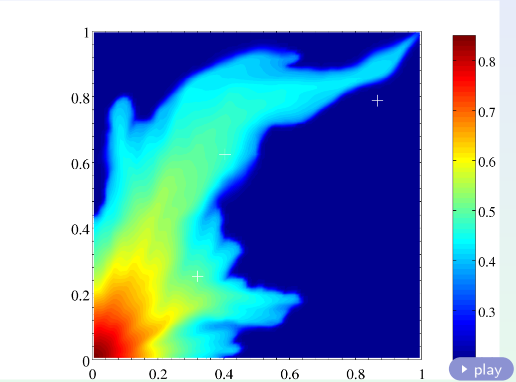


play

Two-Phase Flow



Numerical Results

Exponential case - LABTRAN-GEO

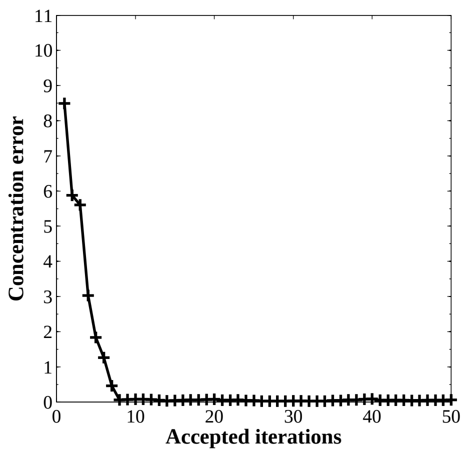


Figure 2: Concentration errors *versus* accepted iterations.

Numerical Results
Exponential case - LABTRAN-GEO

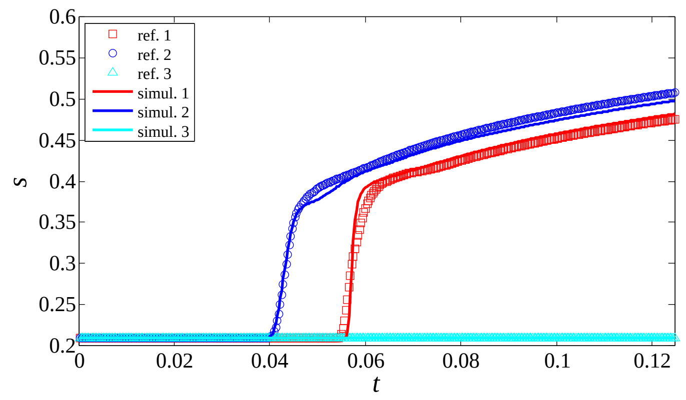
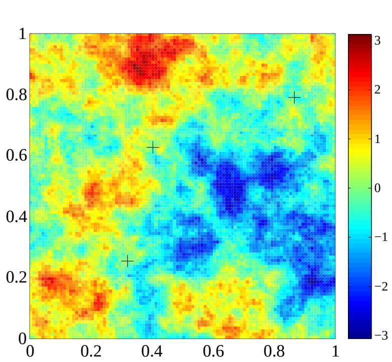


Figure 3: Concentration at different time instances at three sensors.

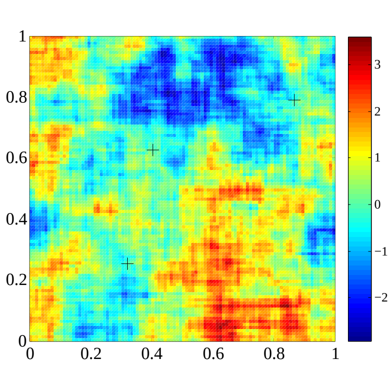
Numerical Results

Exponential case - LABTRAN-GEO

Reference field



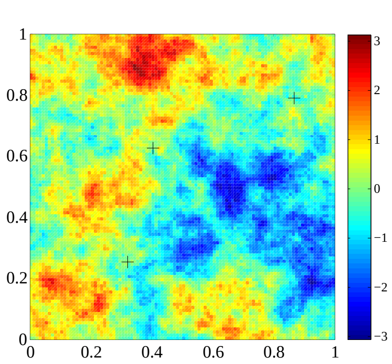
Initial field



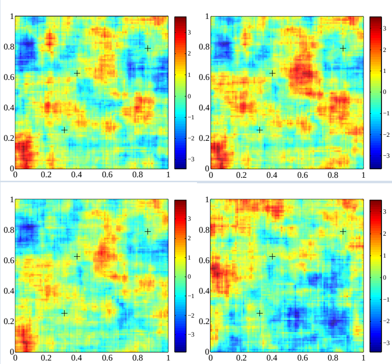
Numerical Results

Exponential case - LABTRAN-GEO

Reference field

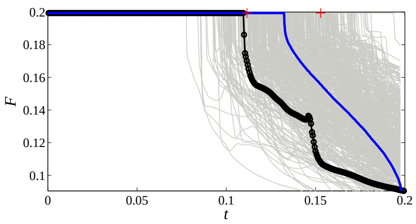


The last 4 accepted realizations

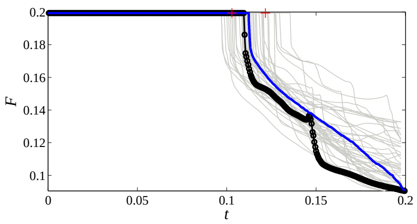


Numerical Results

Exponential case - LABTRANGEO



(a) Mean for ensemble



(b) Mean for selected fields

Figure 4: Reference fractional flow (black), fractional flows corresponding to sampled permeability fields (gray) and mean fractional flow (blue).

Thank you!

References I

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Y. Efendiev, A. Datta-gupta, V. Ginting, X. Ma, and B. Mallick.
An efficient two-stage markov chain monte carlo method for dynamic data integration.
wrr, 41:12423, 2005.

Escoamento Bifásico 3D (água-óleo)

Experimento 1: Slab

- $\mu_w = 1.0$ e $\mu_o = 10.0$
- Domínio $\Omega = [0.4 \times 0.2 \times 1.0]$
- Malha computacional $[40 \times 20 \times 100]$
- Meio heterogêneo

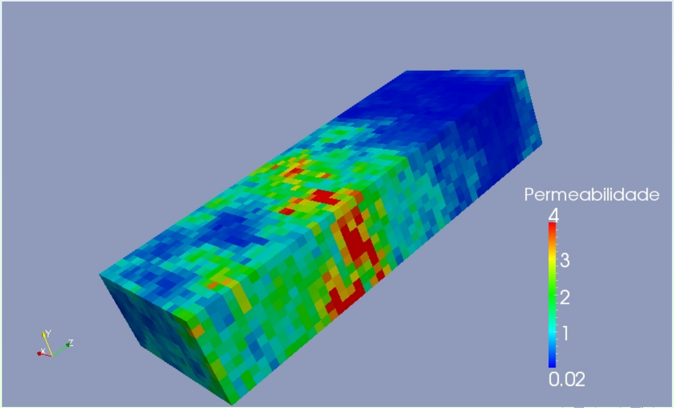
Experimento 2: 1/4 de Five Spot

- $\mu_w = 1.0$ e $\mu_o = 10.0$
- Domínio $\Omega = [0.2 \times 1.0 \times 1.0]$
- Malha computacional $[10 \times 50 \times 50]$
- Meio heterogêneo

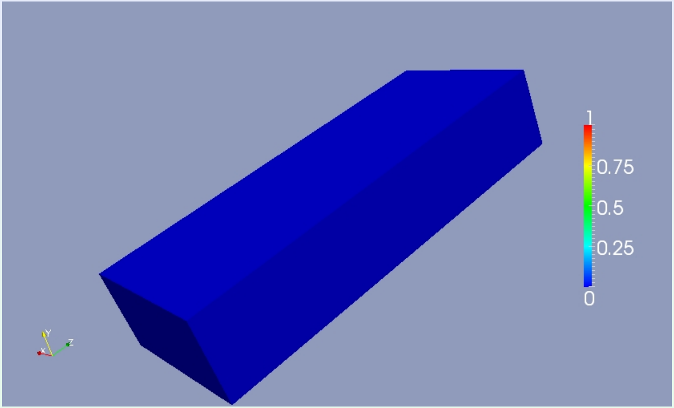
Escoamento Bifásico - Slab

Campo de permeabilidade heterogêneo

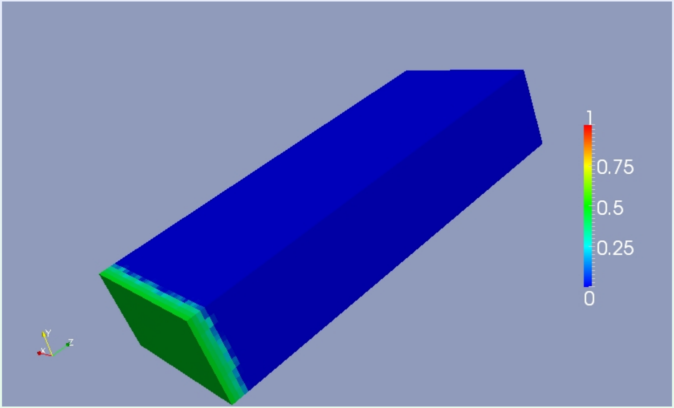
- $\langle k = 1.12146 \rangle$
- $k \in [0.02216, 11.03055]$
- $\sigma_k^2 = 1.62$



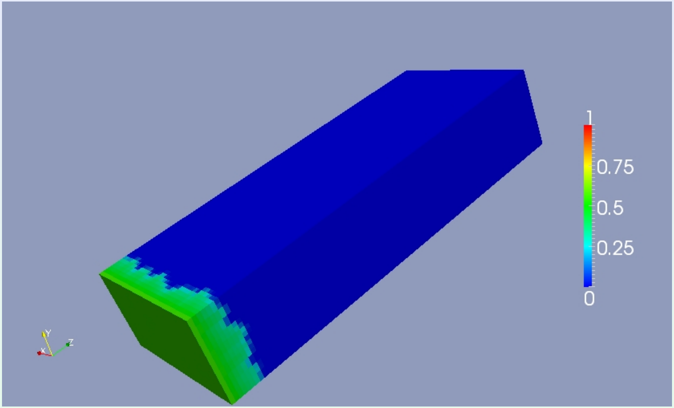
Escoamento Bifásico - Slab - Heterogêneo



Escoamento Bifásico - Slab - Heterogêneo



Escoamento Bifásico - Slab - Heterogêneo



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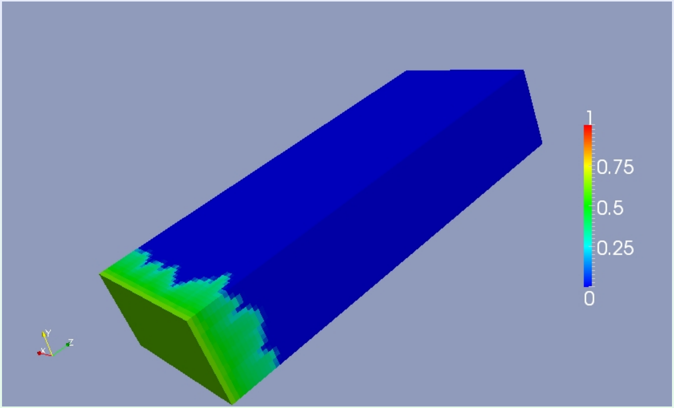
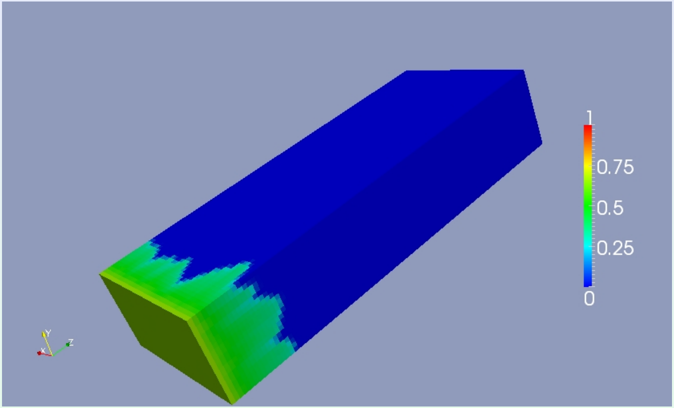


Escoamento Bifásico - Slab - Heterogêneo



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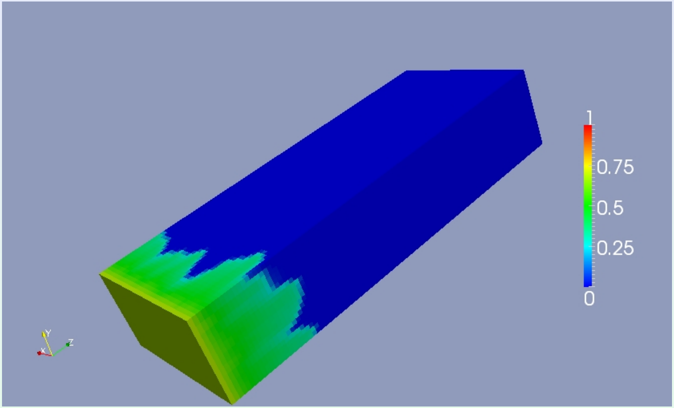

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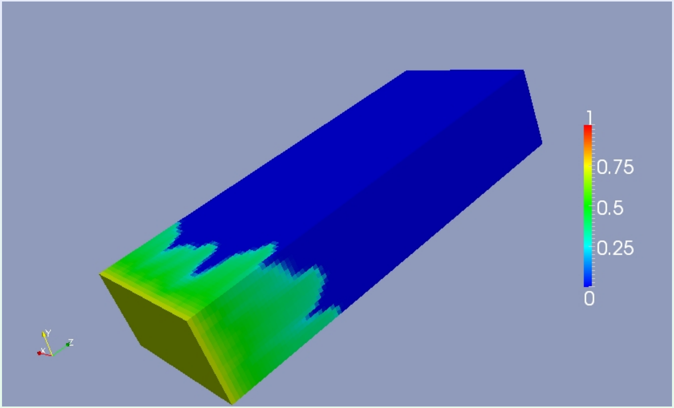

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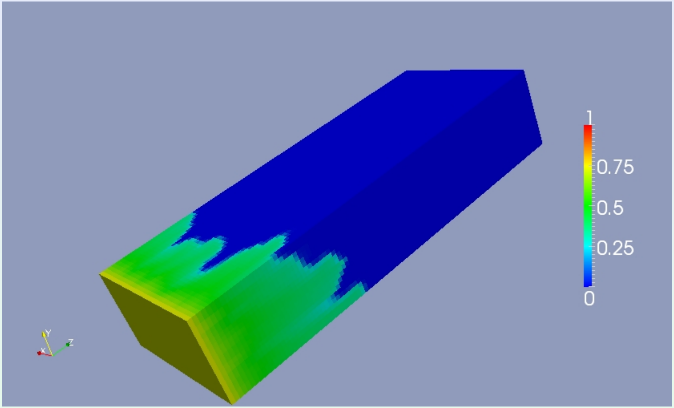

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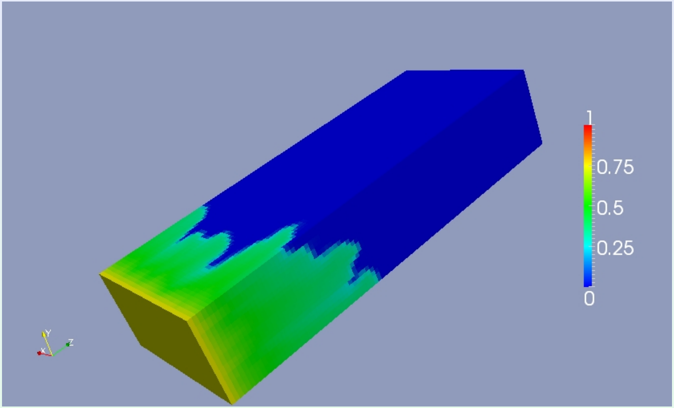

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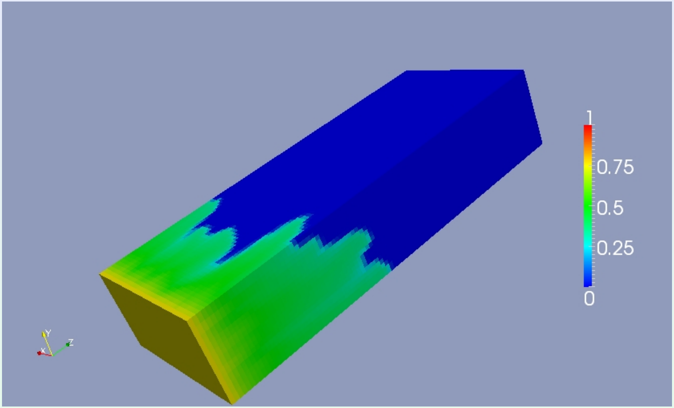

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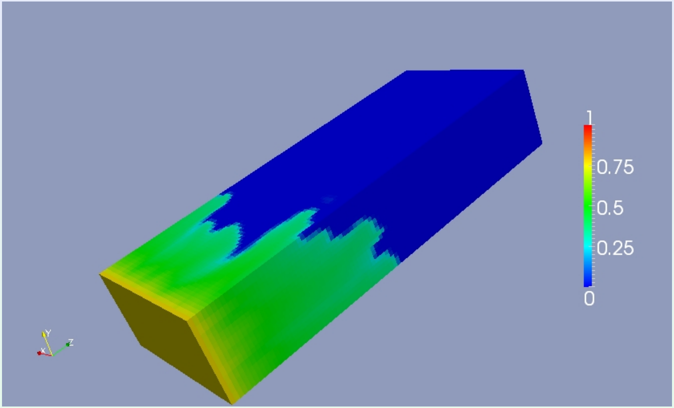

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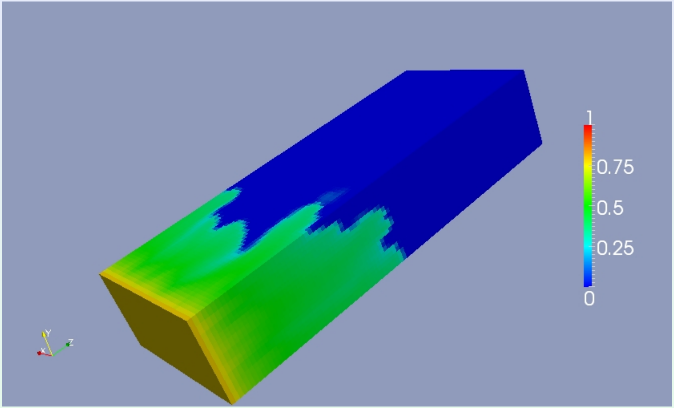
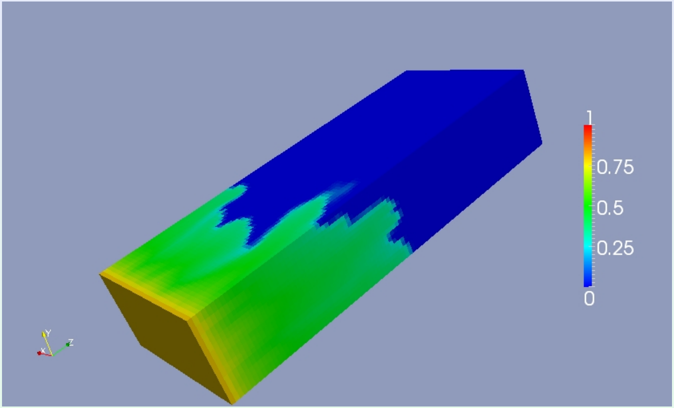


Escoamento Bifásico - Slab - Heterogêneo



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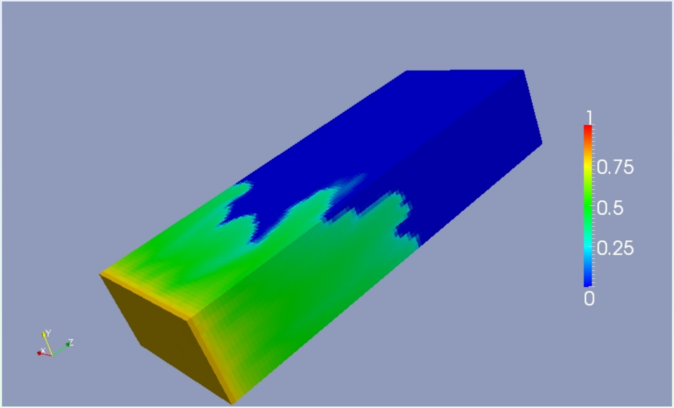

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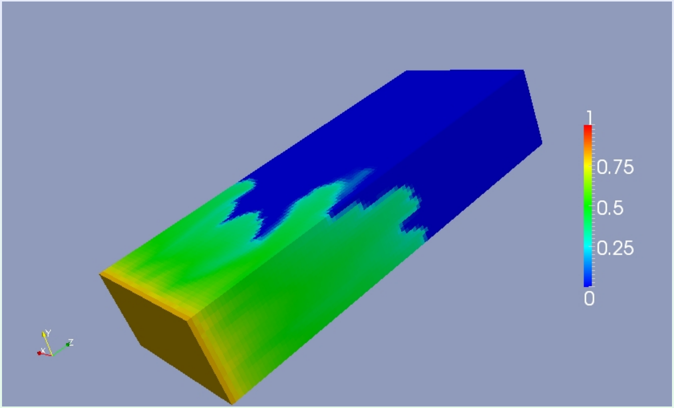

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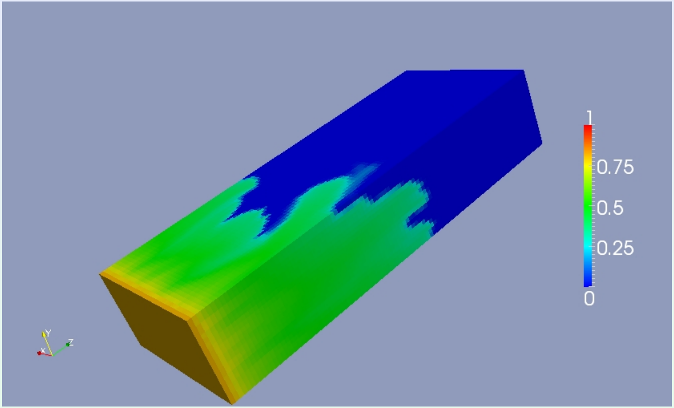

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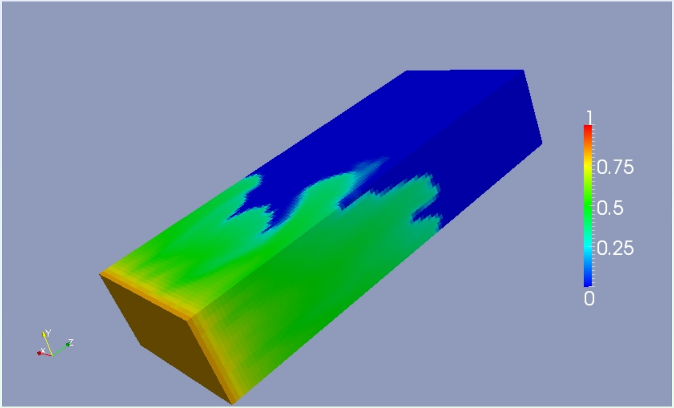

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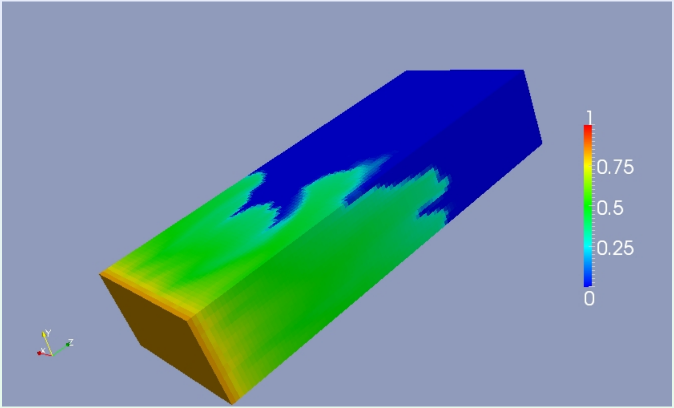

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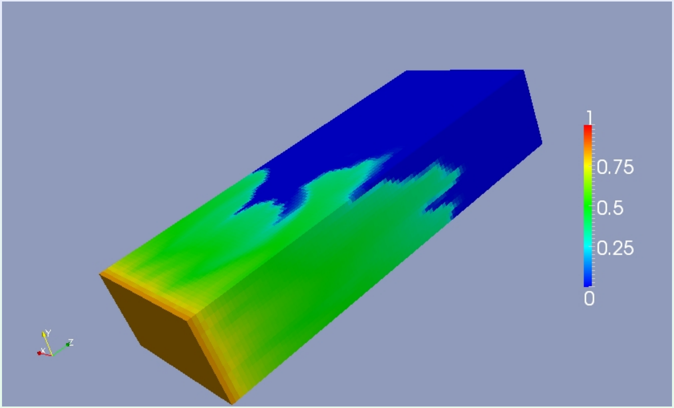

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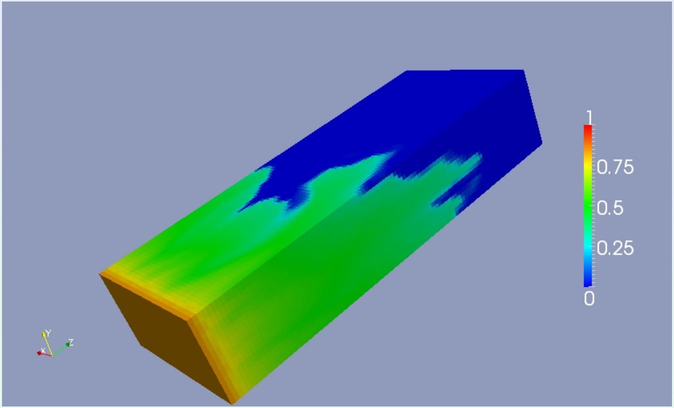

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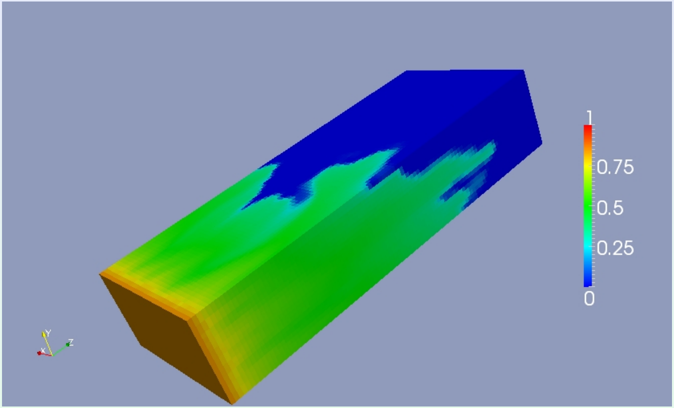

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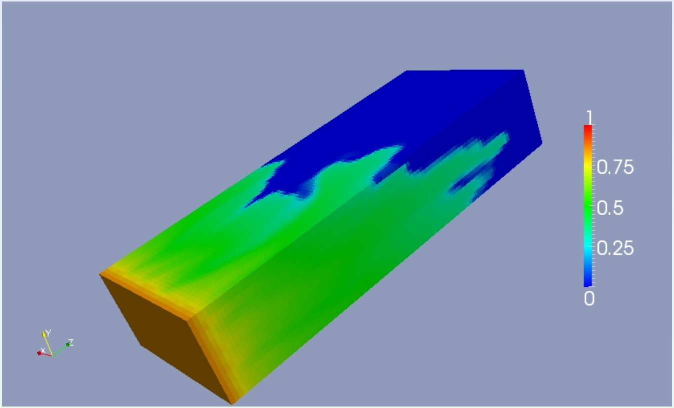

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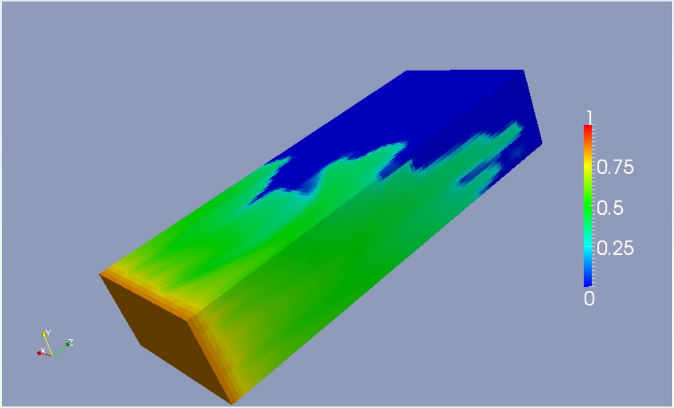

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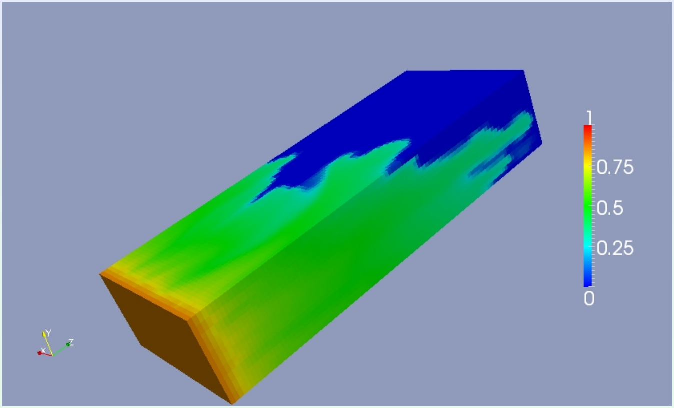

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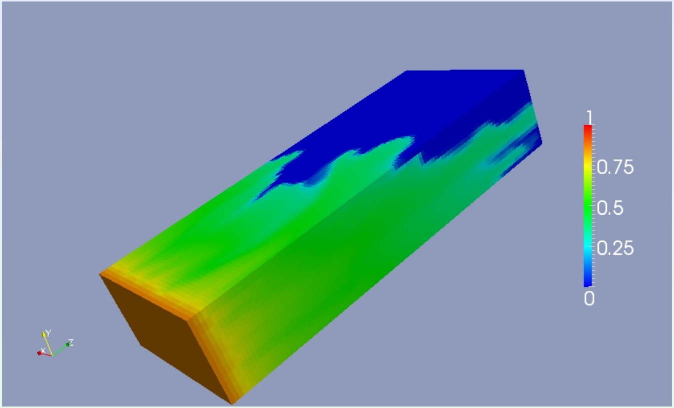

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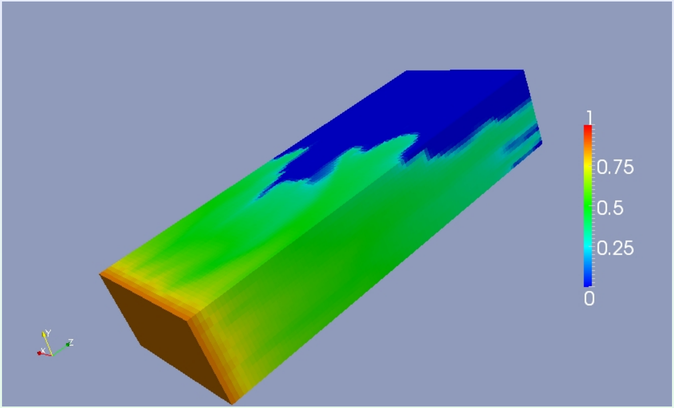

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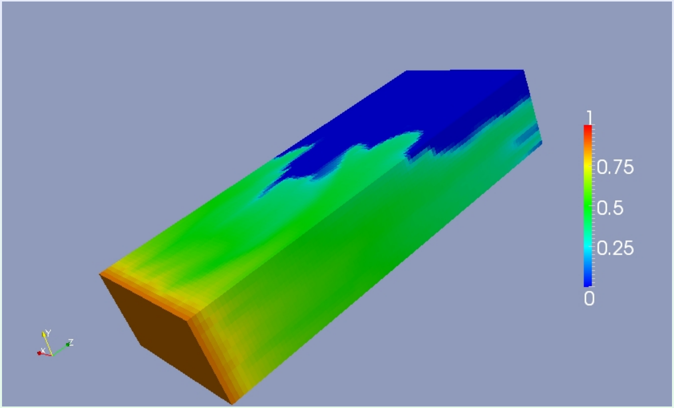

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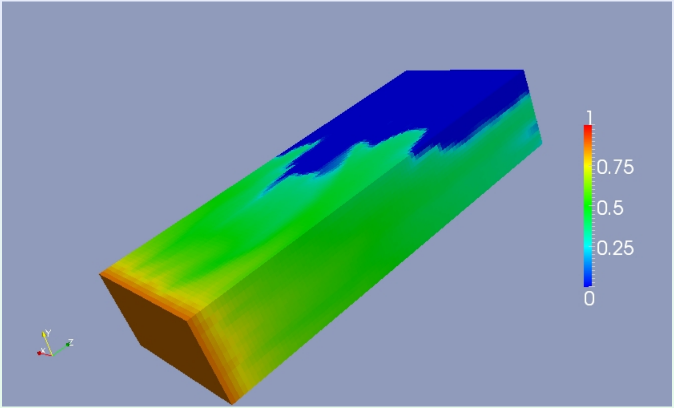

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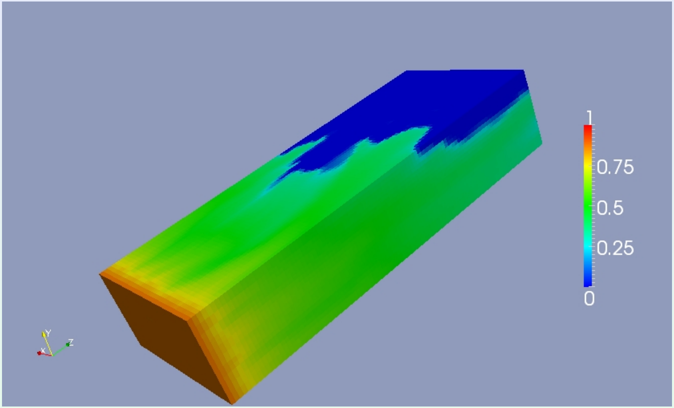

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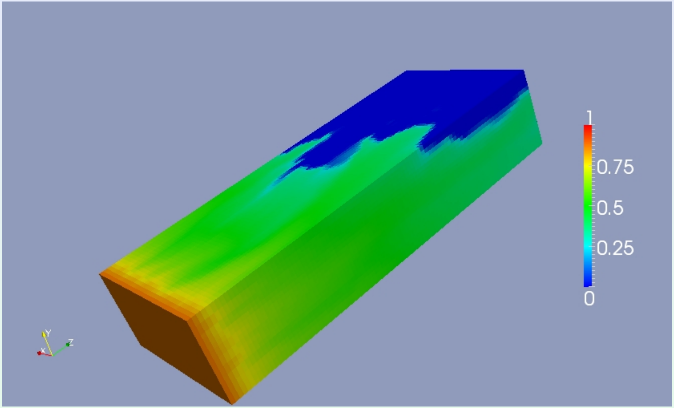

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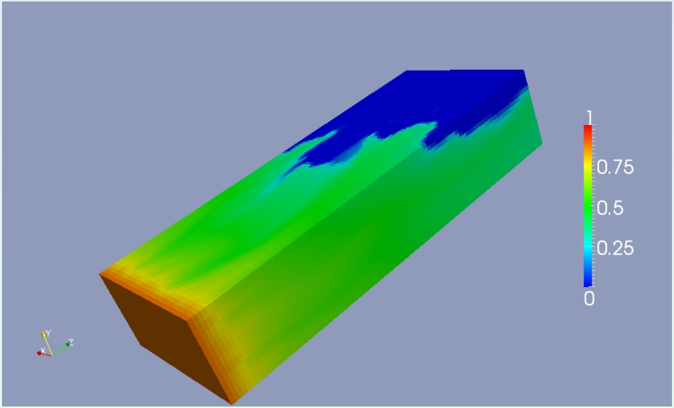

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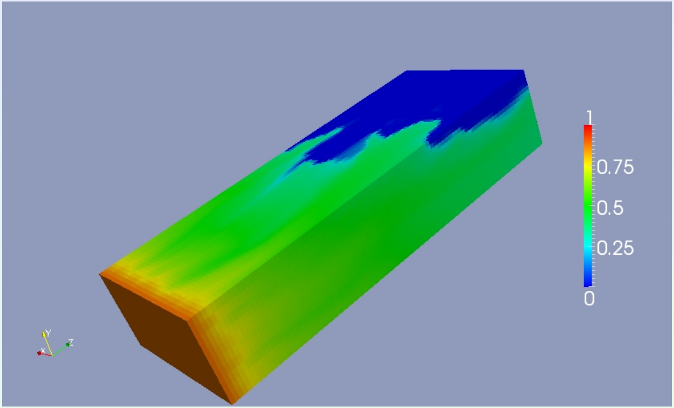

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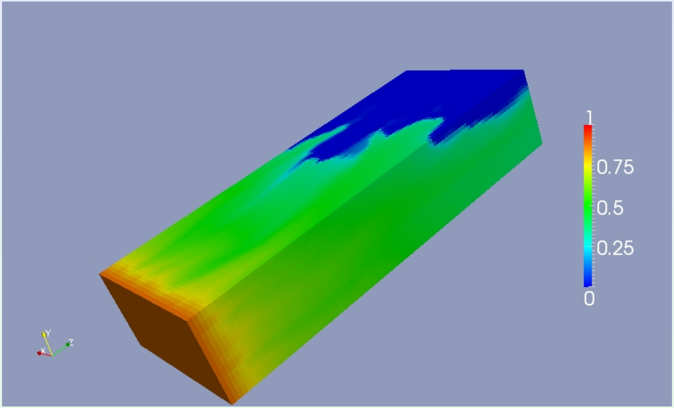
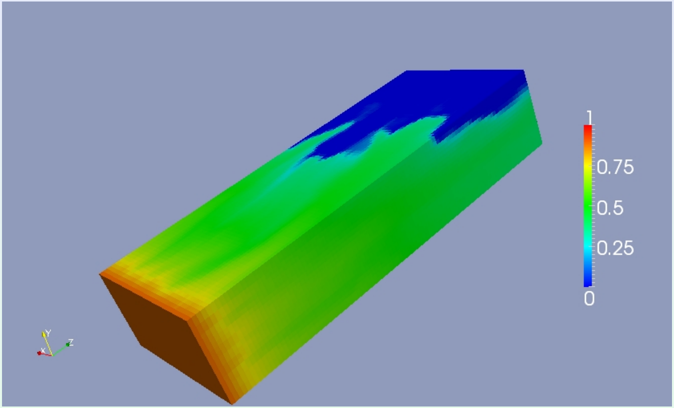
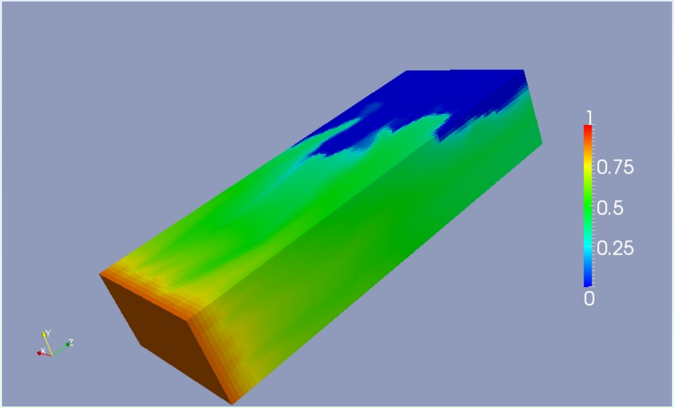


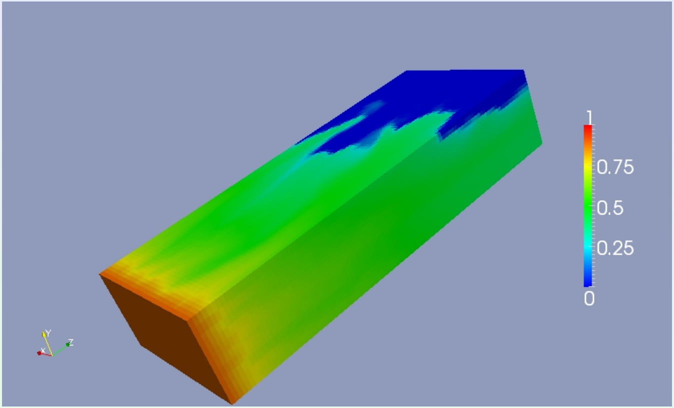
Escoamento Bifásico - Slab - Heterogêneo



Escoamento Bifásico - Slab - Heterogêneo



Escoamento Bifásico - Slab - Heterogêneo



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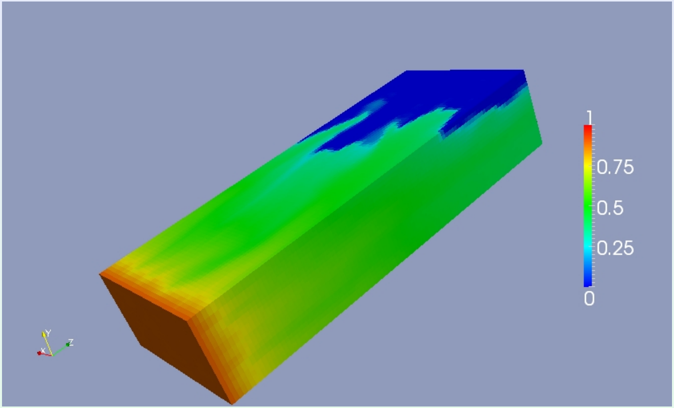

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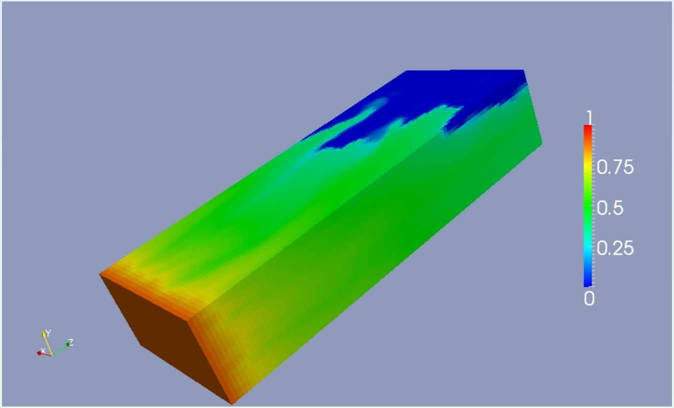

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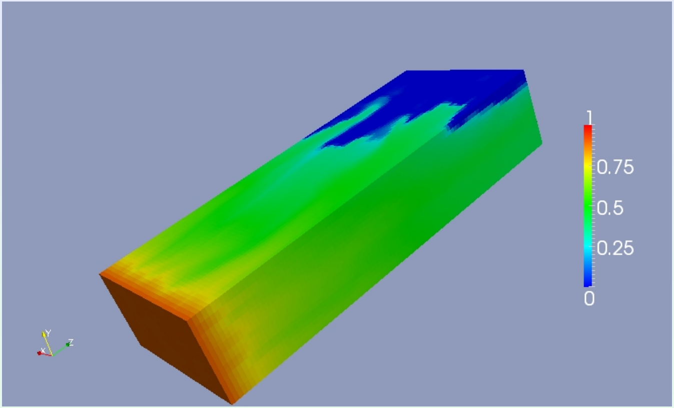
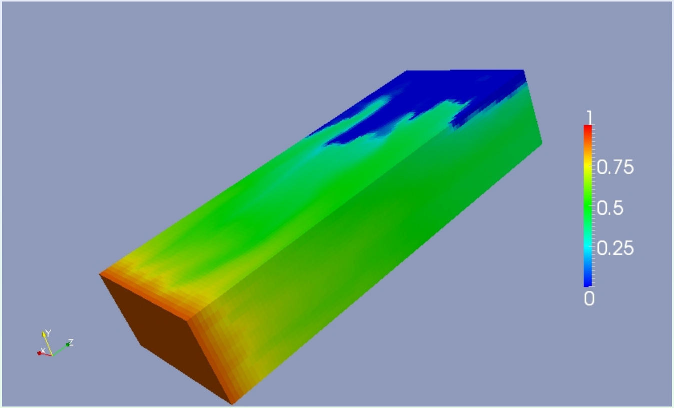
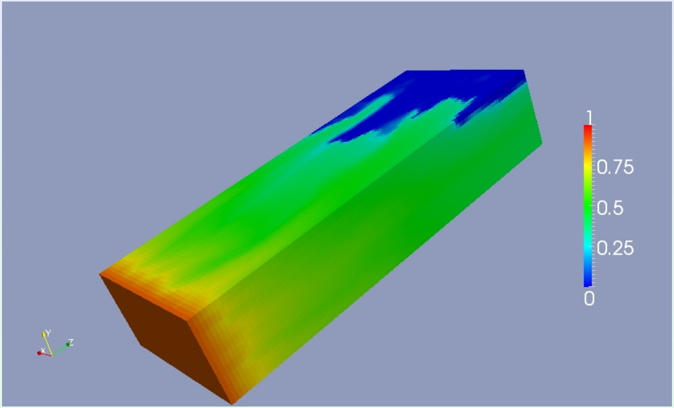


Escoamento Bifásico - Slab - Heterogêneo

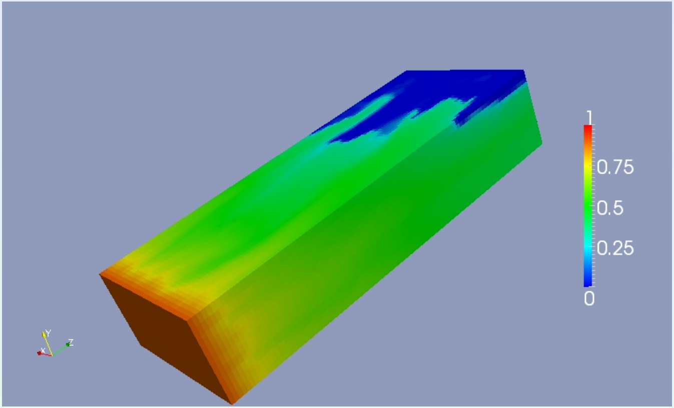


Escoamento Bifásico - Slab - Heterogêneo



play

Escoamento Bifásico - Slab - Heterogêneo

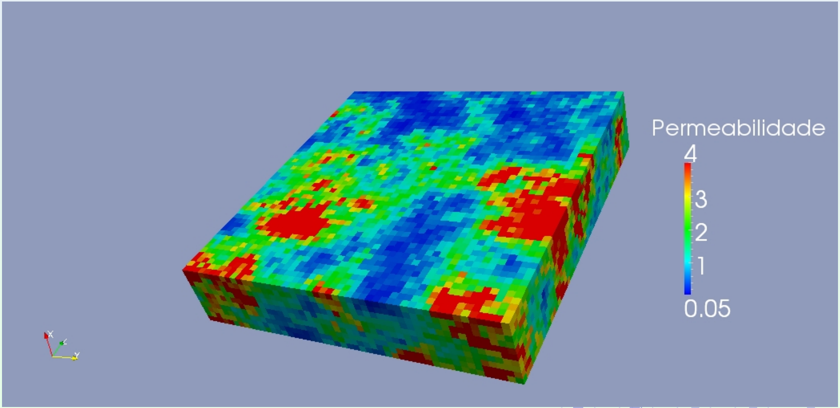


play

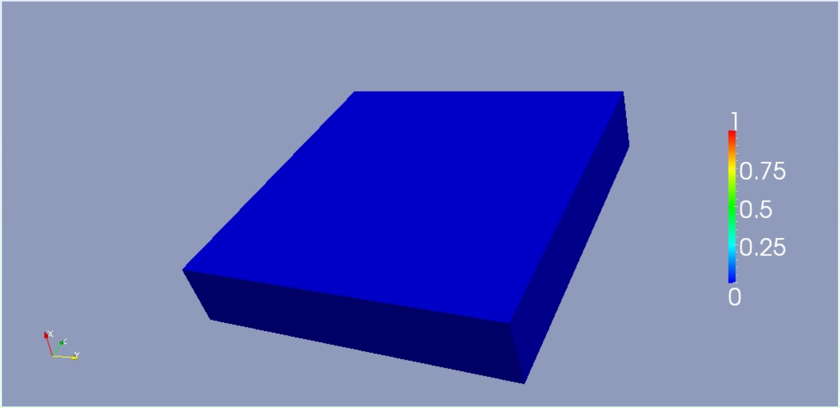
Escoamento Bifásico - 1/4 de Five-Spot

Campo de permeabilidade heterogêneo

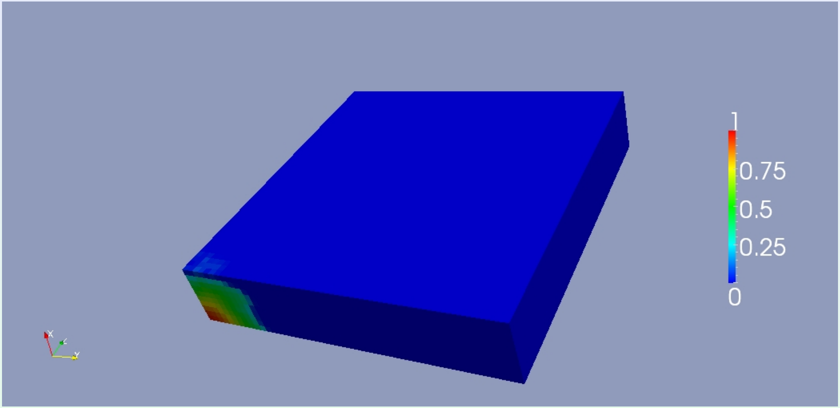
- $\langle k = 1.42886 \rangle$
- $k \in [0.05976, 16.01276]$
- $\sigma_k^2 = 1.98$



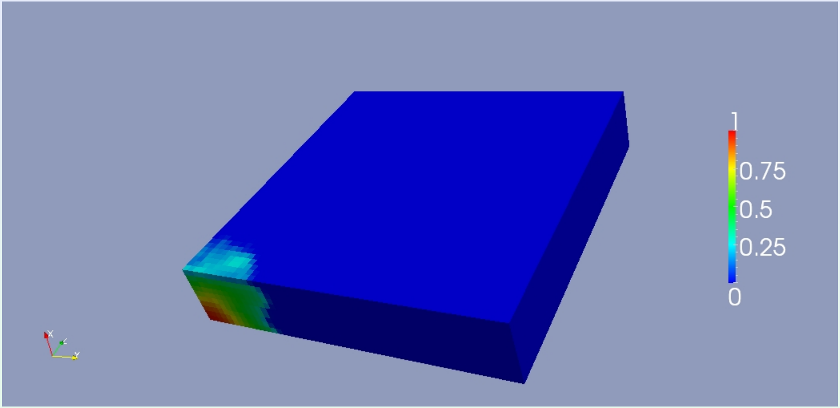
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



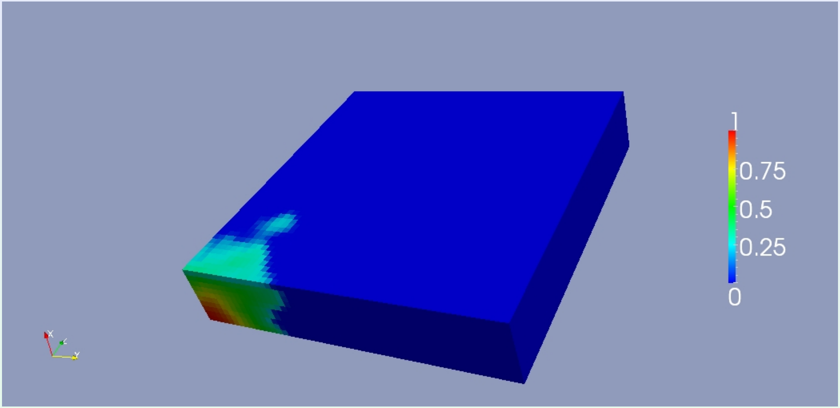
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



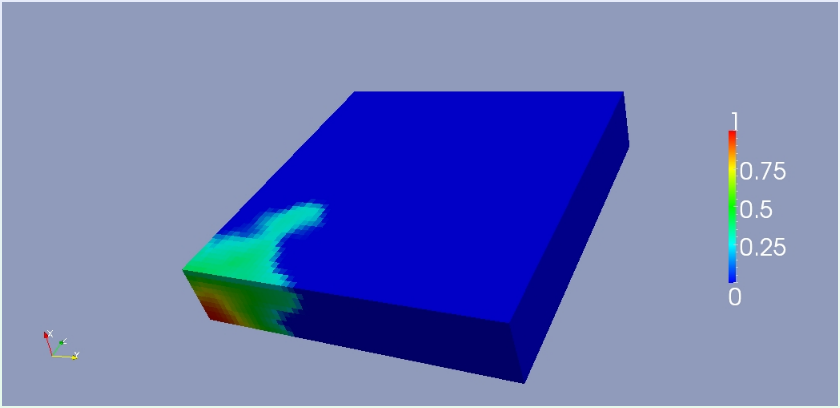
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



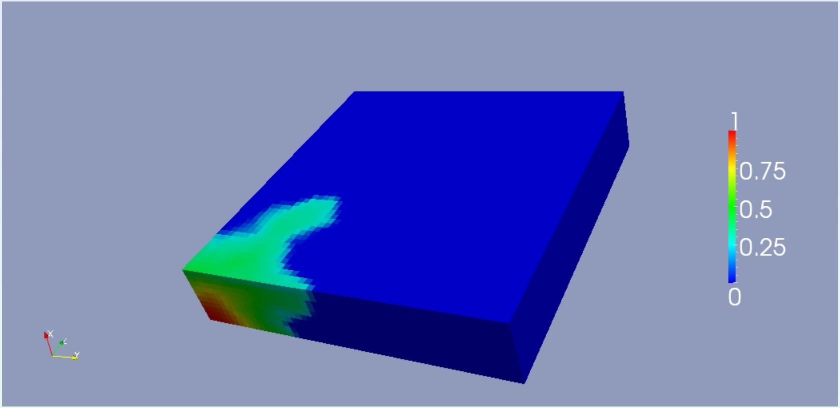
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



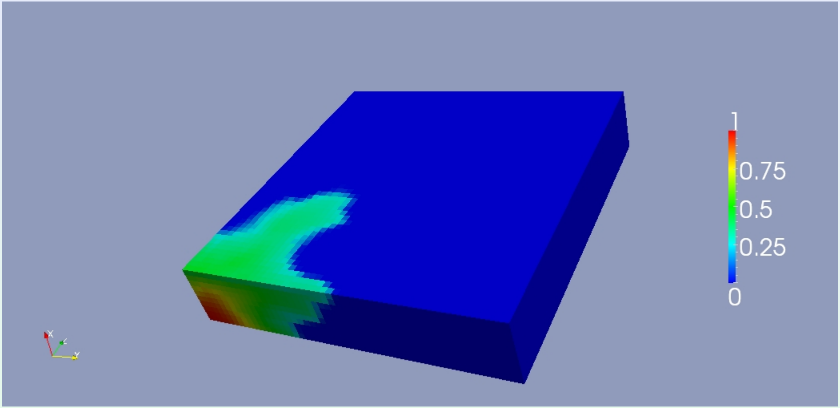
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



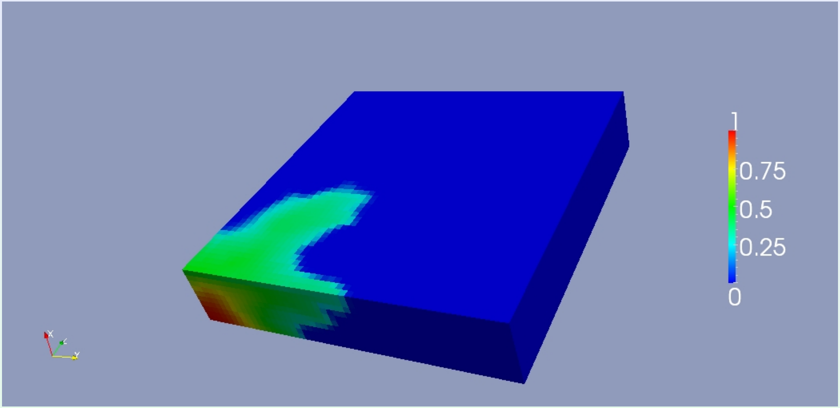
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



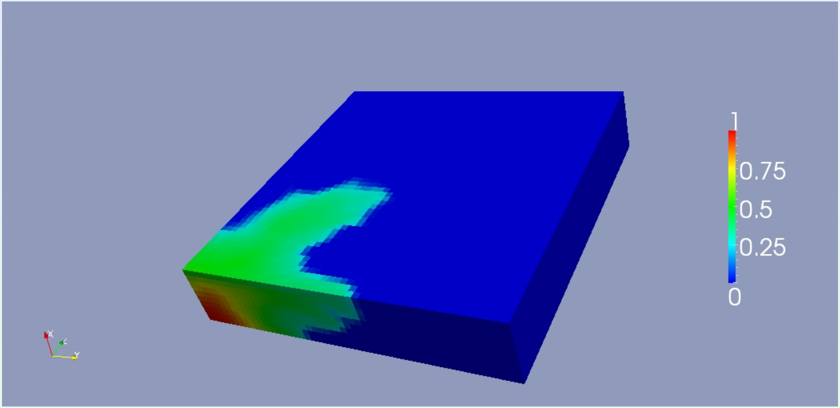
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



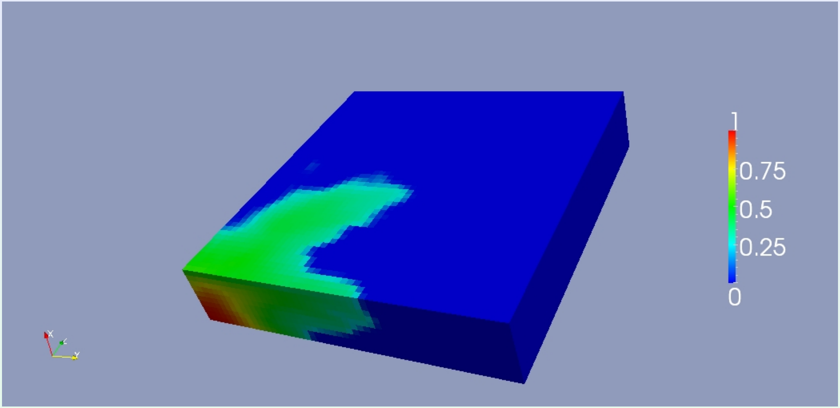
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



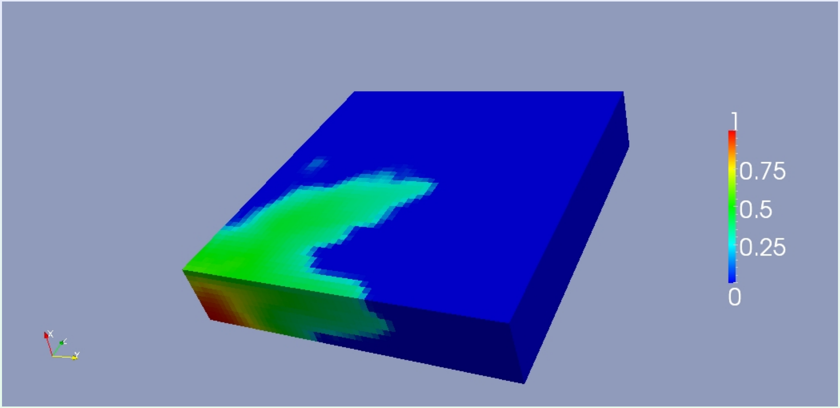
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



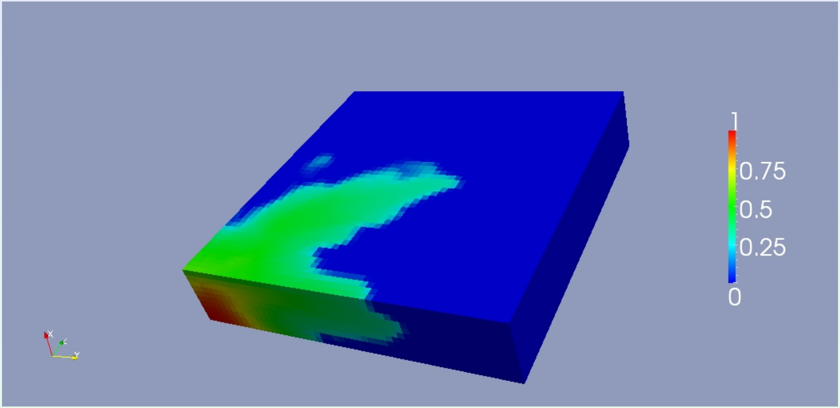
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



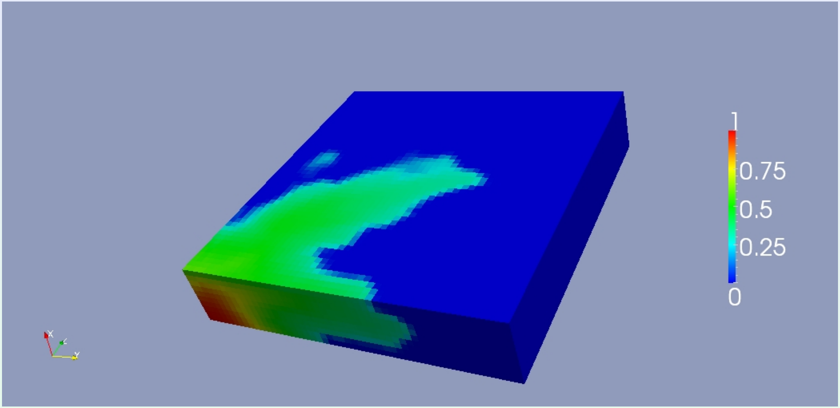
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



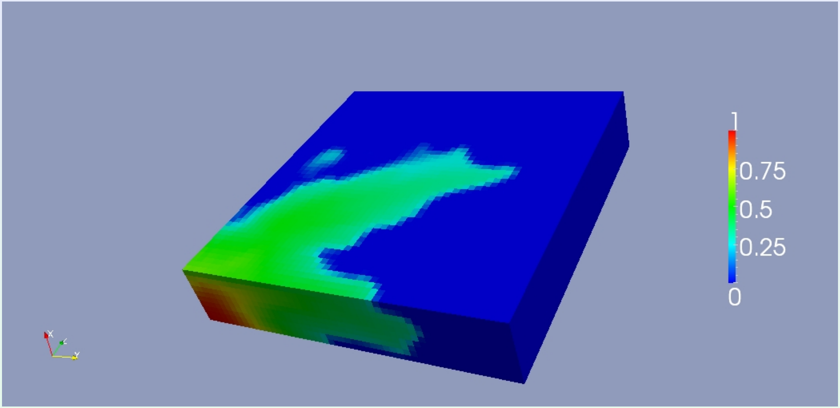
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



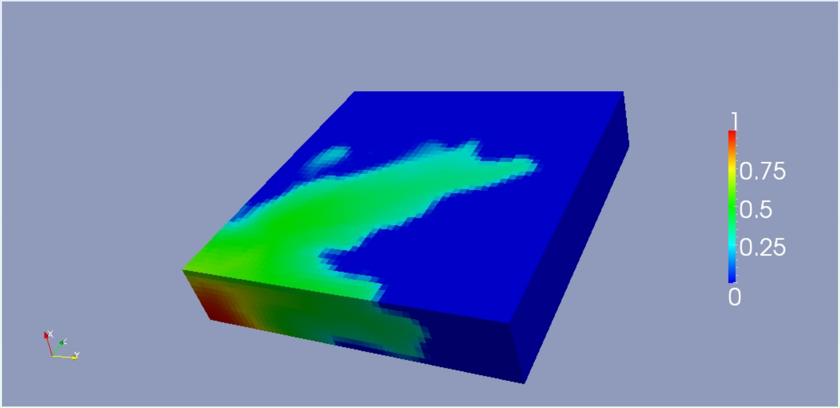
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



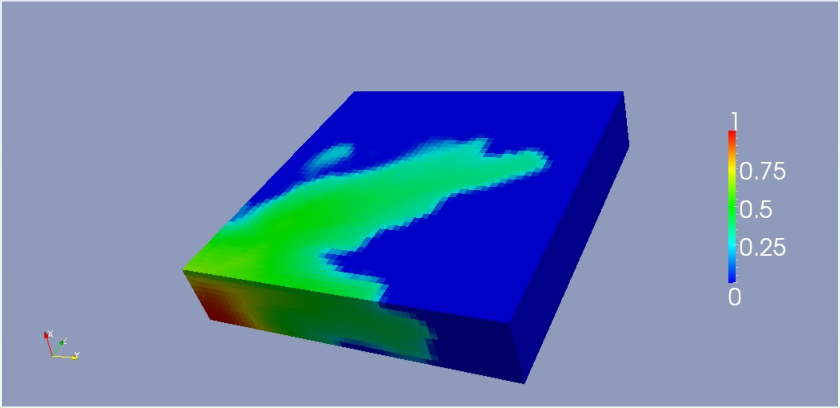
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



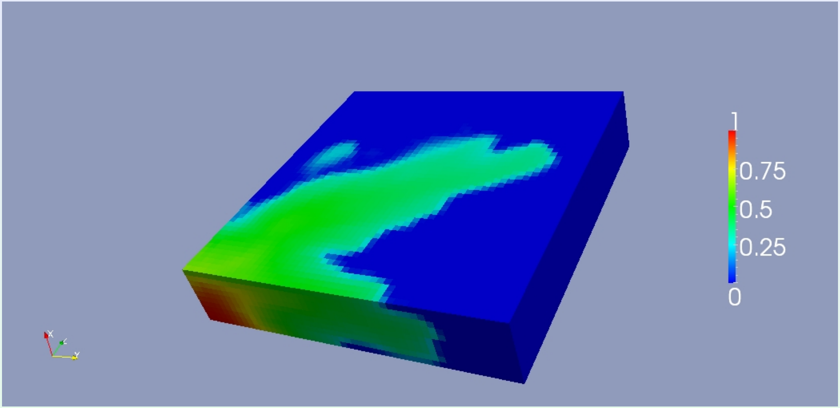
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



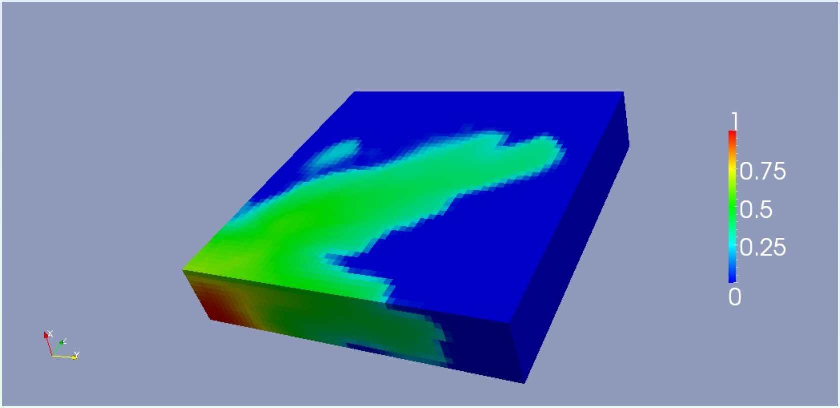
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



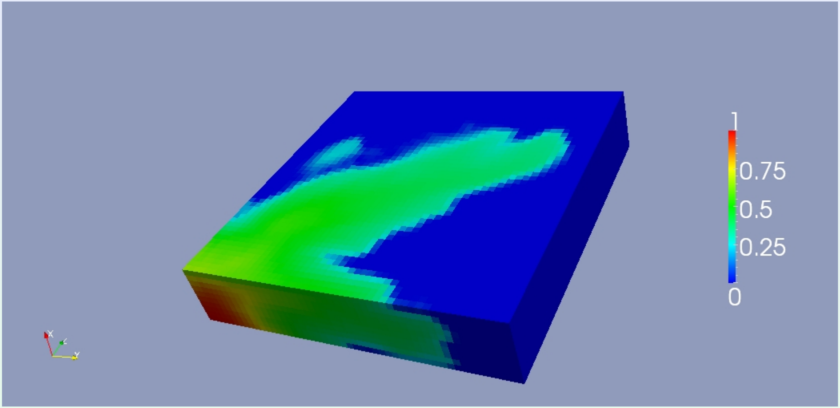
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



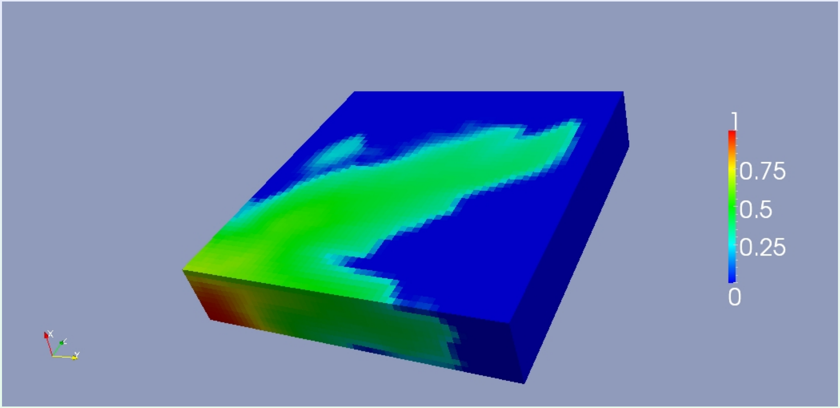
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



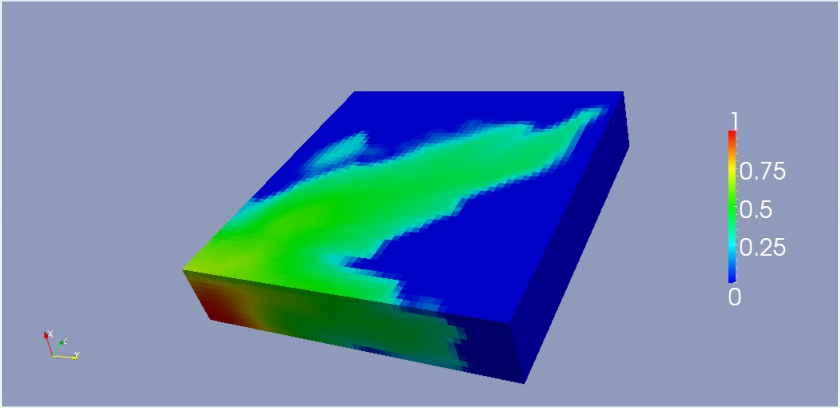
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



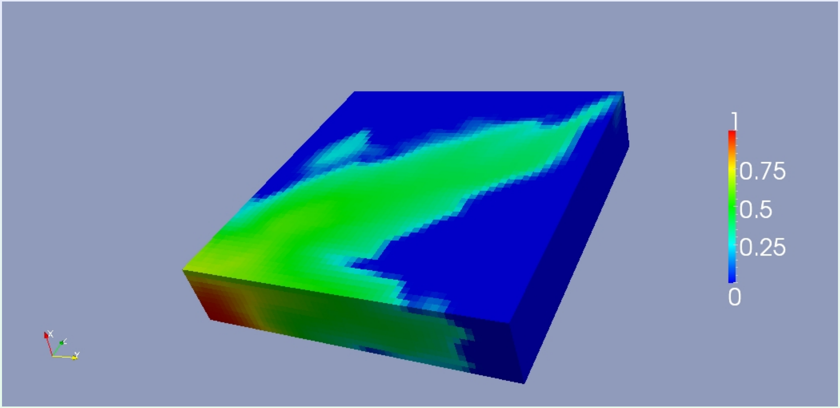
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



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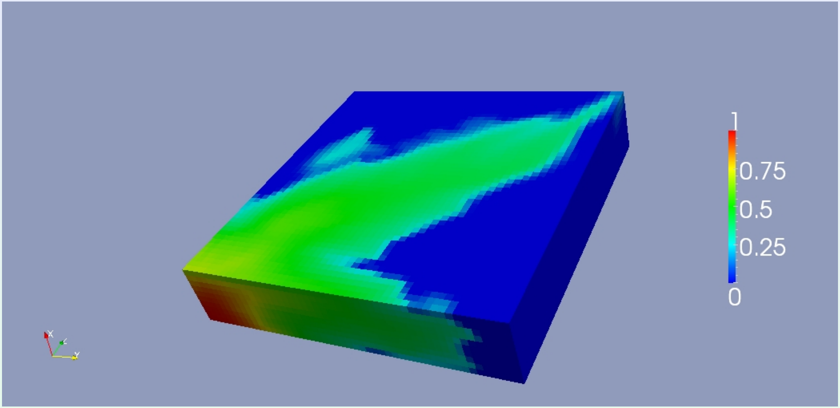
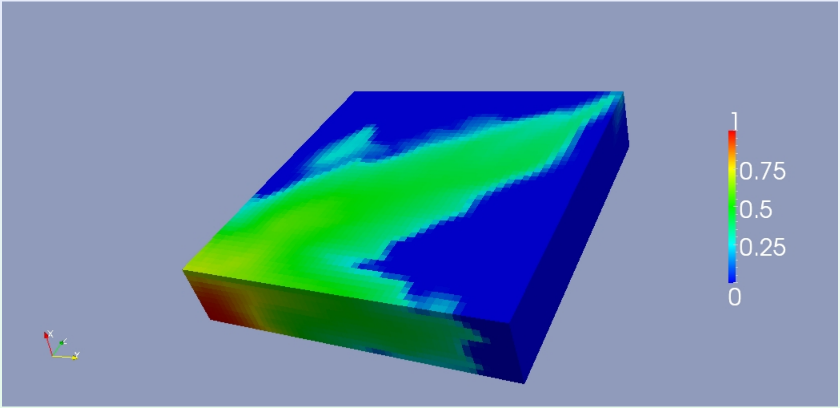
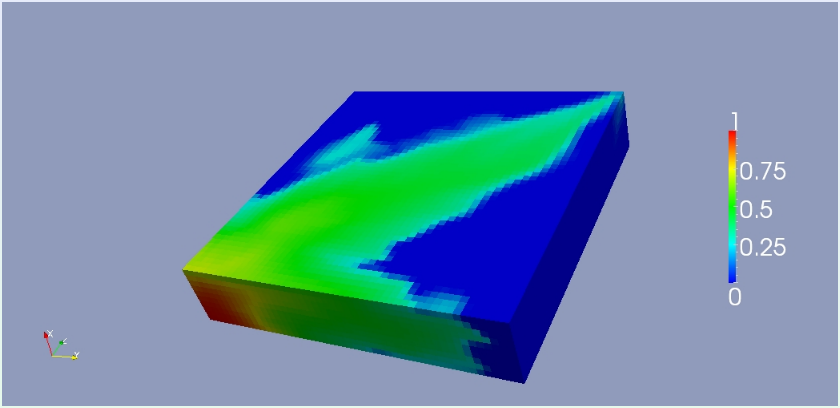


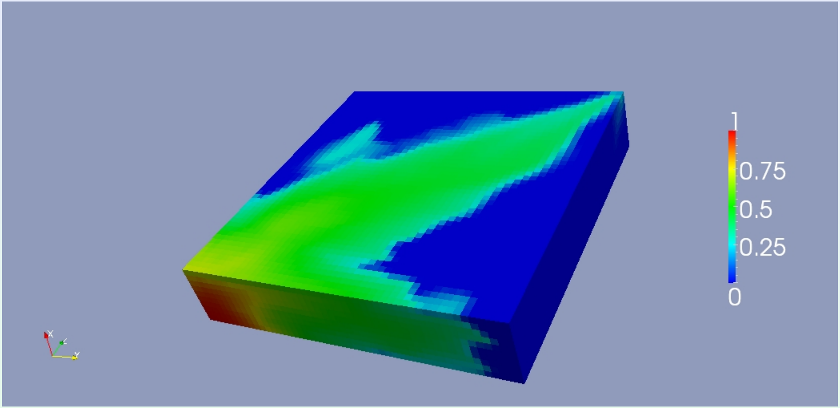
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



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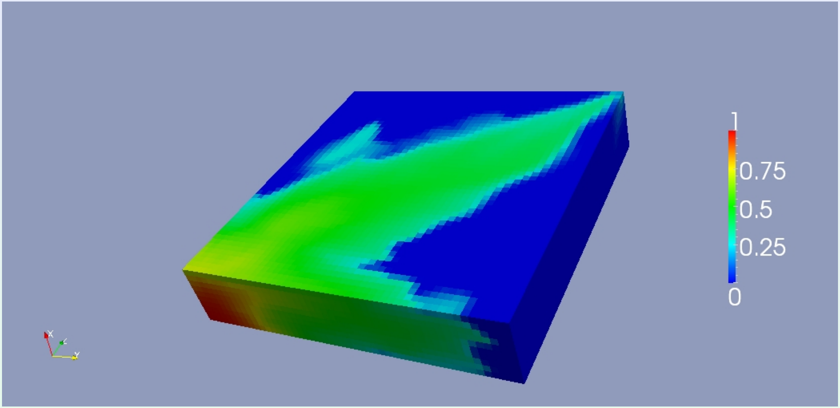
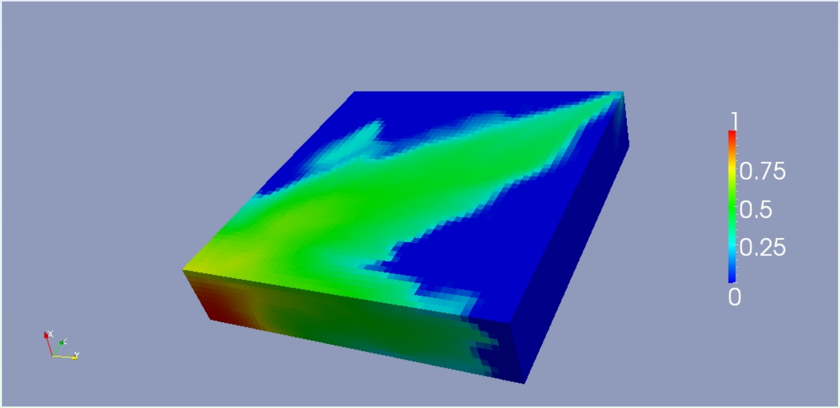
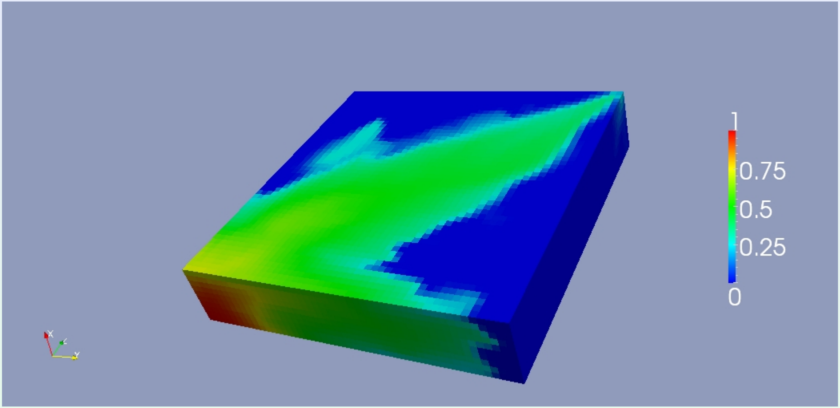


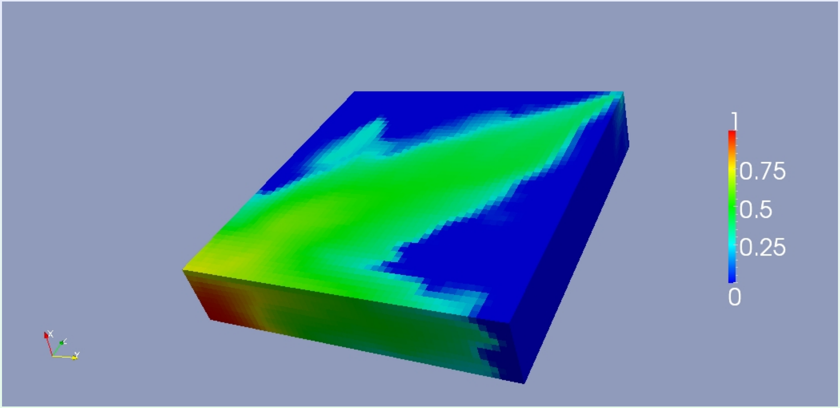
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



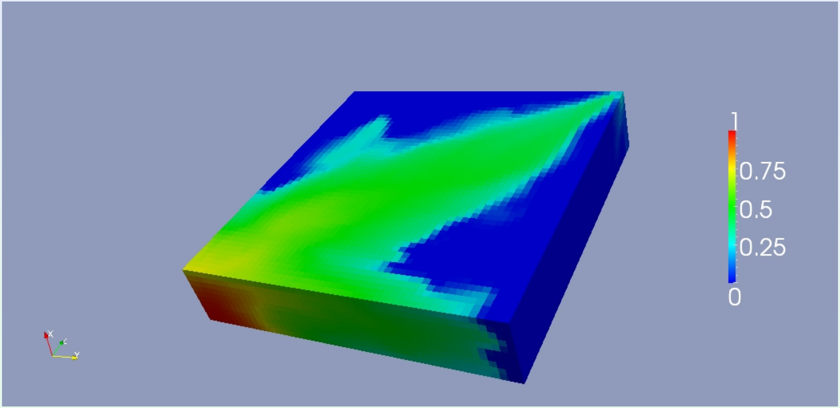
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



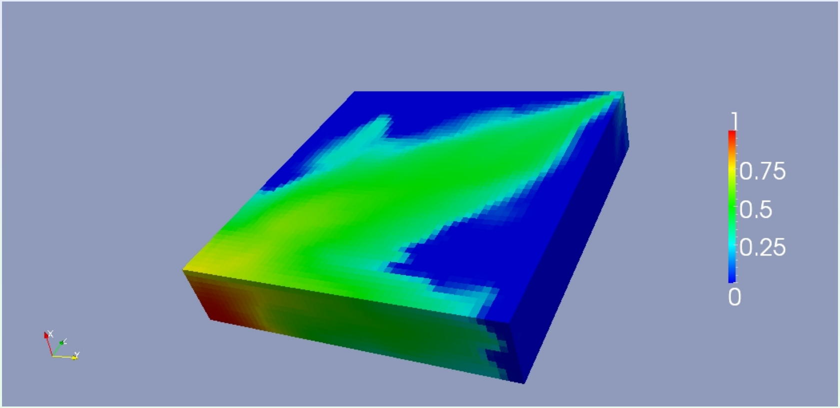
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



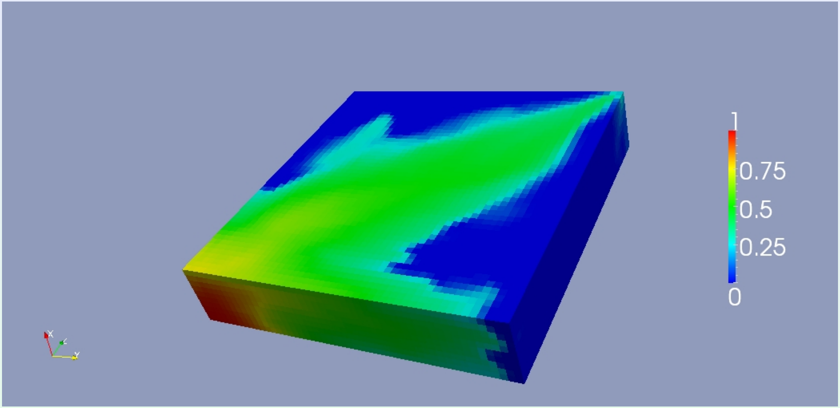
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



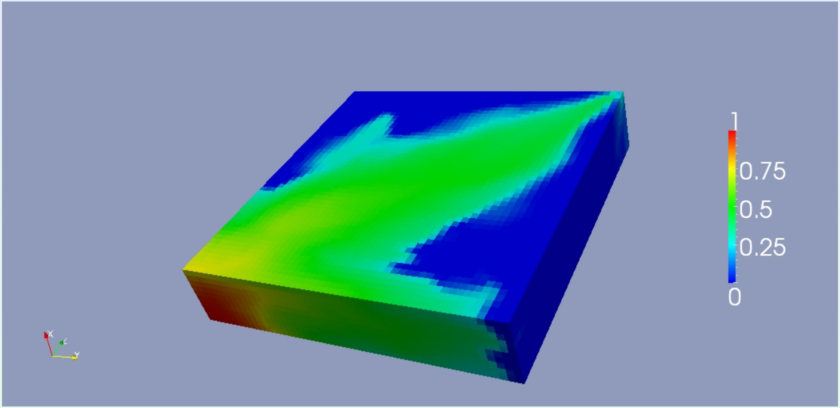
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



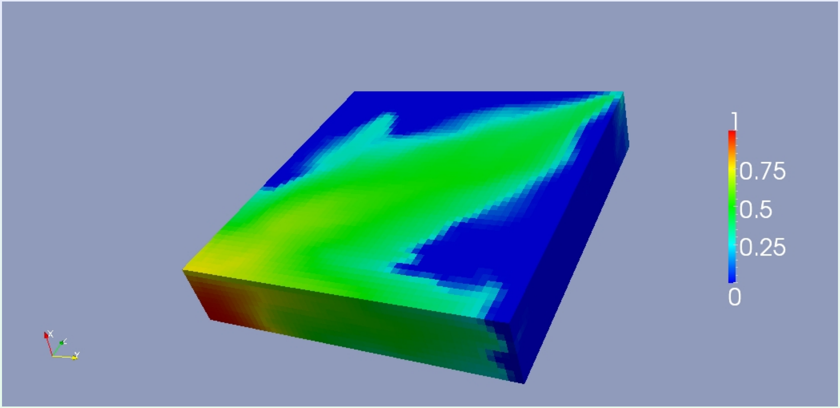
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



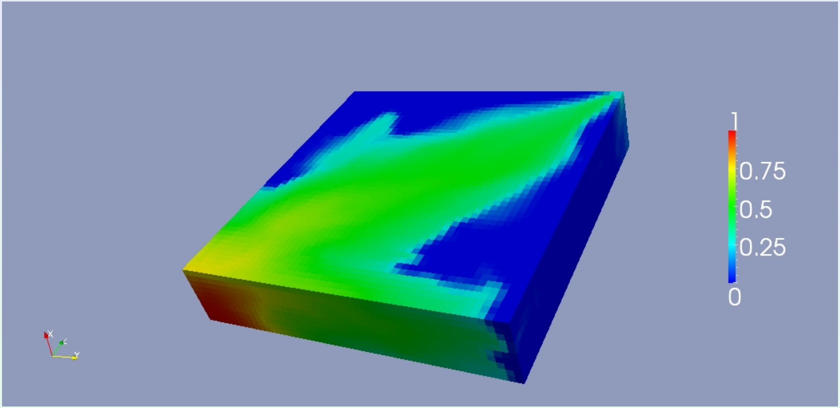
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



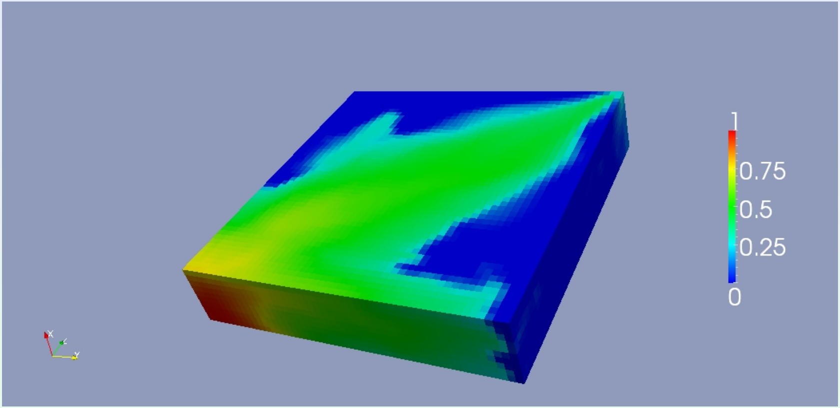
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



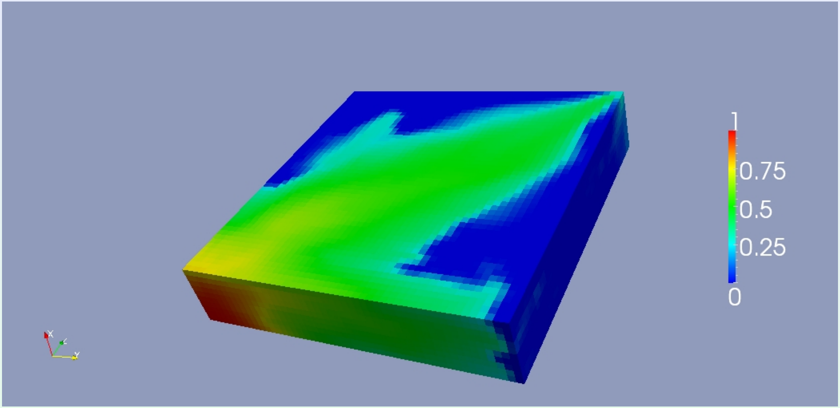
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



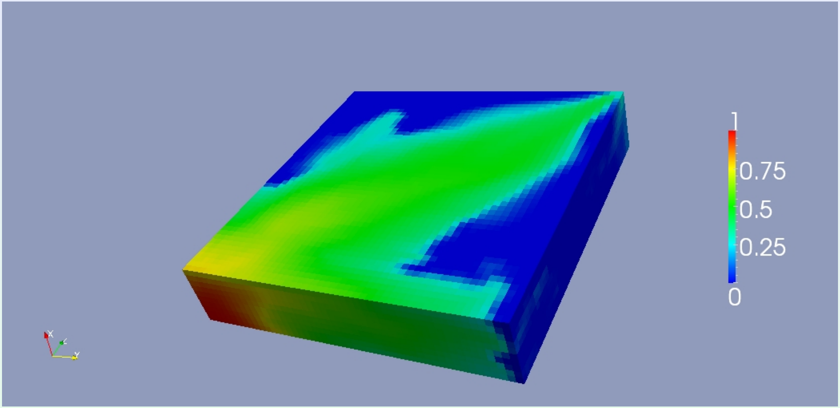
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



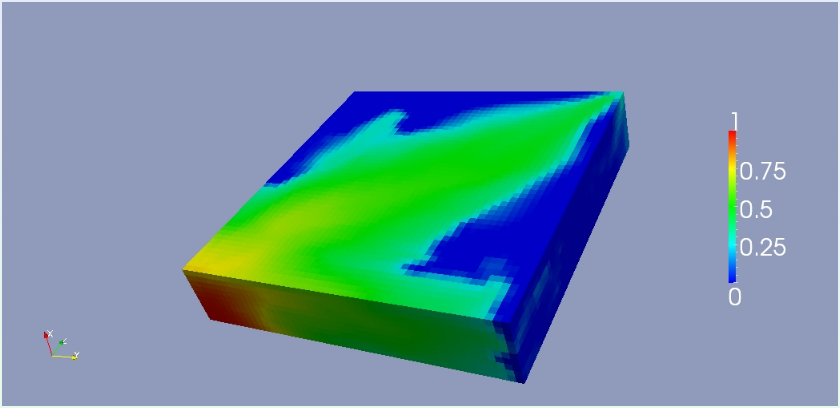
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



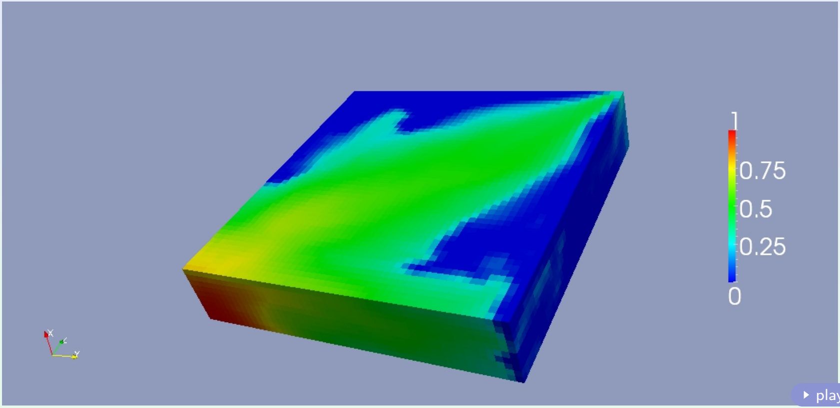
Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo

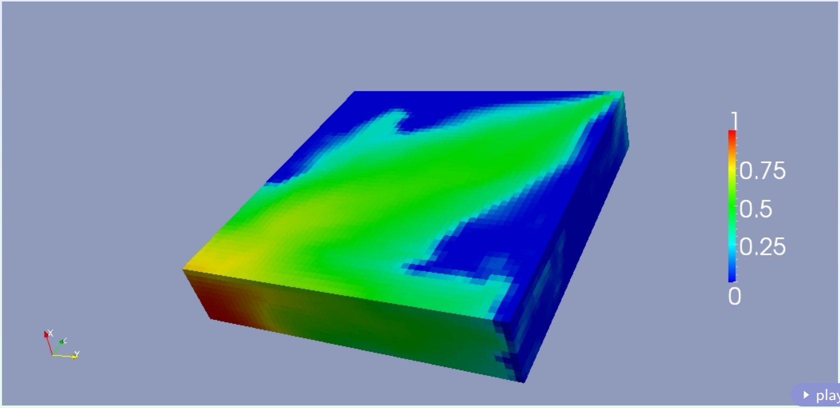


Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



play

Escoamento Bifásico - 1/4 de Five-Spot - Heterogêneo



▸ play

Computação Paralela

- Computador com 12 núcleos de memória compartilhada

	Slab - 40x20x100		
Núcleos	pressão/velocidade	saturação	tempo total
1	1373.11s	205.34s	0h28m
12	343.14s	47.32s (4.33)	0h8m

	1/4 de five spot - 10x50x50		
Núcleos	pressão/velocidade	saturação	tempo total
1	134.78s	9005.52s	2h35m
12	42.60s	827.01s (10.88)	0h14m