CNPq / Inria

PaMPA : Parallel Mesh Partitioning and Adaptation

Cédric Lachat & François Pellegrini & Cécile Dobrzynski

July 25, 2012

Contents

Contents

Context

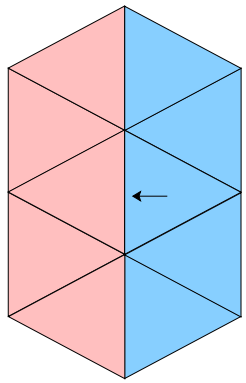
- Distributed meshes
- Subdomain decomposition
- Data exchanges between subdomains

Parallel mesh adaptation algorithms

- Parallel mesh generation (N. Chrisochoides 2005 [3])
  - Delaunay triangulation
  - Frontal method
  - Refinement by subdivisions (B. G. Larwood 2003 [7])
- Communication between subdomains:
  - Data migration
  - Matching algorithms

Problems induced by parallelism

- High complexity to parallelize remesh methods
  - Too much communication on boundaries
  - Boundaries not remeshed:
    - Local remeshing



Sequential algorithms

- Methods:
  - Moving nodes and remeshing locally (O. Hassan 2006 [6])
  - Coarse-grain parallel remeshing through multiple successive sequential remeshings (U. Tremel 2006 [8]):
    - Finding zones to remesh according to error estimator (T. Coupez 2000 [4])
    - Identifying zones to remesh in parallel
    - Remeshing zones in sequential
    - subdomain load balancing
- Main benefits:
  - Scalability of algorithms
  - Re-use of sequential codes

Contents

Generic dynamic load balancing

- General purpose (re)partitioning:
  - Jostle
  - Zoltan (K. Devine 2000 [5])
  - LB_Migrate (R. Chaube 2007 [2])
- Require a lot of extra coding

Specialized dynamic load balancing softwares

- Libraries:
  - DRAMA (A. Basermann 2000 [1])
- Pros and cons:
  - Mainly interfaced with solvers
  - Data structures based on meshes

Contents

Existing tools for handling unstructured meshes

- Partitioners:
  - Chaco
  - MeTiS
  - Mondriaan
  - Patoh
  - Scotch
  - Zoltan
- Intermediate:
  - DRAMA
  - PaMPA
  - PHG
- Advanced:
  - Arcane

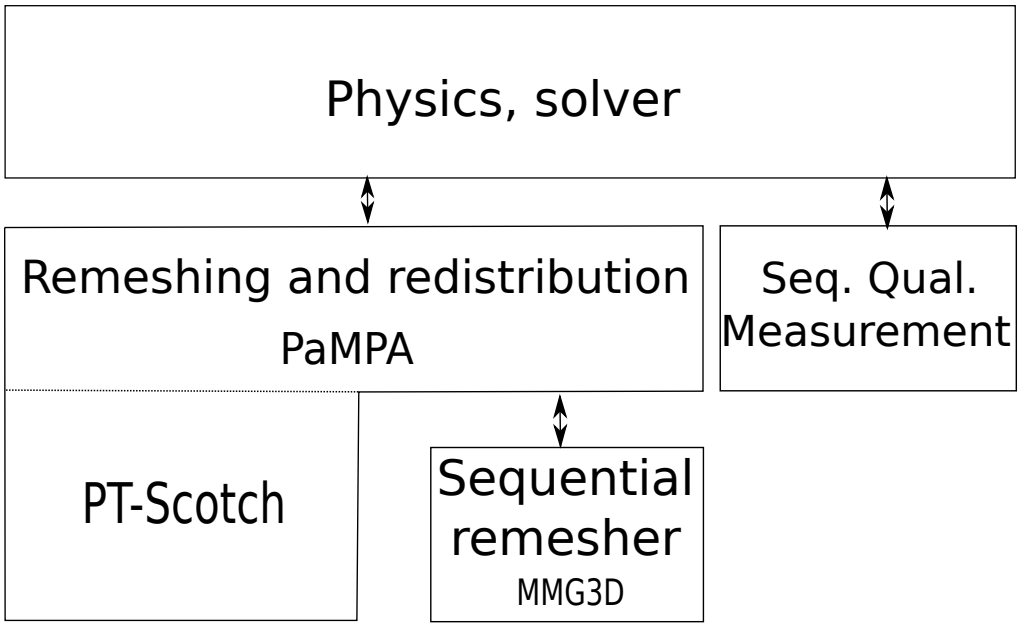
Contents

Common needs of solvers regarding meshes

- Handling of mesh structures
- Distribution of meshes across the processors of a parallel architecture
  - Handling of load balance
- Data exchange across neighboring entities
- Iteration on mesh entities
  - Entities of any kind: e.g. elements, faces, edges, nodes, ...
  - Entity sub-classes: e.g. regular or boundary faces, ...
  - Inner or frontier entities with respect to neighboring processors
  - Maximization of cache effects thanks to proper data reordering
- Dynamic modification of mesh structure
  - Dynamic redistribution
- Adaptive remeshing

What is PaMPA

- PaMPA: “Parallel Mesh Partitioning and Adaptation”
- Middleware library managing the parallel repartitioning and remeshing of unstructured meshes modeled as interconnected valuated entities
- The user can focus on his/her “core business”:
  - Solver
  - Sequential remesher
    - Coupling with MMG3D provided for tetrahedra

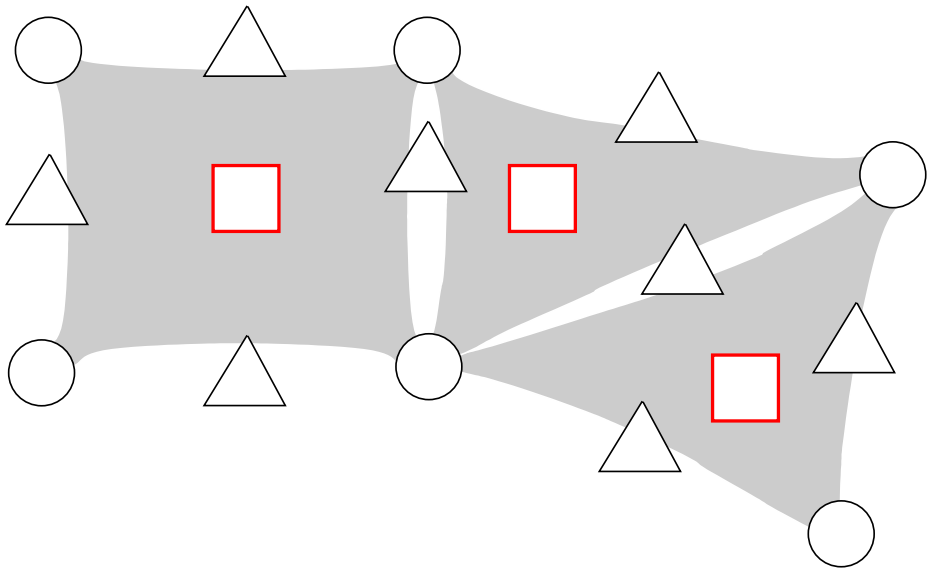


Contents

Definitions

- Mesh:
  - **Element**
  - **Node**
  - **Edge**
    - **Internal**
    - **Boundary**
- PaMPA Mesh:
  - **Vertex**
  - **Relation**
  - **Entity**
  - **Sub-entity**
  - **Enriched graph**

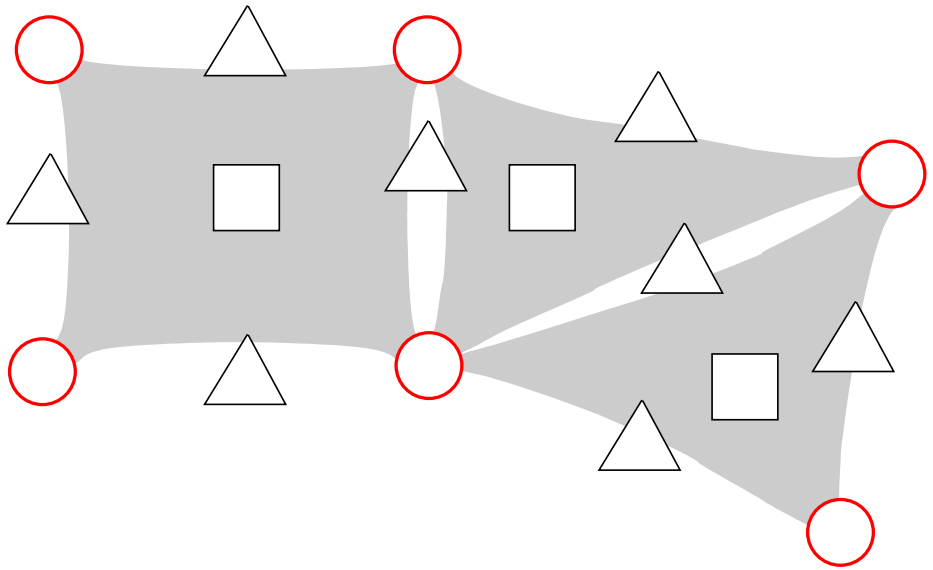
Top-level mesh entity
May bear some data (volume, pressure, etc.)



Definitions

- Mesh:
  - **Element**
  - **Node**
  - **Edge**
    - **Internal**
    - **Boundary**
- PaMPA Mesh:
  - **Vertex**
  - **Relation**
  - **Entity**
  - **Sub-entity**
  - **Enriched graph**

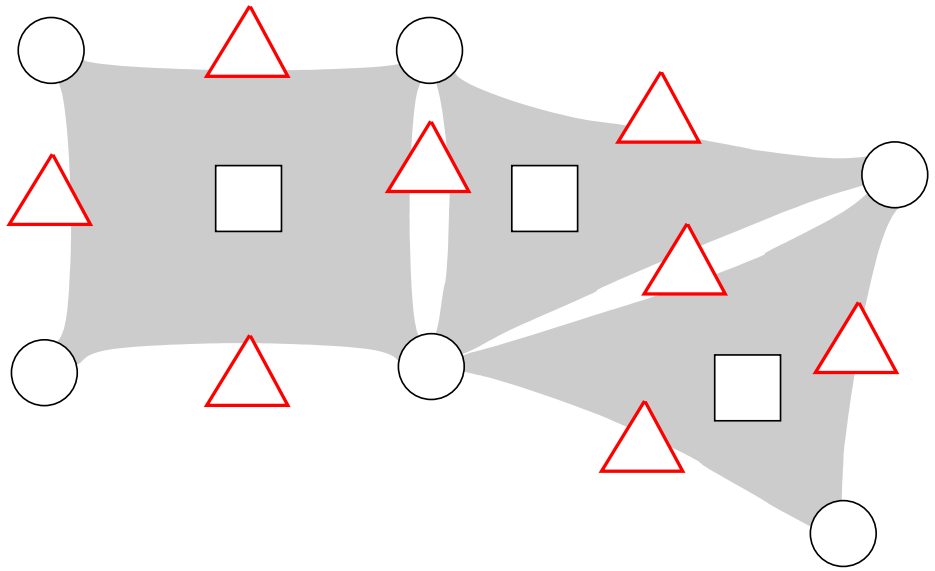
May bear some data (geometry, etc.)



Definitions

- Mesh:
  - **Element**
  - **Node**
  - **Edge**
    - **Internal**
    - **Boundary**
- PaMPA Mesh:
  - **Vertex**
  - **Relation**
  - **Entity**
  - **Sub-entity**
  - **Enriched graph**

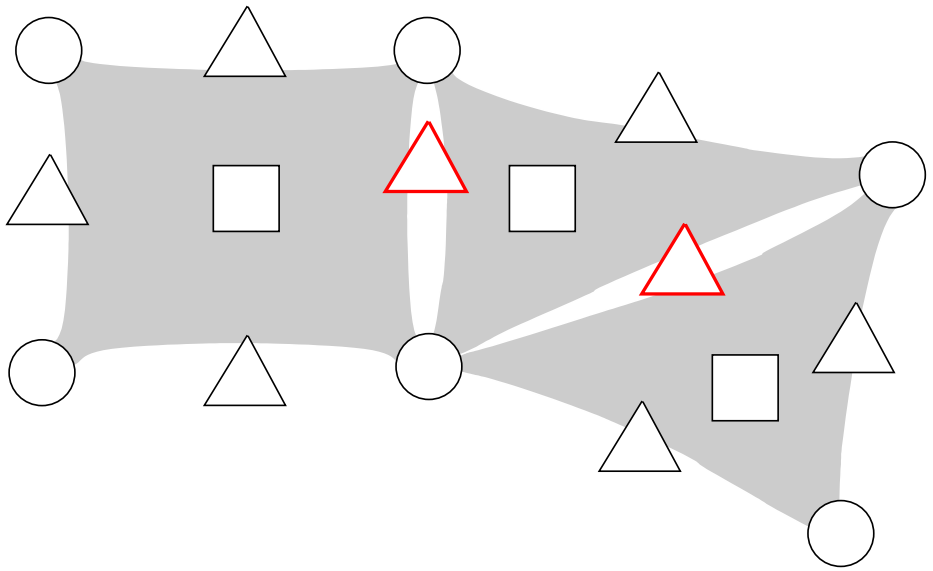
May bear some data (flux, etc.)



Definitions

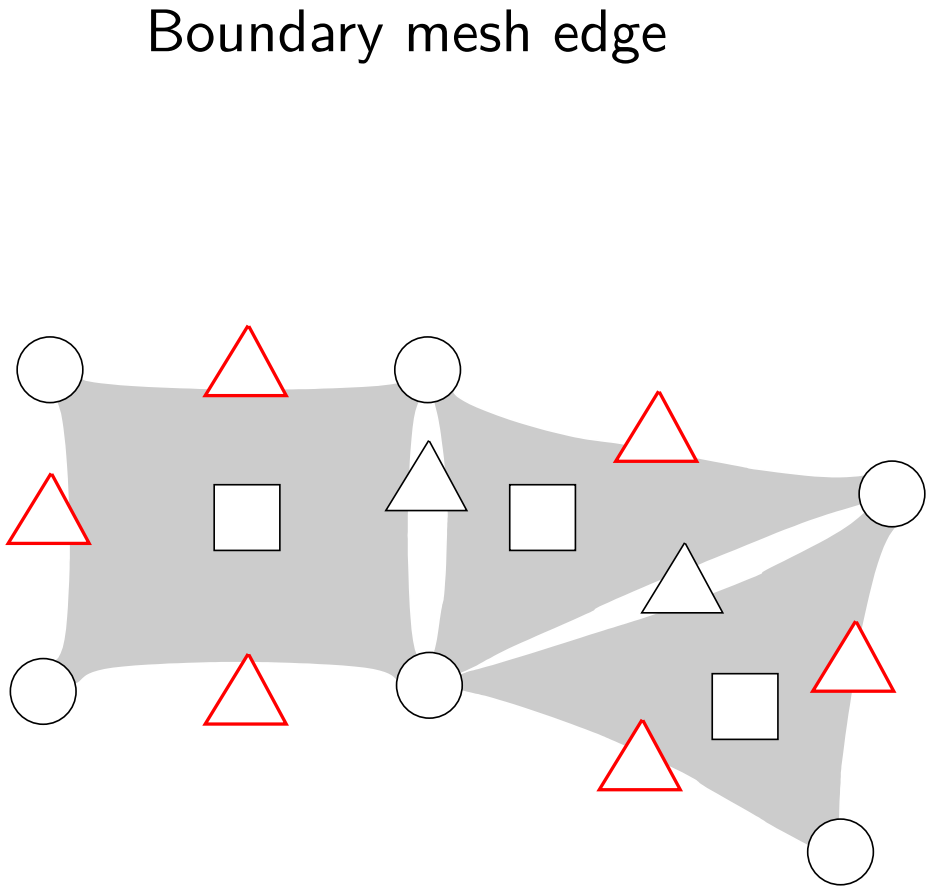
- Mesh:
  - **Element**
  - **Node**
  - **Edge**
    - **Internal**
    - **Boundary**
- PaMPA Mesh:
  - **Vertex**
  - **Relation**
  - **Entity**
  - **Sub-entity**
  - **Enriched graph**

Regular mesh edge



Definitions

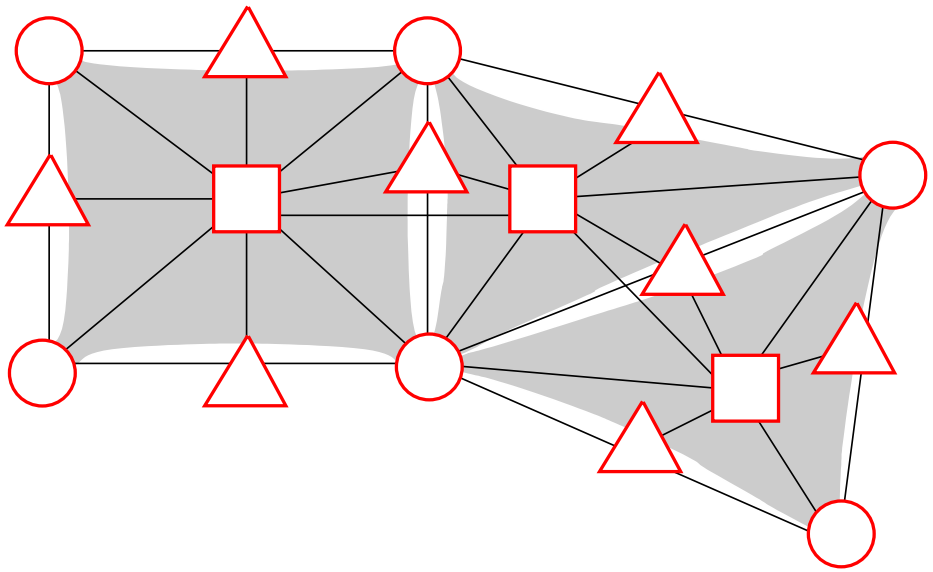
- Mesh:
  - **Element**
  - **Node**
  - **Edge**
    - **Internal**
    - **Boundary**
- PaMPA Mesh:
  - **Vertex**
  - **Relation**
  - **Entity**
  - **Sub-entity**
  - **Enriched graph**



Definitions

- Mesh:
  - **Element**
  - **Node**
  - **Edge**
    - **Internal**
    - **Boundary**
- PaMPA Mesh:
  - **Vertex**
  - **Relation**
  - **Entity**
  - **Sub-entity**
  - **Enriched graph**

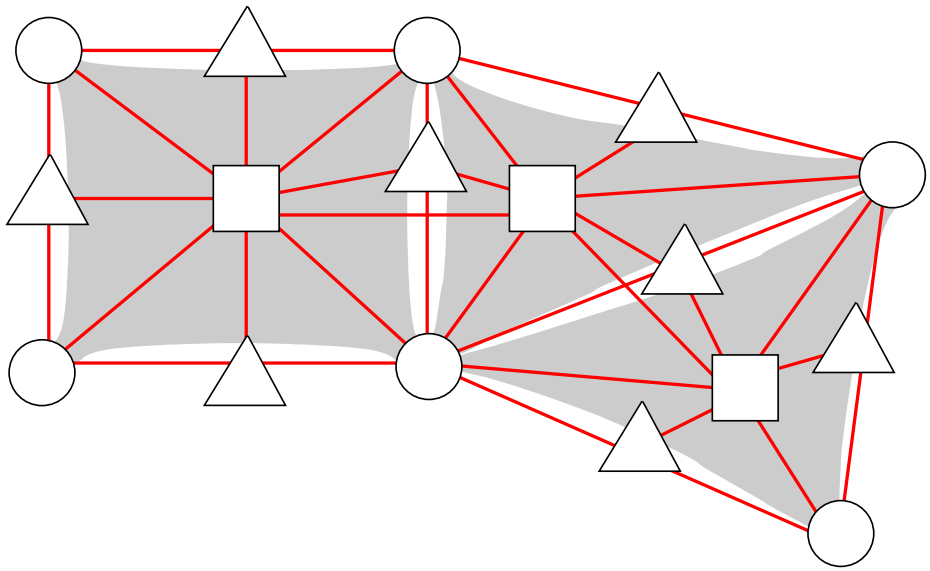
What all entities are in fact...



Definitions

- Mesh:
  - **Element**
  - **Node**
  - **Edge**
    - **Internal**
    - **Boundary**
- PaMPA Mesh:
  - **Vertex**
  - **Relation**
  - **Entity**
  - **Sub-entity**
  - **Enriched graph**

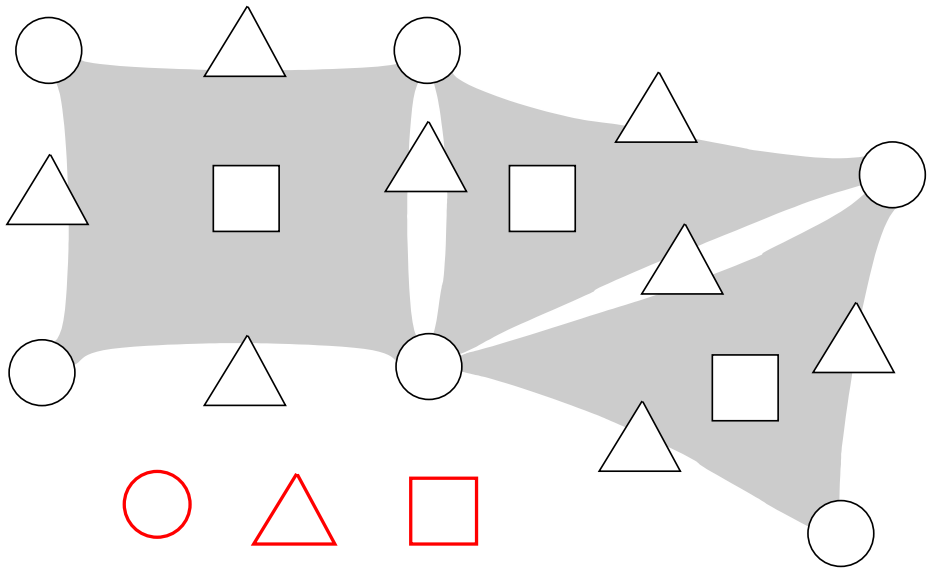
Subset of edges between vertices belonging to prescribed entity types



Definitions

- Mesh:
  - **Element**
  - **Node**
  - **Edge**
    - **Internal**
    - **Boundary**
- PaMPA Mesh:
  - **Vertex**
  - **Relation**
  - **Entity**
  - **Sub-entity**
  - **Enriched graph**

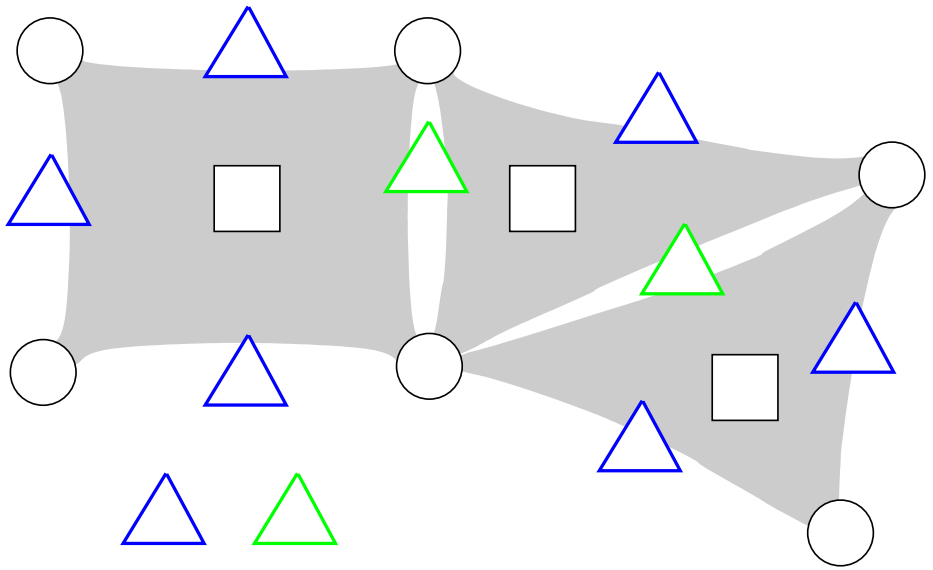
Subset of vertices bearing the same data



Definitions

- Mesh:
  - **Element**
  - **Node**
  - **Edge**
    - **Internal**
    - **Boundary**
- PaMPA Mesh:
  - **Vertex**
  - **Relation**
  - **Entity**
  - **Sub-entity**
  - **Enriched graph**

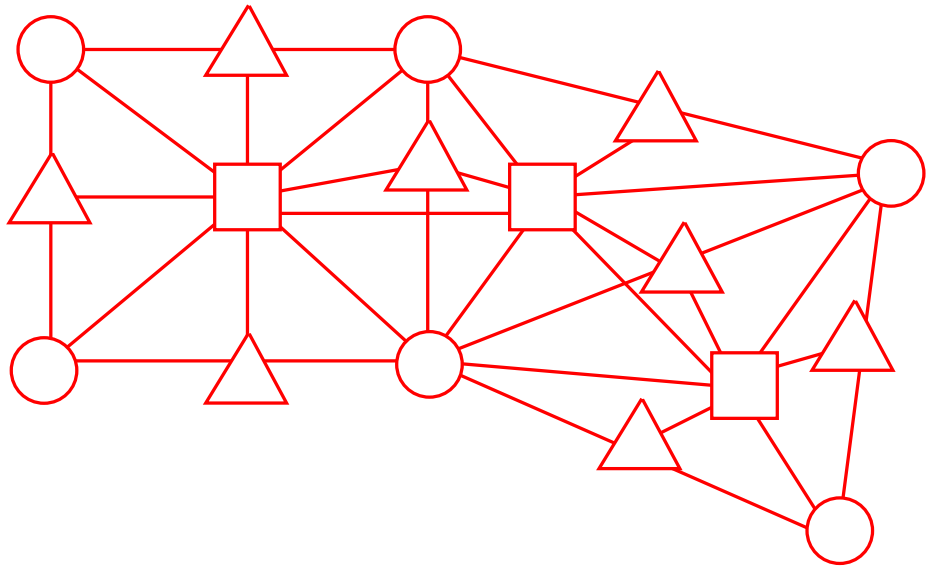
Subset of entity vertices that may bear additional specific data



Definitions

- Mesh:
  - **Element**
  - **Node**
  - **Edge**
    - **Internal**
    - **Boundary**
- PaMPA Mesh:
  - **Vertex**
  - **Relation**
  - **Entity**
  - **Sub-entity**
  - **Enriched graph**

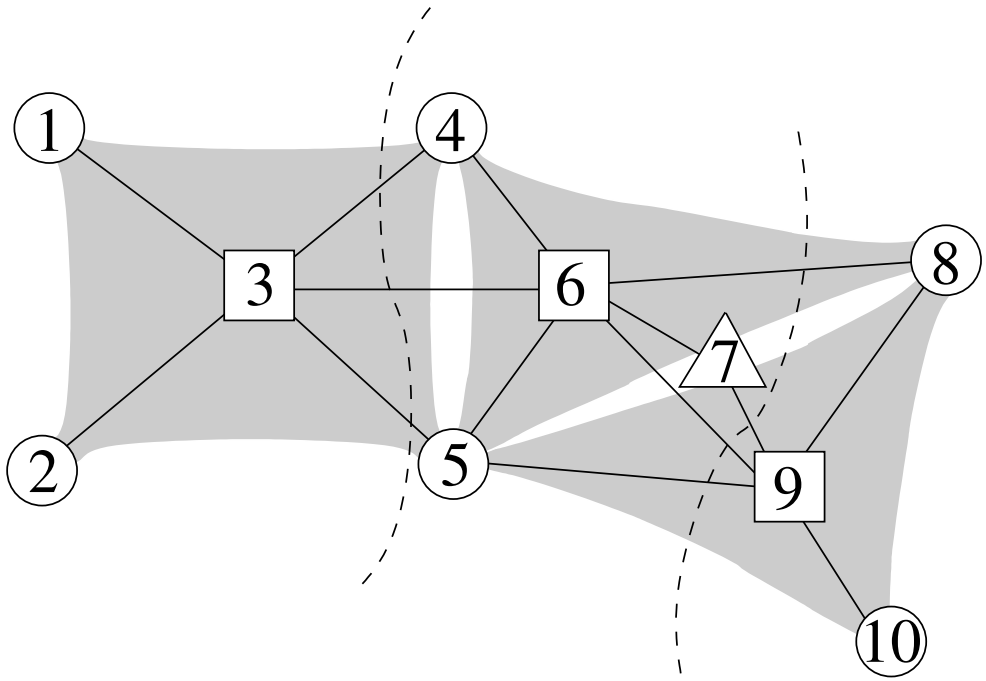
Whole set of vertices and relations
Every vertex belongs to one and only one entity (and sub-entity)



Global vue

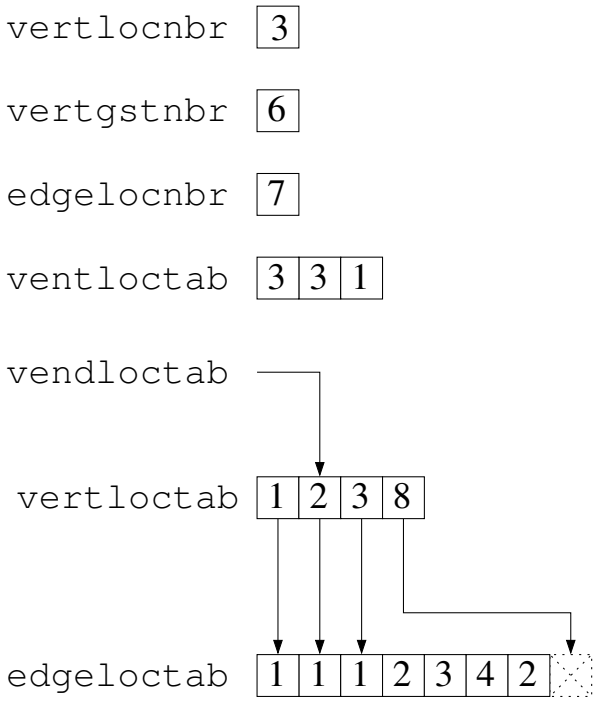
- All vertices have a global unique number

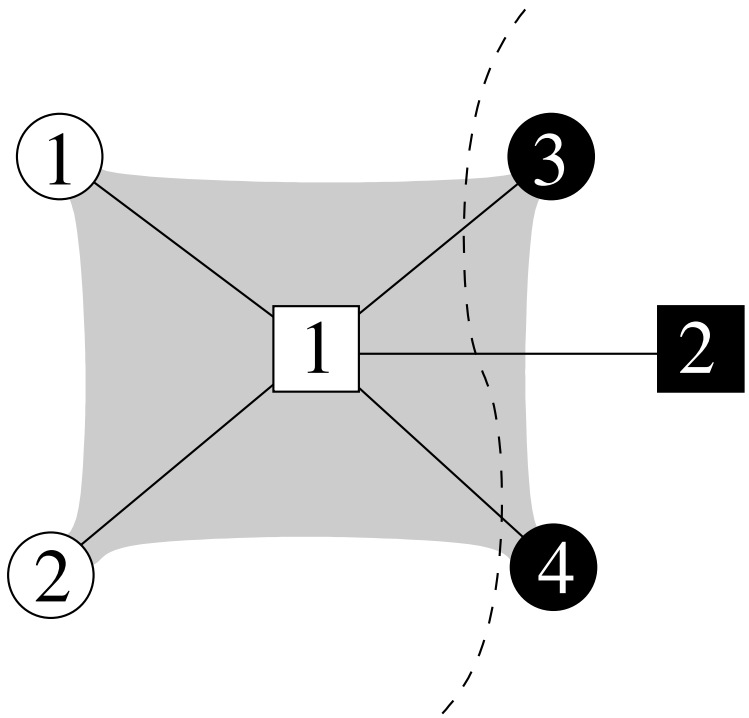
`baseval`	1
`enttglbnbr`	3
`proccnttab`	3 4 3
`procvrttab`	1 4 8 11



Local vision of process 0

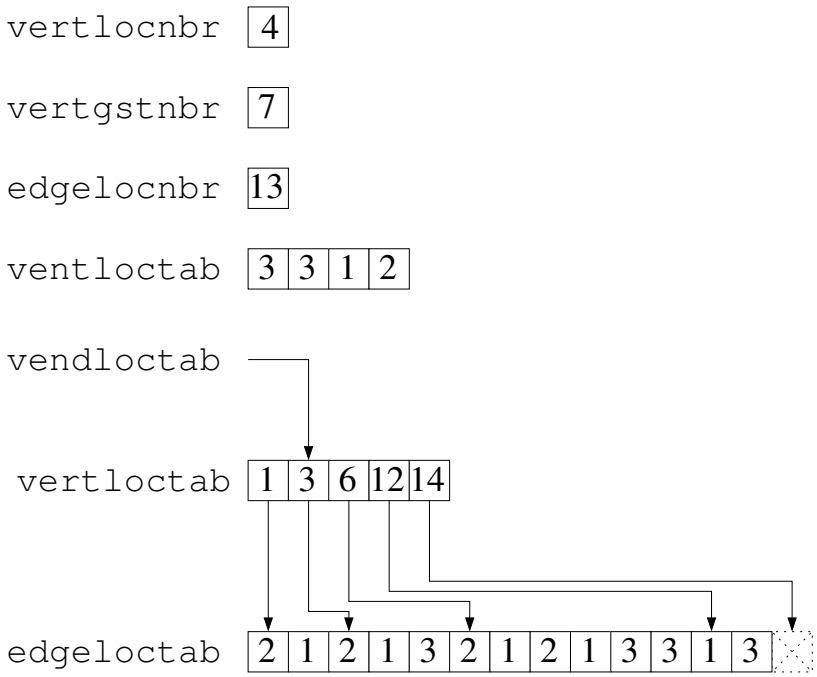
- All local and ghost vertices have a compact local index
  - Per-entity numbering

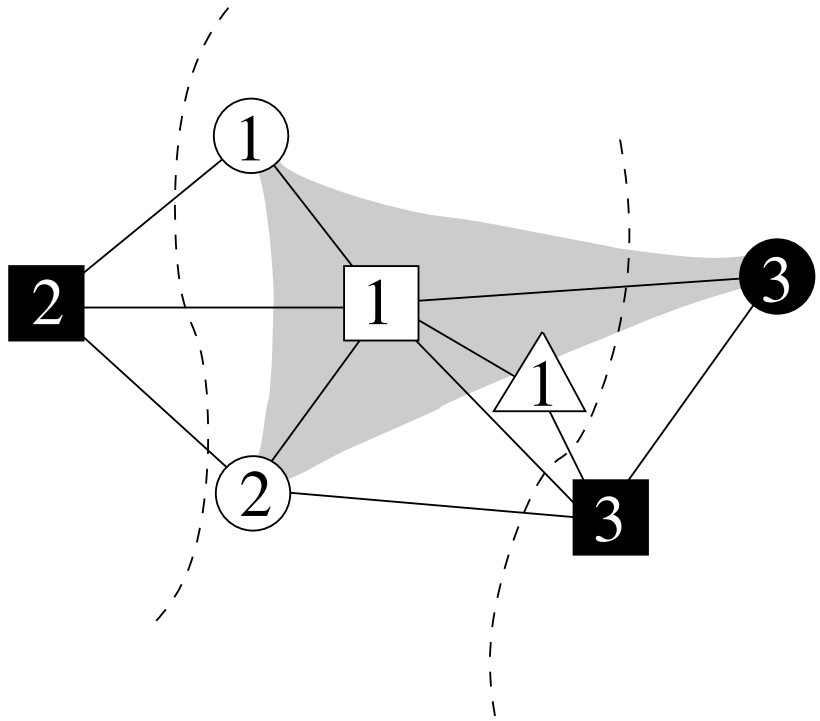




Local vision of process 1

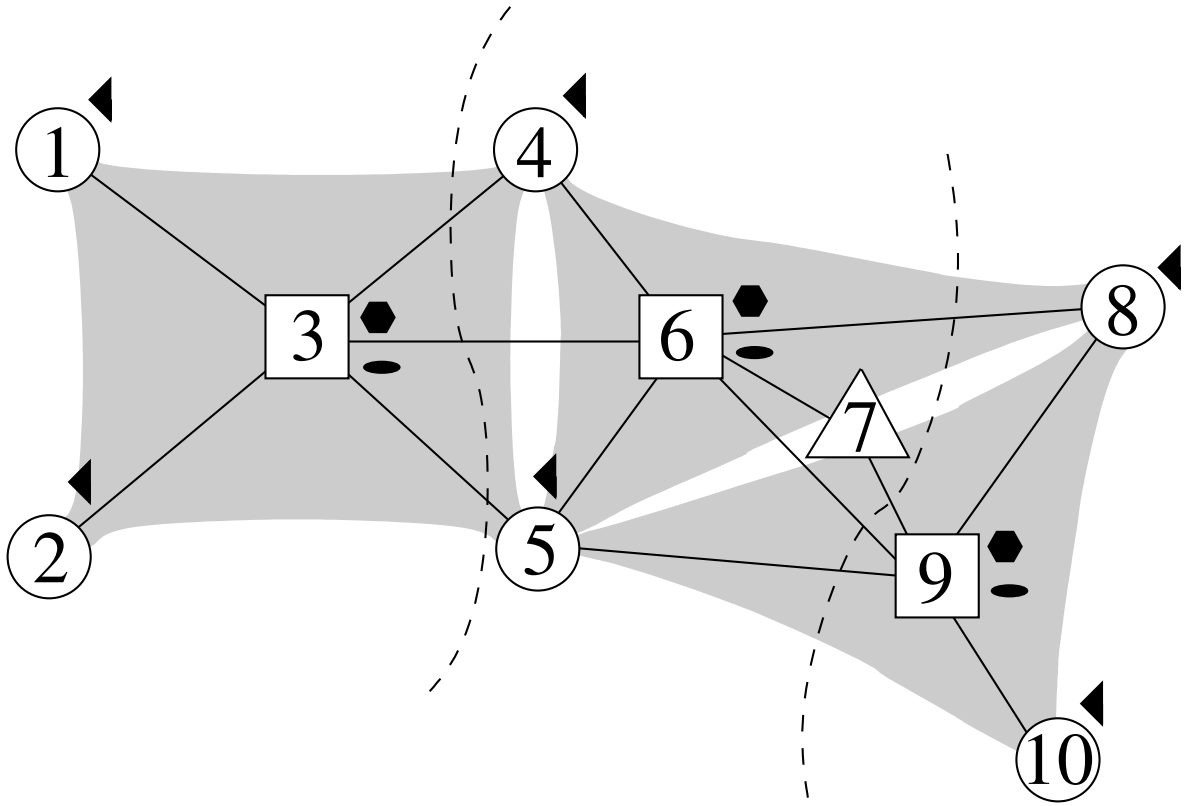
- All local and ghost vertices have a compact local index
  - Per-entity numbering





Linking values to entities

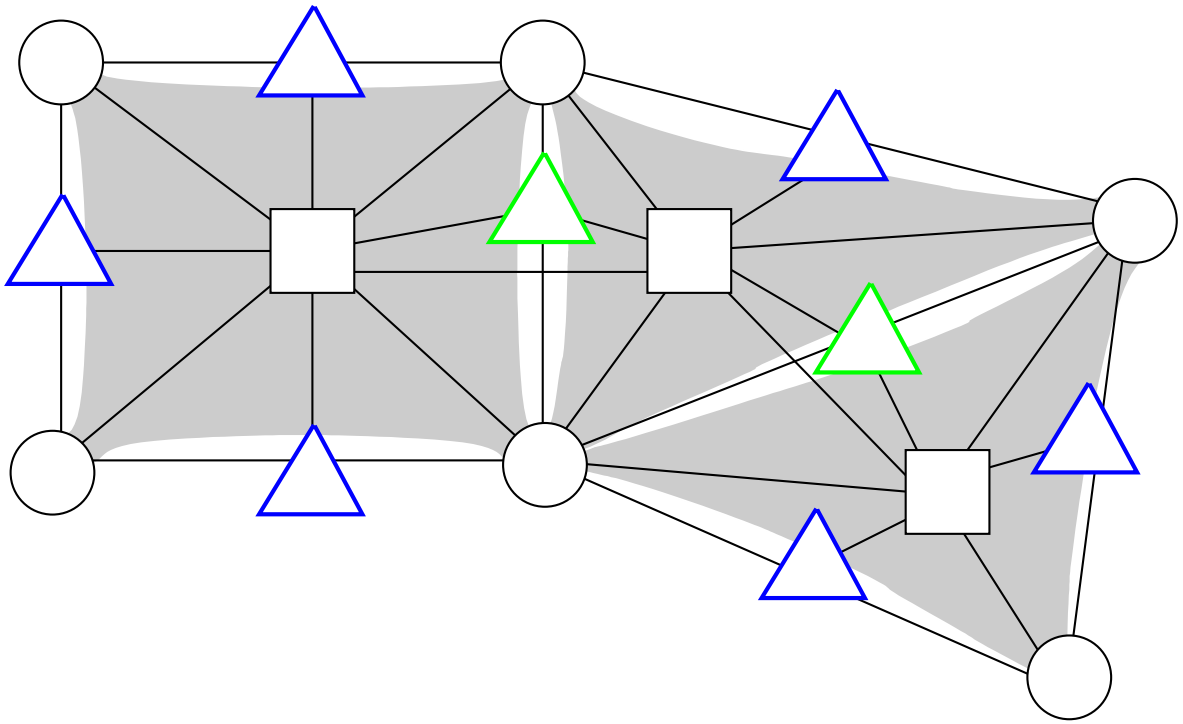
- Multiple value types can be associated with each entity
  - Value data structures can be split to improve cache usage
- Entities without values serve as iterators



Contents

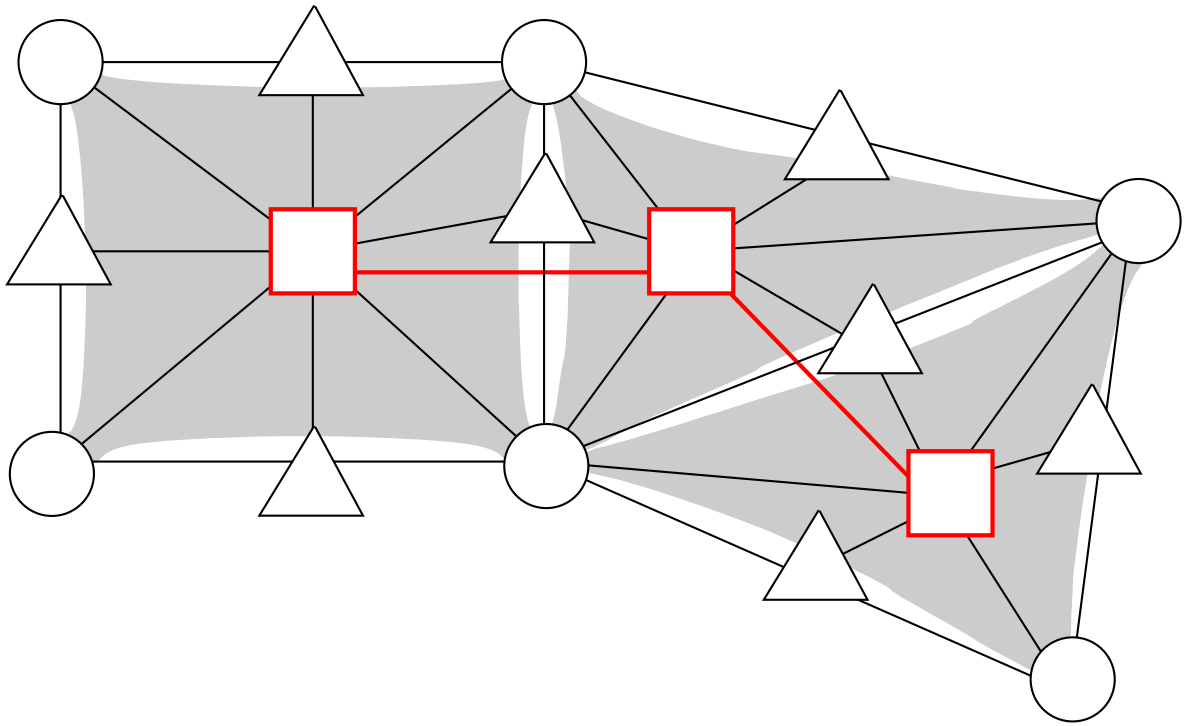
Entities

- Based
  - Smallest array index depends on language
  - 0 in **C** and 1 in **Fortran**
- Stable
  - Neighbor ordering preserved with respect to sub-entities



Partitioning

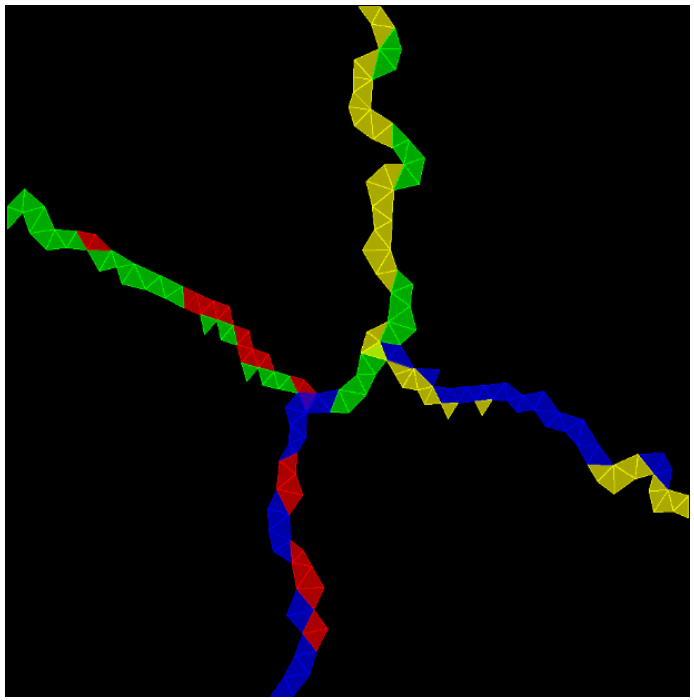
- Performed with respect to top-level entity
- Cache oblivious ordering
  - Overdecomposition into small groups of vertices
  - Vertices in groups ordered consecutively



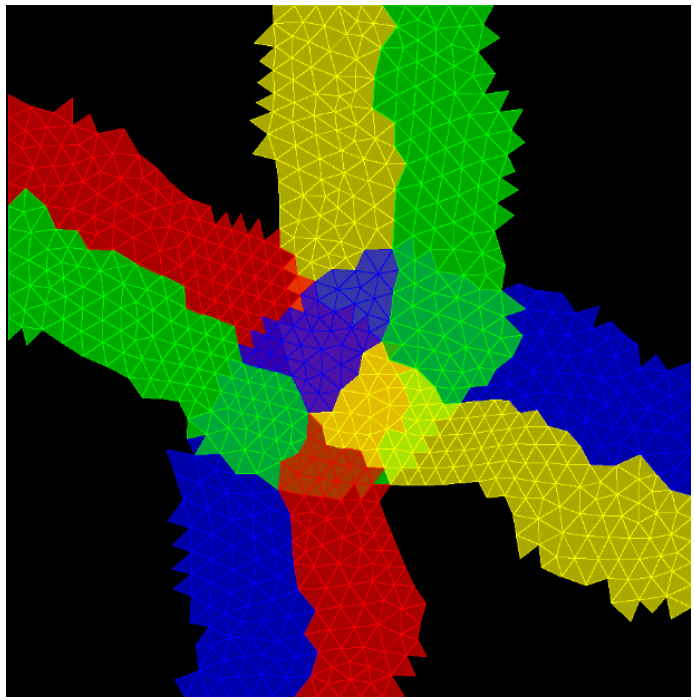
Overlap

- Of size n
- Requires top-level relations

Halo of size 1

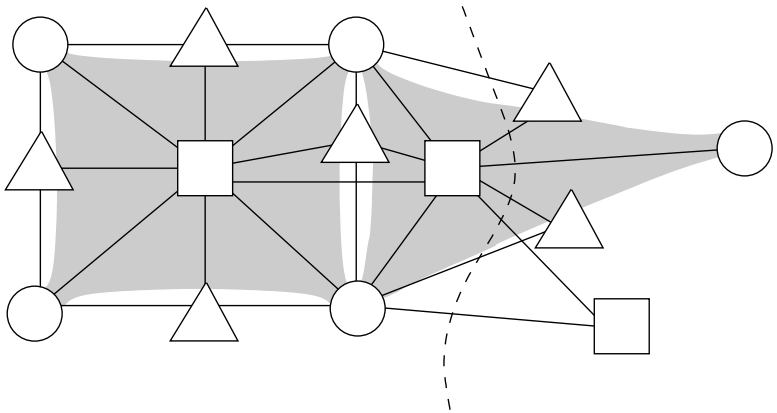


Halo of size 10

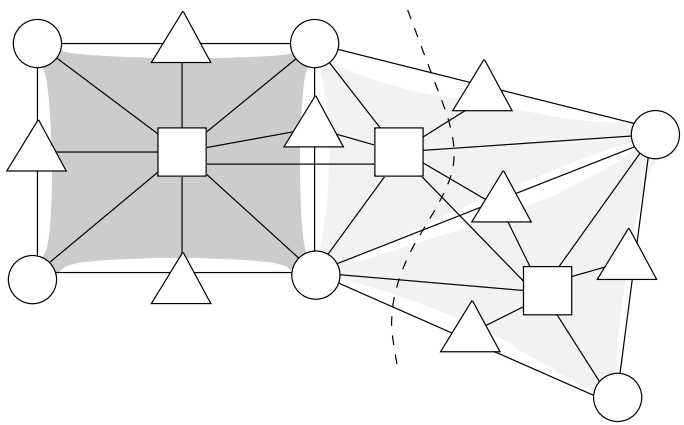


Local vue of process 0

- Halo: local copy of remote vertices linked to local vertices
- Overlap: local copy of remote vertices needed to complete split mesh units
  - Extensible to distance n
- Halo of size 1

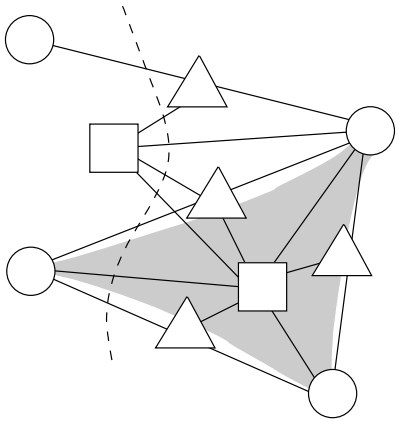


- Overlap of size 1

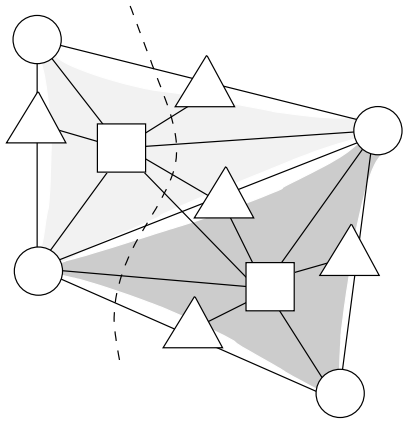


Local vue of process 1

- Halo: local copy of remote vertices linked to local vertices
- Overlap: local copy of remote vertices needed to complete split mesh units
  - Extensible to distance n
- Halo of size 1



- Overlap of size 1



Iterators

- Operate on entities or sub-entities

```
PAMPA_dmeshItInitStart (mesh, ventlocnum, PAMPA_VERT_ANY, &it_vrt) ;
PAMPA_dmeshItInit (mesh, ventlocnum, nentlocnum, &it_ngb) ;

while (PAMPA_itHasMore (&it_vrt)) {
  vertlocnum = PAMPA_itCurEnttVertNum (&it_vrt);
  printf ("vertlocnum : %d\n", vertlocnum);
  PAMPA_itStart (&it_ngb, vertlocnum) ;

  while (PAMPA_itHasMore (&it_ngb)) {
    PAMPA_Num sngblocnum, engblocnum, mngblocnum, nsublocnum;
    mngblocnum = PAMPA_itCurMeshVertNum (&it_ngb);     // Global number of vertex
    engblocnum = PAMPA_itCurEnttVertNum (&it_ngb);     // Local number in entity
    sngblocnum = PAMPA_itCurSubEnttVertNum (&it_ngb); // Local number in sub-entity
    nsublocnum = PAMPA_itCurSubEnttNum (&it_ngb);      // Number of sub-entity
    printf ("[%d] ngb of %d with entity %d : %d (sub : %d) %d (ent : %d) %d (glb)\n",
            rank, vertlocnum, ventlocnum, sngblocnum, nsublocnum,
            engblocnum, nentlocnum, mngblocnum);
    PAMPA_itNext (&it_ngb);
  }
  PAMPA_itNext (&it_vrt);
}
```

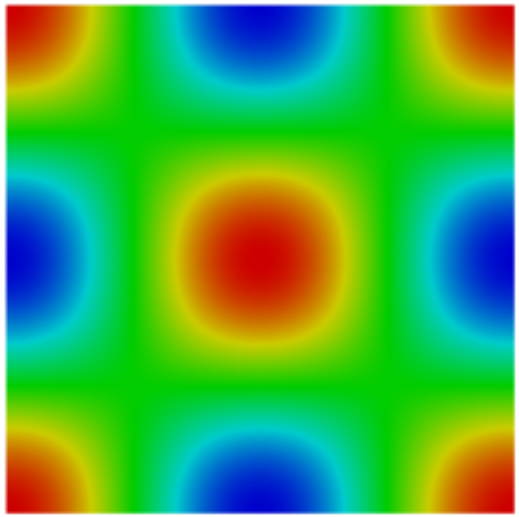
PaMPA software distribution

- Project still in development stage
  - Soon licensed under GPL
  - Objective: create a community
- Already used in development of 2 fluid mechanic solvers:
  - Plato, INRIA Sophia-Antipolis
    - A platform for tokamak simulations
    - Mesh reading in sequential, partitioning and building distributed mesh
  - AeroSol, INRIA Bordeaux
    - High-order methods
    - Sequential and parallel executions
    - Great number of entities, mesh reading and reditributing in parallel

Contents

Definition

- Solving 2D Poisson equation:
  - $\Delta u(x, y) = f(x, y)$
  - $g(x, y) = u(x, y)$ on the boundary Γ
- Test case:
  - $f(x, y) = -2 * \cos(x) * \cos(y)$ in the domain Ω
  - $g(x, y) = \cos(x) * \cos(y)$ on the boundary Γ
  - $u(x, y) = \cos(x) * \cos(y)$



Mesh properties

- Entities:
  - Elements
  - Nodes
  - Boundary edges
- Relations:
  - Element to element
  - Element to node
  - Element to boundary edge
  - Node to node
- Overlap of size 1
- Values:
  - Coordinates and solution on nodes
  - Type on boundary edges
  - Area, volume on elements

All steps

```
! Mesh loading and distribution
!-------------------------------
! Processor 0 only:
if (rank == 0) THEN
      CALL ReadMesh()          ! Read mesh on file
      CALL GraphPampa()        ! Build graph of vertex neighbors for PAMPA
      CALL CentralizedMesh()   ! Build PaMPA global non distributed mesh
END IF

! On all processors:
CALL DistributedMesh()  ! Build PaMPA distributed mesh:
!                         1- Scatter centralized mesh on all processors
!                         2- Call PaMPA mesh partionner
!                         3- Redistribute distribute mesh
CALL ElementVolume()
CALL InitializeMatrixCSR()

! Solution computation
!---------------------

CALL InitSol()
CALL FillMatrix()
CALL SolveSystem()
CALL WriteDistributedMeshAndSolFiles()
```

FillMatrix

```
RHS = 0.
CALL PAMPAF_dmeshItInitStart (dm, ENTITY_ELEM, PAMPAF_VERT_ANY, it_vrt, ierr)
CALL PAMPAF_dmeshItInit (dm, ENTITY_ELEM, ENTITY_NODE, it_ngb, ierr)
DO WHILE (PAMPAF_itHasMore (it_vrt))
  jt   = PAMPAF_itCurEnttVertNum (it_vrt)
  Volt = VolEl (jt)
  ngb  = 0
  CALL PAMPAF_itStart (it_ngb, jt, ierr)
  DO WHILE (PAMPAF_itHasMore (it_ngb))
    ngb = ngb + 1
    is  = PAMPAF_itCurEnttVertNum (it_ngb)
    NuElemt(ngb)      = is
    CoorElemt(:,ngb) = Coor(:,is)
    PAMPAF_itNext (it_ngb)
  END DO
  CALL GradPhi (CoorElemt(:,1), CoorElemt(:,2), CoorElemt(:,3), GrdPhi)
  DO i = 1, Nsmplx
    is = NuElemt(i)
    DO j = 1, Nsmplx
      js   = NuElemt(j)
      JJac = Volt * Sum (GrdPhi(:,i) * GrdPhi(:,j))
      CALL assembly_addCSR (JJac,is,js)
    END DO ! loop on j
    RHS(is) = RHS(is) - Volt * SourceTerm (Coor(1,is), Coor(2,is)) / Nsmplx
  END DO ! loop on i
  PAMPAF_itNext (it_vrt)
END DO
```

Solve system: Jacobi (1/2)

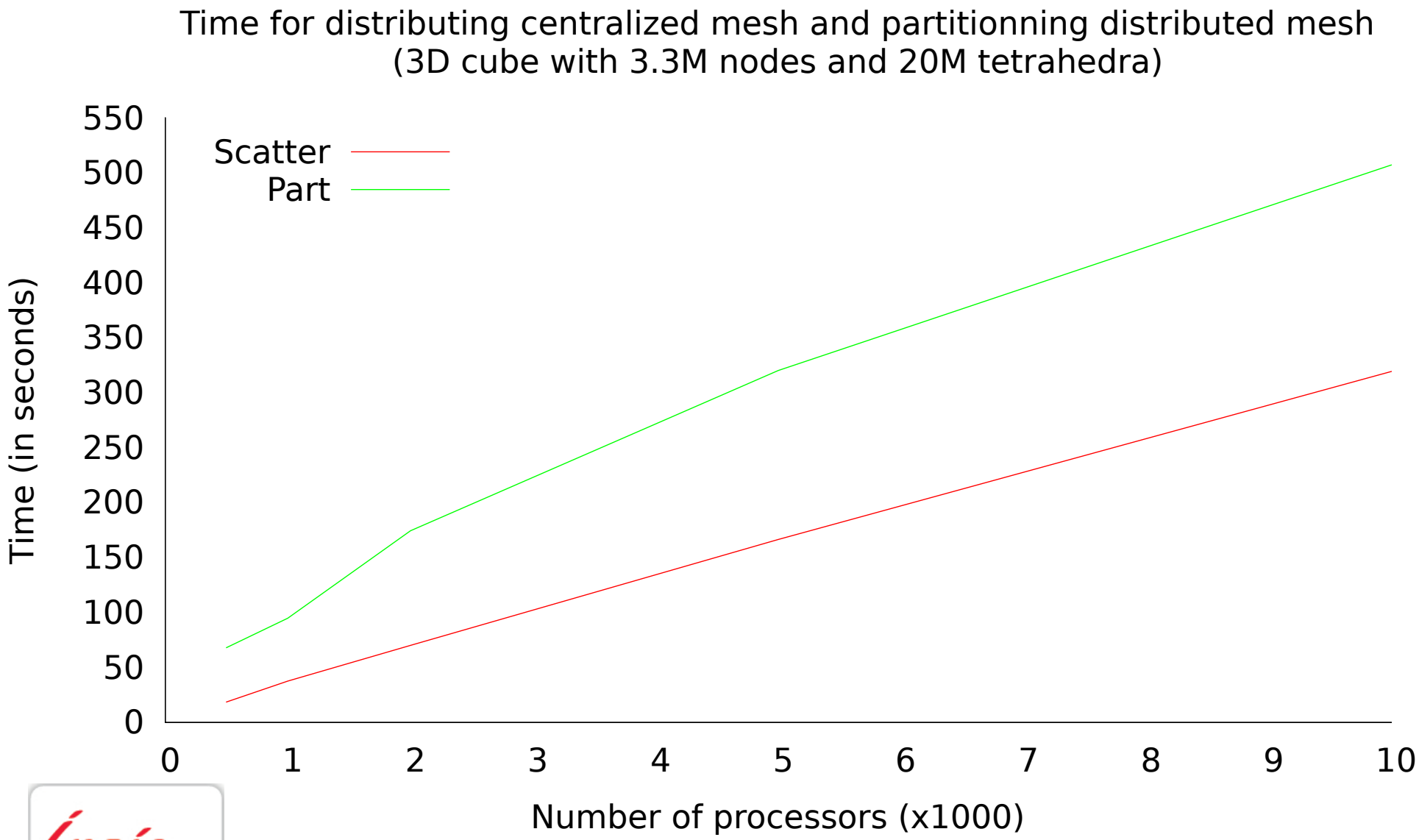
```
UaPrec = 0.          ! Suppose A = L + D + U, system to solve : A x = b
CALL PAMPAF_dmeshItInit(dm, ENTITY_NODE, ENTITY_NODE, it_ngb, ierr)
DO irelax = 1, Nrelax
  res       = 0.
  CALL PAMPAF_dmeshItInitStart(dm, ENTITY_NODE, PAMPAF_VERT_BOUNDARY, it_vrt,  ierr)
  DO WHILE (PAMPAF_itHasMore(it_vrt))
    is = PAMPAF_itCurEnttVertNum(it_vrt)
    CALL PAMPAF_dmeshMatLineData(dm, ENTITY_NODE, is, I1, I1Fin, ierr)
    CALL PAMPAF_itStart( it_ngb,  is,  ierr)
    res0    = RHS(is)                                  ! res0  = b

    iv = i1
    DO WHILE (PAMPAF_itHasMore(it_ngb))
      js = PAMPAF_itCurEnttVertNum(it_ngb)
      PAMPAF_itNext(it_ngb)
      res0  = res0 - MatCSR%Vals(iv) * UaPrec(js) !res0  = b - (L + U) x^n
      iv = iv + 1
    END DO
    Ua(is)  = res0 / MatCSR%Diag(is)                 !x^n+1 = ( b - (L + U) x^n )/D
    PAMPAF_itNext(it_vrt)
  END DO

  CALL PAMPAF_dmeshHaloValueAsync(dm, ENTITY_NODE, PAMPA_TAG_SOL, req, ierr)
```

Solve system: Jacobi (2/2)

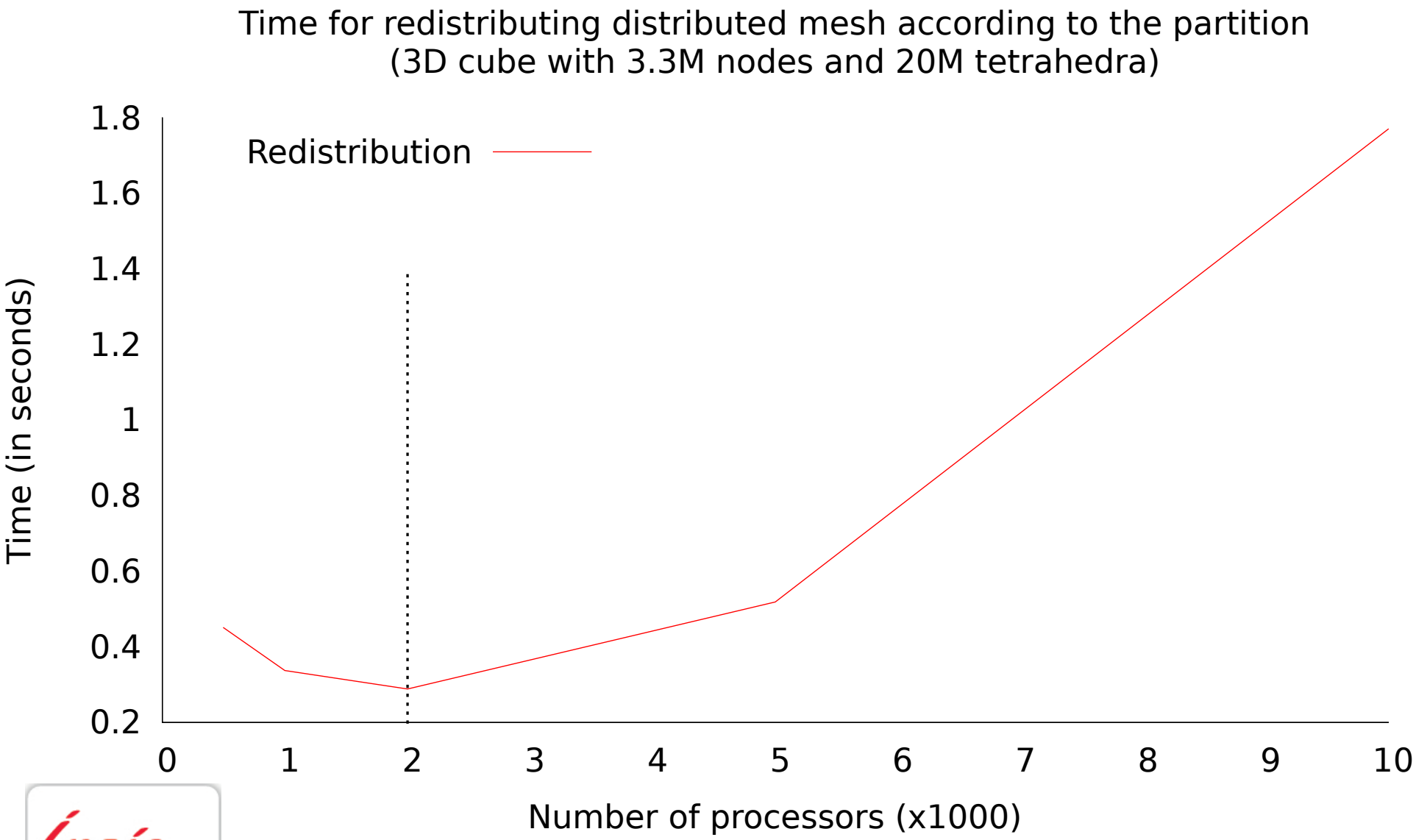
```
CALL PAMPAF_dmeshItInitStart(dm, ENTITY_NODE, PAMPAF_VERT_INTERNAL, it_vrt , ierr)
DO WHILE (PAMPAF_itHasMore(it_vrt))
  is = PAMPAF_itCurEnttVertNum(it_vrt)
  CALL PAMPAF_dmeshMatLineData(dm, ENTITY_NODE, is , I1 , I1Fin , ierr)
  CALL PAMPAF_itStart( it_ngb , is , ierr)
  res0    = RHS(is)                                  ! res0  = b

  iv = i1
  DO WHILE (PAMPAF_itHasMore(it_ngb))
    js = PAMPAF_itCurEnttVertNum(it_ngb)
    PAMPAF_itNext(it_ngb)
    res0  = res0 - MatCSR%Vals(iv) * UaPrec(js)  !res0  = b - (L + U) x^n
    iv = iv + 1
  END DO
  Ua(is)  = res0 / MatCSR%Diag(is)                 !x^n+1 = ( b - (L + U) x^n )/D
  PAMPAF_itNext(it_vrt)
END DO

CALL PAMPAF_dmeshHaloWait(req , ierr)

UaPrec    = Ua
END DO ! end loop on irelax
```

Contents

First results (1/2)



First results (2/2)

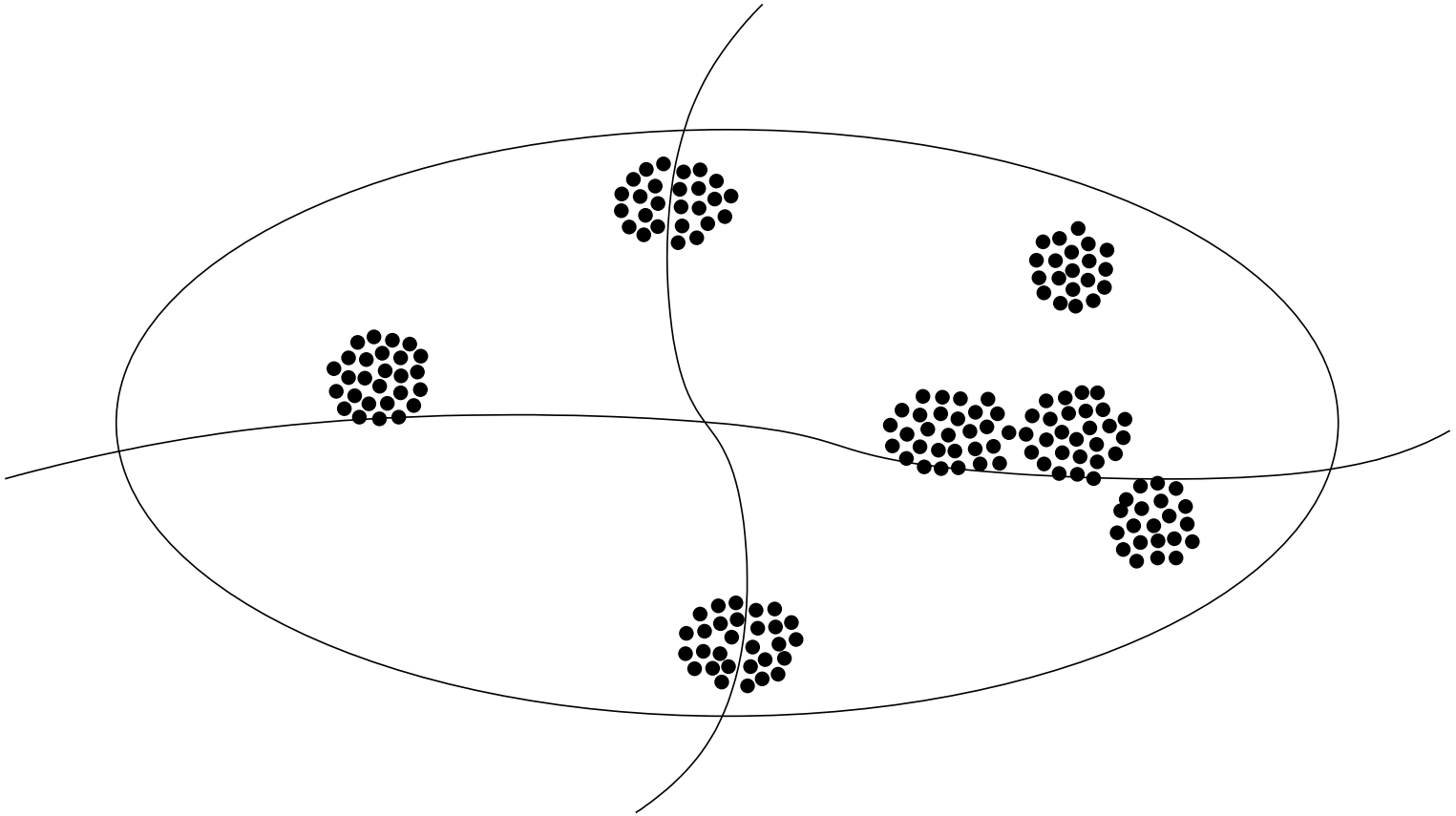


Contents

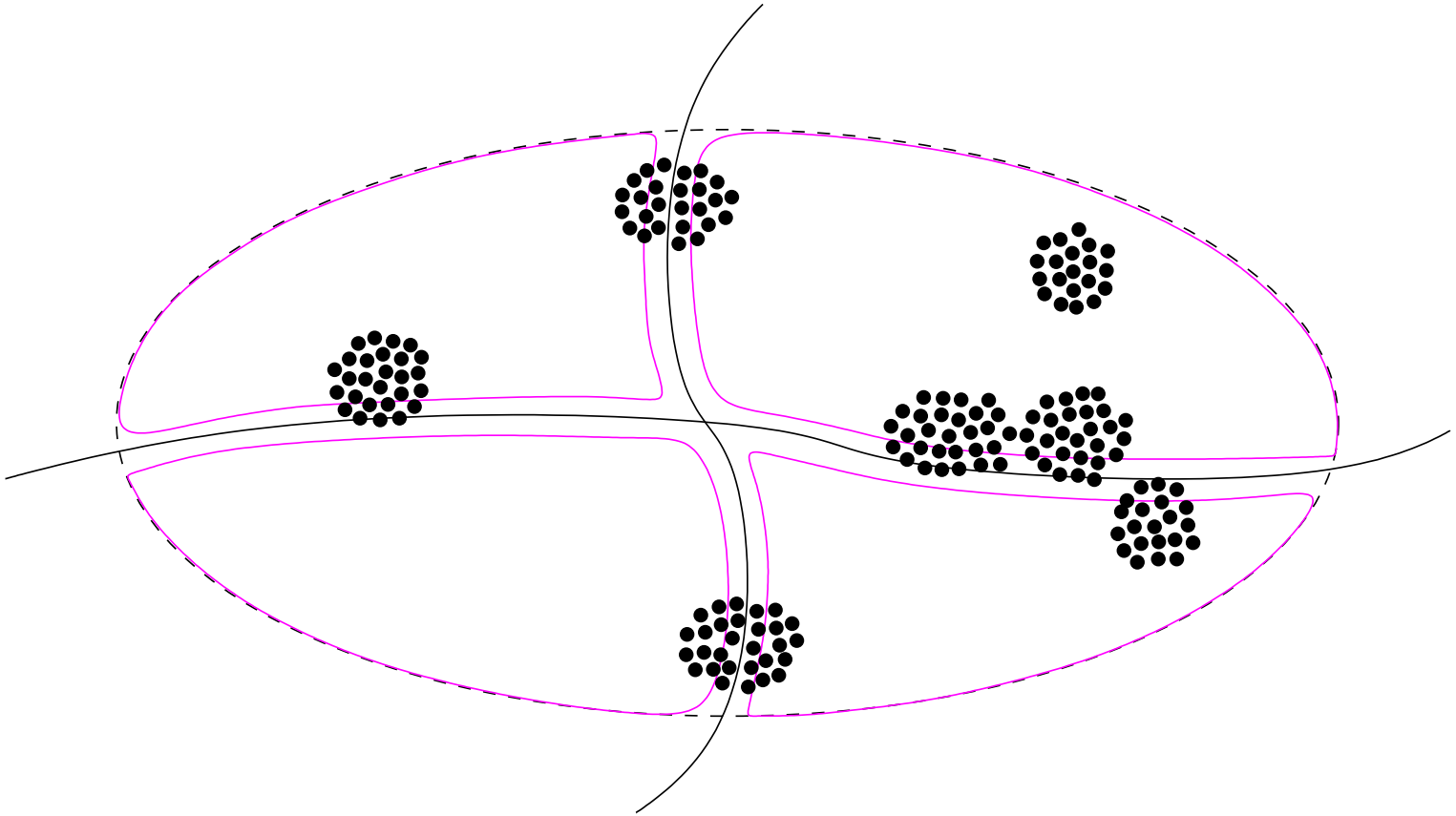
Work in progress

- Release of version 0.1
  - Available soon from Inria Gforge
  - Licensed under GPL
- Parallel mesh adaptation based on sequential remesher

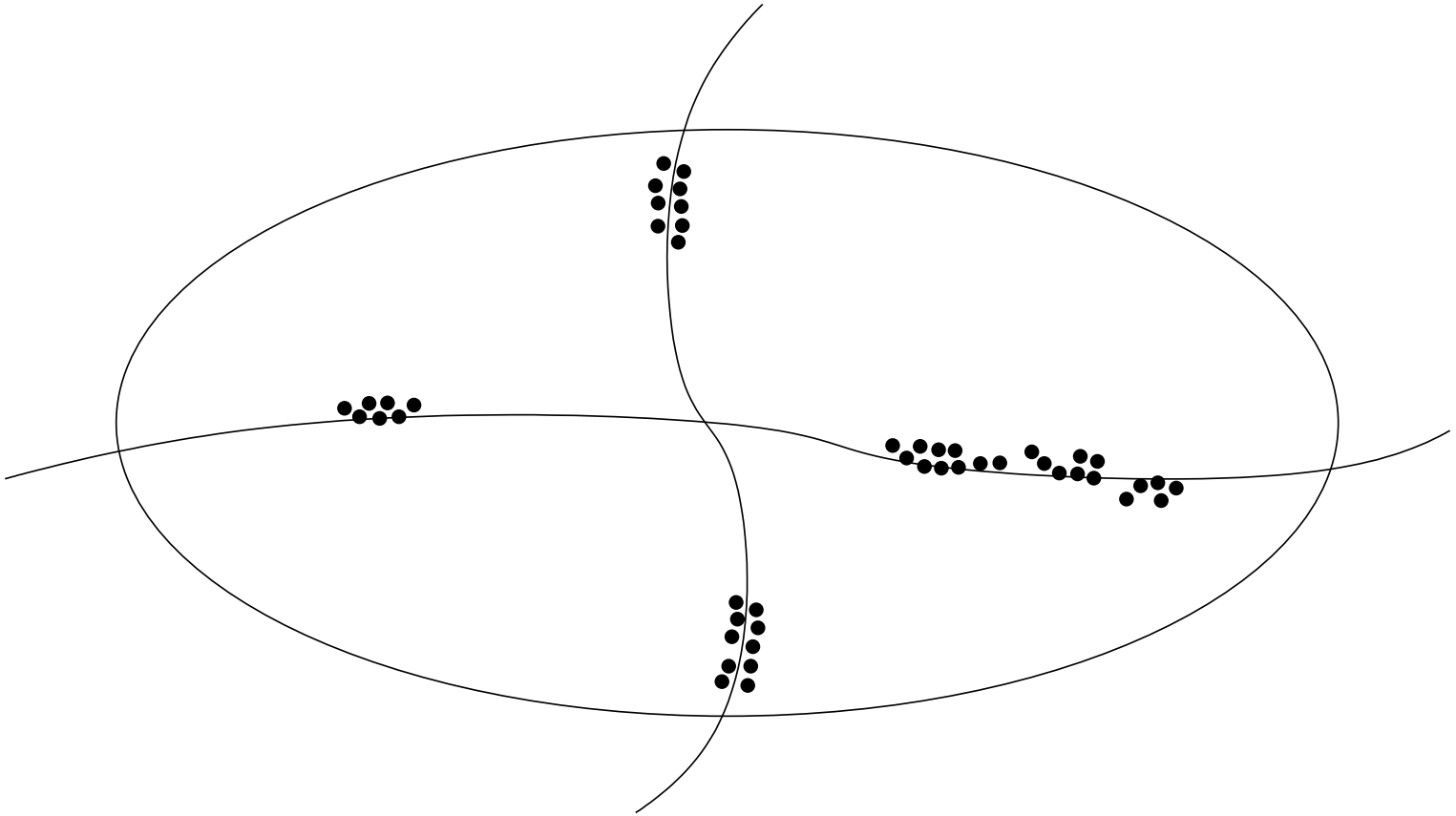
Adaptation: zones to remesh



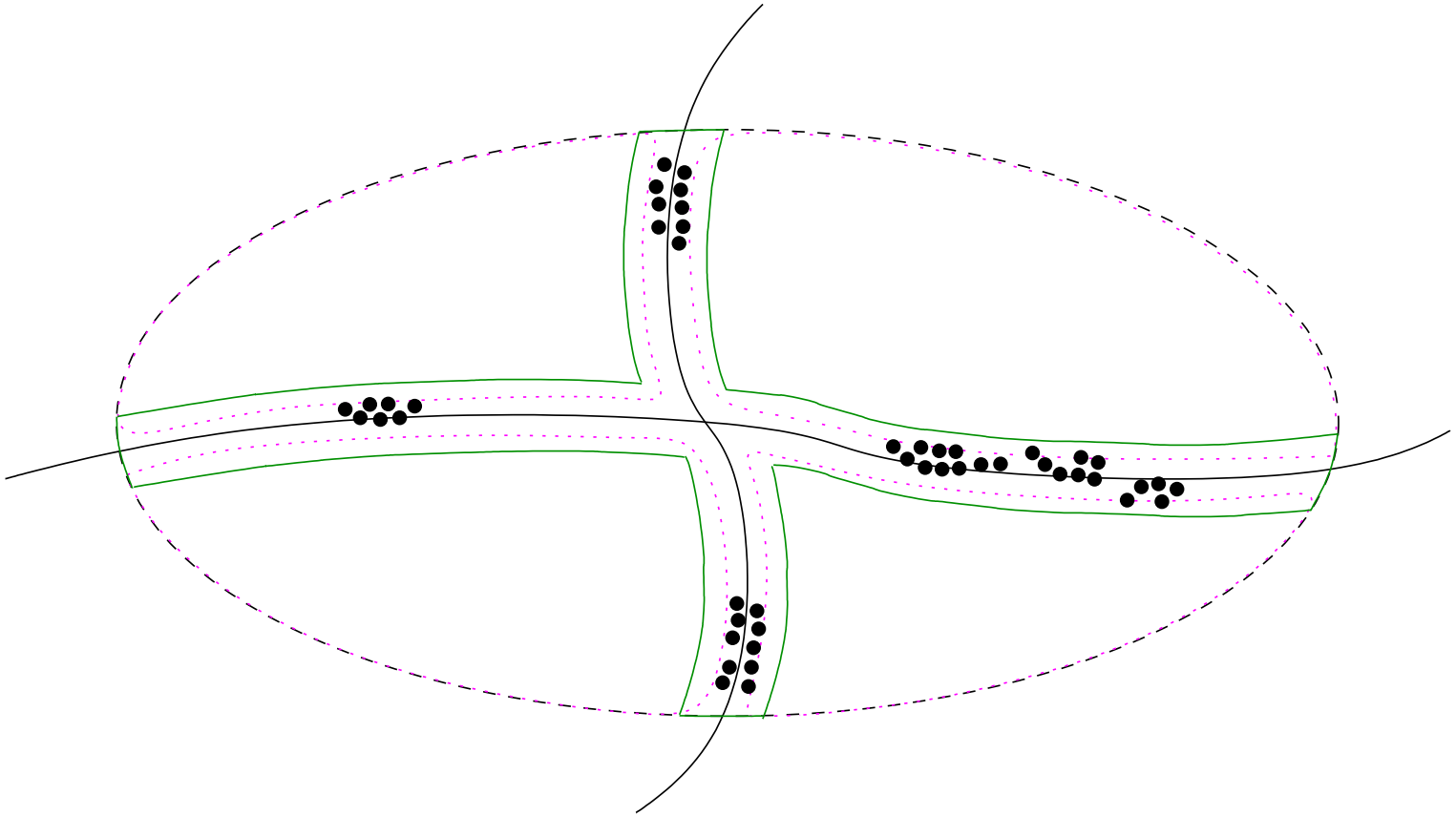
Adaptation: local remeshing



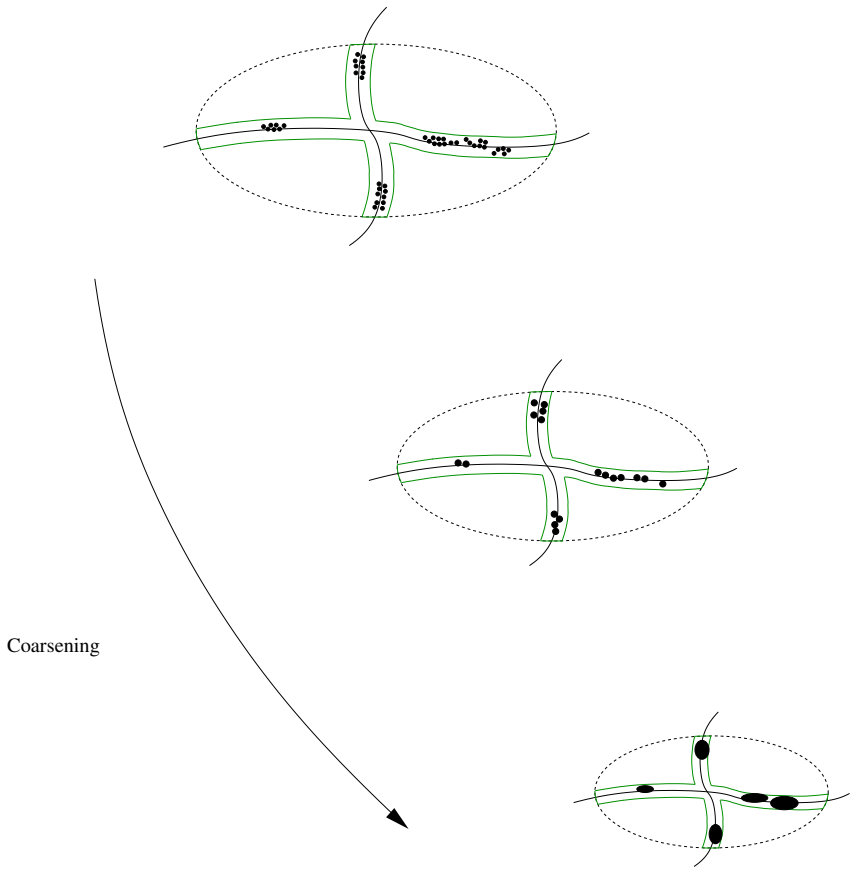
Adaptation: remaining zones



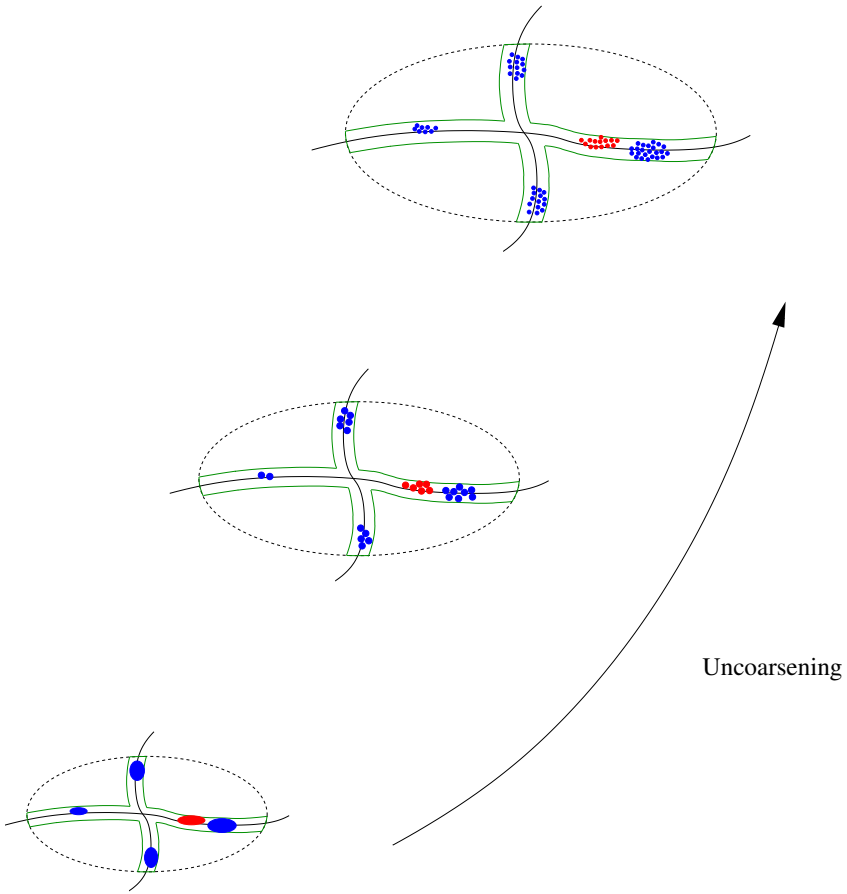
Adaptation: remeshing of frontier zones



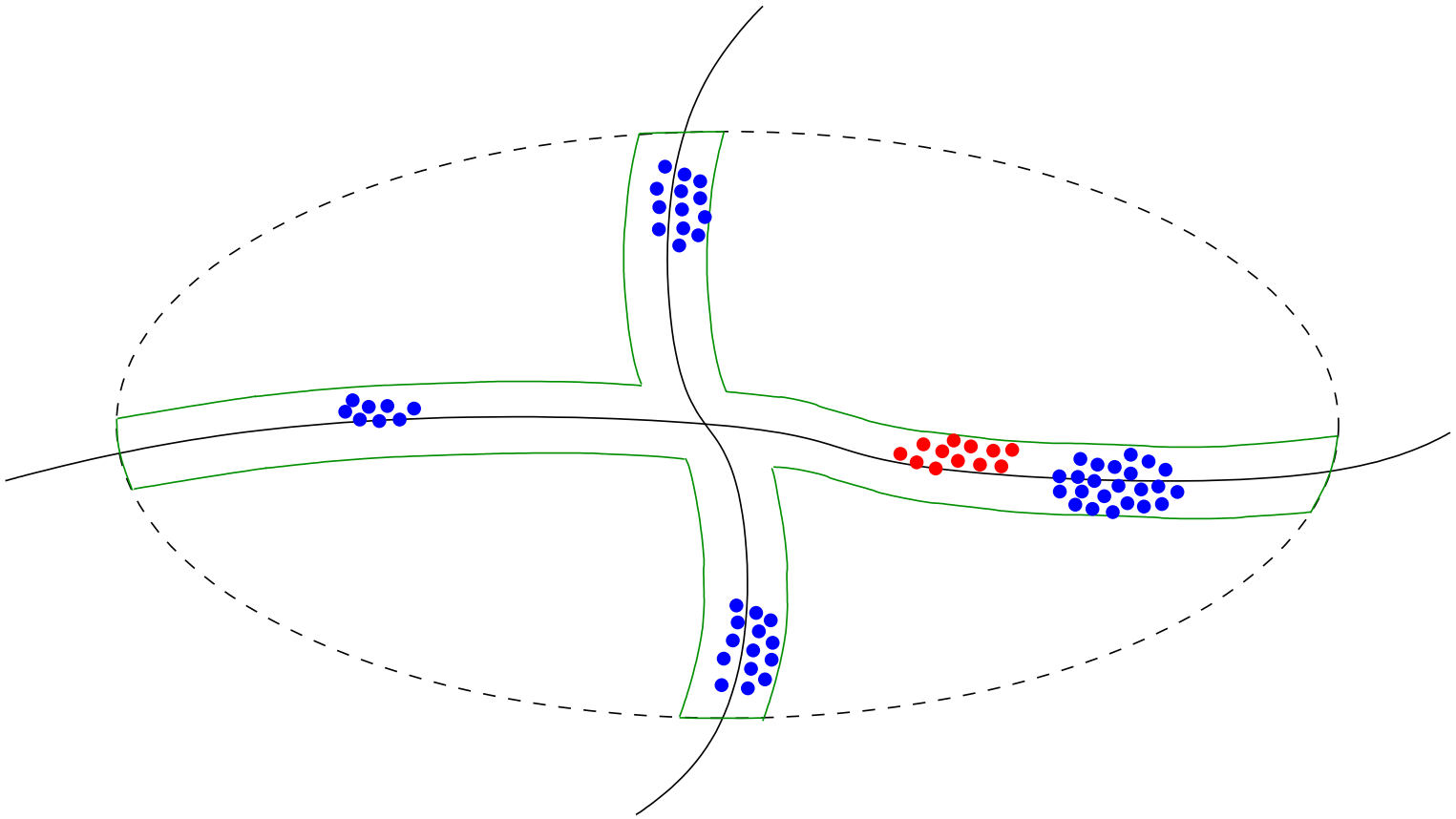
Adaptation: coarsening (iteration 1)



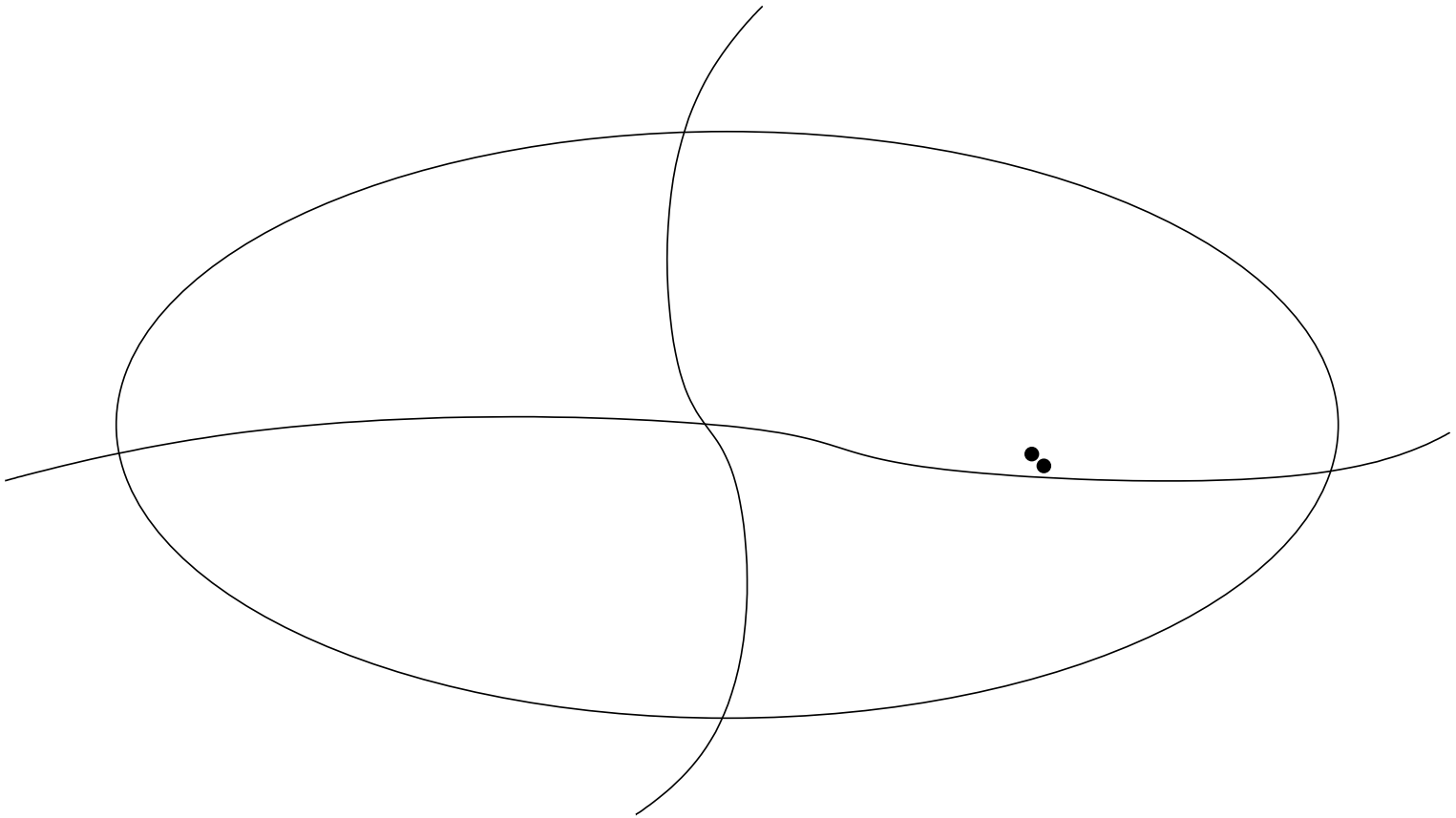
Adaptation: uncoarsening (iteration 1)



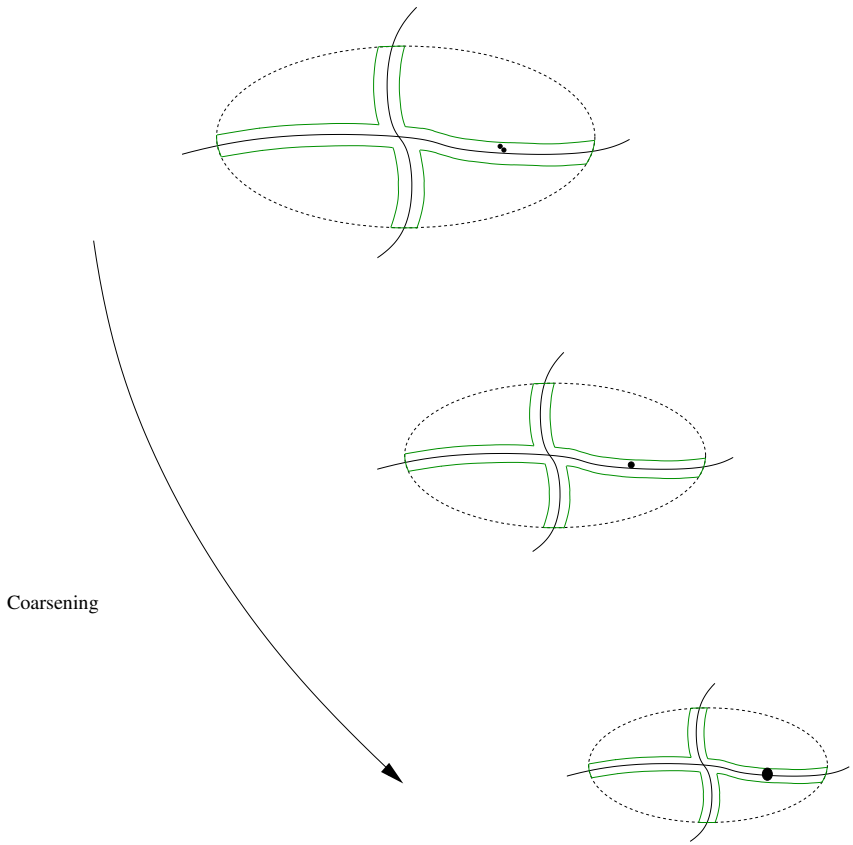
Adaptation: induced subgraphs (iteration 1)



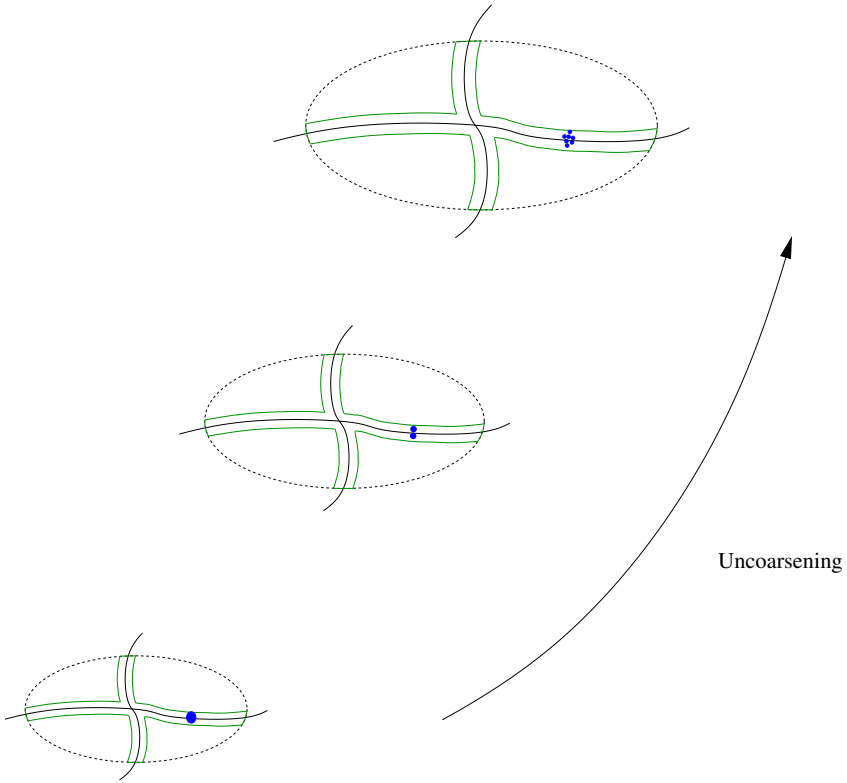
Adaptation: remaining zones



Adaptation: coarsening (iteration 2)



Adaptation: uncoarsening (iteration 2)



Contents

Upcoming features

- Parallel I/O with HDF5
- Unbreakable relations
  - Partitioner will not cut these edges
  - E.g. to implement DG methods
- Source code parser to infer automatically:
  - Overlap and halo sizes
  - Entity and relation types

Thank you


A. Basermann, J. Clinckemaillie, T. Coupez, J. Fingberg, H. Digonnet, R. Ducloux, J.-M. Gratien, U. Hartmann, G. Lonsdale, B. Maerten, D. Roose, and C. Walshaw.
Dynamic load balancing of finite element applications with the drama library, 2000.


R. Chaube, R. L. Carino, and I. Banicescu.
Effectiveness of a dynamic load balancing library for scientific applications.
In *ISPDC '07: Proceedings of the Sixth International Symposium on Parallel and Distributed Computing*, page 32, Washington, DC, USA, 2007. IEEE Computer Society.


N. Chrisochoides.
Parallel mesh generation.
In *in Numerical Solution of Partial Differential Equations on Parallel Computers*. Springer, 2005.


T. Coupez, H. Digonnet, and R. Ducloux.
Parallel meshing and remeshing.
Applied Mathematical Modelling, 25(2):153 – 175, 2000.
Dynamic load balancing of mesh-based applications on parallel.


K. Devine, B. Hendrickson, E. Boman, M. St. John, and C. Vaughan.
Design of dynamic load-balancing tools for parallel applications.
In *ICS '00: Proceedings of the 14th international conference on Supercomputing*, pages 110–118, New York, NY, USA, 2000. ACM.


O. Hassan, K. A. Sørensen, K. Morgan, and N. P. Weatherill.
A method for time accurate turbulent compressible fluid flow simulation with moving boundary components employing local remeshing.
In *International Journal for Numerical Methods in Fluids*, volume 53, pages 1243–1266. John Wiley & Sons, 2007.


B. G. Larwood, N. Weatherhill, O. Hassan, and K. Morgan.
Domain decomposition approach for parallel unstructured mesh generation.
In *International Journal for Numerical Methods in Engineering*, volume 58, pages 177–188, 2003.


U. Tremel, K. A. Sørensen, S. Hitzel, H. Rieger, O. Hassan, and N. P. Weatherill.
Parallel remeshing of unstructured volume grids for cfd applications.
In *International Journal for Numerical Methods in Fluids*, volume 53, pages 1361–1379. John Wiley & Sons, 2007.