Kinematic analysis of the 4-3-1 and 3-2-1 wire-driven parallel crane

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Background

A typical wire-driven parallel crane: `MARIONET-CRANE`

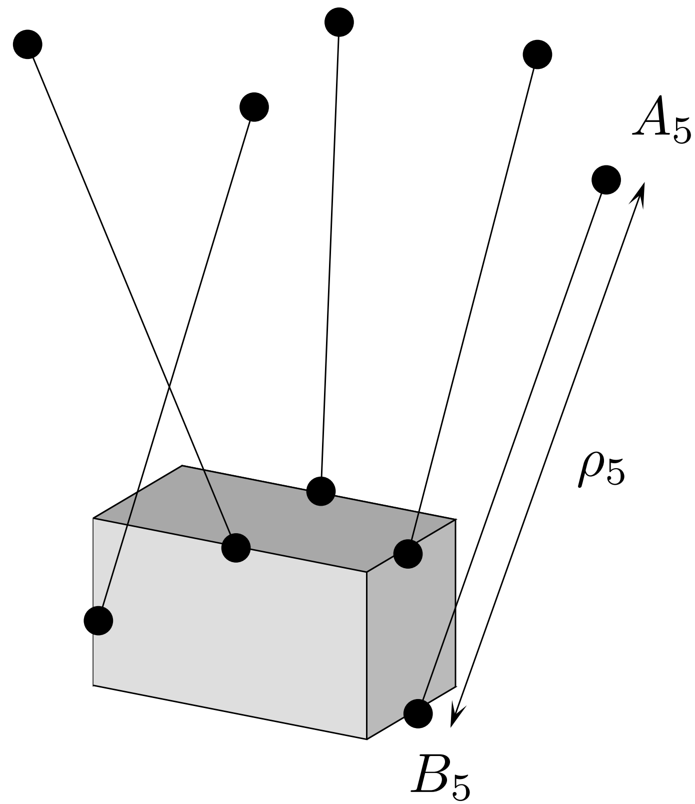
- 6 dof, 6 wires, 200 kg
- lifting ability: 2.5 tons
- deployable in 10 mn

Background

Usual kinematics equations for m wires

- $\rho_j^2 = ||\mathbf{AB}(\mathbf{X})||^2$ (m)
- $\mathcal{F} = \mathbf{J}^{-\mathbf{T}}(\mathbf{X})\tau$ (6)

- IK:
  - m components of $\mathbf{X}$ given
  - 6 unknowns:$\mathbf{X}(6-m), \tau(m)$
- FK:
  - m ρ given
  - $6+m$ unknowns: $\mathbf{X}, \tau$



Background

BUT underlying assumption is:

all wires are under tension which maybe **WRONG**

Real kinematics equations

- $\rho_j^2 = ||\mathbf{AB}(\mathbf{X})||^2$ **and** $\tau_j > 0$
- $\rho_j^2 > ||\mathbf{AB}(\mathbf{X})||^2$ **and** $\tau_j = 0$
- $\mathcal{F} = \mathbf{J}^{-\mathbf{T}}(\mathbf{X})\tau$

$$\Downarrow$$

for a robot with m wires we have to solve **ALL** the IK and FK problems for **ALL** robot with 1 to m wires under tension

Background

This is a difficult and open issue

- 6 wires under tension
  - solve the FK like a classical parallel robot
  - check that $\tau > 0$ by solving the 6×6 linear system $\mathcal{F} = \mathbf{J}^{-\mathbf{T}}(\mathbf{X})\tau$

Background

This is a difficult and open issue

- 6 wires under tension
- 5 wires under tension
  - maximal number of solutions ?
  - maximal number of solutions with $\tau > 0$?
  - maximal number of stable solutions with $\tau > 0$?
  - solving method ?

Background

This is a difficult and open issue

- 6 wires under tension
- 5 wires under tension
- 4 wires under tension
  - maximal number of solutions: ≤ 216
  - maximal number of solutions with $\tau > 0$?
  - maximal number of stable solutions with $\tau > 0$?
  - solving method ?

Background

This is a difficult and open issue

- 6 wires under tension
- 5 wires under tension
- 4 wires under tension
- 3 wires under tension
  - maximal number of solutions: ≤ 158
  - maximal number of solutions with $\tau > 0$?
  - maximal number of stable solutions with $\tau > 0$?
  - solving method ?

Background

This is a difficult and open issue

- 6 wires under tension
- 5 wires under tension
- 4 wires under tension
- 3 wires under tension
- 2 wires under tension
  - maximal number of solutions: ≤ 24
  - maximal number of solutions with $\tau > 0$?
  - maximal number of stable solutions with $\tau > 0$?

Background

More than 6 wires $\Rightarrow$ redundancy

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More than 6 wires $\Rightarrow$ redundancy

Conjecture for **stiff wires**

there will be never more than 6 wires under tension at the same time

Proved for the $N-1$ robot (all wires attached at the same point on the platform)

never more than 3 wires under tension, whatever is $m > 3$

Background

More than 6 wires $\Rightarrow$ redundancy

elastic wires:

- indeed all wires may be under tension
- can we manage the wire tensions distribution ?
  - requires a perfect elasticity model and a perfect knowledge of elasticity parameters
  - a 10% uncertainties on the parameters may lead to a change of 200% in tension values . . .

Contributions

Contributions

- general robot geometry: all attachments point B and output points A are distinct
- specific robot geometries: some points A, B are identical

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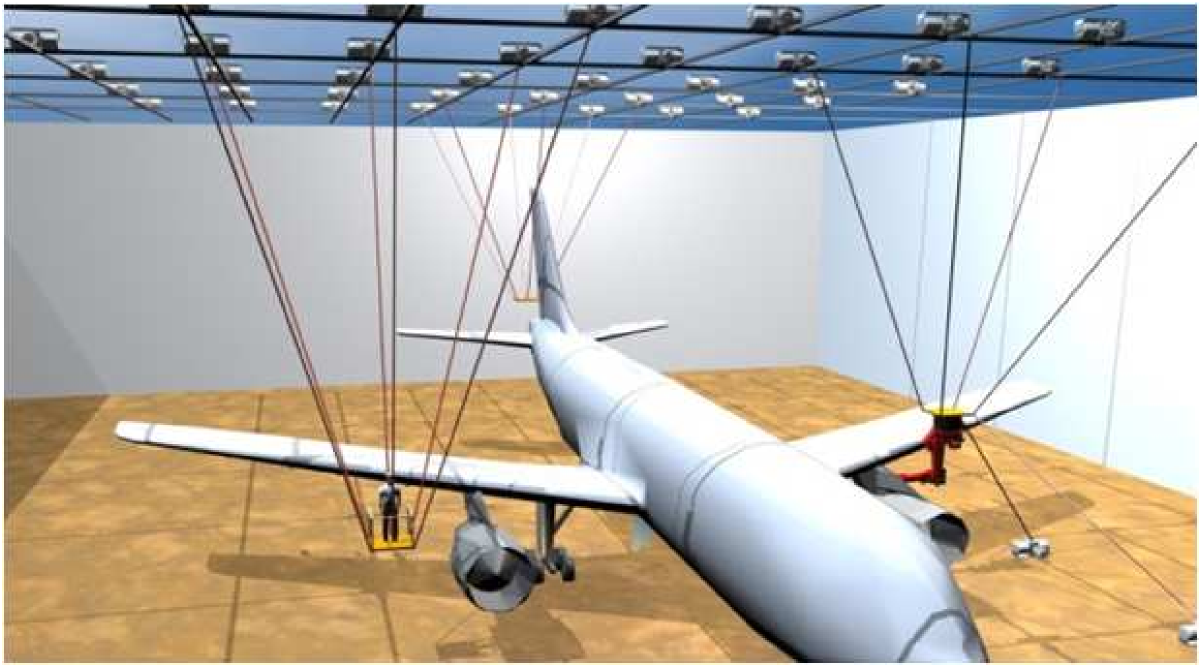
Interests

- may be useful for modular robot



Contributions

- general robot geometry: all attachments point B and output points A are distinct
- specific robot geometries: some points A, B are identical

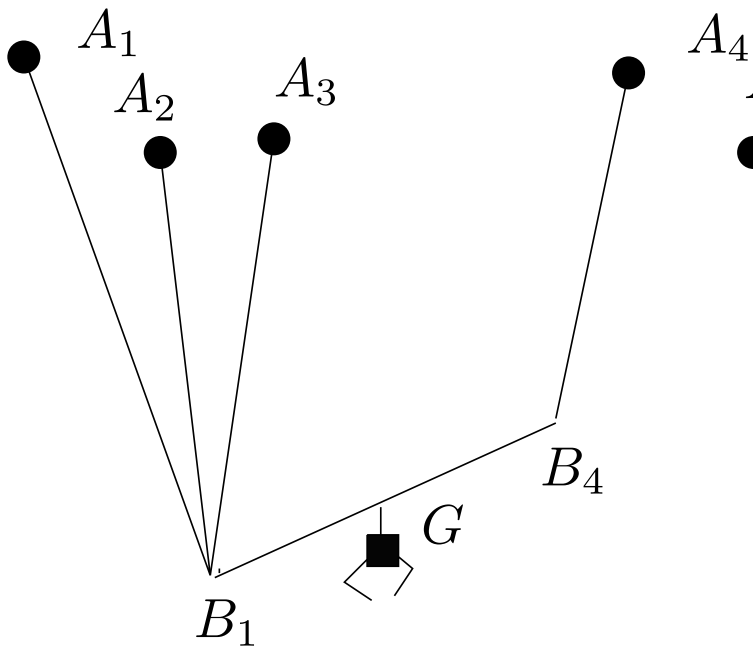
Interests

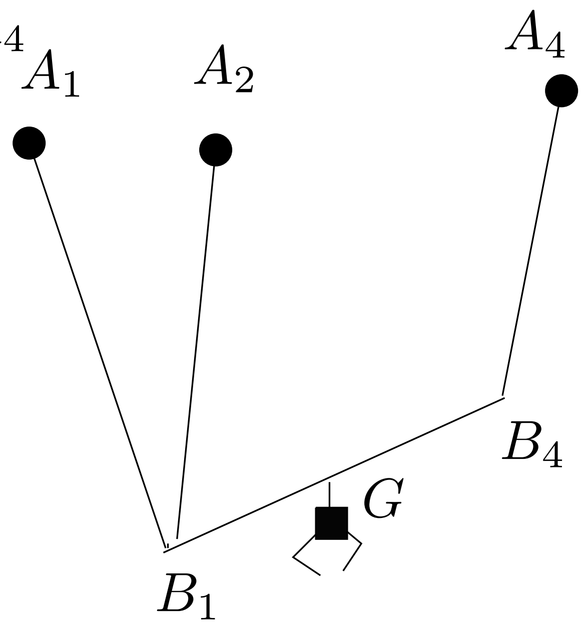
- may be useful for modular robot
- simpler kinematics

Contributions

Analysis of

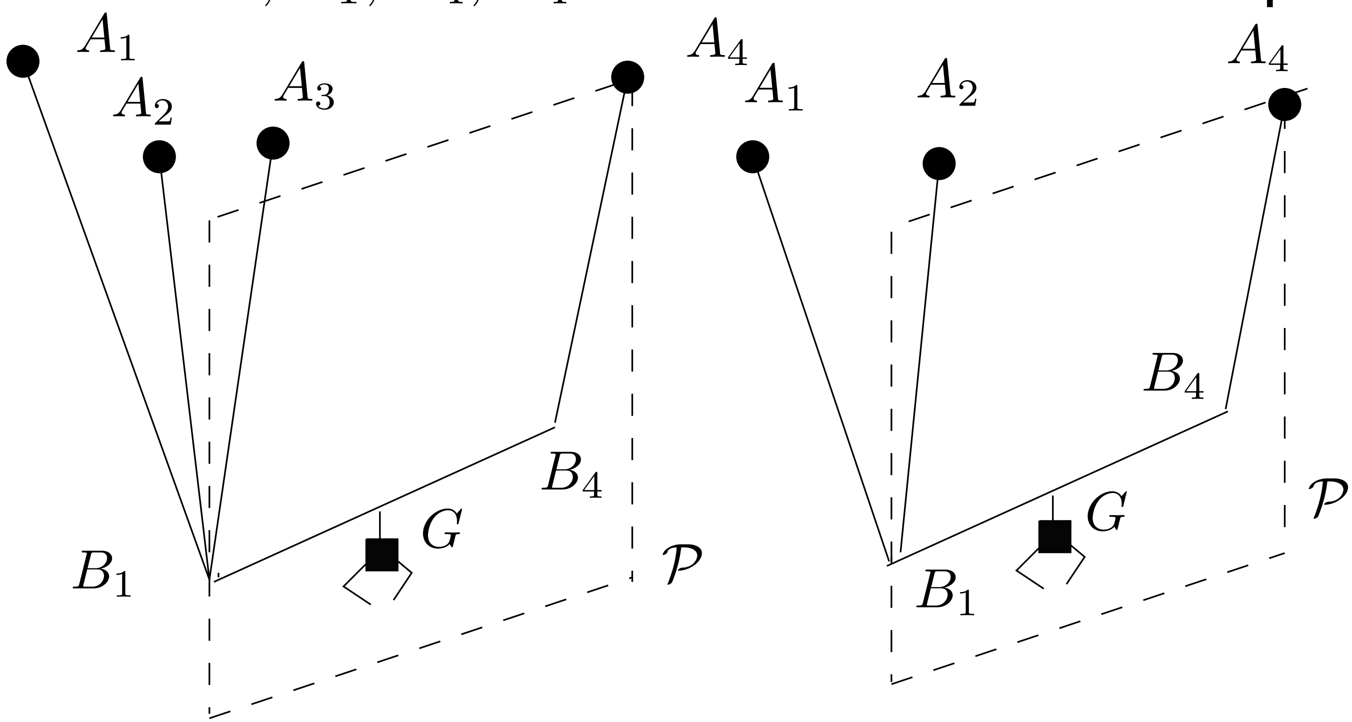
- 4-3-1: 4 wires, 2 anchor points on the platform
- 3-2-1: 3 wires, 2 anchor points on the platform





Contributions

for both robots: G, B_1, B_4, A_4 lie in the same vertical plane $\mathcal{P}$



- 4-3-1 has 4 dof: rotation of $\mathcal{P}$ around the z axis, orientation of the platform in this plane, location of G in this plane
- 3-2-1 has 3 dof: rotation of $\mathcal{P}$ around the z axis, location of G in this plane

Contributions

Important note:

- if wire 4 of the 4-3-1 becomes slack $\longrightarrow$ 3-1 with known kinematics
- if wire 1 or 2 or 3 of the 4-3-1 becomes slack $\longrightarrow$ 3-2-1
- if a pair of wires in (1,2,3) of the 4-3-1 becomes slack $\longrightarrow$ 2-2 with known kinematics
- if wire 1 or 2 in the 3-2-1 becomes slack $\longrightarrow$ 2-2 with known kinematics

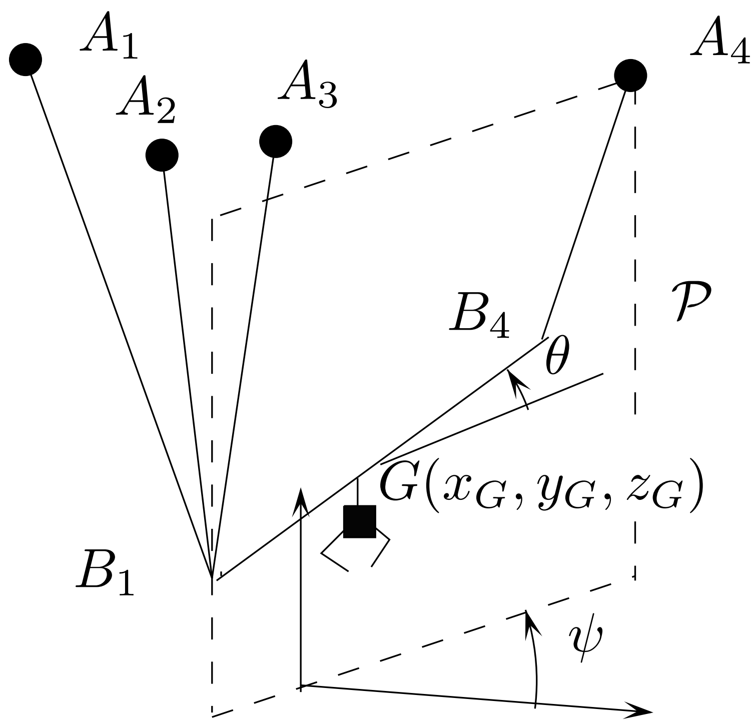
$$\Downarrow$$

all sub-kinematics problems for these robot have been solved

Inverse kinematics

Inverse kinematics

Inverse kinematics of the 4-3-2

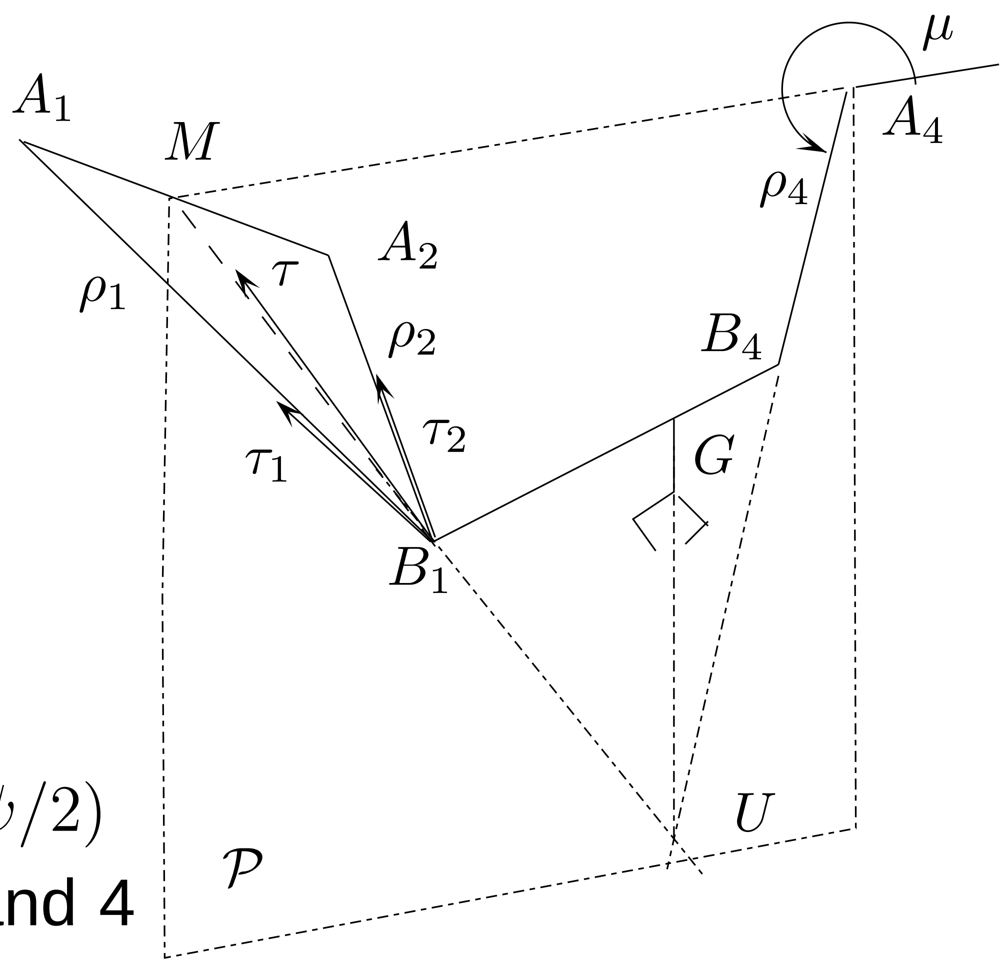


constraints: $(\mathbf{A_4B_4} \times \mathbf{A_4B_1}).\mathbf{z} = 0$

- x_G, z_G, θ, ψ fixed: single solution for y_G
- y_G, z_G, θ, ψ fixed: single solution for x_G
- x_G, y_G, z_G fixed $\Rightarrow$ two solutions for ψ

Inverse kinematics

Inverse kinematics of the 3-2-1



constraints:

$$(\mathbf{A_4B_4} \times \mathbf{A_4B_1}).\mathbf{z} = 0$$

$$\mathbf{A_4U} \times \mathbf{A_4B_4} = \mathbf{0}$$

$$\mathbf{MU} \times \mathbf{MB_1} = \mathbf{0}$$

5 variables: $x_G, y_G, z_G, \psi, \theta$

2 constraint equations

- degree 1 in x_G, y_G, 2 in $\tan(\psi/2)$
- degree 1 in z_G, 2 in x_G, y_G and 4 in $\tan(\psi/2), \tan(\theta/2)$

Forward kinematics

Forward kinematics

Forward kinematics of the 4-3-1

Experimental check: with one of our prototypes `MARIONET-ASSIST`

for arbitrary wire lengths the robot may have 1,2,3 or 4 wires under tension
see `http://www-sop.inria.fr/coprin/prototypes/main.html`

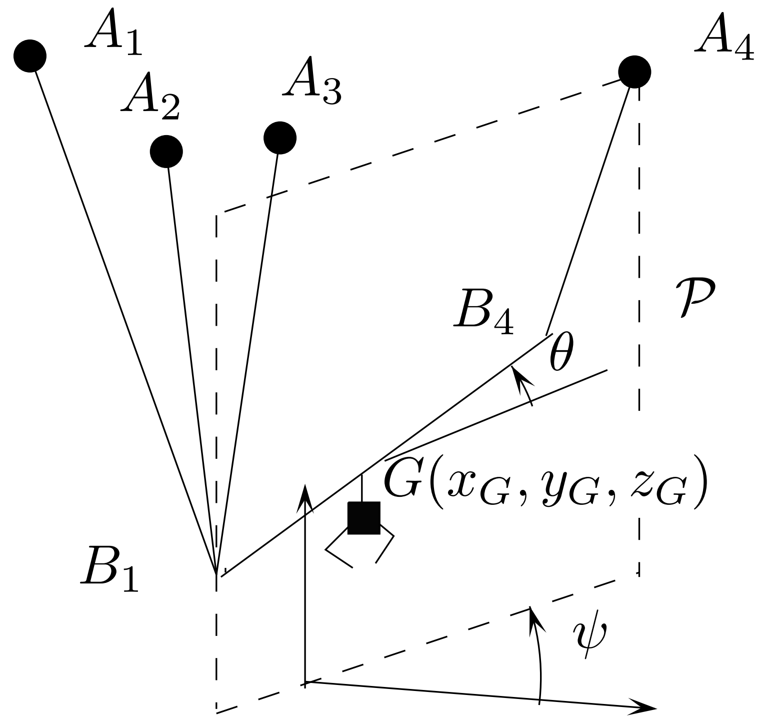
$\Downarrow$

it is essential to check the forward kinematics of all sub-robots

Forward kinematics

Forward kinematics of the 4-3-1

- lengths of 1,2,3 known
  $\Rightarrow B_1$ is known
  (2 solutions but only one is valid)
- B_1 fixed $\Rightarrow \mathcal{P}$ is fixed
  (2 solutions for ψ)
- B_4 at the intersection of 2 circle
  $\Rightarrow$ 2 solutions for B_4
- 4 solutions for the FK



Forward kinematics

Forward kinematics of the 3-2-1: theory

- 2 algebraic constraints in $T = \tan(\psi/2)$, $T_1 = \tan(\theta/2)$
- very large
- resultant leads to polynomial of degree 64 in T_1
- for a given T_1 there will be at most 4 solutions in T
  $\Rightarrow$ at most 256 solutions

Forward kinematics

Forward kinematics of the 3-2-1: symbolic/numerical

several thousands tests with random geometry and wire lengths

- the 64th order polynomial always factor out
- the valid roots are always obtained from a 32th order polynomial

Conjecture: there will be at most 32 solutions to the FK

- examples with
  - 26 real roots to the polynomial,
  - up to 10 solutions with positive wire tensions

Forward kinematics

Forward kinematics of the 3-2-1: numerical

- solving a 64/32th order polynomial is prone to numerical round-off errors
- FK may be formulated as solving a set of 3 equations with angles as unknowns
  $\Rightarrow$ bounded unknowns
- solving with interval analysis: mean computation time less than 0.6s for getting safely all solutions

Numerical analysis for the 4-3-1

Numerical analysis for the 4-3-1

A given robot with:

	x	y	z
A_1	279.04	229.06	305.6
A_2	278.708	10.593	310.5
A_3	11.918	8.368	310.5
A_4	-8.717	217.543	310.5

Two possible platforms

	B_1	B_4
platform 1	(0,-10,0)	(0,10,0)
platform 2	(0,-10,0)	(0,10,10)

Numerical analysis for the 4-3-1

IK check: 4×10^6 poses selected randomly in

$$x_G \in [50, 150] \quad y_G \in [50, 150]$$
$$z_G \in [0, 200] \quad \theta \in [-60, 60](\text{degrees})$$

Percentage of IK with positive wire tensions

- platform 1: 34.8%
- platform 2: 43.31%

Numerical analysis for the 4-3-1

FK check

- for a given $\mathbf{X}$ use the IK to compute the ρ
- use the FK to determine the platform poses $\mathbf{X_r}$

FK distance:

- 0 if $\mathbf{X}$ is valid pose with positive wire tensions
- otherwise maximal distance between $\mathbf{X_r}$ and $\mathbf{X}$

FK distance	min	max	average
platform 1	0	20	9.53
platform 2	0	20	10.59

FK problems for the 4-3-1

number of problems, [polynomial degree], (maximal number of solutions)

- 1 wire under tension: 4 [1](4)
- 2 wires under tension: 6 [2,12](3, 3 $\times$ 24)
- 3 wires under tension: 4 [2,64],(1, 3 $\times$ 64 $\times$ 4)
- 4 wires under tension: 1 [2],(4)

15 FK problems, maximal number of solution: 852

Conclusion

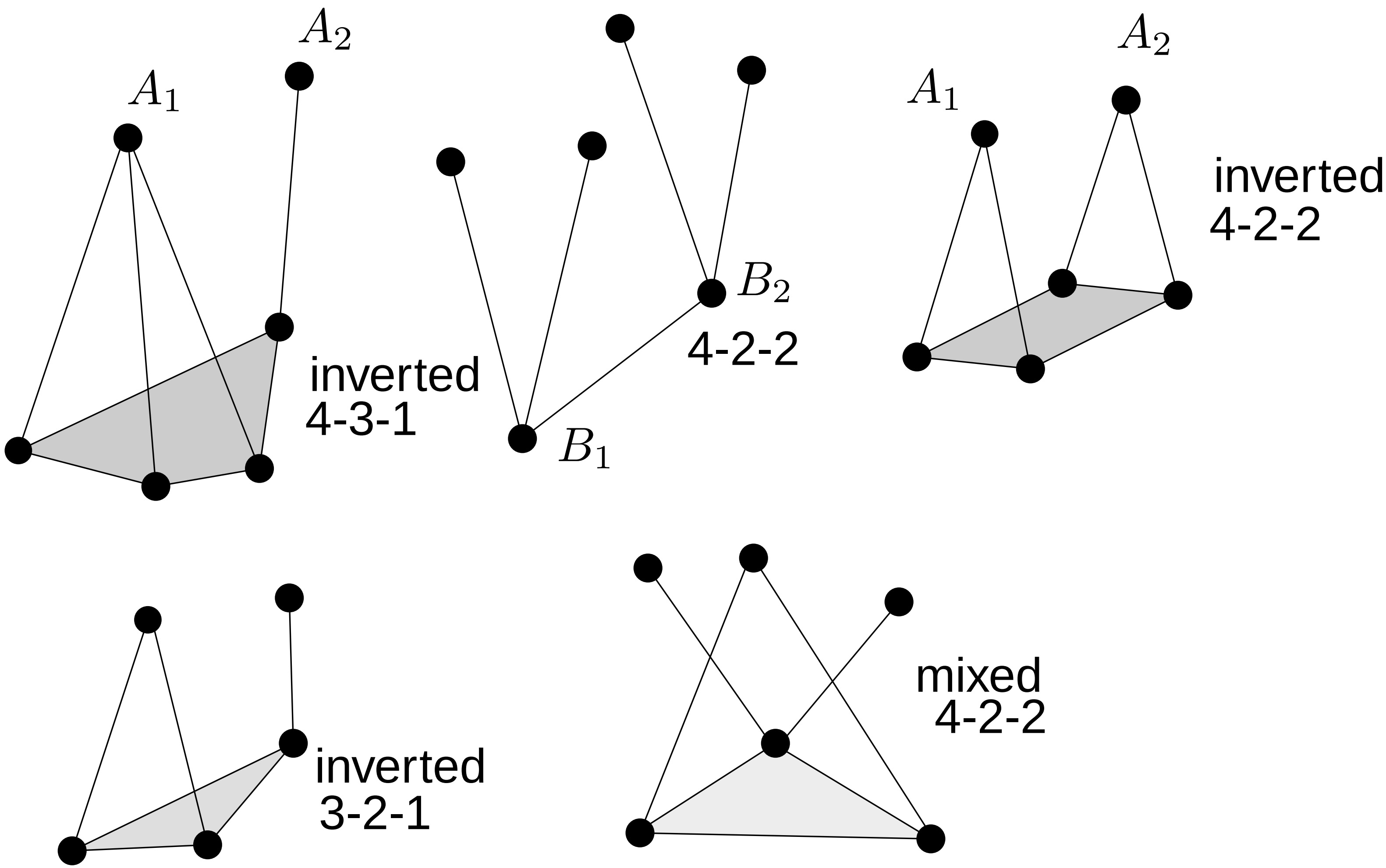
Conclusion

- kinematics for specific cases of WDPR with 3 and 4 wires
- simpler than the general case
- necessity of the study of the kinematics of sub-robots validated:
  in over 50% of the cases the 4-3-1 acts like a 3-2-1

Conclusion

Open issues

- analysis of other specific cases



Conclusion

Open issues

- analysis of other specific cases
- checking if a configuration change 4-3-1 $\leftrightarrow$ 3-2-1 may occur on a trajectory
  - extension of the singularity concept
  - uncertainties have to be taken into account
- what sensing mean will allow to detect a configuration change ?