Unsolved Issues in Kinematics and Redundancy of Wire-driven Parallel Robots

J-P. Merlet
COPRIN project-team
INRIA Sophia-Antipolis
France

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Introduction

Introduction

Focus: robot in a crane configuration

(An old story: L'ARGENT (1928))

Introduction

Examples: MARIONET-CRANE, MARIONET-ASSIST

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Introduction

Notation

- N: number of wires
- τ: tension in the wire (positive if the wire is under tension)
- ρ: length of the wire
- A: output point of a wire on the base
- B: attachment point of a wire on the platform

Introduction

mechanical equilibrium:

$$\mathcal{F} = \mathbf{J}^{-\mathbf{T}}\tau$$

- $\mathcal{F} = (0, 0, -mg, 0, 0, 0)$
- 6 equations, linear in the N τ, non-linear in $\mathbf{X}$

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Inverse Kinematics

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Inverse kinematics (IK), **rigid wires**

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Inverse kinematics (IK), **rigid wires**

- $\underline{N \geq 6 \text{ (spatial)}, N \geq 3 \text{ (planar)}}$:
  - single solution for the ρ

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Inverse Kinematics

Inverse kinematics (IK), **rigid wires**

- $N \geq 6$ (spatial), $N \geq 3$ (planar):
  - single solution for the ρ
  - $N = 6$
    - single solution for the τ

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Inverse Kinematics

Inverse kinematics (IK), **rigid wires**

- $\underline{N \geq 6 \text{ (spatial)}, N \geq 3 \text{ (planar)}}$:
  - single solution for the ρ
  - $\underline{N = 6}$
    - single solution for the τ
    - what should we do if $\exists \tau_i < 0$?: find the "closest" $\mathbf{X_r}$ such that all $\tau > 0$?

Inverse Kinematics

Inverse kinematics (IK), **rigid wires**

- $\underline{N \geq 6 \text{ (spatial)}, N \geq 3 \text{ (planar)}}$:
  - single solution for the ρ
  - $\underline{N > 6}$: theoretically not a single solution for the τ
  - redundancy ?

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Inverse Kinematics

Inverse kinematics (IK), **rigid wires**

- $N < 6$ (spatial), $N < 3$ (planar):
  - only N d.o.f. may be controlled

Inverse Kinematics

Inverse kinematics (IK), **rigid wires**

- $N < 6$ (spatial), $N < 3$ (planar):
  - 6 equations from the mechanical equilibrium
  - unknowns: $6 - N$ components of $\mathbf{X}$, N τ, total: 6
  - mechanical equilibrium provides the system to find the $6 - N$ components of $\mathbf{X}$

Inverse Kinematics

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- $N < 6$ (spatial), $N < 3$ (planar):
  - 6 equations from the mechanical equilibrium
  - unknowns: $6 - N$ components of $\mathbf{X}$, $N\ \tau$, total: 6
  - mechanical equilibrium provides the system to find the $6 - N$ components of $\mathbf{X}$
  - computation may be involved according to the choice of the free variables

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Inverse Kinematics

Inverse kinematics (IK), **rigid wires**

- $N < 6$ (spatial), $N < 3$ (planar):
  - what happen if $\tau_i < 0$?

Inverse Kinematics

Inverse kinematics (IK), **elastic wires**

Inverse Kinematics

Inverse kinematics (IK), **elastic wires**

- $\tau = k(\rho - l)$
  - l: length at rest of the wire (control variable)

Inverse Kinematics

Inverse kinematics (IK), **elastic wires**

- $N = 6$
  - single solution for ρ, τ
  - what happen if $\exists\ \tau_i < 0$?

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Inverse Kinematics

Inverse kinematics (IK), **elastic wires**

- $N > 6$
  - single solution for ρ
  - theoretically multiple solution for τ: redundancy
  - find an "optimal" solution satisfying $\tau_i > 0$?

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Inverse Kinematics

Inverse kinematics (IK), **elastic wires**

- $N < 6$
  - same procedure than for the rigid cases
  - what happen if $\exists\ \tau_i < 0$?

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Forward Kinematics

Forward Kinematics

Even if the IK has provided a solution with all wires under tension, the final pose may have less than N wires under tension

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Forward Kinematics

Even if the IK has provided a solution with all wires under tension, the final pose may have less than N wires under tension

$$\Downarrow$$

the current pose is a solution of the FK with 1 to N wires under tension

$$\Downarrow$$

all FK problems must be solved

VIDEO

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Forward Kinematics

Generic FK with $1, \ldots, m (\leq N)$ wires under tension

wire under tension	slack wires
$\rho_j = \|\|A_j B_j\|\|$ $j \in [1, m]$	$\rho_k \geq \|\|A_k B_k\|\|$ $k \in [m+1, N]$
$\tau_j \geq 0$ $j \in [1, m]$	$\tau_k = 0$ $k \in [m+1, N]$

$$\mathcal{F} = \mathbf{J}^{-\mathbf{T}} \tau \quad \text{with } 6 \times m \; \mathbf{J}^{-\mathbf{T}}$$

$$\tau = k(\rho - l)(\textbf{elastic wires})$$

Forward Kinematics

The generic FK should be solved for:

- all m in $[1, N]$
- all possible combinations of m wires among the N

Forward Kinematics

each FK is always a square system of equations:

- unknowns: **X** (6), τ (m), total: **6+m**
- m geometrical equations $\rho_j = ||A_jB_j||$
- 6 equations from the mechanical equilibrium

Forward Kinematics

FK may always be reduced to a system of 6 equations

mechanical equilibrium $\Rightarrow$ the lines A_iB_i+ the vertical line span a linear complex

$$\Downarrow$$

induces 6-m constraint equations that are τ-free

Forward Kinematics

FK state-of-the-art, **rigid case**

- $N = 6$
  - at most 40 solutions,
  - classical FK solving
  - a posteriori verification of $\tau \geq 0$

Forward Kinematics

FK state-of-the-art, **rigid case**

- $N = 6$: ≤ 40 solutions
- $N > 6$, all wires under tension ?
  - all wires should have **exactly** $\rho_j = ||A_jB_j||$
  - extremely unlikely

Forward Kinematics

FK state-of-the-art, **rigid case**

- $N = 6$: ≤ 40 solutions
- $N > 6$: ≤ 40 solutions
- $N = 5$
  - open issue: no known maximal number of solutions

Forward Kinematics

FK state-of-the-art, **rigid case**

- $N = 6$: ≤ 40 solutions
- $N > 6$: ≤ 40 solutions
- $N = 5$: ?
- $N = 4$: Carricato, ≤ 216 solutions

Forward Kinematics

FK state-of-the-art, **rigid case**

- $N = 6$: ≤ 40 solutions
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- $N = 4$: Carricato, ≤ 216 solutions
- $N = 3$: ≤ 156 solutions

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Forward Kinematics

FK state-of-the-art, **rigid case**

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- $N > 6$: ≤ 40 solutions
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- $N = 4$: Carricato, ≤ 216 solutions
- $N = 3$: ≤ 156 solutions
- $N = 2$: $\leq 2 \times 12$ solutions
- $N = 1$: 1 solution

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Forward Kinematics

FK state-of-the-art, **elastic case**

Much more involved, open issue

- $N = 6$: no more decoupling between geometry and statics
- $N = 3$: with a common $B \rightarrow$ up to 22 solutions (Duffy)

Forward Kinematics

The maximal number of solutions presented above does not take into account:

- that the τ should be positive
- that the solution must be stable
- that the geometry may be specific
  - example for $N = 3$ in the configuration 2-1 (only two B points) $\rightarrow$ no more than 64 solutions instead of 156

Finding the maximum number of stable solutions with $\tau > 0$ is an open issue

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Forward Kinematics

Numerical solving: for all solutions

- the degree of the univariate polynomial for the FK is too high for safe solving
- in many cases we don’t have analytical formulation of the coefficients of the univariate polynomial
- alternate approaches for computing all solutions: homotopy, interval analysis
- some d.o.f. cannot be controlled if one (or more) wire(s) are not under tension

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Forward Kinematics

Numerical solving: for all solutions

- for a given FK problem we have relatively large distances between the solutions with different wire configurations

determining the wire configuration is **crucial**

- for **elastic wires**:
  - solution is sensitive to k
  - τ is very sensitive to k

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Forward Kinematics

Numerical solving: real-time

- certified NR scheme works if we know the wire configuration
- if the wire configuration changes NR may not work because:
  - the system of equations vary according to the wire configuration
  - the initial guess is not good enough

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Forward Kinematics

Possible solution: adding sensory information

- measuring the τ
  - very noisy measurement
  - force sensor must have a large scale: mg+ dynamic effects $\longrightarrow$ poor accuracy
  - very sensitive to mechanical disturbances
  - difficult to implement
  - will it be sufficient to get a single solution ?

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Forward Kinematics

Possible solution: adding sensory information

- measuring the τ
- measuring wire direction (vision, rotary sensors at A, B)
  - relatively easy to implement
  - rough measurements
  - will it be sufficient to get a single solution ?
  - how many sensors are needed ? at which place ?

Forward Kinematics

Other considerations

- sagging
  - induces apparently less positioning errors than error in wire configuration: to be verified

Forward Kinematics

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    - taking sagging into account requires identification of multiple physical, time-varying, parameters: errors in these parameters leads to significant error in the positioning

Forward Kinematics

Other considerations

- sagging
  - induces apparently less positioning errors than error in wire configuration: to be verified
  - taking sagging into account requires identification of multiple physical, time-varying, parameters: errors in these parameters leads to significant error in the positioning
  - correcting sagging effect may lead to worse positioning errors ?

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Forward Kinematics

Other considerations

- sagging
- error in the location of the attachment points: especially if wires are attached at the same point
- time-varying location of the center of mass

Redundancy

Redundancy

- m d.o.f. to be controlled
- $N > m$ rigid wires

Is the robot redundant ?

Redundancy

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- no from a kinematic view point: for a given **x** there is usually a single solution for the IK

Redundancy

Is the robot redundant ?

- no from a kinematic view point: for a given **x** there is usually a single solution for the IK
- from a static viewpoint: for a given **x** can we use the additional wires to adjust the distribution of the τ ?

Redundancy

Some unexpected problems:

- a winch system allows one to control the length of a wire **or** its tension, but not both

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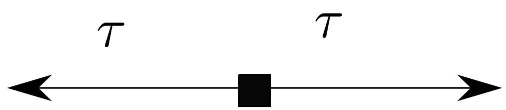
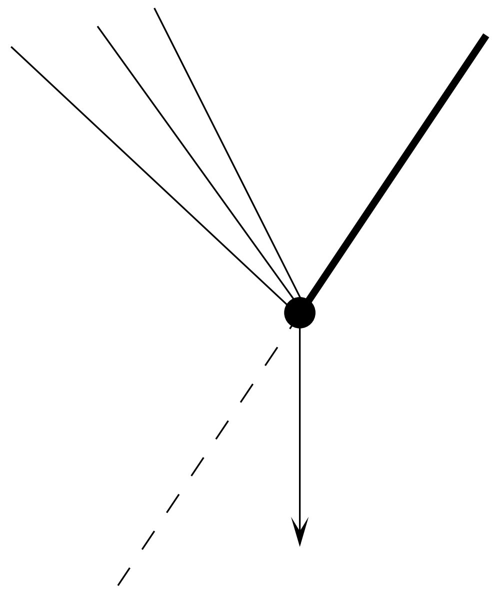
- a winch system allows one to control the length of a wire **or** its tension, but not both
- in a crane configuration there is no antagonistic wire whose tension control may allow to adjust the tension in a given wire





Redundancy

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- for a given pose the length of each wire has a single value

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$$\Downarrow$$

Parallel robot with rigid wires are not statically redundant

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Redundancy

Example: $N-1$ robot i.e. all wires attached to the same B point

- 3 d.o.f.
- whatever $N \geq 4$ there will be at most 3 wires under tension simultaneously
- the robot is <u>not</u> redundant

Redundancy

elastic wires

- multiple control l for the same pose $\mathbf{x}$

$$\Downarrow$$

the robot is redundant

Redundancy

Example: the N-1 robot

- we choose l so that $\sum \tau_j^2$ is minimal (analytical solution)

but

- we have uncertainties on the l, k that will induce positioning errors and imperfect tension distribution

Redundancy

we may solve the FK of this robot (difficult) and perform a sensitivity analysis

- 1% error on the l
- 10% error on the k

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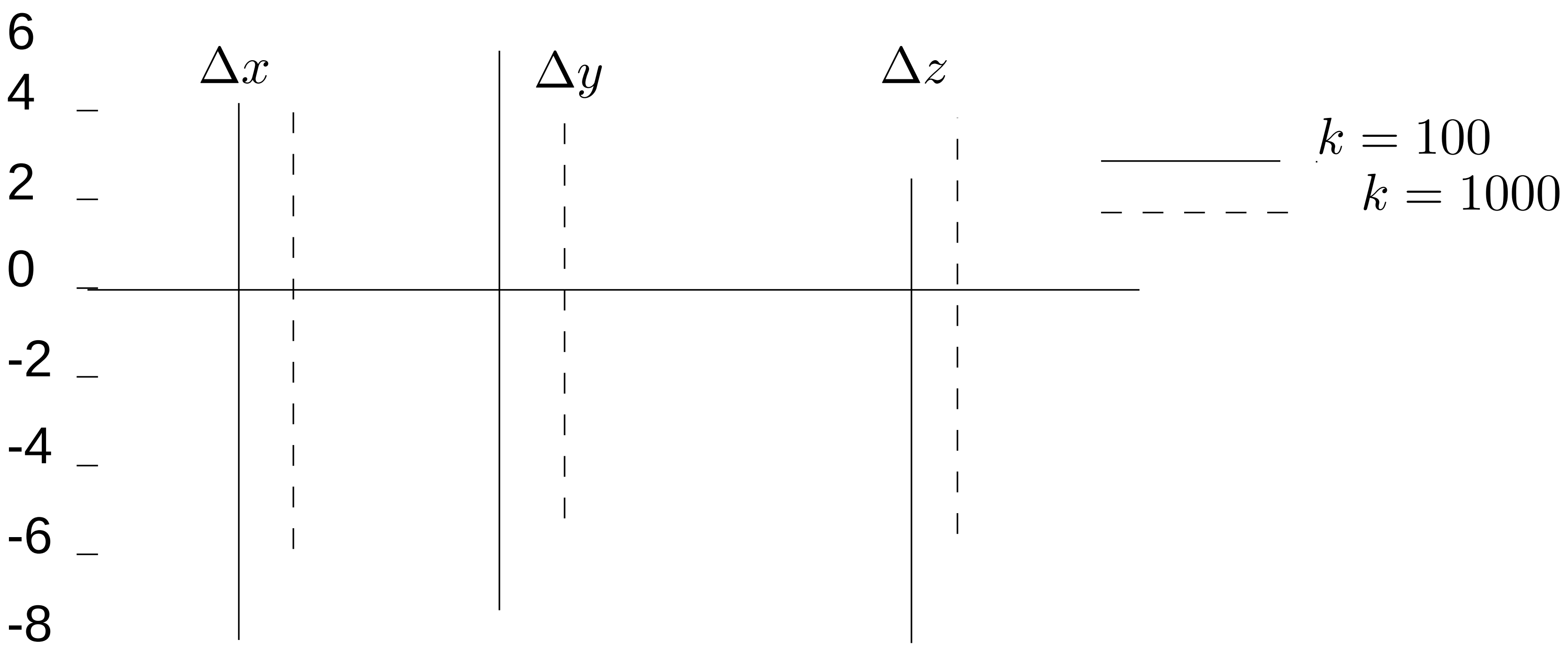
Redundancy



reasonable positioning errors: between 1 and 3 %

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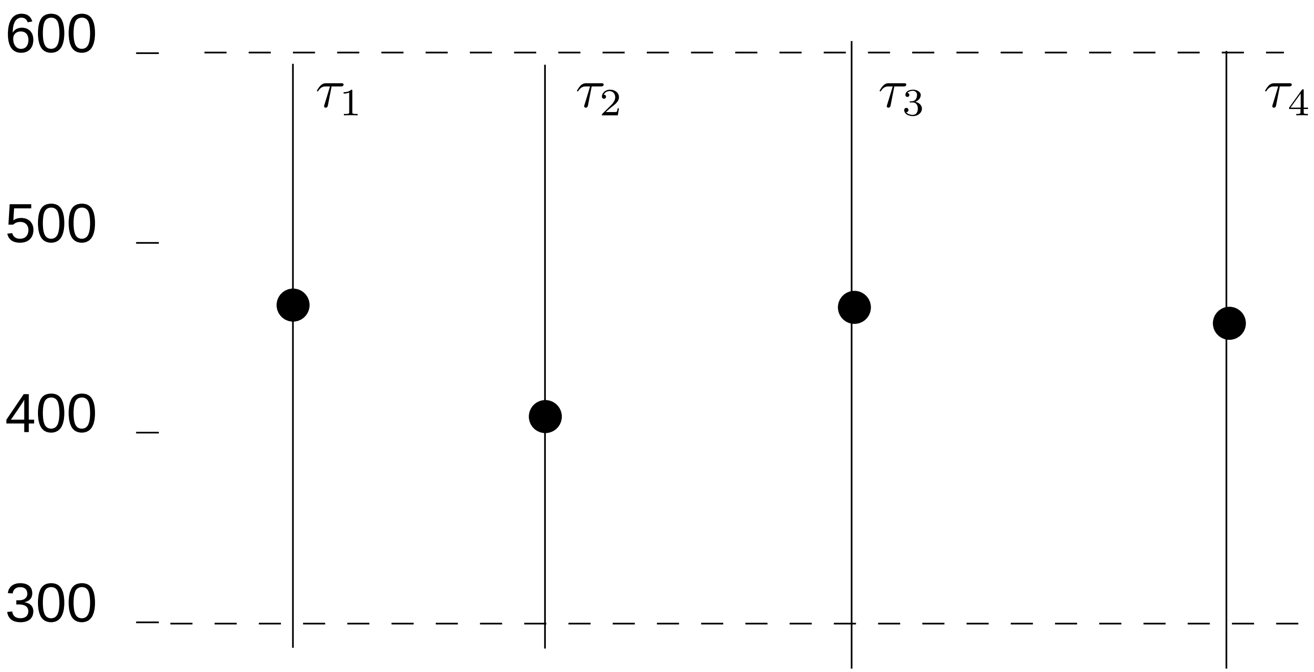
Redundancy

Tension results



●: nominal tension

very large change in the tensions: poor tension management

Modularity

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Modularity: change the geometry of the robot for a better adaptation to the task

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- mechanical modularity

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Modularity

Modularity: change the geometry of the robot for a better adaptation to the task

- mechanical modularity
  - moving the winch systems
  - adding pulleys to change the A location

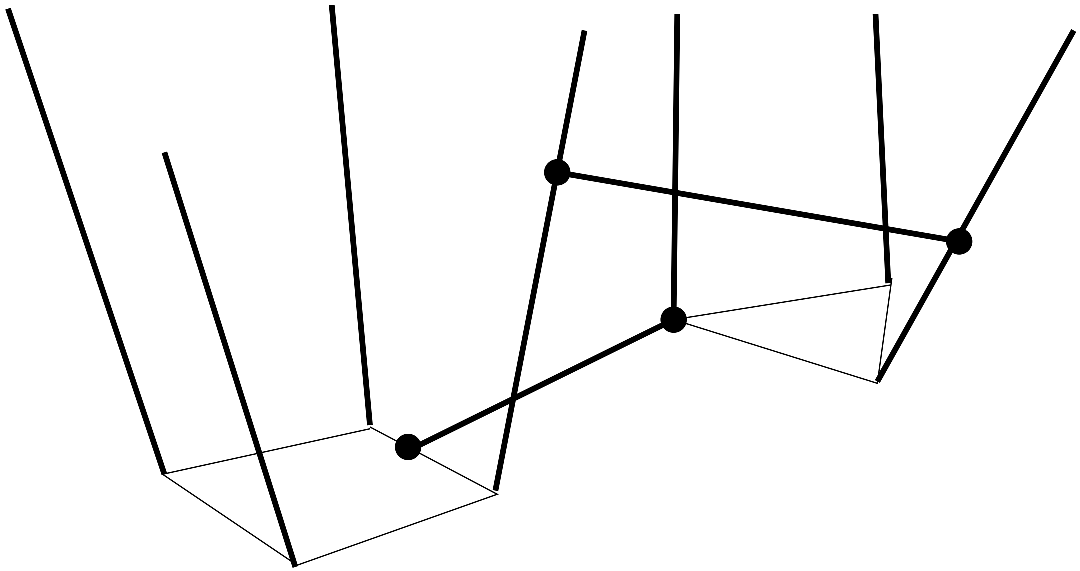
Modularity

Modularity: change the geometry of the robot for a better adaptation to the task

- mechanical modularity
- algorithms for managing modularity

Modularity

An interesting modular concept: multiple WDPRs



- platforms connected by fixed/variable length wires
- inter-connected wires

open issue

Singularity

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- parallel wire-driven robot have the same singularity than parallel robots with rigid legs

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but

- only the singularity that are reachable on a trajectory with $\tau \geq 0$ are of interest: open issue

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Singularity

- parallel wire-driven robot have the same singularity than parallel robots with rigid legs

but

- only the singularity that are reachable on a trajectory with $\tau \geq 0$ are of interest: open issue
- what may be important is not the singularity location itself but its neighborhood ($\tau \leq \tau_{max}$): partially solved issue

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Singularity

- parallel wire-driven robot have the same singularity than parallel robots with rigid legs

but there may be other singularity:

- location where a wire configuration change may occur

$$\Downarrow$$

loss of control

Singularity

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but there may be other singularity:

- location where a wire configuration change may occur: open issue

Conclusion

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Conclusion

- very good point: **WDPR works!**
  - very reasonable accuracy
  - large workspace
  - low cost
  - high modularity: but we don't know yet how to exploit it

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Conclusion

- very good point: **WDPR works!**
  - very reasonable accuracy
  - large workspace
  - low cost
  - high modularity: but we don't know yet how to exploit it
- bad point: **we don't know why!**