DETERMINATION OF 6D-WORKSPACES OF A GOUGH-TYPE 6 D.O.F. PARALLEL MANIPULATOR

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Abstract: We will consider in this paper a Gough-type parallel robot whose leg lengths values are constrained to lie within some given ranges. As a consequence of the coupling between position and orientation of the end-effector the workspace of this type of robot is a variety embedded in a 6 dimensional space. The purpose of this paper is to present algorithms to determine if for a given location of the end-effector it exists a possible orientation of the end-effector such that the leg lengths lie within their limits, determine, with a given accuracy, all the possible locations of the end-effector for which every orientation angles within 3 given ranges leads to valid leg lengths (the *dextrous workspace* is a particular case with the three ranges being $[0, 2\pi]$), determine, with a given accuracy, all the possible locations of the end-effector for which it exists at least one set of three orientation angles within 3 given ranges that leads to valid leg lengths (the *maximal or reachable workspace* is a particular case with the three ranges being $[0, 2\pi]$)

1 Introduction

In this paper we consider a 6 d.o.f. parallel manipulator constituted of a fixed base plate and a mobile plate connected by 6 extensible links. A reference frame (O, x, y, z) is attached to the base and a mobile frame (C, x_r, y_r, z_r) is attached to the moving platform. Let ρ_i be the leg lengths, $\mathbf{X}$ a 6-dimensional vector defining the posture of the end-effector: the three first components of $\mathbf{X}$ are the coordinates of C in the reference frame while the three last components are three rotation angles describing the orientation of the end-effector. The workspace of this type of robot is restricted mainly by the limits on the leg lengths which will be denoted for leg i by $\rho^i_{min}, \rho^i_{max}$. A set of leg lengths will be *valid* if all the lengths lie within their given limits.

As the leg lengths are functions of both the location and the orientation of the end-effector computing the workspace of this type of manipulator is a complex task. This problem has been addressed by fixing three of the 6 posture parameters, either the orientation angles [1],[4],[5] hence computing the workspace for a constant orientation (in numerous papers this computation is done by a discretisation method which is quite inefficient: we mention here only the papers using a geometrical approach which is by far more efficient) or the location the end-effector [6],[8],[9], hence computing the orientation workspace for a particular location of the end-effector. The problem at hand has been addressed only by Kumar [3] for planar robot (a much more simple problem which has been solved in [7]) and Kim [2] which compute a rough approximate of the maximal workspace of 6 d.o.f robot.

2 Preliminary

We define an *extended box* or EB for short as a pair of element: a cartesian box, which represent the possible location of the end-effector, and a set of three ranges, one for each of the rotation angles. An EB is therefore composed of a *location part* (the box) and an *orientation part* and defines a 6D workspace for the robot. For this particular type of workspace we consider the extremal value of the leg lengths over the set of postures defined by the EB. Using interval analysis it is possible to determine an upper bound of the maximal values of the leg lengths and a lower bound of their minimal values. In the case where both platforms are planar it is even possible to compute exactly the extremal values of the leg lengths.

3 Determining if a point belongs to an orientation workspace

Let ψ, θ, ϕ denote the three rotation angles of the end-effector (for example the three Euler angles). An orientation workspace is defined by three ranges S_ψ, S_θ, S_ϕ, one of for each of the Euler angles. The problem we want to solve is to determine for a given location of C if there exists an orientation $\psi \in S_\psi, \theta \in S_\theta, \phi \in S_\phi$ such that the leg lengths for this posture are valid. Note that if the three ranges are defined as $[0, 2\pi]$ then the problem is to determine if C belongs to the maximal workspace of the robot.

Our algorithm start with an EB whose location part is reduced to the location of C and whose orientation part is defined as S_ψ, S_θ, S_ϕ. We then compute the extremal values $[\rho^i_m, \rho^i_M]$ of the leg lengths for this EB. If for all legs $[\rho^i_m, \rho^i_M] \subset [\rho^i_{min}, \rho^i_{max}]$ then the point belongs to the orientation workspace. On the contrary if for one leg we have either $\rho^i_m > \rho^i_{max}$ or $\rho^i_M < \rho^i_{min}$, then the point does not belong to the orientation workspace. Now we have to deal with the case where $[\rho^i_{min}, \rho^i_{max}] \subset [\rho^i_m, \rho^i_M]$. We split each range of the orientation part into two ranges (i.e.

the range $[\psi_1, \psi_2]$ leads to the ranges $[\psi_1, (\psi_1+\psi_2)/2], [(\psi_1+\psi_2)/2, \psi_2])$ and consider the 8 EB build by taking all the possible combinations of the new ranges. We have now a list of EB and we repeat the process with each EB of the list, discarding all the EB for which for at least one leg we have either $\rho^i_m > \rho^i_{max}$ or $\rho^i_M < \rho^i_{min}$, until either for one of the EB of the list we have for all legs $[\rho^i_m, \rho^i_M] \subset [\rho^i_{min}, \rho^i_{max}]$ (the point belongs to the orientation workspace) or we are at the end of the list which means that the point does not belong to the orientation workspace.

On a SUN Ultra 1 workstation the computation time for determining if a point belongs to the maximal workspace ranges from 40ms to 10s (if the point is very close to the border).

Note a variant of this algorithm. Assume that you want to verify the following hypothesis for a point C: all the orientations within the three ranges lead to valid leg lengths. Basically the variant is similar to the previous algorithm except that as soon we find an EB for which for at least one leg we have either $\rho^i_m > \rho^i_{max}$ or $\rho^i_M < \rho^i_{min}$, then the point does not verify the hypothesis.

4 Determining a total orientation workspace

A total orientation workspace (`TOW` for short) is defined as the locations of C for which for any orientation angles within three orientation ranges S_ψ, S_θ, S_ϕ the leg lengths are valid. The *dextrous workspace* is an example of `TOW` with $S_\psi = S_\theta = S_\phi = [0, 2\pi]$. Our algorithm will compute the `TOW` as a set of EB whose orientation part is S_ψ, S_θ, S_ϕ. Each EB of this set have a status which may be: 1 (for any posture within the EB the leg lengths are valid), 2 (the center of the location part of the EB belong to the `TOW` but some point of its location part may not belong to the `TOW`), or -2 (the center of the location part of the EB does not belong to the `TOW` but some point of its location part may belong to the `TOW`). Therefore the resulting workspace will be an approximation of the `TOW` whose accuracy will depend on the size and number of the EB with status 2 and -2. The size of an EB will be here defined as the distance between the center of its location part to one of its vertices.

Our algorithm will start with an EB B_0 whose location part is a bounding box of the overall workspace of the robot. A list $\mathcal{S}$ of EB will be updated during the algorithm: this list is initialized with B_0 and B_i will denote the i-th EB of the list while n denote the number of EB in $\mathcal{S}$. The computation will be done with an accuracy acc, meaning that the EB with status 2 or -2 will have a size less or equal to acc. We start with $i = 0$ and exit if $i > n$:

1. compute the extremal values of the leg lengths for B_i
2. if for all the legs we have $\rho^j_m > \rho^j_{min}$ and $\rho^j_M < \rho^j_{max}$ then B_i has status 1. Update i to $i+1$ and go to step 1
3. if for one leg we have $\rho^j_m > \rho^j_{max}$ or $\rho^j_M < \rho^j_{min}$ then B_i is outside the `TOW`. Update i to $i+1$ and go to step 1
4. if for at least one leg we have $\rho^j_m < \rho^j_{min}$ and $\rho^j_M > \rho^j_{max}$ then:
   (a) if the size of B_i is lower or equal to acc we test if the center of its location part belongs to the `TOW` using the variant algorithm of the previous section. If yes the EB has status 2 otherwise it has status -2. Update i to $i+1$ and go to step 1
   (b) otherwise we split the location part of B_i into 8 new EB using a bisection on the three axis. The new EB are put at the end of $\mathcal{S}$. Update i to $i+1$ and go to step 1

This algorithm has been tested on the INRIA ”left hand” prototype for determining the `TOW` with $S_\psi = 0$, $S_\theta = [0, 20]$, $S_\phi = 0$. The following table indicates the computation time according to the desired accuracy together with the total volume of the EB with status 1,2,-2.

acc	Time	Volume of EB_1	Volume of EB_2	Volume of EB_{-2}
0.74	7mn	226.7	470.8	900.5
0.37	20mn	407.7	283.7	391.7
0.185	1h18mn	537.7	156.6	183.7
0.0925	5h10	612.3	81.7	88.9

Figure 1 present a cross-section of the result for $z = 56$ and a 3D view of the final result.

5 Determining an inclusive orientation workspace

An inclusive orientation workspace (`IOW` for short) is defined as the locations of C for which there exists at least one set of three orientation angles within three orientation ranges S_ψ, S_θ, S_ϕ for which the leg lengths are valid. The *maximal (or reachable) workspace* is an example of `IOW` with $S_\psi = S_\theta = S_\phi = [0, 2\pi]$. Our algorithm will compute the `IOW` as a set of EB whose orientation part is included in S_ψ, S_θ, S_ϕ. Each of the EB will have a status as presented in the previous section with the additional status -1 which mean that the EB will not be part of the

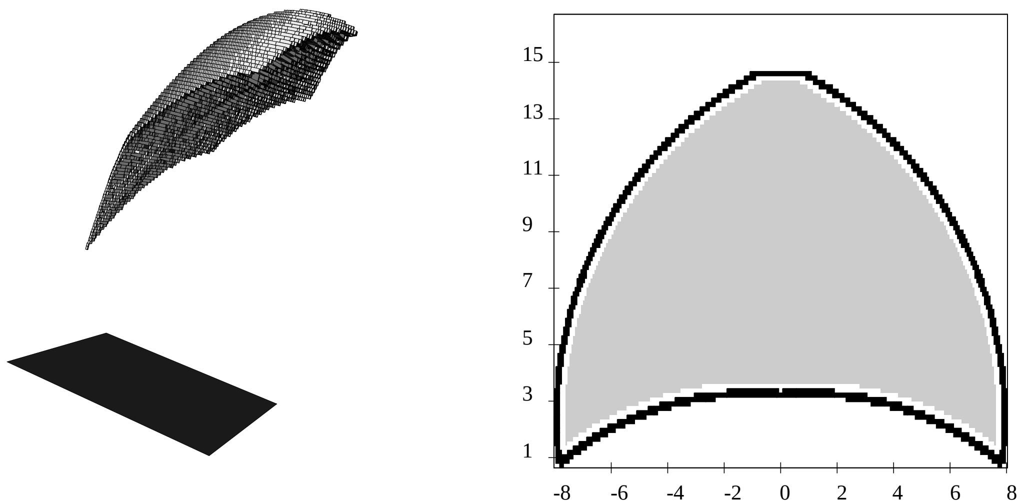

Figure 1: On the left a 3D view of the `TOW` for $S_\psi = 0$, $S_\theta = [0, 20]$, $S_\phi = 0$ (a scale factor of 4 has been applied on the vertical axis). On the left a cross section of this `TOW` for $z = 56$: the EB with status 1 are gray, with status 2 white and with status -2 black.

result. The algorithm is basically similar to the previous one except that now we split also the orientation part of the EB. We start with $i = 0$

1. if $i > n$ then exit

2. if the status of B_i is equal to -1, then $i = i + 1$, go to step 1

3. if among the set of EB=$\{B_0, B_1, ..B_{i-1}\}$ we have an EB with status 1 and whose location part includes the location part of B_i, then B_i has a status -1, $i = i + 1$, go to step 1

4. compute the extremal values of the leg lengths for B_i

5. if for all the legs we have $\rho^j_m > \rho^j_{min}$ and $\rho^j_M < \rho^j_{max}$ then B_i has status 1. If any B_j in $\mathcal{S}$ has a location part included in the location part of B_i, then B_j get the status -1. Update n, then i to $i + 1$ and go to step 1

6. if for one leg we have $\rho^j_m > \rho^j_{max}$ or $\rho^j_M < \rho^j_{min}$ update i to $i + 1$ and go to step 1

7. if for at least one leg we have $\rho^j_m < \rho^j_{min}$ and $\rho^j_M > \rho^j_{max}$ then:

(a) if the size of B_i is lower or equal to acc we test if the center of its location part belongs to the IOW.

   i. If yes the EB has status 2. Then check if any B_j with status -2 in $\mathcal{S}$ has a location part included in the location part of B_i, in which case B_j has status -1.

   ii. If no B_i has status -2.

   iii. Update i to $i+1$ and go to step 1

(b) otherwise we split the location part of B_i into 64 new EB using a bisection on the six parameters. The new EB are put at the end of $\mathcal{S}$, $i = i+1$ and go to step 1

Note an interesting variant of the previous algorithm: assume that you want an extensive description of the IOW, meaning that for any location part for C you want also all the orientation parts such that the corresponding EB has status 1, 2 or -2 (for example we may get a full description of the 6D maximal workspace as a set of EB). To get this description we modify the previous algorithm by removing all the statements in which a status -1 is attributed to an EB.

The computation time of an IOW will be clearly higher than for a TOW as now the orientation part of the EB is no more fixed. The following table indicates the result for the computation of a cross-section of the maximal workspace at $z = 50$.

acc	Time	Area of EB_1	Area of EB_2	Area of EB_{-2}
2.7	1h11mn	1256.07	799.32	1438.8
1.35	1h27mn	1735.66	428.2	999.1
0.338	5h54mn	2059.66	117.75	782.9

Figure 2 presents cross-sections of the maximal workspace of the "left hand" for $z = 50$ with various accuracies. Figure 3 presents an IOW for $z \in [50, 60]$ and $S_\psi = S_\theta = S_\phi = [0, 20]$ with an accuracy of 0.38. The volumes are $EB_1 = 1930.5$, $EB_2 = 425.86$, $EB_{-2} = 387.5$.

6 Conclusion

The algorithms presented in this paper constitute a first approach to solve the remaining problems regarding the workspace computation of 6 d.o.f parallel robot. Although the computation time is large this type of computation is in general done only once. This type of algorithm has been presented for the Gough-type parallel robot but may be extended to other types of parallel robot. Further improvements will have to take into account leg interference and mechanical

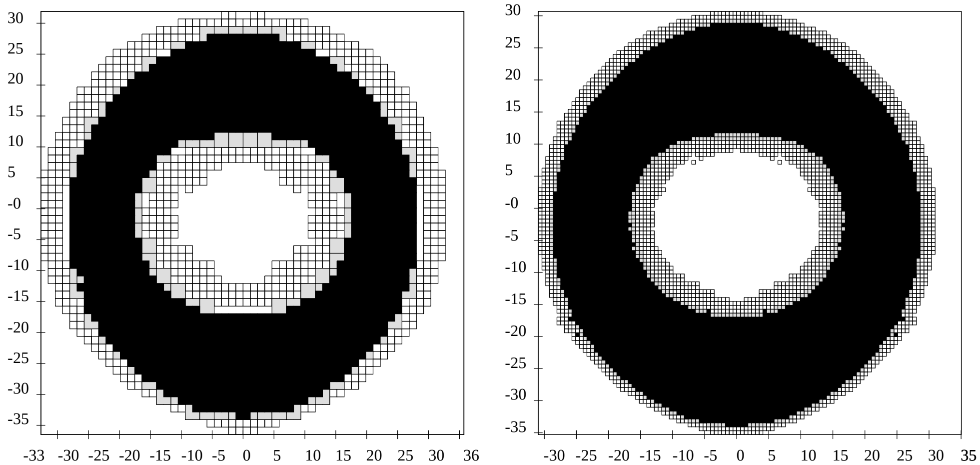

Figure 2: Cross-sections at $z = 50$ of the maximal workspace with accuracy 0.84, 0.42. The black area lie fully in the workspace, EB with status 2 are gray while EB with status -2 are white.

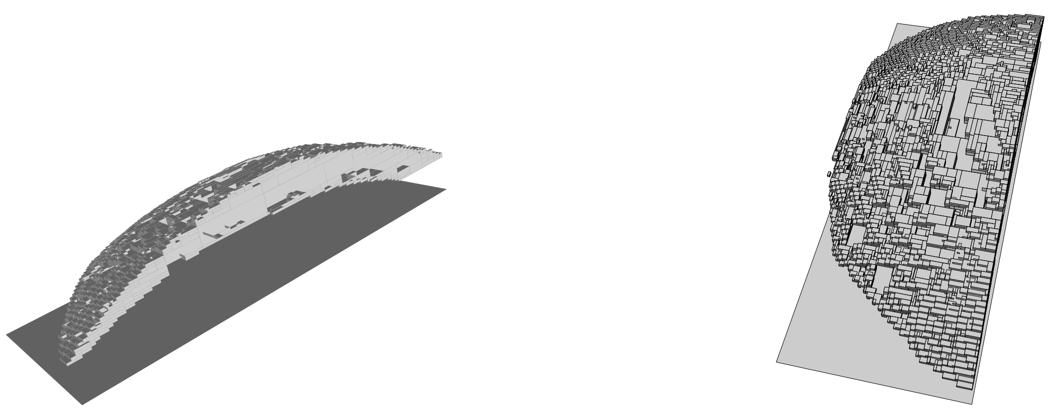

Figure 3: `IOW` workspace for $z \in [50, 60]$ and $S_\psi = S_\theta = S_\phi = [0, 20]$, accuracy 0.38

limits on the passive joints. We believe that a full description of the 6D workspace as a set of EB will be useful to determine the performances of a parallel robot over its whole workspace as the determination of the performances over one EB seems to be a reasonable objective.

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