# Parallel Robots: Open Problems

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**Abstract**

In the past recent years parallel robots have drawn a lot of interest in the robotics community and in many applicative domains: medical, machine-tools, pick-and-place,etc, where the advantages (e.g. accuracy, rigidity, high velocity) of closed-loop chains may be useful. The purpose of this paper is to identify important open theoretical problems in this field.

## 1 Introduction

In the past recent years parallel robots have drawn a lot of interest in the robotics community. This is exemplified by a large increase in the number of papers published on this subject together with the application of parallel robots in very different domains[1]. However in many cases unexpected difficulties in the design and control of such system have led to performances which, although still better than conventional mechanical architectures, were not exactly what was expected. In the following sections we will identify open problems in this field.

## 2 Mechanical designs

A large number of mechanical designs for parallel robots with 2 to 6 DoF have been proposed. A survey of 82 mechanical architectures proposed in the literature shows that 40% have 6 DoF, 3.5% 5 DoF, 6% 4 DoF, 40% 3 DoF, the remaining having 2 DoF. Some systematic approaches for finding all the possible layouts of the joints and actuators that leads to robots with specified number and type of DoF have been proposed [5, 9, 16, 23, 43, 51]. In my opinion these approaches have still to be improved as they are based on simplifying assumptions that discard potentially interesting architectures. This is exemplified by the small number of robots with 4 and 5 DoF which have been proposed: here one difficulty is that no robot with identical chains will have such number of DoF [36].

Another interesting aspect of parallel robots is that they enable to use unconventional actuators like, for example, wire with winches [1] or *binary* linear actuators (having only two states, fully extended or retracted) [7]. This enable to extend the application area of parallel robot but at the same time induces additional constraints on the theoretical problems to be solved (e.g. computing the workspace not only taking into account the limits on the stroke of the actuators but also the preservation of the tension in the wires). Another promising field is the study of *reconfigurable* robots in which the location of the joints may be changed at will to obtain the best robot for the task at hand [30, 50], a problem that we will address in the *synthesis* section.

Parallel robots may use complex joints like multiple ball-and-socket and universal joints, whose restricted motions have a large influence on the workspace volume. New joint designs have been proposed: with large motion capabilities [20], flexible for parallel micro-robot [41]. Still improvements of these joints have received scant attention.

## 3 Direct kinematics

The problem is to determine the pose of the end-effector being given the articular coordinates. Two approaches may be distinguished according to the number of sensors which are used: *minimal*, i.e. strictly equal to the number of actuated articular coordinates, or *redundant*. i.e. greater than this number.

### 3.1 Minimal number of sensors

This is clearly an area where a cooperative work with mathematicians has produced in the last 10 years the most beautiful results. If we measure only the articular coordinates the direct kinematics will have, in general, multiple solutions. As we are interested mainly in the current pose of the platform we may rely on an iterative scheme, e.g. Newton-Raphson, to calculate this pose. These schemes need an initial estimate of the solution (which is in general available) but may experience convergence problem or, worse, may lead to a solution which does not correspond to the current pose. Alternate scheme has been proposed to solve this problem [4, 11, 13], but without improvement in the computation time.

Another approach is to first determine all the possible solutions of the problem and, then, to sort the solutions in order to determine the current pose. The com-

[1] http://www.inria.fr/saga/personnel/merlet/merlet_eng.html

plexity of finding all the solutions increase, in general, with the number of DoF of the end-effector. Note that we may encounter numerical problems even for simple robot as shown by Guglielmetti [19] for the *Delta* robot. For complex robots the solutions may be determined either by using a pure numerical method like the *continuation method* [42] or by *elimination* [14], i.e. by manipulating the equations of the inverse kinematics in order to reduce the problem to the solution of an univariate polynomial, which real roots enable to determine all the possibles poses of the end-effector. Although the former method was able to solve the problem in some cases, its efficiency seems to be lower than the latter method, on which we will focus. A drawback of the elimination method is that it can be performed in many different ways, not all of them leading to the same degree for the resulting polynomial. Therefore to determine the best univariate polynomial, i.e. the one having the lowest possible degree, it is necessary to determine a bound on the number of real solutions of the direct kinematics. Such bounds have been obtained using either classical, but often forgotten, geometrical theorems or the most recent results in algebraic geometry. Basically these bounds are obtained by using analysis theorems, like Bezout's theorem, which enable to determine the maximum number of roots of a given system just by inspection and subtracting from this number all the roots that are at infinity. This number gives an upper bound of the number of real and complex roots of the system. Strangely, in many cases it has always been possible to find a configuration of the robot such that the number of real solutions of the direct kinematics is exactly the bound. This is exemplified by the case of the general Gough platform for which a bound of 40 has been found in 1992 [45] while an example with 40 real solutions has been found in 1998 [12].

In the elimination method, finding the univariate polynomial is either tedious and/or mathematically complex. Fortunately the calculation done for a particular mechanical architecture may sometime be used for another architecture (e.g. the direct kinematics of the *Hexa* robot may be solved by using the algorithm for the Gough platform).

Still there is some work to be done in this area as the theoretical algorithms are difficult to use in practice: for example in the case of the Gough platform finding the 40th order polynomial for any geometry and any leg lengths is still a difficult job [25]. Furthermore the manipulation leading to this polynomial are so complex that they prevent any symbolic factorization, which will lead to a simplified, faster solution. But this simplification may be possible as shown for the Stewart platform: the elimination method leads to a 12th order polynomial but this polynomial is (at least) the product of two 6th order polynomials [32]. Clearly this area has not been sufficiently investigated.

Up to now we have considered only the first step of the solution of the direct kinematics. Indeed as we are interested mainly in the *current* pose of the end-effector, we have to sort the set of solutions. Possible sorting criterion are that the solution should be reached from the *initial assembly mode*, i.e. the pose of the end-effector when it was first assembled, without crossing a singularity and without links interference. At one time it was thought that the former criteria was sufficient to determine an unique solution. It is now known that this is false as shown by Innocenti for planar robot [28] and by Wenger for spatial robot [49]: there exist singularity-free trajectories joining different solutions of the direct kinematics. Still singularity analysis may help to eliminate solutions, but showing that the singularity and interference criterion will lead to an unique solution is an open problem, together with an algorithm implementing these criterion.

### 3.2 Redundant sensors

Another approach to solve the direct kinematics is to add extra sensors to the robot. Indeed the $n$ articular sensors provide a system of equations in the $n$ pose parameters: hence each extra sensor will provide an additional equation, leading to an over-constrained system which, hopefully will have an unique solution. The problem is here to determine the minimal number of sensors and their location in order to have an unique solution with the simplest analytic form and quite robust with respect to the sensor errors. Some of these problems have been addressed in [2, 6, 21, 36, 46], but this issue is far from being solved. Note that adding sensors may play also an important role in the robot calibration (see the next section).

## 4 Calibration

Practical use of the inverse and direct kinematics requires a perfect knowledge of certain geometric elements of the robot, particularly for accurate robots. Thus, position control of a Gough platform needs the locations of the passive joints (a full model requires 132 parameters [35]). Even if a quite accurate estimates of these parameters are available, a calibration may be necessary. Although this problem has been solved for serial robots, this is not the case for parallel robot. Indeed, for a serial robot, small errors in the geometric parameters of the robot lead, in general, to a large difference between the real pose of the end-effector and the expected one. This difference may be evaluated by measuring the pose of the end-effector and then be used in an optimization procedure which will determine values of the parameters decreasing the positioning errors. Applied to parallel robot this method leads to calibration result that are in general disastrous. We pay here for one of the advantages of parallel robot: a large error in a parameter may lead to a quite small error in the pose of the end-effector. Furthermore the measurement noise has a large influence on the result of the calibration process: a rule of thumb for the accuracy of the poses measurement system is given by Vischer [47]: this accuracy should be at least ten times lower that the expected gain in the location of the joints. This type of calibration method may be called *external calibration* as it relies on an external measurement system. Specific methods of external calibration have been proposed [15, 38, 40, 47, 53], and in some cases theoretical difficulties have been identified. For example Innocenti has shown that, even in absence of noise in the measurements, the method proposed by Zhuang may lead to up to 20 different values for the geometric parameters [27]. Another problem is to determine the best measurement poses for the calibration: this problem has been addressed by Nahvi [38] but has led to impractical result as the poses should be near-singular.

Another approach is the *auto-calibration* methods [10]. In that case either extra sensors are used or mechanical constraints are imposed on the legs of the robot (e.g. by clamping a leg so that its direction remains fixed during the calibration). These methods seem to have a large potential but have received little attention.

## 5 Workspace analysis and trajectory planning

The main difficulty of workspace analysis for parallel robot is that, as the reachable locations of the end-effector are dependent on its orientation, a complete representation of the workspace should be embedded in a 6-dimensional workspace for which there is no possible graphical illustration. Only subsets of the workspace may therefore be represented. The most investigated workspace is the 3D *constant orientation workspace*, which describe the possible location of the origin of the end-effector for a constant orientation: geometric or algebraic approaches may be used [22, 17, 31], the former being faster and the latter more general. But many other types of workspace are of interest, for example the *reachable workspace* (all the locations that can be reached by the origin of the end-effector), the *orientation workspace* (all the orientations of the end-effector for a given location of the origin of the end-effector) or the *inclusive orientation workspace* (all the locations that can be reached by the origin of the end-effector with every orientation in a given set). Although the determination of these types of workspace has been addressed for planar robot [36], they remain largely ignored for spatial robots. Furthermore they can be complexified at will by adding constraints (e.g. singularity-free workspace or workspace with a lower bound on the transmission factor). A related problem is to find the volume swept by an object lying on the end-effector [22, 34].

A companion problem to workspace analysis is the trajectory planning problem. This may be understood as to determine first if a given trajectory between two poses fully lie in the workspace of the robot and is singularity-free, and, if the answer is negative, find an alternate trajectory that join the two poses. An interesting variant of this problem for robots having more DoF than necessary (e.g. for a 6 DoF milling machine where the rotation around the normal of the end-effector is not used) is to determine the possible ranges of the extra DoF which ensure that a given trajectory lie in the workspace of the robot, with the further problem of determining the value in these ranges which optimize another criteria (e.g. for which the maximal value of the articular forces over the trajectory is minimal).

## 6 Singularity analysis

This remains an important topic of study although many progress have been made in this field e.g. the geometrical classification of the singularities or algorithms for detecting singularities in a given workspace [36]. Still a global analysis of singularity in relation with the workspace and trajectory planning is needed, for example, to determine if singularity surfaces split the workspace of a robot into connected components, a problem which has been addressed only for planar robot [49] or for special cases of spatial robots [24]. Another interesting field of study is parallel robots which are always in a singular configuration. This type of robot may be of practical interest, but have been studied only at a theoretical level [26].

## 7 Balancing

Parallel robots may be used with their main axis not directed along the vertical. Hence for equilibrating the gravity force the actuators will have to provide forces which may be quite large. Using counterweights or springs for statically balancing the robot may be of interest to decrease the size of the actuator. Here the problem is to determine the location and mass or stiffness of the balancing elements. This problem has been solved for planar robots [29] but remains largely open for spatial robots although it has be proven that it was impossible to balance a Gough platform with counterweights [33].

## 8 Dynamics

Another advantage of the parallel structure is that it enable to design very fast robot by combining the action of the actuators, while the low mass of the moving elements induces small inertia forces. This is exemplified by the performance of the *Delta* robot which may reach a peak acceleration of 500 m/s$^2$ [37]. A first problem here is to determine a tractable dynamic model of the robot: various formulations may be used [3, 39, 44, 48, 52] although simplifying assumption have to be made. A second problem is to implement the algorithm so that the use of the dynamic model will really improve the motion control of the robot, compared to more classical control laws. Specific hardware may have to be used [3, 8, 18].

## 9 Optimal design

A drawback of parallel manipulator is that their performances are heavily dependent upon the dimensions of the robot and the current pose of the platform. For example the maximal stiffness over a given workspace may increase by a factor 7 by only doubling the size of the mobile platform and similar or larger ratio may be observed between two different poses. Two problems may be distinguished in this field: performance evaluation and synthesis.

### 9.1 Performance evaluation

Having designed a robot, it is necessary to evaluate its main characteristics: for example it may be of interest to determine what will be the extremal values of the articular forces, for a given load on the end-effector, for any pose within the reachable workspace. This is clearly a difficult optimization problem: the analytical expressions of the articular forces as a function of the pose parameters are complex expressions involving thousands of terms. We may rely on a discretisation method, as we are usually able to compute numerically the articular forces for a given pose. But this method is computer intensive and does not provide an error bound on the result. Alternate methods are based on the principle that, in general, it is not necessary to compute exactly the extremal values of the characteristic. Indeed, in our example, we need to determine the extreme articular forces to choose the most appropriate actuators among a discrete set of possible choices. Therefore a method which is able to provide these values with a guaranteed error will be in general sufficient, as soon as the error bound may be fixed before running the algorithm. Although this type of methods are now becoming available ( for example it is possible to verify efficiently the absence of singularity within a given workspace of a given robot [36]) this subject is worthy of study. Thus, being given a robot and its workspace, finding the extremum of the articular forces, the passive joint motions, the generalized velocities of the end-effector for bounded articular velocities, the stiffness and the forces/torques that can be applied on the end-effector for bounded articular forces are very important practical problems. Some of these problems are equivalent to the determination of the maximal value of the sum of the absolute value of the pose-dependent elements of a row of the jacobian matrix of the robot over a given workspace.

### 9.2 Synthesis

A second problem is to determine the geometry of the robot which is the most suitable for the task at hand. The classical methods of optimal design, like the cost-function approach, have difficulties to deal with this problem. The first difficulty is due to the large number of parameters that is involved. But more importantly, the main difficulty come from the criterion to consider: they are difficult to evaluate (see previous section), some of them are antagonistic, or not continuous (for example no singularity within the workspace). This is clearly a very open problem.

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