Checking the cable configuration of cable-driven parallel robots on a trajectory

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***Abstract*— This paper addresses the concept of cable configuration of cable-driven parallel robots (CDPR). At a given pose a cable configuration describes the state of cables, i.e. under tension or slack, and plays a major role for the cable tensions and the platform positioning errors. Being given a CDPR with non-elastic cables and a model for the coiling system we describe how the cable configuration can be determined in a guaranteed way for a desired trajectory of the platform. We show that if the CDPR has more than 6 cables it is extremely unlikely that more than 6 cables will be under tension simultaneously even under perfect conditions. It appears that on a trajectory the cable configuration exhibits multiple changes, leading to large variations in the cable tensions.**

I. INTRODUCTION

Cable-driven parallel robot (CDPR) uses a cable coiling mechanism to change the cable lengths for controlling the pose of the platform. Although the study of CDPR has started about 30 years ago, there is currently a renewal of interest in this field because several new possible applications have emerged e.g. large scale maintenance studied in the European project Cablebot [1], rescue robot [2], [3] and transfer robot for elderly people [4] to name a few. We are interested here in CDPR that allows to control all the dof of the platform. If the CDPR is suspended (figure 1) (i.e. the cables cannot exert a downward force) then it must have at least 6 cables otherwise if gravity is not used it must have at least 7 cables.

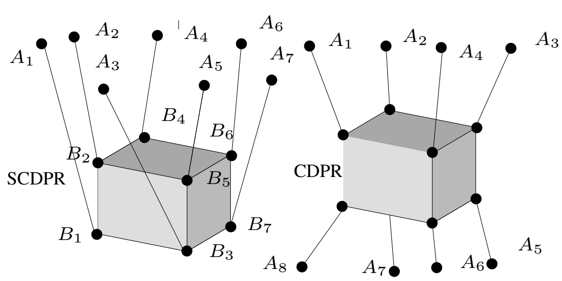

Fig. 1. Cable driven parallel robots: on the left the suspended version. To allow the control of the 6 dof of the platform the suspended version must have at least 6 cables while the non-suspended version must have at least 7 cables.

Cables are attached on the platform at point B_i while the output point of the coiling mechanism is A_i. Let l_i be the length of the cable between the coiling system and B_i. The length l_i may be written as $l_i = a_i + \rho_i$ where a_i is a constant length corresponding to the amount of cable between the coiling system and A_i and ρ_i is the cable length between A_i, B_i. In this paper we will assume that that there is no sagging and no elasticity of the cables. If cable i is under tension, then it exerts a force $\mathbf{f_i}$ on the platform such that

$$\mathbf{f_i} = -\frac{\mathbf{A_i B_i}}{\rho_i}\tau_i \tag{1}$$

where τ_i is the positive tension in the cable. As a cable cannot exert a pushing force $\mathbf{f_i}$ is 0 if the cable is not under tension. Consider a CDPR with n cables and let τ_j denotes the tension in cable j while $\mathcal{F}$ will be the external wrench applied on the platform with the torques applied around a point C. The mechanical equilibrium condition are:

$$\tau = \{\tau_j > 0\} \;\; \text{if } ||\mathbf{A_j B_j}|| = \rho_j \quad \mathcal{F} = \mathbf{J^{-T}}(\mathbf{X})\tau \tag{2}$$

where $\mathbf{J^{-T}}$ is the transpose of the pose dependent inverse kinematic jacobian matrix of the robot, whose j-th column is $((\mathbf{A_j B_j}/\rho_j \;\; \mathbf{CB_j} \times \mathbf{A_j B_j}/\rho_j))$.

A CDPR is usually called *redundant* if it has more cables than the strict minimum to control the dof of the platform. A clear interest of this redundancy is that additional cables, if appropriately located, may drastically increase the size of the workspace. But it is also claimed that redundancy allows one to modify the tensions in the cables without changing the pose of the platform. Hence several papers describe algorithms to calculate a tension distribution that satisfy some optimality condition [5], [6], [7], [8], [9], [10], [11], [12], [13], [14]. However we are not convinced that such schemes may really fulfill their objective. A first reason is that a cable coiling mechanism is a single input- single output system (SISO) and hence can be controlled in force or in cable length but not in both. Therefore a pure cable tension control scheme cannot guarantee that the cable lengths will not change so that the platform is kept at the same pose.

But another reason for the difficulty of implementing a control scheme for managing tension distribution is that discrete time controllers are used with the consequence that it seems difficult to ensure that the constraint $||\mathbf{A_j B_j}|| = \rho_j$ for having all cables under tension simultaneously is satisfied at all time. Hence in spite of tension control some cables will become slack. We will call a *cable configuration* (CC) for a given pose the set of cable numbers that are under tension at this pose. and a *n-cables configuration* is a CC with n cables under tension.

The importance of configuration changes has been illustrated during an experiments with our 6 cables CDPR `MARIONET-CRANE` (figure 2). This robot is a very large scale manipulator with a workspace of 100m$\times$ 100m $\times$ 50m, can lift up to 2.5 tons and it is designed to be portable

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by a team of rescuer (its total weight, including the power source, is 200 kg which is distributed in 20kg subparts). It is intended to be used as a lifting crane during an emergency (earthquake, road accidents). The task assigned to this robot was to move a mannequin along an horizontal trajectory with the mannequin being in vertically seated pose (top left image). During the motion, although the cable lengths were calculated to keep the mannequin posture, two cables have suddenly become slack, which has led the platform in an unstable pose (top right image) and then to a stable pose with a ground-looking posture of the mannequin (bottom image) that was not desired. Intuitively it must be understood that the robot moves along different *kinematic branches* which correspond to different solutions of the forward kinematics with different set of cables under tension or a different pose. When the cable lengths change the robot may move along different manifolds and its pose at a given time depend on the cable lengths and the history of the system. If two manifolds cross, then the robot may continue moving on the manifold it was in before the crossing or may move on the second manifold, while only measuring the cable lengths does not allow to detect this change. In our example the CDPR was moving initially on a given kinematics branch with 6 cables under tension, has crossed another branches with 4 cables under tension, in which it was unstable, leading to a new pose that lies on a branch with 6 cables under tension, but different from the initial one, although no singularity were encountered.. During the sequel of the motion the mannequin has kept this ground looking posture, thereby exhibiting a large orientation error. It must be emphasized that configuration change are extremely important because they induce large positioning error and possibly drastic changes in cable tensions. We aim at calculating exactly

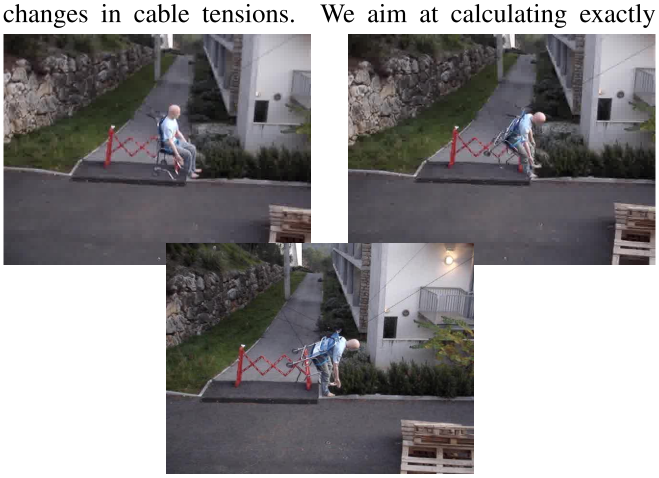

Fig. 2. An experiment showing a change of cable configuration leading to a very large orientation error

what are the cable configurations over time when the robot performs a trajectory

II. Calculating cable configurations

The basic idea in this section is to develop a full model of the system robot/controler/control laws and when the robot moves along a trajectory to determine exactly the times at which a change of configuration will occur and the new configuration. Consequently a first step is to establish a model for the coiling mechanism and for the control.

A. Model of the coiling mechanism and of the control

We assume that the coiling mechanism velocity is a first order so that $V = V_c + (V_0 - V_c)e^{-t/t_a}$, where V_c is the desired velocity, V_0 the actuator velocity at time $t = 0$ and t_a a constant that is motor dependent. Hence if ρ_1, V_1 are the cable length and the velocity of the actuator at time $t = t_1$ the amount of cable length change $\Delta\rho$ at time $t_1 + \Delta t$ is:

$$\Delta\rho = V_c\Delta t - (V_1 - V_c)t_a(e^{-(t_1+\Delta t)/t_a} + e^{-t_1/t_a}) \quad (3)$$

The controller is a loop with a sampling time of Δt_h that gets the current value of the cable length ρ_m (supposed to be perfectly measured), computes what should be the value ρ_c of the cable length at the next sampling time and then calculates a desired actuator velocity V_c using a simple P controller so that $V_c = K(\rho_c - \rho_m)$, where K is a constant gain. There is then an inner control loop with sampling time Δt_i (with Δt_h being a multiple of Δt_i and $\Delta t_i < \Delta t_h$), that control the motor velocity by using a P controller. For the simulation this loop includes the calculation of the CDPR state according to time.

B. Pose parametrization and constraints

We will call *dominant cables* at one pose the cables that are under tension at this pose. The lengths of the dominant cables must verify $||\mathbf{A_jB_j}|| = \rho_j$, while for the non dominant cables we should have $||\mathbf{A_jB_j}|| < \rho_j$. If τ is the tension of the dominant cables the mechanical equilibrium constraint is $\mathcal{F} = \mathbf{H}\tau$ where $\mathbf{H}$ is a matrix whose columns are the normalized Plücker vectors of the dominant cables.

To parametrize the pose of the platform, supposed to be non planar, of a robot with m cables we will use the coordinates of four non coplanar B_i points and we will assume that these points are B_1, B_2, B_3, B_4. At a given pose if these coordinates are known, then the coordinates of the points $B_5, \ldots, B_m$ are obtained as:

$$\mathbf{OB}_k = l_1\mathbf{OB}_1 + l_2\mathbf{OB}_2 + l_3\mathbf{OB}_3 + l_4\mathbf{OB}_4$$
$$\mathbf{OC} = n_1\mathbf{OB}_1 + n_2\mathbf{OB}_2 + n_3\mathbf{OB}_3 + n_4\mathbf{OB}_4$$

where the l_i, n_i are constants that can be determined beforehand. Note that this parametrization (whose choice will be motivated later on) allows us to calculate $\mathbf{A_jB_j}$ for all cables and the Plücker vectors of the dominant cables and hence matrix $\mathbf{H}$. By using twelve unknowns to parametrize the pose of the platform we are redundant and we have to introduce 6 additional constraints which are that the distances between any pair of points in the set B_1, B_2, B_3, B_4 are known:

$$||\mathbf{B_iB_j}||^2 = d_{ij}^2 \quad i, j \in [1, 4], \ i \neq j \quad (4)$$

where d_{ij} is the distance between the points B_i, B_j. For the dominant cables we have:

$$||\mathbf{A_jB_j}||^2 = \rho_j^2 \quad (5)$$

In summary for a CC with n cables under tension we have 12+n unknowns (the coordinates of the selected B_i and the n tensions) and 12 +n constraints (6 equations (4), n equations (5) and the 6 statics equations (2)). Note that if we have a 6-cables configuration, then the system of geometrical equations is square.

C. Study of geometrical conditions

We will assume that at certain time t the robot is in a known 6-cables configuration C_i with the platform pose $\mathbf{X}$ and with velocities $\mathbf{V}$ for the actuators and that there is no singularity in the vicinity of $\mathbf{X}$. We consider the 12 geometrical equations (4) and (5) for the dominant cables and the $m-6$ inequalities $\rho_j > ||\mathbf{A_j B_j}||$ for the non dominant cables during the time interval $[t, t+\Delta t]$, where Δt is a small time increment. Our objective is to determine if this geometrical constraints will be verified for any time in the time interval. Using the cable model (3) we are able to determine a range $[\rho_m, \rho_M]$ for each ρ such that it includes all the possible values of ρ during the time interval.

A first problem to solve is to determine if the equations (4) and (5) always admit a single solution whatever the time is in the time interval. For that purpose we use the Kantorovitch theorem [15][1]. We may calculate the Jacobian matrix of the system at $\mathbf{X}$, which is a constant matrix, and let A_0 be the norm of its inverse Γ_0. We then consider the equation values at $\mathbf{X}$: for equations (4) these values are 0 while for (5) they have interval values because of the interval nature of the ρ. The equation values at $\mathbf{X}$ are summed up in an interval vector $\mathbf{F}$ and we define $\mathbf{U} = \Gamma_0 \mathbf{F}$ which is therefore an interval vector. Using classical interval arithmetic we calculate the norm of $\mathbf{U}$ and we denote by $\overline{U}$ the upper bound of this norm. We then define B_0 as $\overline{U}/2$ so that $||\Gamma_0 \mathbf{F}|| \leq 2B_0$. The theorem also requires the value of the maximum of the norm of the Hessian of the system in the ball centered at $\mathbf{X}$ with radius $2B_0$. However in our case the equations are all quadratic in the unknowns so that the Hessian is a constant matrix with a constant norm H_c (this property is the main motivation for the chosen parametrization of the pose). If $l = 12$ is the the number of equations in the system, then Kantorovitch theorem states that if $2lA_0B_0H_c \leq 1$, then the system admits a single solution, located in a ball centered at $\mathbf{X}$ with radius $2B_0$ and this solution can be found by using the Newton-Raphson scheme, that is guaranteed to converge toward the solution. In our case we note that the value of $2lA_0B_0H_c$ depends on time only through the value of B_0, which decrease if Δt decreases. Hence if the condition $2lA_0B_0H_c \leq 1$ is not satisfied for a given Δt we may always decrease Δt until this condition is fulfilled.

Assume now that the condition $2lA_0B_0H_c \leq 1$ is satisfied for some Δt. This implies that the solution of the geometrical equations for a given time lies inside the interval vector $[\mathbf{X}-2B_0, \mathbf{X}+2B_0]$. This interval vector is used to determine an interval evaluation for the B_i coordinates of the non dominant cables, which in turn allows us to calculate an interval evaluation $[u_j, v_j]$ of $||\mathbf{A_j B_j}||$. Note that u_j, v_j will converge toward $||\mathbf{A_j B_j}(\mathbf{X})|| < \rho_j$ when Δt converge to 0.

[1] see www-sop.inria.fr/coprin/logiciels/ALIAS/ALIAS-C++/

We then define a status for the configuration C_i over the time interval:

- C_i is *geometrically feasible* if $2lA_0B_0H_c \leq 1$ and $v_j < \rho_j$ for all non dominant cables
- C_i is *geometrically unfeasible* if $2lA_0B_0H_c \leq 1$ and there is a non dominant cable such that $u_j > \rho_j$
- C_i is *geometrically uncertain* if $2lA_0B_0H_c > 1$ or if there is a non dominant cable such that $u_j < \rho_j < v_j$

If C_i is geometrically unfeasible, then this configuration cannot hold over the time interval and a configuration change must occur. A geometrically uncertain configuration occurs if Δt is too large: as the configuration is geometrically feasible at $\mathbf{X}$ we are guaranteed to change the status of the configuration from uncertain to feasible by decreasing Δt. Finally if C_i is geometrically feasible, then it may be maintained over the time interval provided that the mechanical equilibrium is maintained with positive tensions.

D. Study of the mechanical equilibrium

Let us assume that C_i is geometrically feasible over a given time interval, which implies that we have interval values for the coordinates of the B_i that allow us to calculate an interval matrix $\mathbf{H_i}$ for the matrix $\mathbf{H}$. We are now interested in the solution in τ of the linear interval system $\mathcal{F} = \mathbf{H_i}\tau$. The interval Gauss elimination method can be used to determine ranges for τ that are guaranteed to include all solutions of $\mathcal{F} = \mathbf{H}\tau$ for all $\mathbf{H}$ in $\mathbf{H_i}$. However the $\mathbf{H}$ matrix is poorly conditioned for interval arithmetic and it is better to pre-condition the system. Let $\mathbf{H_i^m}$ be the mid matrix of $\mathbf{H_i}$, i.e. the scalar matrix whose elements are the mid-point of the ranges in $\mathbf{H_i}$. Gauss elimination is then used on the system $\mathbf{H_i^m}^{-1}\mathcal{F} = \mathbf{H_i^m}^{-1}\mathbf{H_i}\tau$. Note however that the interval matrix $\mathbf{H_i^m}^{-1}\mathbf{H_i}$ includes scalar matrices $\mathbf{H_n}$ such that $\mathbf{H_i^m}\mathbf{H_n}$ has not the structure of a jacobian matrix (i.e. its column are not Plücker vectors) and thus may be removed from the interval matrix. We have used this property to decrease the ranges of the elements of the interval matrix but we cannot elaborate on this method for lack of space. Note also that the interval Gauss elimination do not always lead to a range for the τ: indeed the pivot method that is used implies a division by an interval, an operation that is not possible if this interval includes 0. However this situation will not occur for a sufficiently small Δt.

Like for the geometrical equations we may confer a status to the configuration C_i from the statics viewpoint:

- C_i is *statically feasible* if the interval solutions of $\mathcal{F} = \mathbf{H}\tau$ have all positive lower bound
- C_i is *statically unfeasible* if there is an interval solution of $\mathcal{F} = \mathbf{H}\tau$ that has a negative upper bound
- C_i is *statically uncertain* if the Gauss elimination scheme cannot determine the solution or if there is an interval solution of $\mathcal{F} = \mathbf{H}\tau$ that has a negative lower bound and a positive upper bound

As for the geometrical equations we are sure that a statically uncertain configuration will become feasible if Δt is sufficiently small. We may now state the following theorem for a configuration C_i over a given time interval:

Theorem: if C_i is geometrically and statically feasible over the time interval, then the CDPR will remain in this configuration over the time interval

Proof: assume that the CDPR may be in configuration C_t, different from C_i, at some time t_1 in the time interval. Let j be a cable belonging to C_t but not to C_i. Hence as j does not belong to C_i we must have $||\mathbf{A_j B_j}|| < \rho_j$ but as j belong to C_t we must have $||\mathbf{A_j B_j}|| = \rho_j$, hence a contradiction.

E. Extension of geometrical and statical feasibility

The feasibility has been defined for 6-cables configuration with the advantage that geometrical and statical feasibility may be studied independently. But it may occur that the robot is in a n-cables configuration of the robot with $n < 6$. In that case the system of geometrical equations is no more square. On the other hand if we consider both the geometrical and statical equations we have always a square system with $12+n$ equations. However having the statics equations as part of the system requires to have a look at the Jacobian and Hessian matrices of these equations that play a role in Kantorovitch theorem. Without going into details these matrices are no more constant because of the statics equations but become interval matrices. This does not prohibit their use except that the calculation of the inverse jacobian may not be possible because the interval for the pivots shall not include 0. Consequently an additional feasibility condition will be the success of the calculation of this inverse Γ_0 but basically we can extend the feasibility concept to the case of any cable configuration, the geometrical and statical feasibility being determined in a single step.

F. Configuration change

Let us assume that at a given time t the robot is in a known cable configuration C_i at a pose $\mathbf{X}$ with known velocities of the actuators. As we have seen previously we know that it is always possible to find a Δt such that configuration C_i is geometrically and statically feasible over the time interval $[t, t+\Delta t]$. But this Δt may be very small and it may be of interest to consider a larger Δt and to examine if a configuration change may occur in the time interval. Such a change may occur in the following cases:

1) the length ρ_j one of the non dominant cables of C_i satisfies $||\mathbf{A_j B_j}|| = \rho_j$ at some time in the time interval (i.e. the robot gains one dominant cable)
2) the length ρ_j one of the dominant cables become such that $||\mathbf{A_j B_j}|| < \rho_j$ at some time in the time interval (i.e. the robot loses one dominant cable)
3) the cable tension of one dominant cable of C_i becomes 0 (i.e. the robot loses one dominant cable)

We will now present necessary conditions for configuration change and sufficient one will be given in a later section.

G. Finding possible configuration changes

We examine if the platform at configuration C_i at time t may move to another configuration in the time interval $[t, t+\Delta t]$ by gaining a dominant cable j. A necessary condition for that is that $||\mathbf{A_j B_j}|| = \rho_j$ at some time t_1 included in the time interval.

For taking the time into account we substitute the ρ's by their time functions (3). In the general case if C_i is a n-cables configuration with $n < 6$ we are looking at a time t_1 where the platform may move to a $n+1$-cables configuration. Note that by continuity the tension in the new dominant cable should be 0 at the time t_1. We add the equation $||\mathbf{A_j B_j}|| = \rho_j$ of the non dominant cable to the $12+n$ equations valid for C_i and we add the time as unknown. Consequently we get a square system $\mathcal{D}$ of $13+n$ equations and we are looking for a method that allow us to determine if there is no solution to $\mathcal{D}$ in the time interval or to determine in a guaranteed way its solution(s). For that purpose we must emphasize that the use of interval analysis is a key element because it's the only method that allows one to manage numerical round-off errors, detect that a given problem cannot be solved using the usual computer accuracy and provide certified solutions: hence we use extensively our interval analysis library `ALIAS` [16]. Certification of a solution is obtained by using Kantorovitch theorem but it must be noted that the Hessian matrix of $\mathcal{D}$ is somewhat complex because of the substitution of the ρ's by equations (3). As for the bound for the unknowns, time has to lie in the time interval and as Δt has a small value we may assume that the solution(s) in the B_i coordinates are close to $\mathbf{X}$: hence we choose as intervals for these unknowns an interval centered at $\mathbf{X}$ with a wide amplitude. If a solution is found, then we check that at time t_1 we have $||\mathbf{A_k B_k}|| < \rho_k$ for all the non dominant cables of C_i except cable j. This procedure is repeated for all non dominant cable(s) of C_i and the eventual solutions are stored in a list $\mathcal{T}$ ordered by increasing time t_1. However checking the geometrical constraint is not sufficient: the platform may move at t_1 in a pose such that $||\mathbf{A_j B_j}|| = \rho_j$ with a tension $\tau_j = 0$ but immediately afterward to another one such that $||\mathbf{A_j B_j}|| < \rho_j$.

A change of configuration may also occur if the platform looses a dominant cable j. A necessary condition for this to happen is that the tension in this cable become 0 at a time t_1 lying in the time interval, while we still have $||\mathbf{A_j B_j}|| = \rho_j$. In that case we set $\tau_j = 0$ and we have still the $12+n$ constraint equations where the ρ's are substituted by their time functions (3). The unknowns are the 12 coordinates of the B_i, the $n-1$ dominant cable tensions and the time. Therefore we end up with a square system of $12+n$ equations, that may be solved exactly using the `ALIAS` procedure. If a solution is found it is necessary to check that at this solution $||\mathbf{A_k B_k}|| < \rho_k$ for all the non dominant cables of C_i. This procedure is repeated for all dominant cables of C_i and the eventual solution(s) are added to the list $\mathcal{T}$, which is reordered by increasing time. However checking the tension is not sufficient: indeed the tension of the dominant cable j may decrease until it reaches 0 at time t_1 but may increase after this time.

We may also imagine that the robot gains or looses more than one dominant cable at the same time. If we apply the method described above we will get an overdetermined system of equations. The ALIAS solving procedure may determine that it has no solution but if there are solution(s), then the procedure cannot certify them and will rise a non-certification flag (however note that by solving for each cable the time solutions will be very close to each other). .

In a similar way we have considered up to now only the case where the robot may gain a dominant cable with 6 as a limit to the number of dominant cables. The question that arises is: can we obtain a configuration with 7 (or more) cables under tension ? Let us assume that at time t the robot is in a 6-cables configuration. A necessary condition for the robots to moves to a n-cables configuration with $n \geq 7$ is that the geometrical conditions are satisfied for a time interval included in the interval $[t, t+\Delta t]$. In terms of equations this amounts to that the system of $6+n$ geometrical constraints in the 13 unknowns (the 12 coordinates of the B_i's and the time) should admit a non 0-dimensional solution variety. In that case the ALIAS solving procedure cannot certify the solutions and will rise the non-certified flag. A time interval for which the non-certified flag of ALIAS has been raised will be called an *overconstrained time interval*.

H. Finding the first configuration change

If no possible configuration change has been determined, then the configuration C_i at time t will be preserved in the time interval $[t, t+\Delta t]$. Assume now that we have established a time-ordered list $\mathcal{T}$ of possible configuration changes with l elements. We will denote by t_i the time stamp of the i-th event in the list and we define $t_0 = t$ and $t_{l+1} = t + \Delta t$.

A first lemma is that if a cable k is non dominant at time t_i, then it will stay non dominant in the time interval $[t_i, t_{i+1}[$. This results from the fact that the list records all events $||\mathbf{A_k B_k}|| = \rho_k$ and hence we are sure that no such event has occurred in $[t_i, t_{i+1}[$. Similarly if a cable is non dominant at time t_{i+1}, then it is also non dominant in the time interval $]t_i, t_{i+1}]$

In the previous section we have seen necessary conditions for the platform to gain or loose dominant cables at a given time t_i. They were not sufficient because they describe a status change that may exist only at this time and will vanish immediately after it. But if such change lasts the status will be preserved in the time interval $[t_i, t_{i+1}[$. Hence a configuration change from C_i to C_t at time t_i may be verified if at any time in $[t_i, t_{i+1}[$ configuration C_t is geometrically and statically feasible. We select a time t_α close to, but larger than t_i and test the feasibility at this time. If this test succeeds the configuration C_t will be called *feasible*.

As consequence is that a configuration change in the time interval can be detected by looking at the first event in $\mathcal{T}$ that leads to a feasible configuration different from C_i.

I. Determining cable configuration over a trajectory

We assume that at the start of the trajectory the platform is in a given CC C_i, at a known pose and that the actuator velocities are 0.

At time $t = 0$ the top level loop calculates the new desired pose and actuator velocities V_c, which is executed by the inner loop. We choose as initial value for Δt the inner loop sampling time Δt_i and we check if the CC C_i is geometrically and statically feasible in the time interval $[0, \Delta t]$ by using the methods of sections II-C, II-D, II-E. If this is the case the robot is in the CC C_i at time Δt and we go on by verifying if C_i is valid on the time interval $[\Delta t, 2\Delta t]$. If C_i is uncertain in $[0, \Delta t]$ we divide Δt by 2:

- if Δt is lower than a given threshold ϵ we check if a configuration change may occur in the current time interval by using the results of section II-H. If a configuration change is detected we store the time of the configuration change and set the time to t_α. We then set Δt to the smallest positive value of $k\Delta t_i - t_\alpha$ where k is an integer and we start again. Note that in that case we check if the time interval is overconstrained: if this is the case the algorithm returns a failure flag,
- otherwise we start again the feasibility check of C_i over the new time interval.

When the time reaches $k\Delta t_i$ where k is an integer we go back to the inner loop which will update the command for the actuator velocities, unless $k\Delta t_i = l\Delta t_h$, where l is an integer, in which case we go back to the top level loop.

We have implemented this algorithm in the special case of a CDPR with 8 cables. This was a difficult task because in many cases computer accuracy is barely enough to guarantee the correctness of the calculation. The Kantorovitch test is usually not sensitive to numerical round-off errors and an initial approximation of the solution may be obtained by using the standard ALIAS C++ procedure, but not with the necessary accuracy. In that case we use a specific fast version of the Newton algorithm implemented in Maple, that takes as input the approximation of the system solution as calculated by the solving procedure of ALIAS C++ and returns another approximation with n digits that are guaranteed to be correct, where n is an arbitrary integer (fixed to 100 in our case). It must be said that the algorithm is computer intensive and the simulation of a full trajectory may require several hours.

III. EXAMPLE

We consider the large scale robot developed by LIRMM and Tecnalia as part of the ANR project Cogiro [17] which is a CDPR with 8 cables and we are using meter as length units. We use as test trajectory a circle centered roughly at the middle of the workspace (0,0,2) with radius 1 meter, while the platform have a constant orientation. The trajectory has to be performed in 20 seconds. Preliminary tests done by LIRMM for this trajectory have shown that there were large changes in the motor torques, thereby leading us to believe that configuration changes were occurring. We start at the pose (1,0,2) with the CC 345678 while we add 5 cm to the length of cables 1 and 2 obtained for this pose in order to ensure that they are slack.

Our algorithm has indeed confirmed that several configuration changes were occurring during the trajectory. This is illustrated by figure 3 that shows the cables tensions during the first 0.2 second of the trajectory and by figure 4 which shows the tension of cable 1. A short time history of the

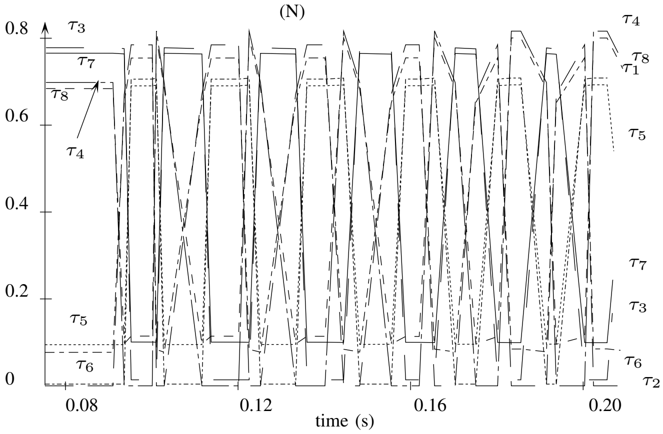

Fig. 3. Cable tensions during the first 0.2s of the trajectory

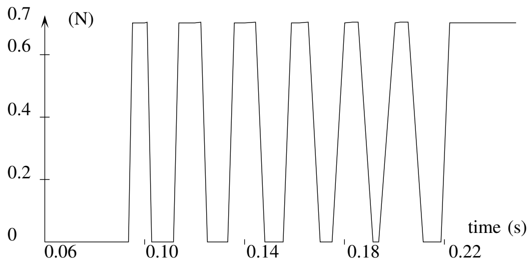

Fig. 4. Tension of cable 1 during the first 0.2s of the trajectory. This cable switches between slack and under tension states.

configuration changes is presented in the following table:

Time(s)	0	0.0936	0.0952	0.1010
Configuration	345678	235678	125678	145678
Time (s)	0.1028	0.1116	0.1137	0.122
Configuration	345678	235678	125678	145678

On this particular trajectory there was never an overconstrained time interval and all CC's were 6-cables configurations. However, as shown in the figures, a configuration change leads to major changes in the cable tensions.

IV. CONCLUSIONS

Configuration change are very important for CDPR has they lead to major changes in the cable tensions and possible control errors. In this paper we have considered a perfect CDPR with perfect measurements and non elastic cables. Even under that condition we have shown that unavoidable configuration changes will occur and that computing them is a demanding task. This raises several issues: can we determine the current CC in a robust way by adding sensors and what kind of sensors (cable tension sensors does not look like a promising way) ? are cable tension management algorithm really effective or shall we adopt another control strategy (e.g. imposing voluntary to cables to be slack to end up in a well defined cable configuration) ? for 6 dof CDPR n-cables configuration with $n < 6$ implies a loss of control: how can we plan a recovery motion so that the CDPR regains as quickly as possible it's 6 dof ? Similar issues have to be addressed if we have elastic cables, the major one being if configuration change may occur. In spite of all these important issues CDPR prototypes are working surprisingly well but the the above issues will most probably give a lot of work to the community.

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