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Computing safe trajectories for an assistive cable-driven parallel robot by selecting the cables under tension and using interval analysis

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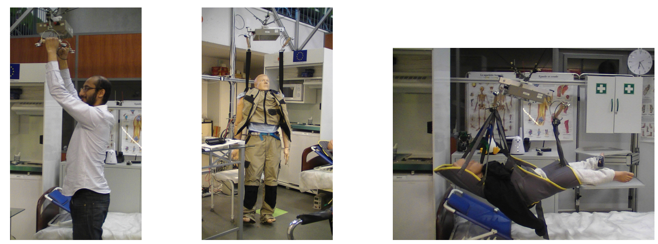
***Abstract*— `Marionet-Assist` is a CDPR designed for transfer operations. Our crane robot has a configuration that provides three translational d.o.f.. Four wires are used to control the position of the end-effector, but as they have no elasticity, only three of them at most will be under tension. For a given position, different triplet of cables under tension are possible, and for a given trajectory, transition between two cable configurations may occur, which can lead to a loss of controllability. Therefore we propose a scheme that rank all the possible sequences for a given trajectory, using interval analysis to guaranty our computing. When a triplet of wires under tension is selected, the fourth one will be forced to be slack by adding purposely extra length. It allows to control changes of cable configurations and thus to have safer trajectories.**

I. Introduction

Daily life can be quite a challenge for our elderly, as well as for people with different levels of handicap. Simple tasks such as moving from an area to another, lifting-up from a wheelchair, or even from the bed, going to the bathroom or the bathtub, can be very effort-demanding and leads often to falls and/or fractures ; more people are dying each year from falls than from car crashes (the ratio being higher than 3). Moreover, fragility is acting like an autocatalyst. For example, people suffering in one of their articulations will transfer too much effort on an other, allowing the fragility to spread. Most of all, physical fragility often leads to social and psychological fragilities, which make it not only a medical issue, but a real and major social concern. So, preventing or identifying the unsafe situations, as well as the aggravating factors, is an important step to improve the quality of life of these communities, which is the objective of the Large-Scale Initiative Action PAL[1] that gathers several scientific and industrial partners in order to propose an alternative approach to assistive robotics.

After the Coprin team, a PAL member, decided to investigate the field of assistance robotics, a two-years period interviews were performed with concerned people, caregivers, gerontologists and territorial authorities in order to fulfill the mentioned requirements. Then, a full-scaled apartment was builded for experimentations. As mobility was a major requirement, it was decided to install a Cable-Driven Parallel Robot (CDPR) in the apartment to allow for transfer operations. CDPR are constituted of a set of winches that can coil/uncoil cables that are attached to the platform. They are known for their large workspace [4], [7], their loading capacity and their mechanical simplicity compared to serial mechanisms [1]. This particular robot has the advantage to be discrete and has a very low intrusivity. Its design also allows to be adapted to the motor skills of the user, from elderly having little difficulties to walk to poly-handicapped people unable to move by themselves (see Figure 1).



(a) Light walking aid (b) Case of an hemiplegic user (c) Case of a paraplegic user

Fig. 1: `Marionet-Assist` being used in three different situations of motor skills

For controlling the 3 translational degrees of freedom, a set of three wires would be sufficient in a crane configuration [5]. But a fourth one was added in order to increase the total workspace (the actual prototype covering almost a $4m$x$3m$x$3m$ cube). However, it has been shown that at most three wires will be in tension and thus support the total load [2]. The set of cables under tension (with at most three elements) will be called a *cable configuration*. Indeed, at a given pose, mechanical equilibrium with positive tension in the cables is possible as soon as the pose lies within the volume obtained by lifting vertically the trangle constituted by the output points of the winches. On a trajectory, the cable configuration of the robot may change at any time. A change in cable configuration induces a mechanical disturbance and, if less than three wires are under tension, a loss of controllability

We recently added visual-servoing functionalities in order to grasp daily-life objects. When an object has felt on the floor or is too high to be easily reached, `Marionet-Assist` will be able to track the target and bring it to the user. But when changes of configurations are occurring, there is an oscillating motion of the camera that is slowing down the processes. Sometimes the target is lost and we have to wait for the equilibrium to be restored in order to pursue. At this moment, the robot does not fit in the perceptual frame of the user, it could be perceived as erratic, strange, and maybe even be dangerous, the displacement being partially uncontrolled.

For these reasons, we developed a scheme allowing

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to compute more acceptable and safer trajectories. When several possible cable configurations exist, we rank them according to criteria that will be presented in Section IV. If the trajectory is such that the cable configuration has to change (because of some point of the trajectory, the current cable trajectory does not satisfy the mechanical equilibrium constraint), we choose the best sequence avoiding under-actuated situations and minimizing the number of cable configuration changes.

Several methods have already by studied. In [3], the authors proposed criteria based on the tension distribution but relies on a dynamical control of a fully constrained robot. [8] show very interesting results, but their criteria relies on architectures allowing a finer estimation of the tension than we can provide in our context. Moreover, the accuracy of the CDPR is limited by several mechanical and modelling uncertainties whose influence may be relatively large because of the size of the workspace. We use interval analysis [6] in order to validate the provided sequences: it allows us to distinguish between absolutely safe sequences, uncertain or unwanted ones.

II. Kinematic analysis of `Marionet-Assist`

Cable-driven parallel robots are parallel mechanisms with non-rigid legs. Each wire is attached to the base frame at a point A_i and to the end-effector at a point B_i (in our case, all the wires are attached on the same point B_0). By controlling the length of each leg, we should be able to move the end-effector along a given trajectory. Four wires whose winches are installed in the top corners of the flat are used to control `Marionet-Assist` (Fig. 2). Hence, the robot has three d.o.f. and allows to perform transfer operations in almost any point of the flat. Very low elasticity Kevlar wires are used, that can be coiled and uncoiled on motorized drums. Wire lengths are estimated through the rotation of the drum motors (see Fig. 2d). Furthermore, small aluminium foils have been glued at regular known points on the wire and can be detected at the output point of the winches: this allows one to update the current wire length as the lengths estimation based on the rotation of the drum may diverge because of the variability of the coiling process.

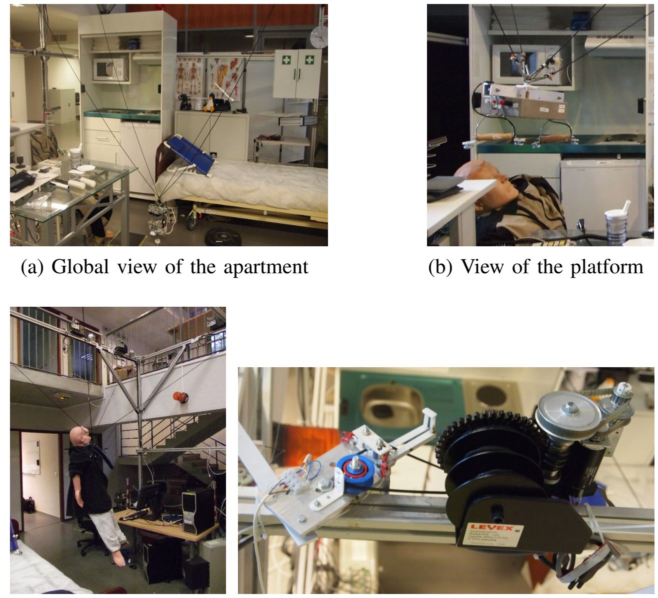
A valid cable configuration with n cables verify the following constraints:

$$\rho_i = ||A_iB_i||, i \in [1, n] \quad (1)$$
$$\rho_i \geq ||A_iB_i||, i \in [n+1, 4] \quad (2)$$
$$\mathcal{F} = J^{-T}(\tau_1, \tau_2, \dots, \tau_n)^T \quad (3)$$

where ρ_i is the effective length of the wire, $\mathcal{F}$ stands for the vector of external forces applied to the platform, and τ the vector of tensions in wires. The matrix $m \times n$ matrix $\mathbf{J^{-1}}$ is called the inverse kinematic Jacobian.

For a given pose, valid cable configurations can be easily obtained by considering all possible cable configurations (see examples in Fig.3,4). Checking if a cable configuration with n cables is valid can be done as follows:

- if $n = 3$, (3) allows one to obtain the τ_i. If all of them are positive, then the cable configuration is valid.



(a) Global view of the apartment (b) View of the platform

(c) Lifting operation (d) View of the coiling system

Fig. 2: The MARIONET-ASSIST wire-driven parallel robot used for transfer operations

- if $n < 3$, (3) allows one to obtain the n τ_i and lead to $3 - n$ constraints on the pose parameters.

Two strategies may be adopted here:

- we use external sensors to verify which wires have a positive tension. In practice, verifying if a cable is under tension is very difficult. Furthermore, this strategy does not allow to control the configuration changes.
- knowing the different possible configuration cables, we force one of them by purposely adding some length to the chosen wires in order to make them slack. This counter-intuitive strategy allows to determine in which cable configuration the robot is and adapt the control law to it.

Then, by selecting whenever it it possible a fully-constrained configuration, we can enhance the quality of the trajectory and avoid unnecessary perturbations.

Yet, nothing is solved, as uncertainties have to be taken into account. For an uncertain robot/measurement, different cable configurations are possible for a given set of ρ measurements.

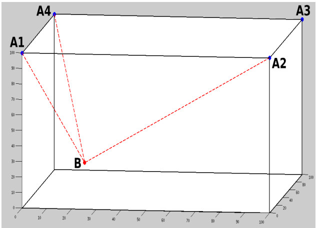
Again, two strategies may be adopted:

- one can use extra sensors in order to determine the current cable configuration
- one can use interval analysis in order to calculate what are the possible cable configuration and possibly what should be the minimal value ρ of the slack cable to decrease the number of possible cable configuration

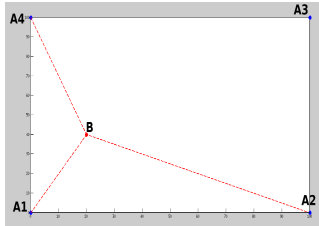
III. Using interval analysis

A real interval $[A]$ is given by the bounded, closed subset of the real numbers defined by:

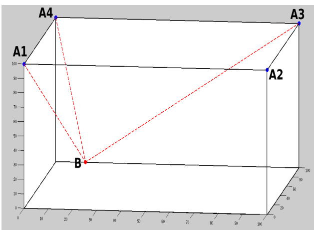
$$[A] = [\underline{a}, \overline{a}] = \{x \in \mathbb{R} | \underline{a} \leq x \leq \overline{a}\}$$



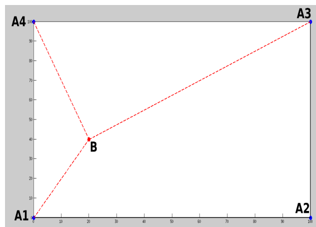
(a) config. (A_1, A_2, A_4), 3D view



(b) config. (A_1, A_2, A_4), top view

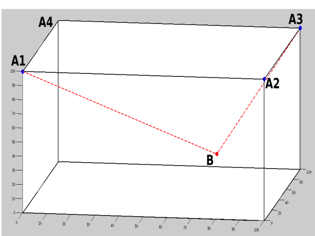


(c) config. (A_1, A_3, A_4), 3D view

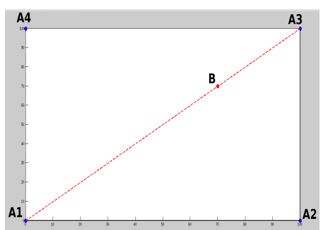


(d) config. (A_1, A_3, A_4), top view

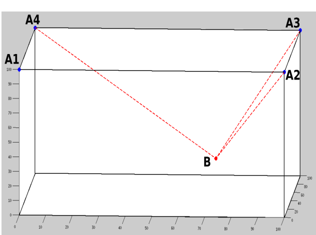
Fig. 3: With $B_0 = (20, 40, 20)$: figures on top show a configuration where wires 1, 2 and 4 have positive tension, figures on bottom show a configuration with wires 1, 3 and 4.



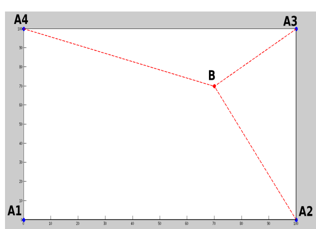
(a) config. (A_1, A_3), 3D view



(b) config. (A_1, A_3), top view



(c) config. (A_2, A_3, A_4), 3D view



(d) config. (A_2, A_3, A_4), top view

Fig. 4: With $B_0 = (70, 70, 20)$: figures on top show a configuration where wires 1 and 3 have positive tension, figures on bottom show a configuration with wires 2, 3 and 4.

where $\underline{a}, \overline{a} \in \mathbb{R}$ and $\underline{a} \le \overline{a}$. We then define the midpoint of $[A]$ as $\text{Mid}([A] = \frac{1}{2}(\overline{a} + \underline{a})$ and its diameter as $\text{Diam}([A]) = (\overline{a} - \underline{a})$. A real number a is then a degenerate interval such as $\underline{a} = \text{Mid}(a) = \overline{a}$ and $\text{Diam}(a) = 0$. $\mathcal{I}(\mathbb{R})$ will denote the set of real intervals.

The basic operators of a real interval arithmetic are given by the following maps functions $Op : \mathcal{I}(\mathbb{R}) \times \mathcal{I}(\mathbb{R}) \to \mathcal{I}(\mathbb{R})$:

$$[A] + [B] = [\underline{a} + \underline{b}, \overline{a} + \overline{b}]$$
$$[A] - [B] = [\underline{a} - \overline{b}, \overline{a} - \underline{b}]$$
$$[A] * [B] = [\min(\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}), \max(\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b})]$$
$$[A]/[B] = [A] * [1/\overline{b}, 1/\underline{b}] \text{ if } 0 \notin I_b$$

For every function $f : \mathcal{I}(\mathbb{R})^m \to \mathcal{I}(\mathbb{R})^n$, we define $[F]$ as the extension of the domain of the function in $\mathcal{I}(\mathbb{R})$, and $\Box F$ as its overestimation.

Two kinds of methods can be used in order to make $\Box F$ converge toward $[F]$:

- **bisection**: an interval $[A]$ is divided in two intervals $[A]_- = [\underline{a}, c]$ and $[A]_+ = [c, \overline{a}]$, with $\underline{a} < c < \overline{a}$
- **contraction**: the boundaries of the interval are reduced, for example by using redundancy of variables.

Using those two processes alternatively, it will be possible to reduce the estimation. Also, interval analysis has the nice property of easily use parallel computing.

IV. Computing Trajectories

Depending on the situation, we may want to either enhance stability of the system or its accuracy.

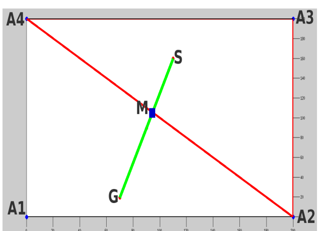
A. Method 1: ensuring stability

For any point M with coordinates (M_x, M_y, M_z), we define $\widetilde{M} = (M_x, M_y, M_y, M_T)$ with M_t denoting a possible configuration for the point M. Any change in the space coordinates will correspond to a translation, and a modification of the fourth coordinate will indicate a change of cable configuration.

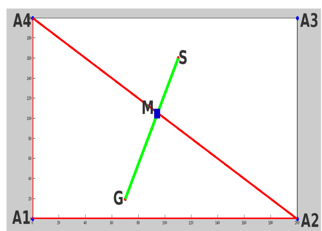
For a given pose, the errors in measurement of each ρ_i will have a different influence on the positioning error: the longer the length of the wire is, the bigger will be its influence on the positioning error (which is quantified by $Max(|\Delta x|, |\Delta y|, |\Delta z|) < 0.05m$): this allows us to rank the different cable configurations for any pose. Also, for a given trajectory, depending of the initial cable configuration, cable configurations changes cannot be avoided. In order to reach the aimed position, the number of transitions can be important and classified in two kinds: either the point lies with certainty inside each workspace defined by the two cable configurations T_k and T_l, or it could happen on a boundary shared by two workspaces (in our cases, the diagonals A_1A_3 and A_2A_4) and then in an under-constrained configuration. Therefore, transitions can also be ranked according to this criterion. Thus, for any computed sequences, we will score both configuration and transitions, and the final rank will reflect both criterion

As each attached point A_i has the same third coordinate, the workspace reachable with a given configuration can be identified with the triangle defined by the three considered wires. We define $O(0, 0, 0)$ as the referenced point in the fixed frame $\mathcal{O}(x, y, z)$, S as the beginning of the trajectory, and G as its goal. For a given configuration C, three cases can happen: C is defined by only one wire with a positive tension, two wires, or fully-constrained with three wires. We also define IM as the interval evaluation of a point M and δ_a as the uncertainty in the position, then $IM = ([M_x - \delta_a, M_x + \delta_a], [M_y - \delta_a, M_y + \delta_a], [M_z - \delta_a, M_z + \delta_a])$, and IA_i the interval evaluation of the localization of the attached points A_i and δ_b the uncertainty on their localizations, and then $IA_i = ([A_{i_x} - \delta_b, A_{i_x} + \delta_b], [A_{i_y} - \delta_b, A_{i_y} + \delta_b], [A_{i_z} - \delta_b, A_{i_z} + \delta_b])$.

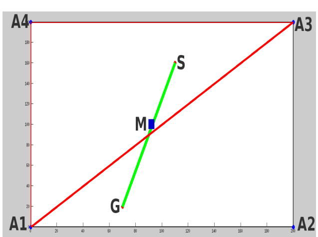
1) if a possible cable configuration for S has less than three wires under tension, we add a point M_0 such that it lies with certainty in a fully-constrained configuration.
2) we check if the trajectory SG (or M_0G) intersects a new fully-constrained configuration, if $\exists M_l, \exists\lambda_1, \lambda_2$ such that $M_l = S + \lambda_1 SG$, $M_l = A_i + \lambda_2 A_i Aj$ and $0 < \lambda_1, \lambda_2 < 1$, A_iA_j being one of the three edges of a fully-constrained cable configuration. We use an interval evaluation to give a measure of the certainty. We then have to ensure that SM_l (or M_0M_l) can entirely happen in the current cable configuration, as two situation may arise: if the cable configurations are possibly sharing a boundary, the localization of M_k is uncertain (see Fig. 5). ; if not the case (see Fig. 5), then we can move safely.
3) when several similar (according to the two previous steps) transition happen, we rank them according to its position in the new cable configuration, using the ρ_i.
4) If M_lG can entirely go on in the now current cable configuration, the sequence is closed. If not, S:=M_l and we go back to the first step.



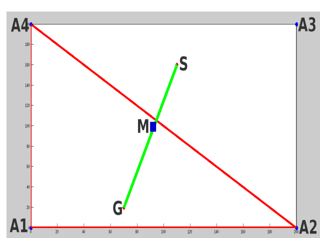
(a) case1: sub-trajectory **SM** with wires $A_2A_3A_4$



(b) case1: sub-trajectory **MG** with wires $A_1A_2A_4$



(c) case2: sub-trajectory **SM** with wires $A_1A_3A_4$



(d) case2: sub-trajectory **MG** with wires $A_1A_2A_4$

Fig. 5: With $S = (1.60, 1.10, 0.20)$ and $G = (0.20, 0.70, 0.20)$. a) and b) show a transition occurring at $M = (90, 90, 20)$, but on the shared boundaries of both workspaces. c) and d) show a transition occurring at $M = (0.93, 0.91, 0.20)$ with a situation allowing a safe transition.

We thus exhaustively compute the set of sequences for a given trajectory and order them according to the two criterion: the localization in the actual cable configuration and the safety of each transition (meaning the absence of an under-constrained situation).

B. Method 2: ensuring accuracy

Given the uncertainties of the coiling process, we want to select the cable configuration that will minimize the uncertainties on the position. Using the kinematic relation:

$$\Delta\rho = \mathbf{J}^{-1}\Delta\mathbf{X} \tag{4}$$

where ρ and $\mathbf{X}$ are respectively the articular coordinates and the position. Using interval analysis, we compute the different jacobian matrices associated to computed possible cable configurations. Then, for a given uncertainty on the articular coordinates, we chose the cable configuration minimizing the maximum diameter for the evauated interval vector $\square X$ obtained :

$$\square(\Delta\mathbf{X}) = \square\mathbf{J}\ \mathcal{I}(\Delta\rho) \tag{5}$$

$$\text{score} = \max_{i=1,23} \text{Diam}(\square\mathbf{X}_i) \tag{6}$$

where $\square\mathbf{J}$ is the evaluation of the kinematic jacobian by inverting $\mathcal{I}\mathbf{J^{-1}}$, $\mathcal{I}(\Delta\rho)$ the interval vector denoting the uncertainty on the wire lenghtes, $\square(\Delta\mathbf{X})$ the computed evaluation of the uncertainties on the position and Diam($\square\mathbf{X}_i$) the diameter of the i^{th} component of this last vector.

By selecting the case with a minimal score, an error on the articular coordinates will have a minimal impact on the final positioning.

V. Simulations

The coordinates of the attached points are $A1 = (0.12, 0.08, 3.10)^2$, $A2 = (2.78, 0.11, 3.10)$, $A3 = (2.79, 2.29, 3.10)$ and $A4 = (-0.08, 2.18, 3.10)$. A_{ijk} will denote a configuration where wires $i, j, k \in 1, 2, 3, 4$ have a positive tension. The accuracy of their localization is $\delta_a = \pm 0.02$, and the accuracy of the localization of the end-effector is chosen to be $\delta_b = \pm 0.05$. For each point M_k, $d_i = \sqrt{(M_{k_x} - A_{i_x})^2 + (M_{k_y} - A_{i_y})^2}$, the location score is computed as $Sc = 5\left(1 - \frac{\min(d_i,d_j,d_k)}{\max(d_i,d_j,d_k)}\right)$ rounded to the lower integer – thus $Sc \in [0, 4]$ – and the transition score as 4 when there is an under-constrained cable configuration involved, 0 otherwise.

A. First method

We choose $S = (0.60, 0.80, 0.80)$ and $G = (1.90, 1.10, 0.50)$. The algorithm provides three distinct sequences, plotted in Fig.6.

The points of the first sequence are

- $\widetilde{S} = (0.60, 0.80, 0.80, A_{134})$
- $\widetilde{M_{11}} = (1.14, 0.92, 0.80, A_{134})$
- $\widetilde{M_{12}} = (1.14, 0.92, 0.80, A_{123})$
- $\widetilde{G} = (1.90, 1.10, 0.50, A_{123})$

and its scores are detailed in Tab.I.

For the third sequence, we have :

- $\widetilde{S} = (0.60, 0.80, 0.80, A_{124})$
- $\widetilde{M_{31}} = (1.40, 0.98, 0.80, A_{124})$
- $\widetilde{M_{32}} = (1.40, 0.98, 0.80, A_{123})$
- $\widetilde{G} = (1.90, 1.10, 0.50, A_{123})$

and the scores are detailed in Tab.II.

The second sequence with points:

[2] all units are expressed in meter

Position scores			
Point	Config	(d_i, d_j, d_k)	Score
S	$A_1A_3A_4$	$(0.87, 2.65, 1.54)$	3
M_1	$A_1A_3A_4$	$(1.32, 2.14, 1.75)$	1
M_1	$A_1A_2A_3$	$(1.32, 1.83, 2.14)$	1
G	$A_1A_2A_3$	$(2.05, 1.32, 1.49)$	1
Transition scores			
Point	Transition		Score
M_1	$A_1A_3A_4 \rightarrow A_1A_2A_3$		4
Total score			10

TABLE I: Scores of sequence 01

Position scores			
Point	Config	(d_i, d_j, d_k)	Score
S	$A_1A_2A_4$	$(0.87, 2.29, 1.54)$	3
M_3	$A_1A_2A_4$	$(1.56, 1.63, 1.91)$	0
M_3	$A_1A_2A_3$	$(1.56, 1.63, 1.91)$	0
G	$A_1A_2A_3$	$(2.05, 1.32, 1.49)$	1
Transition scores			
Point	Transition		Score
M_3	$A_1A_2A_4 \rightarrow A_1A_2A_3$		0
Total score			4

TABLE II: Scores of sequence 03

- $\widetilde{S} = (0.60, 0.80, 0.80, A_{124})$
- $\widetilde{M}_{21} = (1.51, 1.01, 0.80, A_{124})$
- $\widetilde{M}_{22} = (1.51, 1.01, 0.80, A_{2.5})$
- $\widetilde{G} = (1.90, 1.10, 0.50, A_{2.5})$

has a total score of 10.

Thus the third sequence is chosen. One can observe that this is the only one amongst the three that has a safe transition.

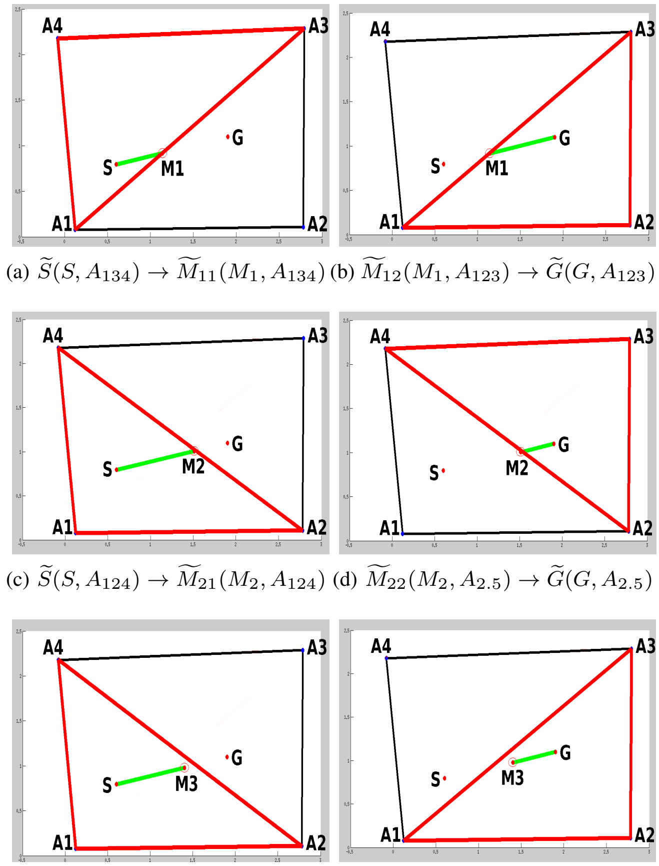
B. Second method

We choose $S = (1.85, 1.40, 1.60)$ and $G = (1.85, 0.80, 1.30)$. Here, S and G are close to an under-constrained configuration (respectively A_1A_3 and A_2A_4). As it is shown in Fig.7, the selected trajectory remains safe, as the provided cable configurations are fully-constrained and the transition occurs in $M_1 = (1.85, 0.88, 0.38)$ without having to cross a shared boundary.

Such a criterion allows to improve the accuracy, but, relying on the kinematic jacobian, it also ensures the stability of the platform by avoiding under-constrained cable configurations.

VI. Experimentations and results

For the two trajectories tested here, we used a Romer Sigma 2018 arm, whose accuracy is $1\mu m$ to measure the pose of the platform. First the robot is asked to go to the starting point S of the trajectory, which we measure, then we force the desired cable configuration by adding $0.20m$ to the length of the wire that we want to be slack. Then, the same operation is done for the point M when the transition occurs, and finally for the goal point G. We then compare the trajectory given by the three measurements to the theoretical values.



(a) $\widetilde{S}(S, A_{134}) \rightarrow \widetilde{M}_{11}(M_1, A_{134})$ (b) $\widetilde{M}_{12}(M_1, A_{123}) \rightarrow \widetilde{G}(G, A_{123})$

(c) $\widetilde{S}(S, A_{124}) \rightarrow \widetilde{M}_{21}(M_2, A_{124})$ (d) $\widetilde{M}_{22}(M_2, A_{2.5}) \rightarrow \widetilde{G}(G, A_{2.5})$

(e) $\widetilde{S}(S, A_{124}) \rightarrow \widetilde{M}_{31}(M_3, A_{124})$ (f) $\widetilde{M}_{32}(M_3, A_{123}) \rightarrow \widetilde{G}(G, A_{123})$

Fig. 6: Figures at the top show a sequence with a score $\omega = 10$ (one insecure transition) ; figures at the middle show also a sequence whose score $\omega = 10$ (one insecure transition) ; figures at the bottom show a safe transition, whose total score $\omega = 4$.

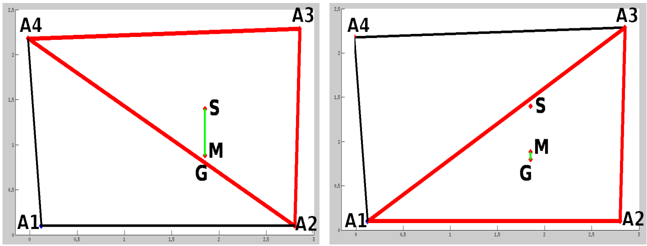
A. First trajectory with method 1

We chose $S = (0.86, 0.66, 1.80)$ with cable configuration $A_1A_2A_4$ and $G = (1.64, 0.84, 1.80)$ with cable configuration $A_1A_2A_3$. The transition point is $M = (1.40, 0.78, 1.80)$. We can see in Fig.8a,8b that the other possible cable configurations are insecure, as both S and G could be on their respective edges. Results of the measurements are given in Tab.IV.

Measurements		
S	M	G
$(0.86, 0.66, 1.80)$	$(1.39, 0.76, 1.75)$	$1.65, 0.88, 1.76)$
Theoretical trajectory		
$S \rightarrow M$	$M \rightarrow G$	$S \rightarrow G$
$(0.54, 0.12, 0.0)$	$(0.24, 0.06, 0.0)$	$(0.78, 0.18, 0.0)$
Measured trajectory		
$S \rightarrow M$	$M \rightarrow G$	$S \rightarrow G$
$(0.53, 0.10, -0.05)$	$(0.26, 0.12, 1)$	$(0.79, 0.22, -0.04)$

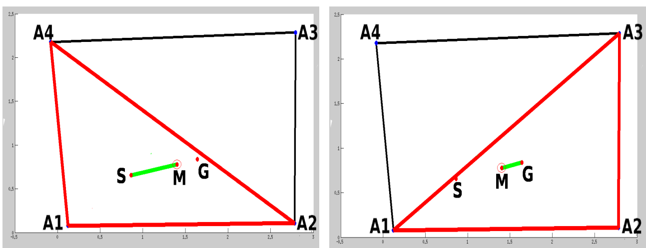
TABLE III: Measurements and comparison of the theoretical and measured trajectories

Knowing that the accuracy of the `Marionet-Assist` has a $0.05m$ radius, we can conclude that the measured

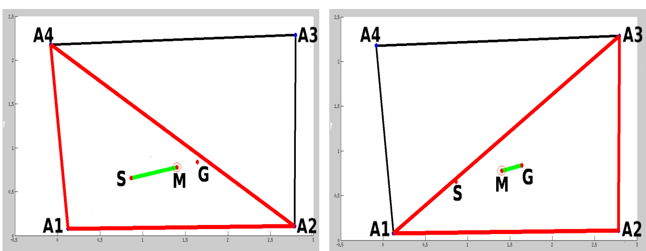


(a) $\widetilde{S}(S, A_{2.5}) \rightarrow \widetilde{M}_{11}(M_1, A_{2.5})$ (b) $\widetilde{M}_{12}(M_1, A_{123}) \rightarrow \widetilde{G}(G, A_{123})$

Fig. 7: Trajectory selected according to the influence of the articular uncertainties on the positioning error



(a) $\widetilde{S}_1(A_{124}) \rightarrow \widetilde{M}_{11}(A_{124})$ (b) $\widetilde{M}_{12}(A_{123}) \rightarrow \widetilde{G}_1(A_{123})$



(c) $\widetilde{S}_2(A_{124}) \rightarrow \widetilde{M}_{21}(A_{124})$ (d) $\widetilde{M}_{22}(A_{123}) \rightarrow \widetilde{G}_2(A_{123})$

Fig. 8: Top figures show the first theoretical trajectory, figures at bottom show the second one

trajectory is sufficiently close to the theoretical one, and thus controlling the cable configuration did not affect negatively the accuracy. By choosing the better ranked sequence, we enhanced our controllability, choosing when and where the transition occur allows to prevent unnecessary ones and the safety of the change of cables configurations.

B. Second trajectory with method 2

We chose $S = (1.85, 1.40, 1.60)$ with cable configuration $A_2A_3A_4$ and $G = (1.85, 0.70, 1.30)$ with cable configuration $A_1A_2A_3$. The transition point is $M = (1.85, 0.88, 1.38)$. We measured the trajectory with our algorithm, and without, in order compare the accuracy of the final positioning. Results of the measurements are given in Tab.IV.

When we did not use our method, two changes in cable configurations occured, both before reaching the M. As we can see, in this experiment, we provide a better accuracy. Along the X-axis (where there should be no variation), there is at least a displacement of $0.06m$ between S and M, when it never exceeds $0.03m$ using our second criterion.

Measurements when our method is used		
S	M	G
$(1.85, 1.40, 1.60)$	$(1.87, 0.91, 1.36)$	$(1.88, 0.73, 1.30)$
Measurements without our method being used		
S	M	G
$(1.85, 1.40, 1.60)$	$(1.90, 0.93, 1.38)$	$(1.89, 0.78, 1.30)$
Theoretical trajectory		
$S \rightarrow M$	$M \rightarrow G$	$S \rightarrow G$
$(0.00, -0.52, -0.22)$	$(0.00, -0.18, -0.08)$	$(0.00, -0.70, -0.30)$
Measured trajectory with our method		
$S \rightarrow M$	$M \rightarrow G$	$S \rightarrow G$
$(0.02, -0.49, -0.24)$	$(0.01, -0.18, -0.08)$	$(0.03, -0.67, -0.32)$
Measured trajectory without our method		
$S \rightarrow M$	$M \rightarrow G$	$S \rightarrow G$
$(0.05, -0.47, -0.22)$	$(-0.01, -0.15, -0.08)$	$(0.04, -0.62, -0.30)$

TABLE IV: Measurements and comparison of the theoretical and measured trajectories with and without our method

VII. Conclusions and Further Works

We showed that by computing different sequences of cable configurations for a given trajectory, we can rank them and have a better control of the robot, reducing sources of disturbances. However, we did not consider here the case where the trajectory crosses the intersection of the two diagonals. In this particular situation, adding a fifth wire may allow a fully-constrained cable configuration in this point, or one could perform a non-linear trajectory avoiding its close neighborhood. We aim now to extend this scheme to other configurations (m wires and n d.o.f.) and evaluate our performances on the quality of the visual servoing process.

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